General Comments

This paper presents a novel method for retrieving ground surface temperatures under snowpack in Arctic permafrost regions using Soil Moisture and Ocean Salinity L-band brightness temperature observations. The study is well-motivated and addresses a critical gap in monitoring Arctic permafrost dynamics. The authors develop and evaluate two microwave emission models to retrieve ground surface temperatures and validate their results against in situ measurements and ERA5 reanalysis data.

The use of two models shows a thoughtful approach to handling complex Arctic environments. Additionally, the optimisation of surface roughness parameters is a key strength in their methodology as it improves the accuracy of retrievals, especially in areas with significant water fractions. The decision to validate against 21 sites provides an effective evaluation of the method's performance. The paper is well-organized, and generally, figures and tables are well-designed and effectively support the text.

Overall, the methodology is sound, and the results are promising, particularly for regions with low water fractions. However, the paper would benefit from a more detailed discussion of its limitations, broader implications, and uncertainties. With some revisions, this paper will significantly contribute to the field of remote sensing and cryosphere studies.

I will recommend this paper be accepted for publication after addressing the major and minor revisions outlined below.

Major Revisions

The discussion section could be expanded to address the broader implications of the study for Arctic climate research and operational monitoring. This would enhance the paper's impact and relevance to a broader community.

The limitations of the method, particularly for sites with high water fractions, should be addressed more thoroughly. The authors could propose specific strategies for improving the model in these regions.

Would it be possible to introduce site-specific roughness optimization or incorporate additional auxiliary datasets? This will allow for a broader understanding of site-specific limitations.

Could a sensitivity analysis be added to assess how variations in permittivity affect retrieval accuracy?

A more detailed uncertainty analysis, including the impact of RFI, atmospheric contributions, and snow property variability, would provide more information on the model's capabilities.

A table summarizing performance across all sites such as median bias, R, etc. would be helpful.

Minor Revisions

The assumptions underlying the models such as constant ground permittivity, snow transparency etc. should be clearly stated and justified in the Methods section.

Ensure consistency in terminology eg: sometimes "frozen ground temperature" is used instead of "Tg"

Line 3 – "reference sites providing with in situ" should be "reference sites providing in situ"

Line 9 – "European Centre for Medium-Range Weather Forecasts reanalysis" should be European Centre for Medium-Range Weather Forecasts reanalysis v5

Line 10 - "retrieved and in situ temperature" should be "retrieved and in situ temperatures"

Line 15 – "Arctic environments is excessively promising" consider using "highly promising instead"

Some sentences are overly technical and could be simplified to a broader audience eg:

line 44 – "By considering that the Arctic ground surface remains frozen throughout winter, the ground emissivity remains constant, and the BT depends mostly on Tg" can be simply "Since the Arctic ground remains frozen in winter, its emissivity is stable, making Tg the primary driver of brightness temperature".

Line 47 - "microwave microwave" should be "microwave"

Line 351: "different for the" should be "different from the".

Some figures like 5 - 7 are generally a bit overly complicated, as is the colour scheme. Try to simplify.

Figure 8 is a really interesting figure for this paper. It could be improved by adding a legend or annotations to clarify the different lines. I understand shading the text to represent it, but this might not to be intuitive to readers.