


# An Augmented Reality Mathematics Serious Game

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## Abstract

This article presents the results obtained from an experiment using an Augmented Reality (AR) serious game for learning mathematical functions in middle school, in contexts that resort to Game Based Learning. A serious game was created specifically for this purpose and allowed to conduct an exploratory study with a quantitative and qualitative methodological approach, with two groups of teachers of different subjects: mathematics and informatics. The game, called FootMath, allows the visualization, manipulation and exploration of linear, quadratic, exponential and trigonometric mathematical functions, through the simulation of a 3D football game, in which the user can change the function parameters with different values, in order to score a goal. It was tested the potential use of AR technologies in learning scenarios, considering the teacher's perspective. According to the findings, FootMath was considered to be a promising and innovative tool to be incorporated in real mathematics teaching scenarios.

**2012 ACM Subject Classification** Computing methodologies → Mixed / augmented reality

**Keywords and phrases** Serious Game, Augmented Reality, Mathematics, Functions

**Digital Object Identifier** 10.4230/OASICS.ICPEC.2020.6

**Acknowledgements** 2Ai – Applied Artificial Intelligence Lab, Polytechnic Institute of Cávado and Ave, Portugal

## 1 Introduction

Although still in early stages, research has shown that Augmented Reality (AR) has become increasingly popular [9] as a promising innovative technology that offers a new way to blend virtuality and reality [5]. The number of mobile applications with AR targeting multiple areas of education has rapidly increased [17, 21] and it is thriving. The interest in AR is not limited to education but encompasses other areas of knowledge such as science, engineering, business and entertainment. To some extent, this growth results from the adaptability of AR to the technological changes of mobile platforms [14], such as smartphones and tablets (usually equipped with a camera, gyroscope and accelerometer) as well as the increase of the Internet speed, local networks, and ubiquitous access. The fast pace of technological changes has made AR-based resources appealing to different agents of education due to their potential and the new pedagogical opportunities [26], particularly on account of their level of interaction, and



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First International Computer Programming Education Conference (ICPEC 2020).

Editors: Ricardo Queirós, Filipe Portela, Mário Pinto, and Alberto Simões; Article No. 6; pp. 6:1–6:8

OpenAccess Series in Informatics



OASICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

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how the information is exchanged, the spatial integration of the surrounding environment, and the possibility of connecting with one's daily life. While facing this new paradigm, new tools and pedagogical strategies that allow students to visualize the world in a new way, are imperative. AR technology can be experienced through earphones, glasses, and headsets or Head Mounted Displays (HMD). The Microsoft Hololens and the Magic Leap are examples of AR projects based on HMD [9]. However, AR is mostly applied in mobile devices such as tablets and smartphones. In the future, other (wearable) devices such as smartwatches and contact lenses could possibly be other promising targets of AR technology. Concerning mobile devices, the virtual objects are included in the image that is given by the device's camera (lens), in real-time, while realistically blending with the tangible world. In this context, we set out to promote new spaces for learning while considering the challenges students face when learning mathematics. This study was carried out with the aim of understanding and answering the following Research Questions: - RQ1. What are the challenges that AR poses when learning basic mathematical functions in a digital game? - RQ2. What do teachers think about this new learning opportunity based on a digital AR game? The investigation revolved around an exploratory study on the impact and challenges of using (i) a serious game for learning mathematical functions, and (ii) a component of tangible interaction that is mediated by AR technology for a mobile platform, such as Android or iOS. The choice of these platforms stems from the fact that the majority of middle school students have a cell phone and/or tablet (sometimes equipped with superior technology than the resources that are available in schools). Through a playful (gameplay) approach, the game intends to help problem solving, and the understanding of some mathematical concepts associated with basic functions, such as  $y = ax + b$  (linear function) and  $y = ax^2 + bx + c$  (quadratic function). The game environment intends to simplify the processes that are related to both teaching and learning. Currently, the use of serious digital games and AR in education are two important research topics [16, 23]. Research on Game Based Learning (GBL) is by far more comprehensive than research on AR in education, since GBL and its associated technologies are older than the use of AR in education. A search of the terms "Game Based Learning" on Google Scholar, provides 3,5 million results; whereas the terms "Augmented Reality", provided around 700.000 results. Research related to the improvement of the quality of education refers that the adequate integration of technological resources with a particular pedagogy and contents, promotes good practices and the success of learning and teaching experiences [15, 24]. In a brief overview, the literature related to the teaching and learning of mathematics describes positive aspects and multiple advantages of using digital technologies, in particular in face-to-face lessons with the different methods of teaching and learning. The world's largest organization of teachers of mathematics, the National Council of Teachers of Mathematics, with members and affiliates in the United States and Canada, provides a method for the introduction of technology in mathematics lessons, anchored on the notion that technology has the potential to improve the learning of mathematics, to support more effective teaching of mathematics, and to significantly influence what mathematics can teach [20]. In this context and aligned with the challenges of developing tools and methodologies that motivate the students, this work aimed to develop and investigate the potential of an AR serious game for learning mathematical functions in e-contexts. The following statement is particularly relevant.

*"(. . .) Many children find mathematics difficult and boring. But they are curious, and they love to have fun with exciting things around them. Appropriate activities can be found to stimulate them to have fun and love learning mathematics (. . .)" [27, p. 759]*

## 2 Background and related work

When it comes to learning, one cannot question the value and relevance of digital games: gaming is instinctive, and, by playing, one is rewarded with a learning experience [19]. Marc Prensky lists several characteristics that make digital games so engaging and appealing to millions of people: games are a type of entertainment, that is interactive and adaptable. Games have goals, results, feedback, rewards, conflict, competition, challenges, opposition, problems, representation and history [22].

### 2.1 Learning based on games

Educational or serious games can be defined as digital games to improve and promote learning, with the advantage of being a type of entertainment [16]. These types of games (video or digital) seem to increase the player's motivation while allowing the progression and assimilation of new learning contents within a continuous and significant narrative. Malone and Lepper [18] identified four strengths that might potentially promote a learning environment as a game activity, which is intrinsically motivational: fantasy, curiosity, challenge, and control. Thereby, with these four elements, Malone and Lepper show what educational games should enforce, while attempting to define specific principles for the design of these same games: 1) The games should use fantasy to reinforce the learning outcomes and stimulate the pre-acquired interests of the student; 2) Games must create sensorial stimuli (such as through audio-visual media) and develop the cognitive curiosity of the student; 3) Games should pose a challenge; through the achievement of goals and feedback, the student must feel continuously stimulated, and the difficulties must be increased taking into account a balance between the obstacles that are posed by the game and the acquisition of skills, in order to prevent that the student gets bored or frustrated; and 4) The student must feel a sense of control through the feedback that is provided throughout the game; i.e., s/he should feel that the learning outcomes are determined by his/her own actions. These four key elements may potentially enhance learning and must be taken into consideration in order to understand if a game meets what is required of it and if it is adequate to be used as a learning resource [18]. According to the literature, digital games may be successfully used as complementary learning tools [6]. Furthermore, the use of digital games in learning encourages the research of new paradigms of teaching-learning given their advantages over traditional learning materials, such as encouraging students to make decisions and the experimentation of different solutions to solve problems [10]. As previously mentioned, the new technological developments have the potential to create innovative scenarios for learning and teaching in real and virtual environments, as well as those that combine them. In particular, the use of AR digital games in the classroom allows the introduction of different methodologies and learning strategies that are focused on the student/group, while exploring the motivation and attention that is incited by technology (interactive digital environment). In mathematics lessons, educational digital games potentiate immersive experiences (in different degrees) in which the players may retain information, think and solve problems in an amusing way. Therefore, the games that are used in educational contexts involve pedagogical aspects. However, and although the pedagogical elements must be taken into consideration, the element of entertainment must be brought to the forefront. It is precisely the educational component that transforms pure entertainment into a powerful learning tool referred to as serious games. Serious games recur to pedagogy to introduce instruction in the gaming experience [28].

## 2.2 Augmented Reality in Education

The areas of applicability of Virtual Reality, Augmented Reality and Mixed Reality technologies are increasingly more diverse. These types of technologies have been gaining interest and notoriety within the research community. The new AR technology, in particular has appeared in education and in research work, showing that its application may have rather positive learning outcomes [8, 11]. The present research intends to analyze the value and the impact of AR technology in the process of teaching and learning, while presenting AR as part of a new paradigm of education. AR applications are particularly adequate to the visualization of space, offering advantages over other forms of materials (such as books), because they allow to simplify the visualization of 3D objects, by simulating dynamic processes that are not easily visible in real life [7, 12]. Consequently, AR applications contribute to a reduction of the cognitive load of the users [3, 13], by freeing cognitive resources. A systematic review of research work on the use of AR in educational contexts [1] points out motivation, interaction, collaboration, and learning as the main advantages of such use. These studies suggest that AR can be used effectively in educational contexts, while contributing to enhancing the motivation and collaboration of the student, potentially resulting in a better process of learning. Billinghurst and Duenser [2] state that:

*“(...) AR educational media could be a valuable and engaging addition to classroom education and overcome some of the limitations of text-based methods, allowing students to absorb the material according to their preferred learning style (...)”.*

The pedagogical approach to be adopted while facing an AR resource and the alignment between the design/interface of the technology, the methodology of teaching and the experiences of learning are key elements that must be taken into consideration [25]. Although AR offers new opportunities for learning, it also poses new challenges, both for teachers and students.

## 3 Technical Development of FootMath

The FootMath is an AR serious game, following the Game Based Learning strategy (Fig. 1), was developed to run on Android mobile devices. In order to do so, two development platforms were used: the game engine Unity<sup>1</sup> and the augmented reality engine Vuforia<sup>2</sup> platform. FootMath was designed to incorporate AR and to work with 2D physical markers (Fig. 2).

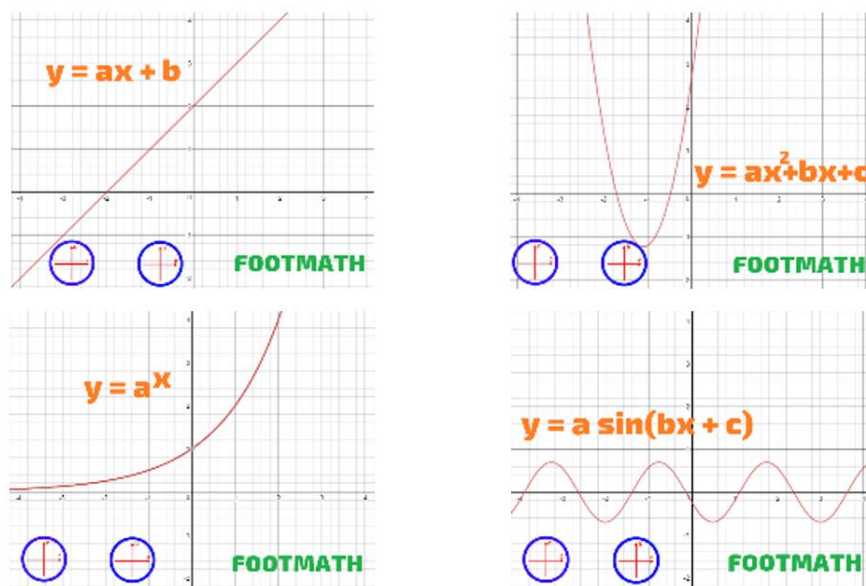
In its execution, the markers are a part of the tangible interface and are used to activate the exploration of the different mathematical functions. Previous experiments with physical AR markers allowed us to explore spatial and mathematical concepts with significant results in the learning process [4]. The database of the physical markers was created in the Vuforia platform and then printed in paper. Vuforia indicated the quality and the degree of precision of each marker, by classifying them according to a particular scale. In order for the physical markers to be successfully detected, each marker must rank high on the scale when it comes to their visual characteristics.

<sup>1</sup> <https://unity.com/>

<sup>2</sup> <https://developer.vuforia.com/>



■ **Figure 1** FootMath.



■ **Figure 2** (clockwise) Markers for the functions: linear,  $y = ax + b$ ; quadratic,  $y = ax^2 + bx + c$ , exponential,  $y = a^x$  and trigonometric sine,  $y = a \sin(bx + c)$ .

#### 4 Exploratory activity with FootMath

In the present study, our sample size consisted of 22 middle school teachers, divided into two groups (created in different dates and places): one group of 11 teachers of mathematics and another group of 11 teachers of informatics. After experimenting with FootMath, the teachers had to fill a questionnaire that was divided into three parts: i) the first part revolved around the profile design of the teacher, including questions regarding their teaching experience, the levels of education they teach, if they had used AR applications and if they had any

type of experience with AR games or programs to teach; ii) the second part was designed to understand the perspective of the teachers on the trial of FootMath; and iii) the third part was created to collect the opinions of the inquired on the challenges that AR poses when applied to educational purposes, particularly in mathematics. 17 of the 22 teachers were female (77,3%). The direct observation of the participants on the execution of the proposed tasks (during the trial and group interviews) and the listening to the groups, were the tools used to collect and fill the grid of results. Some examples of the proposed tasks were: placement and shift of the physical markers (targets), with the use of joysticks (manipulation of the parameters of the functions when looking for the zero of the function); experimenting with changing the slope (positive/negative) of the linear function ( $y = ax + b$ ); attempt to score goals with the linear function in an early phase and, afterward, with the quadratic function ( $y = ax^2 + bx + c$ ). Thus, it was possible to notice that FootMath sparked the interest and the engagement of every single participant.

#### 4.1 Analysis and discussion of the obtained data

Through the experimentation and exploration of FootMath, teachers of different subjects (mathematics and informatics) focused on pointing out the benefits and challenges related to the potential of AR serious games with basic mathematical functions. The view of the agents of teaching was taken into consideration regarding the potential of using this resource as a playful educational tool, capable of motivating and involving the students in problem-solving and logical thinking. After the experimentation and analysis of FootMath, every math teacher agreed on the following aspects: a) it was an interactive and innovative experience; b) if applied in a classroom as a strategic resource, the game can contribute to certain advantages when it comes to attention, interaction, experimentation and engagement; c) the game contributes to an innovative and adjusted observation and exploration of functions such as: linear ( $y = ax + b$ ), quadratic ( $y = ax^2 + bx + c$ ), trigonometric (sine and cosine) and exponential ( $y = a^x$ ); d) AR can be used inside the classroom as a complement to the processes of learning and teaching of items related to basic mathematical functions; e) AR might increase the level of interest of the students for mathematics as a whole; f) it is valuable to use books and interactive worksheets with AR; g) overall, it can increase/enhance the process of learning. Similar results were found for computer science teachers. The teachers of both groups (22) expressed their opinion (agree or completely agree) as follows: a) Game experience – innovative (95%), interesting (91%), motivating (82%), entertaining (91%), dynamic (86%), interactive (95%), and fluid (68%); b) Advantages of using FootMath inside the classroom – attention (100%), interaction (95%), commitment (95%), experimentation (100%), engagement (95%) and learning (86%); c) FootMath might provide a complementary strategy to the formal approaches to the concepts of linear functions ( $y = ax+b$ ) and quadratic functions ( $y = ax^2 + bx + c$ ) on the levels of knowledge (91%), understanding (82%) and with the application (86%); d) The game contributes to an innovative and adjusted observation and exploration of functions such as: linear ( $y = ax + b$ ), quadratic ( $y = ax^2 + bx + c$ ), trigonometric (sine and cosine) and exponential ( $y = a^x$ ); e) The incorporation of the graphs of the basic functions in a 3D world (football field) – correct (86%), feasible (82%) and viable (82%); f) AR increases the level of interest of the students for mathematics as a whole. In a nutshell, the results that were obtained from the questionnaire confirm a positive impact on the possible use of FootMath inside classrooms. This new opportunity of learning based on a digital AR game presents several benefits, and it might also improve the learning process of basic mathematical functions.

## 5 Conclusion

The present study focused on evaluating the use of FootMath, an AR serious game, as a complementary tool in an educational setting. The primary goal of this virtual game is to simplify the visualization and understanding of linear, quadratic, exponential, and trigonometric (sine and cosine) functions by students and also of enhancing their interest in these subjects. It was possible to test and evaluate FootMath through the evaluation of teachers with several years of teaching experience and, therefore, specialists in the subjects that FootMath deals with. The research allowed us to explore the benefits and the challenges related to the potential of AR serious games for learning basic mathematical functions. In conclusion, FootMath is a promising tool that offers students a different way to learn math by presenting information in a different format and allowing innovative interaction. It is a complimentary resource to motivate and engage students in learning with playing.

### 5.1 Future work

In the future, other mathematical functions at different levels of difficulty will be developed. Furthermore, there is a plan to expand the features of this serious game by replacing the physical markers by Google's ARCore technologies in order to explore and manipulate the functions in a more interactive way with the surrounding environment and different angles and distances. Further, there is an intention to explore the application of AR technologies in the Student-Centered Learning of Mathematics, integrated with Artificial Intelligence tools to achieve the ability to interpret student behavior and dynamically adjust or reconfigure their teaching process.

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