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Discrete Time Averaging of Non-Ideal PWM DC-DC Converters Operating in DCM with Feedback

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Discrete Time Averaging of Non-Ideal PWM DC-DC Converters Operating in DCM with Feedback

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I. Introduction

DC-DC converters are widely used in various applications. These converters are usually operated in two modes of operation, namely continuous and discontinuous conduction modes [1]. The discontinuous conduction mode of operation of the converter is most frequently used for light load applications. It is also useful for extracting maximum power efficiently from the solar panel [2, 3].

Due to various applications and needs of dc-dc converter operating in DCM, it is very essential to have a proper analytical model for this mode of operation for the analysis and design. The literature shows that more effort has been taken in this view for past three decades [2, 4].

One of the most widely used techniques in the design procedure in power electronics is averaging technique. This technique provides the basic analytical foundation for the most power electronic design. In fact classical averaging procedure is not suitable when there are state discontinuities. At high switching operations the periodic solution has some amplitude ripple, and these ripples are not considered in the classical averaging theory. Due to this it has been found that the directly obtained averaged models are inaccurate for the converters operated in DCM [4]. This has inculcated to take efforts to obtain more accurate averaging method.

In the continuous-time averaging procedure, each circuit topology is modeled separately and then combined in on approximated model [4, 5]. The duty cycle is a discrete-time variable, but treated as a continuous time variable in the existing continuous-time averaged models. Thus the orbital nature of the periodic solution is not obtained. Intern the periodic solution of the converters is averaged to equilibrium to form a nominal solution. In contrast, no such approximations are made in the sample date model. This provides the most accurate result, which replicates the actual behavior of PWM systems and is also suitable for digital control process. Sampled-data models allow us to focus on cycle -to-cycle behavior, ignoring intra cycle ripples. This makes them effective in general simulation, analysis and design. These models predict the values of signals at the beginning of each switching period, which most of the times represent peaks or valleys of the signals rather than average values. To better understand the average behavior of the system, a discrete-time model for the OCA signals was presented in [6].

In this paper, a sampled-data model for nonideal closed loop PWM converters operating in DCM is formulated. This gives the exact discrete-time mathematical representation of the values of the output and internal signals at constant frequency. A discrete-time model to provide the one-cycle-average (OCA) signals of the non ideal closed loop PWM converters operating in DCM is proposed. This model provides the exact discrete-time mathematical representation of the averaged values of the output signal at each switching period. It also provides the average values of other internal signals with little increase in simulation time. The main motivation for the new model is based on the fact that, in many power electronic applications, it is the average values of the voltage and current rather than their instantaneous values that are of greatest interest. In addition to that, the existing models are extended to accommodate for conduction losses. Numerical simulations show the accuracy of the propose model.

II. Existing Average Models

The modeling method is validated by a non-ideal boost converter with feedback as shown in Figure 1 for the different existing averaging methods and presented in this section.

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Global Journal of Researches in Engineering (F) Volume XIII Issue V Version I Year 2013

a) Switched Model

Let $u(t) \in R^m$ is the input vector, $x(t) \in R^n$ is the state vector, $y(t) \in R^p$ is the output vector while T_s denotes the length of a switching cycle. Then the DCM PWM converter can be described as [7].

$$\dot{x}(t) = \begin{cases} A_1x(t) + B_1u(t) \;; & t \in \tau_1 \\ A_2x(t) + B_2u(t) \;; & t \in \tau_2 \\ A_3x(t) + B_3u(t) \;; & t \in \tau_3 \end{cases} \tag{1}$$

$$y(t) = \begin{cases} C_1x(t) \;, & t \in \tau_1 \\ C_2x(t) \;, & t \in \tau_2 \\ C_3x(t) \;, & t \in \tau_3 \end{cases} \tag{2}$$

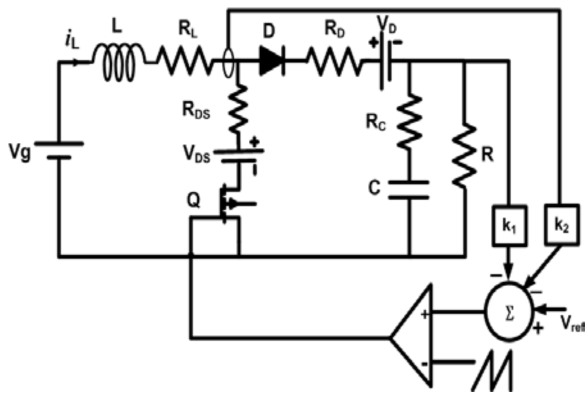

Figure 1 : Boost converter with parasitics

Where The system switches between three topologies (A_1, B_1, C_1), (A, B_2, C_2), and (A_3, B_3, C_3), with switching intervals determined by

$$\begin{aligned} \tau_1 &:= kT_s \le t < kT_s + d_k^1T_s \\ \tau_2 &:= kT_s + d_k^1T_s \le t < kT_s + (d_k^1 + d_k^2)T_s \\ \tau_3 &:= kT_s + (d_k^1 + d_k^2)T_s \le t < kT_s + T_s \end{aligned}$$

Where $(d_k^1 + d_k^2) \in [0, 1]$ are the switch duty ratios, and k is the discrete-time index. All auxiliary inputs will be assumed to be piecewise constants, i.e. $u(t) = u_k$ for all t $\in [kT_s\,(k+1)T_s)$. This assumption is not necessary and is made for convenience only; more general cases would only require more complex notations.

This is the exact switching model which will be used as the base model for comparison of different methods. The control scheme given in is applied, where is $m(t) = V_{ref} - k_1 i(t) - k_2 v(t)$ $V_{ref} = 0.13$, $k_1 = 0.174$, and $k_2 = -0\ 0435$ as in [8].

b) DCM State-Space Average Model (SSA)

In the conventional state-space averaging method the averaged model for DCM has been presented previously in numerous publications [2]. The converters state-space equation in this mode is given by

$$\begin{aligned} \dot{x}(t) &= [d_1A_1 + d_2A_2 + (1 - d_1 - d_2)A_3]x(t) \\ &\quad + [d_1B_1 + d_2B_2 + (1 - d_1 - d_2)B_3]u(t) \end{aligned} \tag{3}$$

The problem with the state-space averaging approach in DCM is that we are averaging just the matrix parameters, and not necessarily the state variable themselves. It is intended that (3) will apply when the true average of each state variable is used, but the average inductor current depends on the parameters and duty ratios. Considering this, the modified state-space averaged model that would correctly predict the behavior in DCM for the boost converter is given as [2].

$$\begin{aligned} \dot{x}(t) &= [d_1A_1 + d_2A_2 + (1 - d_1 - d_2)A_3]Mx(t) \\ &\quad + [d_1B_1 + d_2B_2 + (1 - d_1 - d_2)B_3]u(t) \end{aligned} \tag{4}$$

Where M is the modification Matrix and its given by

$$M = \begin{bmatrix} \frac{1}{d_1 + d_2} & 0 \\ 0 & 1 \end{bmatrix} \tag{5}$$

Where d_1 and d_2 are the duty ratios and d_2 can be determined as follows

$$d_2 = \frac{2Li_L}{d_1T_sV_g} - d_1 \tag{6}$$

Based on (4), (5) and (6) the averaged model can also be derived considering non-ideality in the switching stage components, and is given by

$$\begin{aligned} \frac{di_L}{dt} &= \frac{(R_L + R_{DS})(R + R_C)(V_gd_1^2T_s)}{2L^2(R + R_C)} \\ &\quad - \frac{((R_L + R_D)(R + R_C) + RR_C)(2i_CL - d_1^2V_gT_s)}{2L^2(R + R_C)} \\ &\quad - \frac{(2i_LL - d_1^2V_gT_s)RV_C}{LV_gd_1T_s(R + R_C)} \\ &\quad + \frac{2i_LLV_g - d_1^2V_{DS}V_gT_s - 2i_LLV_D + d_1^2V_gV_DT_s}{LV_gd_1T_s} \\ \frac{dv_C}{dt} &= \frac{1}{C(R + R_C)}\left[Ri_L - V_C - \frac{d_1^2V_gT_sR}{2L}\right] \end{aligned}$$

c) Conventional Discrete-Time Model

The conventional discrete-time mode (CDTM) is given by [4]

$$x_{k+1} = \mathcal{A}(d_k^1, d_k^2)x_k + \mathcal{B}(d_k^1, d_k^2)u_k \tag{7}$$

Where the input nonlinearities $A(d^1, d^2)$ *and* $B(d^1, d^2)$ are given by

$$\mathcal{A}(d^1, d^2) := \Phi_3\Phi_2\Phi_1 \tag{8}$$

$$\mathcal{B}(d^1, d^2) := \Phi_3(\Phi_2\Gamma_1 + \Gamma_2) + \Gamma_3 \tag{9}$$

The arguments d^1T_s, d^2T_s, and $(1-d^1-d^2)T_s$ *for* $(\Phi_1, \Phi_2, \Phi_3, \Gamma_1, \Gamma_2$ and $\Gamma_3)$, respectively are omitted from the above equations for notation simplicity. Where

Global Journal of Researches in Engineering (F) Volume XIII Issue V Version I Year 2013

$$\begin{aligned}\Phi_i(t) &:= e^{A_i t}\\ \Gamma_i(t) &:= \int_0^t e^{A_i \tau} b_i d\tau\end{aligned}$$

III. Proposed Model

This section introduces the new averaged discrete-time model for closed loop PWM dc-dc converter operating in DCM considering conduction losses. Description of the original system and derivation of the proposed model are discussed here. The one-cycle average (OCA) representation of the output signal [6] is given by

$$y^*(t) := \frac{1}{T_s}\int_{t-T_s}^{t} y(\tau)d\tau. \qquad (10)$$

The signal, $y^*(t)$ is used to develop a new discrete-time model for PWM converters operating in DCM. This model provides the basis for discrete-time simulation of the averaged value of any state in the DCM PWM system, even during transient non-periodic operating conditions.

a) Proposed OCA Discrete-Time Model

It is desired to compute, without approximation, the evolution of all system variables at the sampling instants, $t = kT_s$ assuming three different topologies for the system. Since the state and output equations (1) - (2) are piecewise-linear with respect to time t, the desired discrete-time model can be obtained symbolically. Using the notation, $x_k := x(kT_s)$ and $y_k^* := y^*(kT_s)$, the result is the OCA large signal model

$$\begin{aligned}x_{k+1} &= \mathcal{A}(d_k^1, d_k^2)x_k + \mathcal{B}(d_k^1, d_k^2)u_k \qquad (11)\\ y_{k+1}^* &= \mathcal{C}(d_k^1, d_k^2)x_k + \mathcal{D}(d_k^1, d_k^2)u_k \qquad (12)\end{aligned}$$

Where the input nonlinearities $A(d^1, d^2)$, $B(d^1, d^2)$, $C(d^1, d^2)$ and $D(d^1, d^2)$ are given by

$$\begin{aligned}\mathcal{A}(d^1,d^2) &:= \Phi_3\Phi_2\Phi_1\\ \mathcal{B}(d^1,d^2) &:= \Phi_3(\Phi_2\Gamma_1+\Gamma_2)+\Gamma_3\\ \mathcal{C}(d^1,d^2) &:= C_1\Phi_1^* + C_2\Phi_2^*\Phi_1 + C_3\Phi_3^*\Phi_2\Phi_1\\ \mathcal{D}(d^1,d^2) &:= C_1\Gamma_1^* + C_2(\Phi_2^*\Gamma_1+\Gamma_2^*)\\ &\quad + C_3(\Phi_3^*(\Phi_2\Gamma_1+\Gamma_2)+\Gamma_3^*)\end{aligned}$$

The arguments d^1T_s, d^2T_s, and $(1-d^1-d^2)\,T_s$ for $(\Phi_1, \Phi_1^*, \Gamma \text{ and } \Gamma_1^*)$, $(\Phi_2\ ,\ \Phi_2^*,\ \Gamma_2 \text{ and } \Gamma_2^*)$ and $(\Phi_3, \Phi_3^*, \Gamma_3$ and $\Gamma_3^*)$ respectively are omitted from the above equations for notation simplicity. Where

$$\begin{aligned}\Phi_i(t) &:= e^{A_i t}\\ \Gamma_i(t) &:= \int_0^t e^{A_i \tau} b_i d\tau\end{aligned}$$

$$\begin{aligned}\Phi_i^*(t) &:= \frac{1}{T_s}\int_0^t \Phi_i(\tau)d\tau\\ \Gamma_i^*(t) &:= \frac{1}{T_s}\int_0^t \Gamma_i(\tau)d\tau.\end{aligned}$$

Note that the averaging operation adds "sensor" dynamics to the system; as a consequence, the largesignal model (11) - (12) is not in standard statespace form. By defining the augmented state vector $x^* \in R^{n+p}$ such that

$$x_{k+1}^* := \begin{bmatrix} x_{k+1} \\ \mathcal{C}(d_k^1, d_k^2)x_k + \mathcal{D}(d_k^1, d_k^2)u_k \end{bmatrix} \qquad (13)$$

An equivalent (but standard form) representation of the OCA large-signal model is given by:

$$\begin{aligned}x_{k+1}^* &= \mathcal{A}^*(d_k^1, d_k^2)x_k^* + \mathcal{B}^*(d_k^1, d_k^2)u_k \qquad (14)\\ y_k^* &= \mathcal{C}^* x_k^* \qquad (15)\end{aligned}$$

Where

$$\begin{aligned}\mathcal{A}^*(d^1,d^2) &:= \begin{bmatrix} \mathcal{A}(d^1,d^2) & 0_{n\times p} \\ \mathcal{C}(d^1,d^2) & 0_{p\times p}\end{bmatrix}\\ \mathcal{B}^*(d^1,d^2) &:= \begin{bmatrix} \mathcal{B}(d^1,d^2) \\ \mathcal{D}(d^1,d^2)\end{bmatrix}\\ \mathcal{C}^*(d^1,d^2) &:= \begin{bmatrix} 0_{p\times n} & I_{p\times p}\end{bmatrix}\end{aligned}$$

Note that not only the OCA values of output signal will be available but also the values of the signals (without averaging) at the beginning of every switching period as well.

b) Feedback Computation

The modulation signal for feedback control is $m(t) = V_{ref} - k_1 i(t) - k_2 v(t) = V_{ref} - Kx(t)$ and the duty ratio at each switching period is $d_k = \frac{t^*}{T_s}$. The time instant t* at which the modulation signal crosses

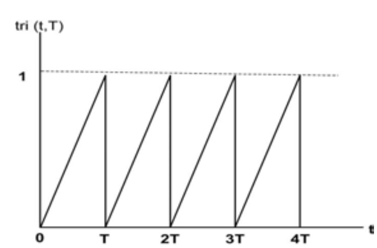

Figure 2 : PWM sawtooth function

the sawtooth is computed by solving the nonlinear equation

$$\begin{aligned}tri(t^*, T_s) &= V_{ref} - Kx(kT_s + t^*)\\ &= V_{ref} - K\{\Phi_1(t^*)x_k + \Gamma_1(t^*)u_k\} \qquad (16)\end{aligned}$$

at each time instant k, where the sawtooth function is shown in Figure 2 and mathematically represented by tri

Global Journal of Researches in Engineering (F) Volume XIII Issue V Version I Year 2013

$t, T_s) = \frac{t}{T_s} -$ floor $(\frac{t}{T_s})$. For reasonably high switching frequency, the value of $x(kT + t^*)$ can be approximated by neglecting the higher order terms in the Taylor expansion of the nonlinear functions Φ_1 and Γ_1. That is

$$\Phi_1(t^*) = I + A_1 t^* + \frac{A_1^2}{2!} t^{*^2} + \ldots \approx I + A_1 t^* \qquad (17)$$

$$\Gamma_1(t^*) = (It^* + A_1 \frac{t^{*^2}}{2!} + \ldots) B_1 \approx It^* B_1 \qquad (18)$$

And hence, a good approximation of (16) becomes

$$tri(t^*, T_s) \approx V_{ref} - K\{(I + A_1 t^*) x_k + B_1 t^* u_k\} \qquad (19)$$

Noting that $tri(t^*, T_s)$ equals to $\frac{t}{T_s}$ for $t^* \in (kT_s, (k+1)T_s]$, we get

$$\frac{t^*}{T_s} \approx V_{ref} - K\{(I + A_1 t^*) x_k + B_1 t^* u_k\} \qquad (20)$$

or

$$d_k = \frac{t^*}{T_s} \approx \frac{V_{ref} - K x_k}{K T_s (A_1 x_k + B_1 u_k) + 1} \qquad (21)$$

Which provides a closed from solution for d_k. The duty ratio d_k can be computed without approximation by solving the nonlinear equation for t^*.

IV. Numerical Example

To compare existing models with DCM OCA model, consider a Boost converter with feedback considering conduction loss shown in Figure 1. The input is u = V_g and state variables are $x_1 = i_L$ and $x_2 = v_C$. Where $R = 30\ \Omega$, $L = 75\ \mu$H, $C = 4.4\ \mu$F, $V_g = 5$V, and $T_s = 50\ \mu$s.The values of the parasitics used in the example are $R_L = 0.0176\ \Omega$, $R_C = 30\ m\Omega$, $R_{DS} = 0.17\ \Omega$, $R_D = 0.15\ \Omega$, $V_{DS} = 0.17$ V, and V_D =0.4V. The three periodical switching operating stages and the state equation are given below. In the resulting operating mode, these three operating stages are periodically switched and they are looped correspondingly.

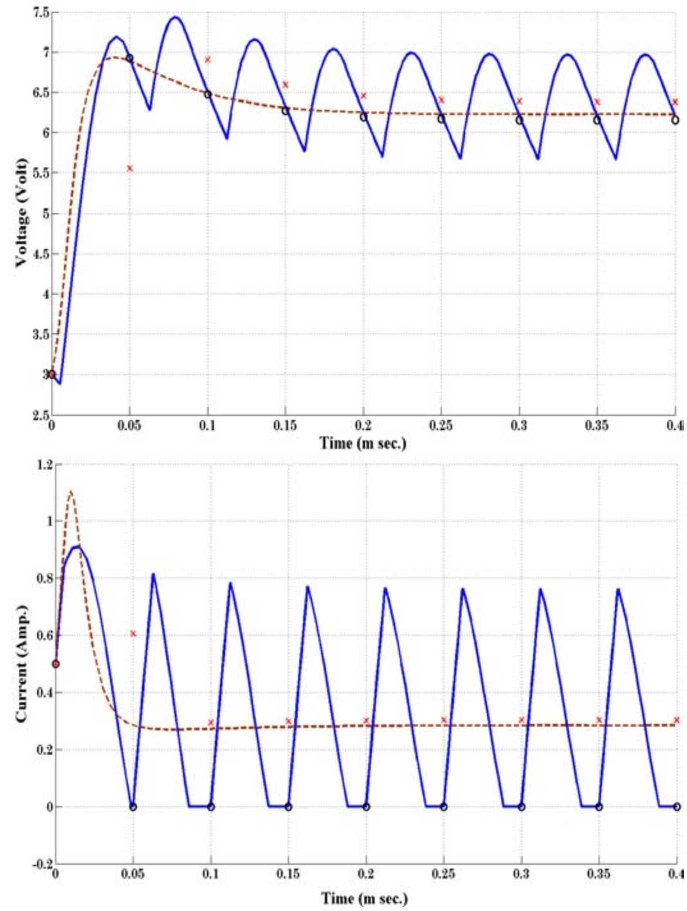

Figure 3 : Comparison of DCM simulation of non-ideal closed loop boost converter

—: switched mode ; --: DCM SSA
○: CDTM ; ×: DCM OCA

The state space matrices $A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2$, and C_3 are defined as

$$A_1 = \begin{bmatrix} -\frac{(R_L+R_{DS})}{L} & 0 \\ 0 & -\frac{1}{(R+R_C)C} \end{bmatrix};$$

$$A_2 = \begin{bmatrix} -\frac{(R_L+R_D)(R+R_C)+RR_C}{L(R+R_C)} & -\frac{R}{L(R+R_C)} \\ \frac{R}{C(R+R_C)} & -\frac{1}{C(R+R_C)} \end{bmatrix};$$

$$A_3 = \begin{bmatrix} -\frac{(R_L+R_{DS})}{L} & 0 \\ 0 & -\frac{1}{(R+R_C)C} \end{bmatrix};$$

$$B_1 = \begin{bmatrix} \frac{1}{L} & -\frac{1}{L} & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad B_2 = \begin{bmatrix} \frac{1}{L} & 0 & -\frac{1}{L} \\ 0 & 0 & 0 \end{bmatrix};$$

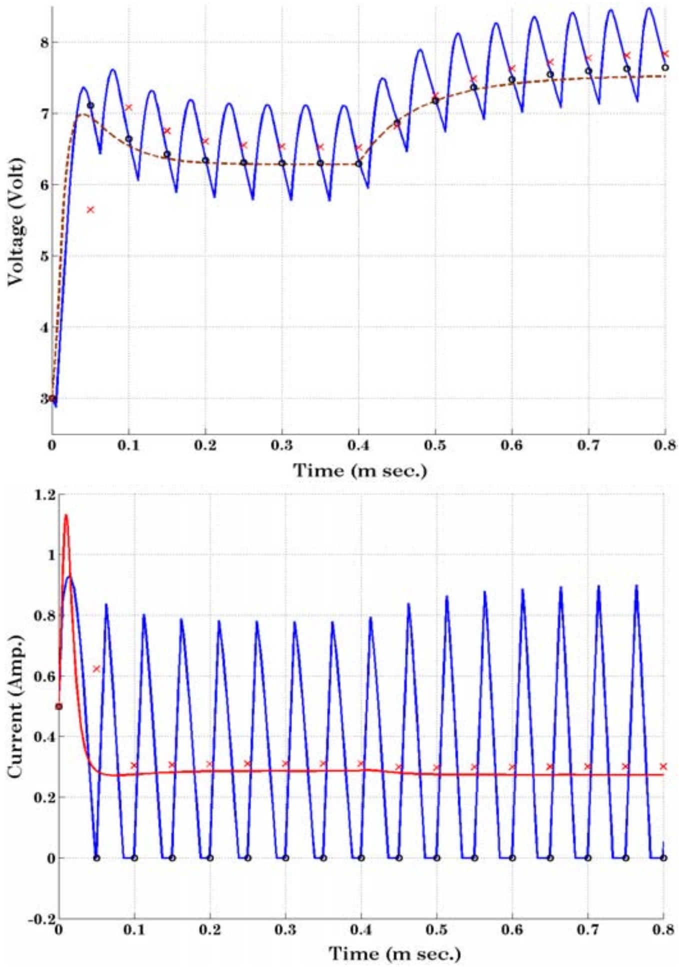

Figure 4 : Comparison of DCM simulation of non-ideal closed loop boost converter with variable\ load resistance

–: switched mode ; -- : DCM SSA

∘ : CDTM ; × : DCM OCA

$$B_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad C_1 = \begin{bmatrix} 0 & \frac{R}{R+R_C} \end{bmatrix};$$

$$C_2 = \begin{bmatrix} \frac{RR_C}{R+R_C} & \frac{R}{R+R_C} \end{bmatrix}; \quad C_3 = \begin{bmatrix} 0 & \frac{R}{R+R_C} \end{bmatrix}$$

and $u(t)$ is given by

$$u(t) = \begin{bmatrix} V_g & V_{DS} & V_D \end{bmatrix}' \tag{22}$$

It should be noted that no approximation is made in deriving the new discrete-time model, and all simulations were performed using Matlab. To show the accuracy and speed of the proposed model, the steady state average values of the proposed model are compared to most commonly used existing models (switched, SSA and CDTM). The results of switched model, SSA, CDTM and the proposed OCA model for the boost converter operating in DCM are shown in Figure 3. In the figure, current and voltage waveforms are represented. The steady-state values of output voltage for different models are $v_C = 6.225V$ for SSA, $v_C = 6.15V$ for CDTM, and $v_C = 6.375V$ for OCA model. It is obvious that the steady state values computed by the proposed method is by far closer to the steady state average values of interest to PWM converter design.

a) Effect of change in load

To study the effect of load resistance on the simulation results, a step change on the load resistance R from 30 Ω to 45 Ω at time instant, t = 0.4 ms has been simulated and the results are shown in Figure 4. It can be observed that the average values produced by SSA model depends on the load resistance. On the other hand the proposed model provides the same accuracy of waveforms regardless of the change in load resistance.

V. Conclusion

This paper presents a conventional discrete-time model and a one-cycle-average discrete-time model for closed loop PWM converters operating in DCM with non-ideality in the switching stage components is formulated. These models provide the exact discrete time mathematical representation of the instantaneous / averaged values of the output signal respectively. The DCM OCA model proposed here also provides the average values of other internal signals with little increase in simulation time. It is also compared to existing models extended to accommodate for conduction losses, through a numerical example of boost converter. The numerical simulation results show the accuracy and speed of the OCA discrete-time model for PWM converter operating in the DCM for non ideal condition. Table I summarizes the normalized simulation times for boost converter operating in DCM for different simulation methods.

Table 1 : Simulation times for closed loop boost converter

Method	Normalized Simulation Time
Switched	45
SSA	1
CDTM	8.54
DCM OCA	9.12

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