Strategic Payments in Financial Networks

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Abstract

In their seminal work on systemic risk in financial markets, Eisenberg and Noe [13] proposed and studied a model with n firms embedded into a network of debt relations. We analyze this model from a game-theoretic point of view. Every firm is a rational agent in a directed graph that has an incentive to allocate payments in order to clear as much of its debt as possible. Each edge is weighted and describes a liability between the firms. We consider several variants of the game that differ in the permissible payment strategies. We study the existence and computational complexity of pure Nash and strong equilibria, and we provide bounds on the (strong) prices of anarchy and stability for a natural notion of social welfare. Our results highlight the power of financial regulation – if payments of insolvent firms can be centrally assigned, a socially optimal strong equilibrium can be found in polynomial time. In contrast, worst-case strong equilibria can be a factor of $\Omega(n)$ away from optimal, and, in general, computing a best response is an NP-hard problem. For less permissible sets of strategies, we show that pure equilibria might not exist, and deciding their existence as well as computing them if they exist constitute NP-hard problems.

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1 Introduction

The last major financial crisis and its aftermath have highlighted the systemic risks and resulting hazards for society that arise in financial markets, which are characterized by different, highly interconnected financial institutions. Over the last decade an increased research effort has been underway to analyze, understand, and manage the systemic risks in financial markets. Main aspects of interest are contagion effects and cascading defaults, as well as recommendations for suitable regulation on a national and international level.

A prominent approach in the area of systemic risk stems from the seminal work by Eisenberg and Noe [13]. Here the set V of financial institutions (or *firms*) is the node set of a directed graph $G = (V, E)$. The directed and weighted edges $e \in E$ express the debt

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relations among firms. In addition, each firm has non-negative *external assets*, which capture the value of property rights (such as real-estate, gold, business and mortgage loans, etc.) that the firm has acquired from non-financial institutions. Eisenberg and Noe discuss a clearing mechanism for such a market, in which every firm v uses its available assets to pay its debt. The clearing follows basic balance sheet identities and evolves rather mechanically. It is commonly assumed that payments are distributed among all creditors in a pro-rata fashion, i.e., for each interbank debt (i.e., each outgoing edge) firm v allocates a proportional share of its available assets. Similarly, a firm receives the payments for its incoming edges (so-called *internal assets*), which in turn are also paid proportionally to outgoing edges (until all debt is paid or all assets are distributed). In this way, clearing payments constitute a fixed point of the system. Eisenberg and Noe discuss existence and uniqueness issues of such fixed point payments.

While the model has been studied intensively over the last two decades, there are a plethora of open questions, especially towards understanding strategic and game-theoretic issues. In this paper, our focus are the strategic incentives and their consequences for clearing mechanisms in financial markets. For example, rather than using pro-rata payments, financial institutions often put priority on clearing certain debts first. In this way, a firm v could profit substantially when using a suitable priority-based payment scheme – by clearing certain debt first, the money propagates through the network and returns to v in the form of additional internal assets. In the Eisenberg-Noe model, it is easy to see that the intuitive pro-rata clearing mechanism is not always incentive compatible, and strategic incentives of this kind can arise frequently.

In this paper, we study the properties of priority-based clearing mechanisms for financial markets. We focus on payment schemes that constitute a pure Nash or even a strong equilibrium in the underlying strategic payment game for the firms. In this way, firms have no unilateral or coalitional incentives to deviate from the payment schemes proposed by the clearing mechanism. Depending on the granularity of priorities, the resulting games have different properties. In particular, if priorities are implemented over debt contracts, existence of a pure Nash or a strong equilibrium becomes strongly NP-hard to decide. Instead, if firms can assign each unit of money arbitrarily to any open debt, a strong equilibrium always exists and can be computed in strongly polynomial time. Moreover, in this case, there is even a strong equilibrium that maximizes the sum of all assets of all firms.

Our results imply interesting insights for bankruptcy settlement of insolvent firms. It turns out that only insolvent firms face a potential strategic decision about where to allocate money in order to maximize the internal assets through network effects (or, equivalently, minimize the remaining debt after clearing the network). In case there is a benevolent and centralized bankruptcy settlement, we show that it can implement a clearing mechanism with monotone payment strategies that leads to a socially optimal clearing state. It comes with the additional guarantee of giving no coalition of firms an incentive to pay their debts differently.

Instead, if clearing payments are determined by suitable negotiation in a decentralized fashion resulting in some arbitrary Nash or strong equilibrium, the total amount of internal assets available to the firms in the system can deteriorate drastically. Similar problems arise if a centralized clearing mechanism is restricted to payment schemes based on priorities over single loans (rather than units of money). This can lead to non-existence of pure equilibria in the resulting games. Even if equilibria exist, they can be undesirable since the total amount of internal assets of all firms can be drastically smaller than in an optimal solution. This shows a marked contrast between centralized and decentralized bankruptcy settlement and highlights how the structure of permissible payment strategies impacts the performance and the structural properties of the clearing mechanism.

1.1 Contribution

In this paper, we study the properties of priority-based clearing mechanisms for financial markets. We assume the network of liabilities is given, but insolvency resolution is driven by strategic considerations. In particular, we analyze payment schemes that constitute a pure Nash or even a strong equilibrium in the underlying strategic payment game for the firms.

Below, we introduce basic preliminaries of the Eisenberg-Noe model. In addition, we introduce two classes of priority-based payment strategies for the firms and analyze the resulting clearing mechanisms. For an *edge-ranking strategy*, a firm ranks its debt contracts and assigns its assets in order of the ranking. As a superset of strategies, we consider *coin-ranking strategies*, where money is considered in units ("coins"). Instead of contracts, each firm ranks single coin payments to the contracts. By letting the value of a coin approach zero, coin-ranking strategies become equivalent to monotone strategies, where the payments of a firm are simply a monotone function of its total available assets.

In Section 2 we present structural insights on the clearing states for a strategy profile in an edge- or coin-ranking game. In such a profile, each firm is choosing an edge- or coin-ranking strategy to pay its debt. For each such profile, we prove that the possible clearing states form a lattice with respect to the vector of assets of each firm (Theorem 3). In particular, there is a unique clearing state that pointwise maximizes the assets available to each firm (given this strategy profile). In the full version [6] we show that it can be computed in strongly polynomial time. We assume that this state defines the assets and, thus, the utility of each firm in the strategy profile. Similar properties were shown in [13] for profiles composed of pro-rata payment strategies.

In Section 3 we study coin-ranking games and strategic choice of payments. Our interest lies in the existence, computational complexity, and social quality of equilibria. We show that there always is a strategy profile that represents a strong equilibrium, in which no coalition of firms has an incentive to deviate. Furthermore, it maximizes social welfare, i.e., the sum of all assets or total revenue available to all firms. Such a strong equilibrium can be computed in strongly polynomial time (Theorem 6). It can be represented compactly, even though the strategy shall rank all (possibly pseudo-polynomially many) coins that a firm might have available. In contrast, it is strongly NP-hard to find a best-response strategy for a single firm in a given arbitrary strategy profile of a coin-ranking game (Theorem 9).

For worst-case equilibria and the strong price of anarchy, we show that the deterioration of social welfare in a strong equilibrium compared to a social optimum is tightly characterized by the min-max length of cycles in any social optimum (Theorem 11). This implies that networks with optimal money circulation composed of small cycles yield a small inefficiency in strong equilibria. In contrast, a worst-case Nash equilibrium, which is stable only against unilateral deviations, can be arbitrarily worse than a social optimum, even in simple games with a constant number of firms (Proposition 10).

In Section 4 we study equilibria in edge-ranking games, where all firms are restricted to play edge-ranking strategies. Restricting the strategy space to rankings over contracts can have devastating consequences for the existence and social quality of equilibria. In edge-ranking games, pure Nash and strong equilibria can be absent, and deciding their existence is strongly NP-hard (Theorem 14). The same hardness applies for computing a social optimum, and for computing a pure Nash or strong equilibrium when it is guaranteed to exist. Even the best strong equilibrium can be a factor of $\Omega(n)$ worse than the social optimum in terms of social welfare (Proposition 16). For pure Nash equilibria, even the best one can be arbitrarily worse than a social optimum (Proposition 17).

1.2 Related Work

Financial Networks. On a conceptual level, we study issues of strategic choice and computational complexity in financial networks. There have been works addressing computational complexity of diverse issues, such as pricing options with [3] and without information asymmetry [7], finding clearing payments with credit default swaps [27], or estimating the number of defaults when providing a shock in the financial system [20]. In addition, many extensions to the model by Eisenberg and Noe have been proposed in the literature on financial markets. However, even models including cross-holdings of equity [28], default costs [26], or debt contracts of different maturities [15] follow the idea of the basic approach that all contracts have to be cleared consistently, i.e., clearing payments locally adhere to the rather mechanical clearing rule and constitute a fixed point solution globally. Indeed, Barucca et al. [5] have recently shown that many of the above models can be unified in terms of self-consistent network valuations. A well-known result of such models is the "robust-yet-fragile" property exhibited by financial networks, i.e., contagion arises in an all-or-nothing fashion akin to the formation of a giant connected component in random graph models [17]. This provides important insights into systemic risk and advises the need for macro-prudential regulation.

Accordingly, the rather mechanical pro-rata payments are also usually presumed in models studying contagion effects arising from overlapping portfolios [8, 9]. In this case, distressed firms are selling assets which in turn decreases the value of these assets by market impact. Here, it is commonly assumed that firms mechanically sell all their assets in a pro-rata fashion. In turn, the resulting market impact is modeled as a known function parametrized by the market depth or liquidity of each asset. In contrast, especially decisions regarding the portfolio composition of financial firms yield substantial potential for strategic consideration in reality.

To our knowledge, strategic aspects are currently reflected mostly in models of network formation [14, 1]. A three period economy is assumed where firms can invest into risky assets. To do so, they strategically decide to borrow funds from outside investors as well as other firms. Thereby a network of financial cross-holdings is endogenously formed as each firm maximizes their expected profit. The results show that risk-seeking firms tend to over-connect leading to stronger contagion and systemic risk as compared to the socially optimal risk-sharing allocation. Note that in this case, strategic aspects only play a role in the formation of inter-bank relations whereas the clearing mechanism is assumed to follow the same process as in [13]. Another strategic variant of the model by Eisenberg and Noe has been considered in [2] in the form of a two-period model. Firms strategically store some amount from their first period endowment with an interest rate in order to increase the available assets in the second period. The clearing in both rounds is based on pro-rata payments. Each firm optimizes a function of the remaining debts in both periods.

Flow Games. On a more technical level, our game-theoretic approach is related to a number of existing game-theoretic models based on flows in networks. In cooperative game theory, there are several notions of flow games based on a directed flow network. Existing variants include games, where edges are players [12, 21, 22, 18, 11, 4], or each player owns a source-sink pair [25, 24]. The total value of a coalition C is the profit from a maximum (multi-commodity) flow that can be routed through the network if only the players in C are present. There is a rich set of results on structural characterizations and computability of solutions in the core, as well as other solution concepts for cooperative games. In contrast to our work, these games are non-strategic. Instead, here we consider each player as a single node with a strategic decision about flow allocation.

More recently, a class of strategic flow games has been proposed in [23, 19]. There is a capacitated flow network with a set of sources nodes. At each source node, a given amount of flow enters the network. Each node of the network is owned by a single player. Each player always owns a designated sink node, as well as one or more additional nodes from the network. A player can choose a flow strategy for each of her nodes. The flow strategy specifies, for every node v and every $x \geq 0$, how an incoming flow of x at v is distributed onto the outgoing edges (if any). Each flow strategy needs to fulfill flow conservation constraints at every node, subject to capacity on the outgoing edges. Each player aims to maximize the incoming flow at its sink node.

For these games there exist a number of Σ_2^p-completeness results for, e.g., determining the value of a game in a two-player Stackelberg variant, or determining the existence of a pure Nash equilibrium in a multi-player variant. In the latter game, computing a best response can also be NP-hard. Our approach is related to these games. However, motivated by financial networks we assume each firm is a single (source) node. The firm optimizes the incoming flow at its node (without it being a sink node). We study the computational complexity and social quality of equilibria. Moreover, strategic incentives arise mainly from cycles in the network – a condition absent in the existing work on max-flow games [23, 19] where the network is assumed to be acyclic.

The problem of calculating a clearing state for a given strategy profile in our games is closely related to the notion of a stable flows. In the stable flow problem, each node is equipped with an externally given preference order over both incoming and outgoing arcs. There always exists a stable flow, and the set of stable flows forms a lattice [16]. In fact, there is an augmenting path algorithm for computing a stable flow with polynomial running time [10].

1.3 Financial Networks with Payment Strategies

Network Model. We consider a financial network model due to Eisenberg and Noe [13]. There is a network $G = (V, E)$ with node set V of *institutions* or *firms*. Each firm $v \in V$ has *external assets* of value $a_v^x \geq 0$. Moreover, the firms are related via a set E of *liabilities*. Each liability $(u, v) \in E$ is a directed edge from firm $u \in V$ to firm $v \in V$. The *weight* $c(e) \geq 0$ of some edge $e = (u, v)$ is the amount of money that u owes to v. We follow standard notation in graph theory and denote by $E^+(v) = \{(v, u) \in E\}$ and by $E^-(v) = \{(u, v) \in E\}$ the set of outgoing and incoming edges of $v \in V$, respectively. The *total liabilities* $\ell(v)$ of firm v is the total amount of money firm v owes to other firms, i.e.,

$$\ell(v) = \sum_{e \in E^+(v)} c(e).$$

We strive to understand issues of computational complexity. As such, we will assume that all numbers in the input, i.e., all a_v^x and $c(e)$, are integer numbers.

We consider clearing mechanisms based on strategic payments decisions. A *money flow* g_e on edge e satisfies $0 \leq g_e \leq c(e)$. Given a money flow on each edge, the *internal assets* of firm v are the total incoming money from other firms, i.e.,

$$a_v^i = \sum_{e \in E^-(v)} g_e.$$

The *total assets* of v are the sum of external and internal assets $a_v = a_v^x + a_v^i$. A firm is *insolvent* if its total assets are strictly smaller than its total liabilities, i.e., $a_v < \ell(v)$.

Eisenberg and Noe define a clearing mechanism with money flows given by *pro-rata* payments. In their clearing mechanism, each firm v distributes its total assets a_v proportionally on its outgoing edges until all debt is paid. More formally, every edge $e \in E^+(v)$ is assigned a money flow of $g_e(a_v) = \min\left\{c(e), a_v \cdot \frac{c(e)}{\ell(v)}\right\}$. Firm v keeps the surplus $a_v - \ell(v) \geq 0$ for itself, if any.

Payment Strategies. In this paper, we analyze incentives when firms strategically manipulate their payments. As such, we study *money flow games* defined as follows. Each firm $v \in V$ chooses as a *strategy* a parametrized flow function $f_e(y)$ for every outgoing edge $e \in E^+(v)$ and every $y \geq 0$. The strategy $\mathbf{f}^v = (f_e(y))_{e \in E^+(v)}$ must satisfy for every $y \geq 0$

$$0 \leq f_e(y) \leq c(e) \qquad \text{(capacity constraint)} \tag{1}$$

$$\sum_{e \in E^+(v)} f_e(y) = \min\{y, \ell(v)\} \qquad \text{(no-fraud constraint)} \tag{2}$$

Intuitively, the strategy specifies, for every possible value $y \geq 0$ of total assets available to firm v, how v will allocate these assets to pay its debts. The capacity constraint ensures that no debt is overpaid, the no-fraud constraint requires that v does not embezzle assets as long as there is unpaid debt. This definition includes pro-rata payments as one possible strategy profile. Given a *strategy profile* $\mathbf{f} = (\mathbf{f}^v)_{v \in V}$, a *clearing state* $\mathbf{a} = (a_v)_{v \in V}$ is a vector of assets such that

$$a_v = a_v^x + \sum_{e=(u,v) \in E^-(v)} f_e(a_u) \qquad \text{(fixed point constraint)} \tag{3}$$

holds for all nodes $v \in V$. Equivalently, this ensures that for a given strategy profile $\mathbf{f}$, there is a clearing state $\mathbf{a}$ such that the flow $\mathbf{g}$ defined by $g_e = f_e(a_u)$ is in fact a money flow. The *utility* of firm v is a_v, i.e., v's goal is to choose a strategy to maximize its total assets in the clearing state.

▶ **Proposition 1.** *If all $f_e(y)$ are continuous, there exists at least one clearing state.*

The proof is a straightforward application of Brouwer's fixed point theorem and thus omitted. If strategies are not continuous, it is easy to construct examples where no clearing state exists. Still, even for a continuous strategy profile $\mathbf{f}$, there could be multiple clearing states $\mathbf{a}$ (even for pro-rata profiles). Given sufficiently complex strategy profiles with compact representation, computation of a clearing state might even become computationally difficult.

In the rest of the paper, we focus on a set of rich and meaningful strategy spaces, for which we can single out a unique clearing state with a simple algorithm. An intuitive and well-motivated class of strategies can be derived via *rankings* or *seniorities*.

Edge-Ranking Games. In an *edge-ranking game*, each player $v \in V$ spends its assets to pay its debts according to a strict and total order over $E^+(v)$, which we represent by a permutation $\pi_v = (e_1, e_2, \ldots)$. v first pays all debt of edge $e_1 = \pi_v(1)$, then $e_2 = \pi_v(2)$, etc. until all debt is paid or it runs out of assets. Formally, $f_{e_i}(y) = \min\{c(e_i), \max\{0, y - \sum_{j<i} c(e_j)\}\}$. The *edge-ranking strategy* of v is fully described by the ranking π_v, hence we denote a strategy profile in edge-ranking games by $\boldsymbol{\pi} = (\pi_v)_{v \in V}$.

Coin-Ranking Games. As a strict superset of such strategies, consider the case where each player $v \in V$ can spend its assets to pay its debts in a monotone fashion. In coin-ranking games, we rely on integrality of all values for c_e and a_v^x, and interpret money flow as being discretized into "coins" of value 1. Thus, for a *coin-ranking strategy*, the parametrized flow functions $f_e(y)$ for every outgoing edge $e \in E^+(v)$ are defined on the non-negative integer numbers $f_e(y) : \mathbb{N}_0 \to \mathbb{N}_0$. They are characterized by capacity and no-fraud constraints, and, for every $y, y' \in \mathbb{N}_0$ with $y \geq y'$

$$f_e(y) \geq f_e(y') \qquad \text{(monotonicity constraint)} \tag{4}$$

Note that, by letting the value of a coin tend to 0, coin-ranking strategies become arbitrary *monotone strategies* $f_e(y) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$.

Coin-ranking strategies generalize edge-ranking strategies. Maybe counterintuitively, every coin-ranking game is also a special edge-ranking game – replacing each edge e with weight $c(e)$ many multi-edges of unit weight expands a coin-ranking game into an equivalent edge-ranking game. There is a one-to-one correspondence between coin-ranking strategies in the original game and edge-ranking strategies in the expanded game. Intuitively, for a coin-ranking strategy in the original game, a player v pays the first coin of assets to the multi-edge $\pi_v(1)$, the second coin to $\pi_v(2)$, etc. in the expanded edge-ranking game until all debt is paid or v runs out of assets. The expansion of the game implies a pseudo-polynomial blowup in representation size, but nevertheless, the structural equivalence is very useful for characterizing and analyzing solutions to coin-ranking games.

Note that the representation of a coin-ranking strategy might require pseudo-polynomial size even in the original non-expanded game. We discuss this issue below in Section 3.1. It turns out that in every coin-ranking game, we can restrict attention to a subset of compactly representable coin-ranking strategies.

Clearing States and Utilities. For a given strategy profile $\boldsymbol{\pi}$ in an edge- or coin-ranking game, we determine the utility using a clearing state $\hat{\mathbf{a}}$, where we choose the one that maximizes the total revenue in the network, i.e., the sum of total assets available to all firms

$$\text{Rev}(\boldsymbol{\pi}, \mathbf{a}) = \sum_{v \in V} a_v = \sum_{v \in V} a_v^x + \sum_{e=(u,v) \in E} f_e(a_u) = \sum_{v \in V} a_v^x + \sum_{e \in E^+(v)} f_e(a_v).$$

Note that the choice of $\hat{\mathbf{a}}$ and thus, the resulting revenue significantly depends on the strategy profile $\boldsymbol{\pi}$. In many cases, for a fixed strategy profile $\boldsymbol{\pi}$, the clearing state $\hat{\mathbf{a}}$ is unique, and there is no choice based on maximum revenue. Moreover, in case there are several clearing states for a strategy profile $\boldsymbol{\pi}$ in an edge-ranking game, it turns out that they can be arranged into a lattice with coordinate-wise maximum inducing a partial order. We choose the coordinate-wise maximal clearing state, since it maximizes $\text{Rev}(\boldsymbol{\pi}, \mathbf{a})$ as a natural measure of social welfare. In this sense, the properties of edge-ranking games mirror the conditions shown for pro-rata payments in [13]. We prove these conditions in the subsequent section.

2 Clearing States

2.1 Circulation Structure in Money Flow Games

We observe a useful *circulation representation* of clearing states in money flow games. Given a strategy profile $\mathbf{f}$, the fixed point and no-fraud constraints imply for any clearing state $\mathbf{a}$ the conservation of money flow. Now using an auxiliary source s, we can represent all money flows in $\mathbf{a}$ in the form of a circulation. We build a *circulation network* G' by adding node s, for every $v \in V$ we add an auxiliary edge (v, s) with capacity $c((v, s)) = \infty$, and for every

$v \in V$ with $a_v^x > 0$ we add an auxiliary edge (s, v) with $c((s, v)) = a_v^x$. In this way, external assets of v become internal assets via a flow on edge (s, v). Surplus assets of w become a flow on edge (w, s), i.e., the internal assets of s.

▶ **Proposition 2.** *For every clearing state* $\mathbf{a}$ *of a strategy profile* $\mathbf{f}$ *in a money flow game, the flow in* G' *can be decomposed and represented as a circulation. The auxiliary source* s *has assets of* $a_s = \sum_{v \in V} a_v^x$*, and all auxiliary edges* (s, v) *are saturated.*

Proof. This is a simple consequence of fixed point and no-fraud constraints. Surplus at firm $v \in E$ exists only if v pays all debt

$$\sum_{e \in E^+(v)} f_e(a_v) = \min(a_v, \ell(v)) \enspace .$$

Moreover, the total external assets constitute the total net revenue:

$$\begin{aligned}
\sum_{v \in V} a_v^x \quad &= \quad \sum_{v \in V} a_v^x + \sum_{v \in V} \sum_{e=(u,v) \in E^-(v)} f_e(a_u) - \sum_{v \in V} \sum_{e=(u,v) \in E^-(v)} f_e(a_u) \\
&= \quad \sum_{v \in V} a_v - \sum_{e \in E^+(v)} f_e(a_v) \quad = \quad \sum_{v \in V} \max\{0, a_v - \ell(v)\}
\end{aligned}$$

The net revenue of every firm gets routed to the auxiliary source s and constitutes the assets a_s. Thus $a_s = \sum_{v \in V} \max\{0, a_v - \ell(v)\} = \sum_{v \in V} a_v^x$, and all auxiliary edges (s, v) are saturated. Overall, by routing the surplus assets to s, we obtain exact flow conservation at every node. As such, the flow can be decomposed and represented as a circulation. ◀

2.2 Structure for Monotone Strategies

Lattice Structure. Consider an arbitrary money-flow game and a strategy profile $\mathbf{f}$ of monotone strategies. Let $\mathcal{A}$ be the set of feasible clearing states for a strategy profile $\mathbf{f}$. We show that $(\mathcal{A}, \geq)$ forms a lattice with the coordinate-wise comparison. Formally, $\mathbf{a} \geq \mathbf{a}'$ iff $a_v \geq a'_v$ for all v; and $\mathbf{a} > \mathbf{a}'$ iff $a_v \geq a'_v$ for all v and $a_v > a'_v$ for at least one v.

▶ **Theorem 3.** *For every strategy profile* $\mathbf{f}$ *in a money-flow game with monotone strategies, the pair* $(\mathcal{A}, \geq)$ *forms a lattice.*

Proof. Consider $A = \left\{ \mathbf{a} \;\middle|\; 0 \leq a_v \leq a_v^x + \sum_{e \in E^-(v)} c(e), \quad \forall v \in V \right\}$, a superset of all possible asset vectors. Obviously, $(A, \geq)$ forms a lattice with the coordinate-wise comparison defined above. For a given strategy profile $\mathbf{f}$, the map $g : A \to A$ with $g(\mathbf{a})_v = a_v^x + \sum_{e=(u,v) \in E^-(v)} f_e(a_u)$ is a monotone function for every firm $v \in V$, since for every edge $e = (u, v) \in E$ the strategy of firm u implies that f_e is monotone in a_u. Obviously, the set of clearing states $\mathcal{A}$ is the set of fixed-point asset vectors of g. The result follows by applying the Knaster-Tarski theorem. ◀

Solvent Firms. The previous result implies uniqueness of $\hat{\mathbf{a}}$ for a given monotone strategy profile $\mathbf{f}$. We observe another interesting property, which shows that in monotone money flow games the maximal clearing state $\hat{\mathbf{a}}$ is unique as long as all insolvent firms stick to their strategy. Since every strategy satisfies capacity and no-fraud constraints, the payments of solvent firms remain the same if they receive the same assets, and vice versa. Consequently, strategies of solvent firms have no impact on the asset vector $\hat{\mathbf{a}}$. For any solvent firm v, every strategy constitutes a best response.

▶ **Proposition 4.** *For a given money flow game, consider any monotone strategy profile* $\mathbf{f}$*, the corresponding clearing state* $\hat{\mathbf{a}}$*, and any solvent firm* v *with* $\hat{a}_v \geq \ell(v)$*. Every strategy* $\mathbf{f}'^v$ *is a best response for* v *against the other strategies* $\mathbf{f}_{-v}$ *and results in the same clearing state* $\hat{\mathbf{a}}$*.*

Proof. Consider a deviation $\mathbf{f}'^v$, the resulting state $\mathbf{f}' = (\mathbf{f}'^v, \mathbf{f}^{-v})$ and the resulting revenue-maximizing clearing state $\mathbf{a}'$. Suppose that $\hat{\mathbf{a}} \neq \mathbf{a}'$. Firm v is solvent under $\hat{\mathbf{a}}$, thus $\sum_{e \in E^+(v)} f_e(\hat{a}_v) = l(v)$ and $f_e(\hat{a}_v) = c(e)$. Using Theorem 3 we can assume w.l.o.g. that $\hat{\mathbf{a}} > \mathbf{a}'$ (i.e., $\hat{a}_u \geq a'_u$ for all firms u, and $\hat{a}_w > a'_w$ for at least one firm w). We construct an equivalent game, in which we remove all edges in $E^+(v)$ and instead increase external assets to $\bar{a}^x(u) = a^x_u + c(e)$ for all u with $(v, u) \in E^+(v)$. Observe that $\hat{\mathbf{a}}$ is still a feasible clearing state in the constructed game. However, any clearing state $\mathbf{a}$ in the original game with $a_v \geq l(v)$ induces $f(a_v) = c(e)$ independent of the chosen strategies of v due to the non-fraud condition. Thus, any clearing state $\mathbf{a}$ with $a_v \geq l(v)$ in the new game is still a feasible clearing state in the old game. We conclude that $\hat{\mathbf{a}}$ is a feasible clearing state under $\mathbf{f}'$. This is a contradiction to the maximality of $\mathbf{a}'$. ◀

3 Coin-Ranking Games

3.1 Representation

An instance of a money flow game is given by the network G and integer numbers for edge weights $c(e)$ and external assets a^x_u. Hence, the representation of the instance is logarithmic in input numbers for $c(e)$ and a^x_u. In contrast, if we consider arbitrary coin-ranking strategies $\mathbf{f}^v$ for firm v, this specifies a ranking over all *coins* of value 1. This is linear in $\sum_{e \in E^+(v)} c(e)$ and, thus, only pseudo-polynomial in the instance representation. Our first observation is that in every coin-ranking game, we can restrict attention to threshold-ranking strategies with a polynomial representation. A *threshold-ranking* strategy $\pi^t_v = (\pi_v, \tau_v)$ is composed of a permutation π_v over $E^+(v)$ and a vector of thresholds $\tau_v = (\tau_e)_{e \in E^+(v)}$ with $0 \leq \tau_e \leq c(e)$ for every $e \in E^+(v)$.

The interpretation is as follows. In π^t_v, firm v first pays τ_e to every edge $e \in E^+(v)$, sequentially in the order given by π_v. Then, it pays the remaining $c(e) - \tau_e$ to every edge in the order given by π_v. That is, v first considers edge $\pi_v(1)$ and pays the first $\tau_{\pi_v(1)}$ coins to this edge. The next $\tau_{\pi_v(2)}$ coins are paid to edge $\pi_v(2)$ etc. until $\sum_{j=1}^{|E^+(v)|} \tau_{\pi_v(j)}$ coins are paid to the edges (or v runs out of assets). Then, the remaining $c(\pi_v(1)) - \tau_{\pi_v(1)}$ coins are paid to edge $\pi_v(1)$, then the next $c(\pi_v(2)) - \tau_{\pi_v(2)}$ coins to $\pi_v(2)$ etc.

Indeed, we can restrict attention to threshold-ranking strategies in coin ranking games. The formal proof is deferred to the full version [6].

▶ **Proposition 5.** *For every strategy profile* $\mathbf{f}$ *in a coin-ranking game with clearing state* $\hat{\mathbf{a}}$ *and every firm* v*, there is a threshold-ranking strategy* π^t_v *such that the profile* $(\pi^t_v, \boldsymbol{\pi}_{-v})$ *has the same clearing state* $\hat{\mathbf{a}}$ *and, thus, the same utilities for all firms.*

3.2 Existence and Computation of Equilibria

Our first result is that in every coin-ranking game there is a strong equilibrium that maximizes the total revenue of all firms. This strong equilibrium can be computed in polynomial time. In particular, consider the instance (G, c, a^x) as a money flow game and an arbitrary clearing state, i.e., a circulation of maximum value in the circulation network G'. This circulation is also a clearing state of a strong equilibrium in threshold-ranking strategies.

▶ **Theorem 6.** *For every coin-ranking game, there is a strong equilibrium with money flows that maximize the total revenue in the network. The strong equilibrium can be computed in polynomial time.*

Proof. Consider the circulation network $G' = (V, E')$. An optimal circulation $\mathbf{f}^*$ that maximizes the total flow value saturates all outgoing auxiliary edges from s. Hence, it maximizes the total assets of all firms

$$\sum_{e \in E'} f_e^* = 2 \sum_{v \in V} a_v^x + \sum_{e \in E} f_e^* = \sum_{v \in V} a_v^x + \textsc{Rev}(\mathbf{f}^*) \ .$$

$\mathbf{f}^*$ can be computed in strongly polynomial time [29]. Since all edge weights are integral, we can assume all f_e^* are integral. We can turn this circulation into a clearing state for a strategy profile with threshold-ranking strategies. Every firm v chooses an arbitrary order π_v over $E^+(v)$ and sets thresholds $\tau_e = f_e^*$. Clearly, in this strategy profile the optimal circulation corresponds to the maximum-revenue clearing state $\hat{\mathbf{a}}$.

Let us prove that this strategy profile $\boldsymbol{\pi}$ is a strong equilibrium. A coalition $C \subseteq V$ of firms has a profitable deviation $\boldsymbol{\pi}'_C = (\pi'_v)_{v \in C}$ if upon joint deviation of C to π'_C, the resulting assets $\mathbf{a}'$ in the new profile $(\boldsymbol{\pi}'_C, \boldsymbol{\pi}_{-C})$ are strictly better, i.e., $a'_v > \hat{a}_v$ for every $v \in C$. We will show that no coalition $C \subseteq V$ has a profitable deviation.

Suppose for contradiction that there is a coalition C with a profitable deviation. Examine the new profile $(\boldsymbol{\pi}'_C, \boldsymbol{\pi}_{-C})$ and consider a firm $v \in C$. Since $a'_v > \hat{a}_v$, there must be an edge $(u, v) \in E^-(v)$ that has strictly more incoming flow in the new profile $f'_e(a'_u) > f_e(a_u)$. Now consider node u. If $u \in C$, then $a'_u > a_u$, so there is again some edge $E^-(u)$ that has strictly more incoming flow in the new profile. Otherwise, if $u \notin C$, then u still plays the threshold-ranking strategy obtained from $\mathbf{f}^*$. Since this is a monotone strategy, a higher flow on (u, v) can only occur if u has larger assets. Thus, $a'_u > a_u$, so there is again some edge $E^-(u)$ that has more incoming flow in the new profile.

We can repeat this argument indefinitely. As such, there must be a cycle of edges that all have more flow under $(\boldsymbol{\pi}'_C, \boldsymbol{\pi}_{-C})$ than under $\boldsymbol{\pi}$. Such a cycle, however, can be used to increase the flow circulation. This contradicts that $\hat{\mathbf{a}}$ represents an optimal circulation. ◀

▶ Remark 7. For the profitable deviation, we can even allow arbitrary continuous strategies and any choice of clearing state for the deviation profile. As such, the strategy profile obtained from $\mathbf{f}^*$ is a strong equilibrium even in general money flow games (with suitable choice of clearing state).

▶ Remark 8. If one allows deviations that weakly improve the coalition (i.e., $a'_u \geq \hat{a}_u$ for all $u \in C$ and $a'_v > \hat{a}_v$ for at least one $v \in C$), it is a simple exercise to see that there are coin-ranking games, in which no stable state (termed super-strong equilibrium) exists.

While it is computationally easy to compute a socially optimal strong equilibrium, computing a best-response strategy for a general strategy profile can be strongly NP-hard, since best responses can provide answers to computationally hard decision problems. For the following result, we assume the coin-ranking game is given in the edge-ranking representation as a network with unit-weight multi-edges. Note that the edge weights in our instances of interest can be restricted to the set $\{0, 1\}$. Thus, our construction needs no multi-edges, and the representation incurs no overhead. A proof of the following theorem is given in the full version [6].

▶ **Theorem 9.** *For a given strategy profile* $\mathbf{f}$ *of a coin-ranking game, deciding whether a given firm v has a best response resulting in assets at least k is strongly NP-complete. This holds even in coin-ranking games without external assets and all edge weights in* $\{0, 1\}$.

3.3 Total Revenue of Equilibria

In this section, we analyze the total revenue in pure Nash and strong equilibria. We relate this value to the social optimum, i.e., the total sum of assets for all firms in the best strategy profile. Clearly, since we proved existence of a system optimal strong equilibrium, the price of stability for Nash and strong equilibria are both 1. We bound the prices of anarchy for Nash and strong equilibria. The proof of the next Proposition is given in the full version [6].

▶ **Proposition 10.** *The price of anarchy for Nash equilibria is unbounded, even in coin-ranking games without external assets.*

The total revenue depends crucially on the emergence of cycles in the strategy profile. This requires an effort that is inherently coalitional, as such it might be unsurprising that in general Nash equilibria fail to provide good revenue guarantees.

To analyze the quality of strong equilibria, we again consider the coin-ranking game in the form of unit-weight multi-edges. Consider an optimal circulation $\mathbf{f}^*$ of maximum total revenue in the circulation network G'. Since we have unit-weight edges, we can assume that the optimal circulation has binary flows on each edge. Let $\mathcal{C}(\mathbf{f}^*) = \{C_1, \ldots, C_k\}$ be a decomposition of $\mathbf{f}^*$ into cycles of unit flow. We denote by

$$d = \min_{\mathbf{f}^*, \mathcal{C}(\mathbf{f}^*)} \max_{C \in \mathcal{C}(\mathbf{f}^*)} |C_i|$$

the min-max size of any cycle, in any decomposition $\mathcal{C}(\mathbf{f}^*)$ of any optimal circulation $\mathbf{f}^*$.

▶ **Theorem 11.** *In coin-ranking games, the strong price of anarchy is at most d.*

Proof. Consider an optimal circulation $\mathbf{f}^*$ and a decomposition $\mathcal{C}(\mathbf{f}^*)$ such that all flow cycles $C_i \in \mathcal{C}(\mathbf{f}^*)$ have size at most $|C_i| \leq d$. The total revenue in $\mathbf{f}^*$ is given by

$$\textsc{Rev}(\mathbf{f}^*) = \sum_{C_i \in \mathcal{C}(\mathbf{f}^*)} |C_i| - \sum_{v \in V} a_v^x$$

since the circulation also accounts for the assets of the auxiliary source s.

Now consider a strong equilibrium $\boldsymbol{\pi}$ in the coin-ranking game with clearing state $\hat{\mathbf{a}}$. It yields a binary money flow in the network. Suppose there is a cycle $C_i \in \mathcal{C}(\mathbf{f}^*)$ such that $f_e(\hat{a}_u) = 0$ for all $e = (u, v) \in C_i$. Then the firms in this cycle have an incentive to jointly deviate and place the edges of C_i on first position in their ranking. Then the clearing state $\hat{\mathbf{a}}$ will emerge as before, adding a flow of 1 along the cycle C_i. This is a profitable deviation for the firms of C_i.

Consequently, for every cycle $C_i \in \mathcal{C}(\mathbf{f}^*)$ there must be at least one edge $e = (u, v) \in C_i$ such that $f_e(a_u) = 1$. Thus, the revenue in the strong equilibrium $\boldsymbol{\pi}$ is

$$\textsc{Rev}(\boldsymbol{\pi}, \hat{\mathbf{a}}) \geq \sum_{C_i \in \mathcal{C}(\mathbf{f}^*)} 1$$

and the ratio is at most d. ◀

▶ **Proposition 12.** *For every $d \geq 2$, there is a coin-ranking game with strong price of anarchy of $d - 1$.*

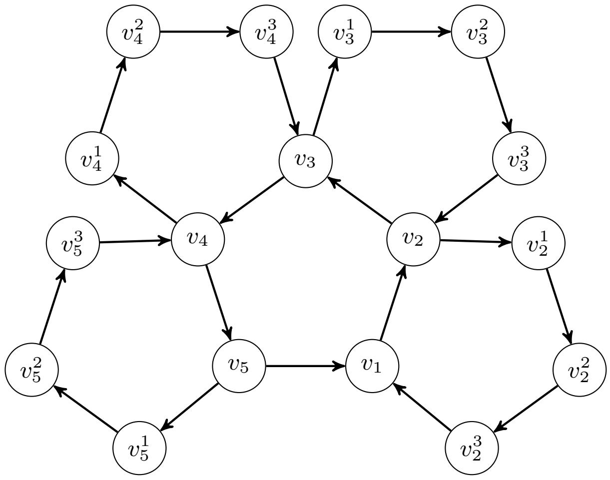

Figure 1 A coin-ranking game with $d = 5$ and a strong price of anarchy of $d - 1 = 4$.

Proof. The game is given by a graph G with $d + (d-1)(d-2)$ firms. G is constructed as follows. The firms $v_1, \ldots, v_d$ are called *central firms* and they form a cycle of length d. For each $i = 1, \ldots, d-1$, there are firms $(v_{i,j})_{j=1,\ldots,d-2}$ that form additional cycles of length d with the edge (v_i, v_{i+1}). Thus, the set of edges is given by

$$E = \{(v_i, v_{i+1}) \mid i \in \{1, \ldots, d-1\}\} \cup \{(v_d, v_1)\} \\ \cup \bigcup_{i=2,\ldots,d} \left((v_i, v_i^1) \cup \{(v_i^j, v_i^{j+1}) \mid j = 1, \ldots, d-3\} \cup (v_i^{d+2}, v_{i-1})\right) .$$

All edges have unit weight. An example of the instance with $d = 5$ is depicted in Figure 1. Observe that only firms v_i, $i = 2, \ldots, d$ have multiple outgoing edges. We claim that $\pi_i = \left((v_i, v_{i+1}), (v_i, v_i^1)\right)$ and $\pi_d = \left((v_d, v_1), (v_d, v_d^1)\right)$ is a strong equilibrium. In order to see this, let $\hat{\mathbf{a}}$ be the clearing state corresponding to $\boldsymbol{\pi}$. The clearing state is given by

$$\hat{a}_v = \begin{cases} 1 & \text{if } v = v_1, v_2, \ldots, v_d \ , \\ 0 & \text{otherwise} \ , \end{cases}$$

that is, $\textsc{Rev}(\boldsymbol{\pi}, \hat{\mathbf{a}}) = d$. Now, assume there is a non-empty coalition of player $S \subseteq (v_2, \ldots, v_d)$ that all strictly increase their assets by a joint deviation. Note that v_d only has a single incoming edge, so $v_d \notin S$. Thus, the cycle $v_d, v_d^1, \ldots, v_d^{d-2}, v_{d-1}$ cannot carry any flow. We conclude that v_{d-1} has only a single edge that can carry flow. Iterating this argument yields $S = \emptyset$, a contradiction.

However, the optimal flow emerges with the strategy profile $\pi_i = \left((v_i, v_i^1), (v_i, v_{i+1})\right)$ and $\pi_d = \left((v_d, v_d^1), (v_d, v_1)\right)$. It is easy to observe that this yields a total revenue of $(d-1)d$. Thus, the strong price of anarchy in this instance is $d - 1$. ◀

4 Edge-Ranking Games

4.1 Existence and Computation of Equilibria

With coin-ranking strategies we assume that firms have flexibility in allocation of single units of assets. In this section, we focus on a more restricted class of strategies, in which firms simply rank their outgoing edges and allocate assets in order of this ranking until they run

Table 1 Utility matrix for firms v_2 and v_3 in the proof of Proposition 13.

π_{v_2} \ π_{v_3}	$((v_3, v_1), (v_3, v_8))$	$((v_3, v_8), (v_3, v_1))$
$((v_2, v_1), (v_2, v_5))$	4 4	3 4
$((v_2, v_5), (v_2, v_1))$	2 5	3 3

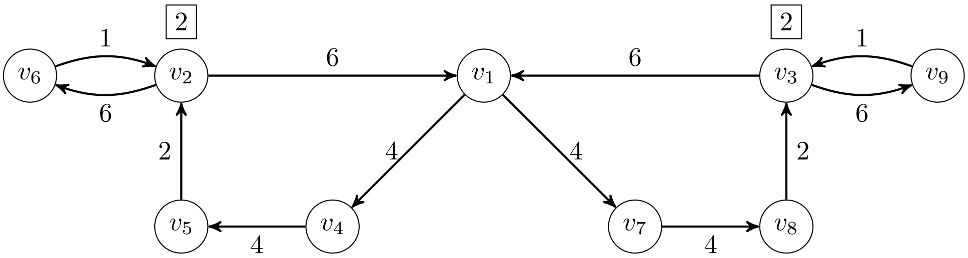

Figure 2 An edge-ranking game without a pure Nash equilibrium.

out of assets of all debts are paid for. In contrast to coin-ranking games, the restriction to rankings over edges (with different weights) can destroy the existence of (optimal) stable states. In fact, there are even games without a single pure Nash equilibrium.

▶ **Proposition 13.** *There is an edge-ranking game without a pure Nash equilibrium.*

Proof. Consider the game in Figure 2. The capacities of the edges are depicted next to the edges. Firms v_2 and v_3 each have external assets of 2, the other firms have 0 external assets. Firms v_1, v_2 and v_3 are the only ones with multiple outgoing edges. The strategy choices of the other firms are fixed. Due to the symmetry of the network, we can assume w.l.o.g. $\pi_{v_1} = ((v_1, v_4), (v_1, v_7))$. There are two possible strategy choices for each of the nodes v_2 and v_3. Checking all four resulting strategy profiles yields the utility matrix shown in Table 1 for firms v_2 and v_3. Inspecting the utilities, we see that there is no pure Nash equilibrium. ◀

The following theorem shows that a number of natural decision and optimization problems in edge-ranking games are indeed computationally intractable. Note that for coin-ranking games, these problems are either trivial (a strong equilibrium always exists) or can be solved in polynomial time (a strong equilibrium, which also represents a profile with maximum total revenue, can be computed in strongly polynomial time). The proof of the following theorem is deferred to the full version [6].

▶ **Theorem 14.** *Given an edge-ranking game the following problems are strongly* NP*-hard:*
1. *Deciding whether a pure Nash equilibrium exists or not.*
2. *Deciding whether a strong equilibrium exists or not.*
3. *Computing a pure Nash equilibrium, when it is guaranteed to exist.*
4. *Computing a strong equilibrium, when it is guaranteed to exist.*
5. *Computing a strategy profile with maximum total revenue.*

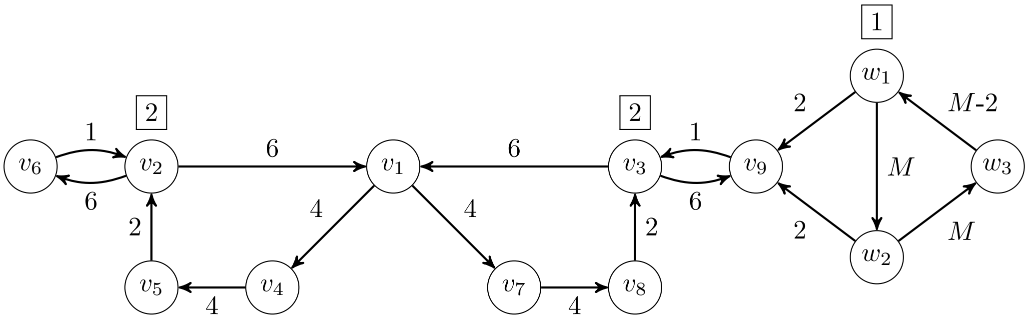

Figure 3 Edge-Ranking Game with unbounded Price of Stability.

▶ Remark 15. It is unclear whether the problem of deciding existence of a pure Nash equilibrium in an edge-ranking game is in NP or not, due to NP-hardness of verification that a firm plays a best response (see Theorem 9). It is easy to see that the decision problem is in Σ_2^p. The problem could be Σ_2^p-complete, similar to related decision problems in strategic max-flow games [23, 19]. Proving such a result is an interesting open problem.

4.2 Total Revenue of Equilibria

For edge-ranking games, the lower bound on the price of anarchy observed for coin-ranking games does apply, i.e., the price of anarchy can be unbounded. The restriction to edge-ranking strategies can have a drastic effect even on the quality of the best equilibrium in case it exists. In particular, in edge-ranking games the strong price of stability can be as high as $\Omega(n)$, and the price of stability might even be unbounded.

▶ **Proposition 16.** *For every $\varepsilon > 0$, there is an edge-ranking game with strong price of stability of at least $n/2 - \varepsilon$.*

Proof. We consider a slight modification of the instance in Proposition 12. In contrast to the instance described in the proof of Proposition 12, the edges $(v_1, v_n), (v_n, v_1), (v_1, v_2)$ have a weight $M + 1$ and all other edges a weight of M. The only node with more than a single outgoing edge is still v_1. If $\pi_{v_1} = ((v_1, v_n), (v_1, v_2))$, player v_1 gets assets of $M + 1$, which is optimal. The total revenue for $\boldsymbol{\pi}$ is $2M + 2$. $\boldsymbol{\pi}$ is the only Nash equilibrium and the only strong equilibrium.

In contrast, for profile $\boldsymbol{\pi}'$ with $\pi'_{v_1} = ((v_1, v_2), (v_1, v_n))$, firm v_1 only gets a revenue of M, but the total revenue is nM. Thus, the strong price of stability is $nM/(2M + 2) = n/2 - n/(2M + 2)$, which is at least $n/2 - \varepsilon$ for $M \geq n/(2\varepsilon) - 1$. ◀

▶ **Proposition 17.** *There is an edge-ranking game with unbounded price of stability.*

Proof. Consider the game in Figure 3, which uses the game without pure equilibrium from Figure 2. We add three firms. w_1 has external assets equal to 1, w_2 and w_3 no external assets. These firms have a cycle C of edges (w_1, w_2) and (w_2, w_3) with weight $M \gg 2$, as well as edge (w_3, w_1) with weight $M - 2$. In addition, there are edges (w_1, v_9) and (w_2, v_9) of weight 2.

In an optimal circulation, w_1 and w_2 prioritize the edges of C, leading to total revenue of $\Theta(M)$. In contrast, a pure Nash equilibrium can only exist if the w-firms ensure that the external assets of w_1 are routed to v_9, in which case a Nash equilibrium can exist (as observed in the proof of Theorem 14). Clearly, both w_1 and w_2 have an incentive to deviate towards C. Hence, if *either* w_1 *or* w_2 places the edge to v_9 in first rank and the other does not, a unilateral deviation suffices to close C – thereby leaving the v-nodes with instability. However, if *both* w_1 *and* w_2 play strategies $\pi_{w_1} = ((w_1, v_9), (w_1, w_2))$ and $\pi_{w_2} = ((w_2, v_9), (w_2, w_3))$, no unilateral deviation can lead to flow along C. In this case, a pure Nash equilibrium evolves. Obviously the total revenue in this equilibrium is at most a constant. Hence, the price of stability is as large as $\Omega(M)$. ◀

5 Conclusions

In this paper, we have studied clearing mechanisms for financial networks and analyzed their properties from a computational and game-theoretic perspective. Our main results show that in these games, solvent firms have no strategic incentives, i.e., the game is played exclusively among insolvent firms. If firms are using coin-ranking strategies, every social optimum that maximizes the sum of all assets in the network constitutes a strong equilibrium. Moreover, it can be computed in strongly polynomial time. This result implies that a centralized bankruptcy settlement can achieve a clearing state, in which the social welfare is maximized and no coalition of firms gets incentivized to deviate. In contrast, when considering decentralized clearing and arbitrary strong equilibria, the social welfare depends on the length of cycles in the money circulation of a social optimum. For pure Nash equilibria, the deterioration in social welfare can be severe due to the lack of coordination among firms. Alternatively, when restricting the strategy spaces to edge-ranking strategies, we show that pure Nash and strong equilibria can be absent, hard to compute, and highly undesirable in terms of social welfare.

There are many open problems that arise from our work. For example, real-life markets involve a number of complex financial products (such as derivatives, credit-default swaps, etc.). Their impact on stability and computational complexity of financial markets is only beginning to attract attention in the literature. In this context, there are a variety of important game-theoretic aspects with respect to pricing, information revelation, or network creation, which are crucial for understanding financial markets and represent interesting avenues for future work.

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