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Performance Analysis of Zero Forcing Equalizer in 2×2 and 3×3 MIMO Wireless Channel

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Index Terms: MIMO system, zero forcing equalizer, 2×2 MIMO channel, 3×3 MIMO channel, inter symbol interference (ISI), bit error rate (BER).

I. Introduction

During the past there has been an explosion in wireless technology. This growth has opened a new dimension to future wireless communication whose ultimate goal is to provide universal personal and multimedia communication without regard to mobility or location with high data rates. To achieve such an objective, the next generation personal communication network will need to be support a wide range of services which will include high quality voice, data, facsimile, still pictures and streaming video. These future services are likely to include applications which require high transmission rates of several Mega bit per seconds (Mbps). The data rate and spectrum efficiency of wireless mobile communications have been improved over the last decade [1].

In mobile communication systems, data transmission at high bit rates is essential for many services such as video, high quality audio and mobile integrated service digital network. When the data is transmitted at high bit rates, over mobile radio channels, the channel impulse response can extend over many symbol periods, which lead to inter-symbol interference (ISI). This paper discuss the performances of equalization techniques by considering 2 transmitting and 2 receiving antenna case (resulting in a 2×2 MIMO channel) and 3 transmitting and 3 receiving antenna case (resulting in a 3x3 MIMO channel). Assume that the channel is a flat fading Rayleigh multipath channel and the modulation is BPSK. The ultimate goal is to provide universal personal and multimedia communication without regard to mobility or location with high data rates. To achieve such an objective we need a strong equalization technique to compensate ISI. Hence, there is need for the development of novel practical, low complexity equalization techniques and for understanding their potentials and limitations when used in wireless communication system characterized by very high rates, high mobility and the presence of multiple antennas [2].

II. System Overview

a) MIMO Systems Basics

Multiple-Input Multiple-Output (MIMO) [5, 8] technology is a wireless technology that uses multiple transmitters and receivers to transfer more data at the same time shown inFig.1. MIMO technology takes advantage of a radio-wave phenomenon called multipath where transmitted information bounces off walls, ceilings, and other objects, reaching the receiving antenna multiple times via different angles and at slightly different times.

Fig. 1 : MIMO technology uses multiple radios to transfer more data at the same time

MIMO technology leverages multipath behaviour by using multiple, "smart" transmitters and receivers with an added "spatial" dimension to dramatically increase performance and range. MIMO allows multiple antennas to send and receive multiple spatial streams at the same time. MIMO makes antennas work smarter by enabling them to combine

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data streams arriving from different paths and at different times to effectively increase receiver signal-capturing power. Smart antennas use spatial diversity technology, which puts surplus antennas to good use. If there are more antennas than spatial streams, the additional antennas can add receiver diversity and increase range.

MIMO (multiple-in, multiple-out) takes advantage of multiplexing to increase wireless bandwidth and range. MIMO algorithms send information out over two or more antennas and the information is received via multiple antennas as well. On normal radio, multiplexing would cause interference, but MIMO uses the additional pathways to transmit more information and then recombines the signal on the receiving end. MIMO systems provide a significant capacity gain over conventional single antenna systems, along with more reliable communication. The benefits of MIMO lead many to believe it is the most promising of emerging wireless technologies.

MIMO system is represented by

$$Y=HX+N \tag{1}$$

Hence,

X= Transmitting signal

Y= Received signal

H=Channel matrix

N= Noise vector.

i. *2×2 MIMO Channel*

In a 2×2 MIMO channel shown in Fig. 2, probable usage of the available 2 transmitting antennas can be as follows:

- Consider that we have a transmission sequence, for example$\{x_1, x_2, x_3 \ldots x_n\}$.
- In normal transmission, we will be sending x_1 in the first time slot, x_2 in the second time slot, x_3 and so on.
- However, as we now have 2 transmitting antennas, we may group the symbols into groups of two. In the first time slot, send x_1 and x_2 from the first and second antenna. In second time slot, send x_3 and x_4 from the first and second antenna send x_5 and x_6 in the third time slot and so on.
- Notice that as we are grouping two symbols and sending them in one time slot, we need only n/2 time slots to complete the transmission. Hence the data rate is doubled.
- This forms the simple explanation of a probable MIMO transmission scheme with 2 transmitting antennas and 2 receiving antennas. The two transmitted symbols interfered with each other and we will use zero forcing equalizer to minimize the interference.

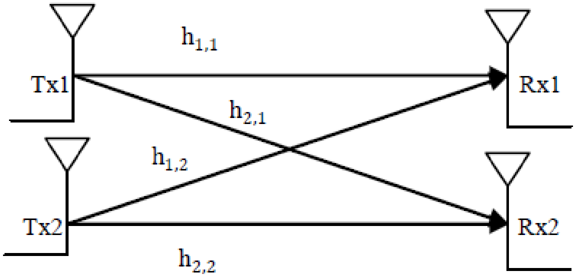

Fig. 2 : 2 transmit 2 receive (2 x 2) MIMO channel

b) *Other Assumptions*

- The channel is flat fading. In simple terms, it means that the multipath channel has only one tap. So, the convolution operation reduces to a simple multiplication [4].
- The channel experienced by each transmitting antenna is independent from the channel experienced by other transmitting antennas.
- For i^{th} the transmitting antenna to j^{th} receiving antenna, each transmitted symbol gets multiplied by a randomly varying complex number $h_{j,i}$. As the channel under consideration is a Rayleigh channel, the real and imaginary parts of hj, i are Gaussian n distributed having mean $\mu_{h_{j,i}} = 0$and variance $\sigma^2 \, h_{j,i} = 1/2$.
- The channel experienced between each transmitter to the receiving antenna is independent and randomly varying in time.
- On the receive antenna, the noise has the Gaussian probability density function with

$$p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-\mu)^2}{2\sigma^2}}$$

With $\mu = 0$ and $\sigma^2 = \frac{N_o}{2}$

- The channel $h_{j,i}$ is known at the receiver.

i. *3×3 MIMO Channel*

In a 3×3 MIMO channel shown in Fig. 3, probable usage of the available 3 transmitting antennas can be as follows:

- Consider that we have a transmission sequence, for example$\{x_1, x_2, x_3 \ldots x_n\}$.
- In normal transmission, we will be sending x_1 in the first time slot, x_2 in the second time slot, x_3 and so on.
- However, as we now have 3 transmitting antennas, we may group the symbols into three groups. In the first time slot, send x_1, x_2 and x_3 from the first, second and third antenna. In second time slot, send x_4, x_5 and x_6 from the first, second and third antenna, send x_7, x_8 and x_9 in the third time slot and so on.4.

Notice that as we are grouping three symbols and sending them in one time slot, we need only n/3 time slots to complete the transmission. Hence the data rate is tripled.

- This forms the simple explanation of a probable MIMO transmission scheme with 3 transmitting antennas and 3receiving antennas. The three transmitted symbols interfered with each other and we will use zero forcing equalizer to minimize the interference.

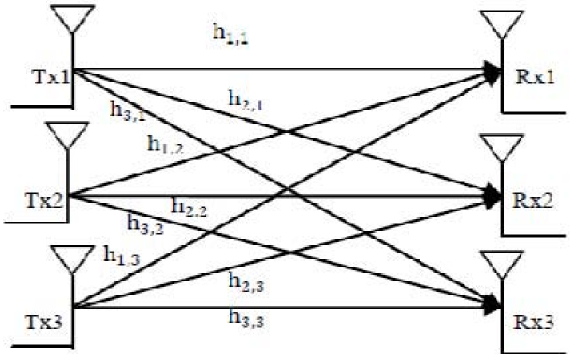

Fig. 3 : 3 transmit 3 receive (3 x 3) MIMO channel

Zero Forcing Equalizer [7] refers to a form of linear equalization algorithm used in communication systems which applies the inverse of the frequency response of the channel. This form of equalizer was first proposed by Robert Lucky. The Zero-Forcing Equalizer applies the inverse of the channel frequency response to the received signal, to restore the signal after the channel. It has many useful applications. For example, it is studied heavily for IEEE 802.11n (MIMO) where knowing the channel allows recovery of the two or more streams which will be received on top of each other on each antenna. The name Zero Forcing corresponds to bringing down the inter-symbol interference (ISI) to zero in a noise free case. This will be useful when ISI is significant compared to noise [3].

For a channel with frequency response (f) the zero forcing equalizer (f) is constructed by $(f) = 1/F(f)$. Thus the combination of channel and equalizer gives a flat frequency response and linear phase $F(f)\ C(f) = 1$. In reality, zero-forcing equalization does not work in most applications, for the following reasons: 1. Even though the channel impulse response has finite length, the impulse response of the equalizer needs to be infinitely long. 2. At some frequencies the received signal may be weak. To compensate, the magnitude of the zero-forcing filter ("gain") grows very large. As a consequence, any noise added after the channel gets boosted by a large factor and destroys the overall signal-to-noise ratio. Furthermore, the channel may have zeroes in its frequency response that cannot be inverted at all. (Gain * 0still equals 0). This second problem is often the more limiting condition. These problems can be addressed by making as mall modification to the denominator of (f):$(f) = 1/(F(f) + k)$where k is related to the channel response and the signal SNR[6].If the channel response (or channel transfer function) for a particular channel is (s) then the input signal is multiplied by the reciprocal of it. This is intended to remove the effect of channel from the received signal, in particular the inter-symbol interference (ISI).

The zero-forcing equalizer removes all ISI, and is ideal when the channel is noiseless. However, when the channel is noisy, the zero-forcing equalizer will amplify the noise greatly at frequencies f where the channel response $(j2\pi f)$ has a small magnitude (i.e. near zeroes of the channel) in the attempt to invert the channel completely. A more balanced linear equalizer in this case is the minimum mean-square error equalizer, which does not usually eliminate ISI completely but instead minimizes the total power of the noise and ISI components in the output.

Let us now try to understand the math for extracting the two symbols which interfered with each other. In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \quad h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \tag{2}$$

The received signal on the second receive antenna is,

$$y_1 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \quad h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \tag{3}$$

where

y_1, y_2 are the received symbol on the first and second antenna respectively,

$h_{1,1}$ is the channel from 1st transmit antenna to 1st receive antenna,

$h_{1,2}$ is the channel from 2nd transmit antenna to 1streceive antenna,

$h_{2,1}$ is the channel from 1st transmit antenna to 2nd receiveantenna,

$h_{2,2}$ is the channel from 2nd transmit antenna to 2nd receive antenna,

x_1, x_2 are the transmitted symbols and

n_1, n_2 is the noise on 1st, 2nd receive antennas.

We assume that the receiver knows $h_{1,1}$, $h_{1,2}$, $h_{2,1}$ and$h_{1,2}$. The receiver also knows $y_1$and y_2. The unknowns are$x1$and x_2. With two equations and two unknowns we cansolve it.

For convenience, the above equation can be represented in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{4}$$

Equivalently,

$$y = Hx + n \tag{5}$$

To solve for, we know that we need to find a matrix Wwhich satisfies WH + I.

The Zero Forcing (ZF) linear detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H \quad (6)$$

This matrix is also known as the pseudo inverse for a general m x n matrix. The term,

$$\begin{bmatrix} h_{1,1}^* & h_{2,1}^* \\ h_{1,2}^* & h_{2,2}^* \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} = \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \quad (7)$$

ii. *Zero forcing (ZF) equalizer for 3×3 MIMO channel*

Let us now try to understand the math for extracting the two symbols which interfered with Each other for 3×3 MIMO channel. In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + h_{1,3}x_3 + n_1 = [\, h_{1,1} \;\; h_{1,2} \;\; h_{1,3}] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + n_1 \quad (8)$$

The received signals on the second and third receive antenna are,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + h_{2,3}x_3 + n_2 = [\, h_{2,1} \;\; h_{2,2} \;\; h_{2,3}] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + n_2 \quad (9)$$

$$y_3 = h_{3,1}x_1 + h_{3,2}x_2 + h_{3,3}x_3 + n_3 = [\, h_{3,1} \;\; h_{3,2} \;\; h_{3,3}] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + n_3 \quad (10)$$

where

y_1, y_2 and y_3 are the received symbol on the first, second and third antenna respectively,

$h_{1,1}$ is the channel from 1st transmit antenna to 1st receive antenna,

$h_{1,2}$ is the channel from 2nd transmit antenna to 1st receive antenna,

$h_{1,3}$ is the channel from 3rd transmit antenna to 1st receive antenna,

$h_{2,1}$ is the channel from 1st transmit antenna to 2nd receive antenna,

$h_{2,2}$ is the channel from 2nd transmit antenna to 2nd receive antenna,

$h_{2,3}$ is the channel from 3rd transmit antenna to 2nd receive antenna,

$h_{3,1}$ is the channel from 1st transmit antenna to 3rd receive antenna,

$h_{3,2}$ is the channel from 2nd transmit antenna to 3rd receive antenna,

$h_{3,3}$ is the channel from 3rd transmit antenna to 3rd receive antenna,

x_1, x_3 and x_3 are the transmitted symbols and n_1, n_2 and n_3 are the noise on 1st, 2nd and 3rd receive antennas respectfully.

We assume that the receiver knows $h_{1,1}$, $h_{1,2}$, $h_{1,3}$,$h_{2,1}$,$h_{2,2}$,$h_{2,3}$,$h_{3,1}$,$h_{3,2}$, and $h_{3,3}$. The receiver also knows y_1, $y_2$and y_3. The unknowns are x_1, $x_3$and x_3. With three equations and three unknowns we can solve it.

For convenience, the above equation can be represented in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & h_{1,3} \\ h_{2,1} & h_{2,2} & h_{2,3} \\ h_{3,1} & h_{3,2} & h_{3,3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Equivalently,

$$y = Hx + n \quad (11)$$

To solve for x, we know that we need to find a matrix which satisfies WH +I.

The Zero Forcing (ZF) linear detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H \quad (12)$$

iii. *BER with ZF equalizer with 2×2and 3×3 MIMO*

Note that the off diagonal terms in the matrix HHH are not zero. Because the off diagonal terms are not zero, the zero forcing equalizer tries to null out the interfering terms when performing the equalization, i.e. when solving for x_1 the interference from x_2 is tried to be nulled and vice versa While doing so, there can be amplification of noise. Hence Zero Forcing equalizer is not the best possible equalizer to do the job. However, it is simple and reasonably easy to implement. Further, it can be seen that, following zero forcing equalization, the channel for symbol transmitted from each spatial dimension (space is antenna) is a like a 1×1 Rayleigh fading channel. Hence the BER for 2×2 and 3×3MIMO channel in Rayleigh fading with Zero Forcing equalization is same as the BER derived for a 1×1 channel in Rayleigh fading [4].

For BPSK modulation in Rayleigh fading channel, the bit error rate is derived as,

$$P_b = \sqrt{\frac{\left(\frac{E_b}{N_0}\right)}{\left(\frac{E_b}{N_0}\right)+1}} \quad (13)$$

III. RESULT AND DISCUSSION

As expected, the simulated results with a 2×2 MIMO system using BPSK modulation in Rayleigh channel is showing matching results as obtained in for a 1×1 system for BPSK modulation in Rayleigh channel shown in Fig. 4. The ZF equalizer helps us to achieve the data rate gain, but not take advantage of diversity gain (as we have two receiving antennas). We might not be able to achieve the two fold data rate improvement in all channel conditions. It can so happen that channels are correlated (the coefficients are almost the same).

BER for BPSK modulation with 2x2 MIMO and ZF equalizer (Rayleigh channel)

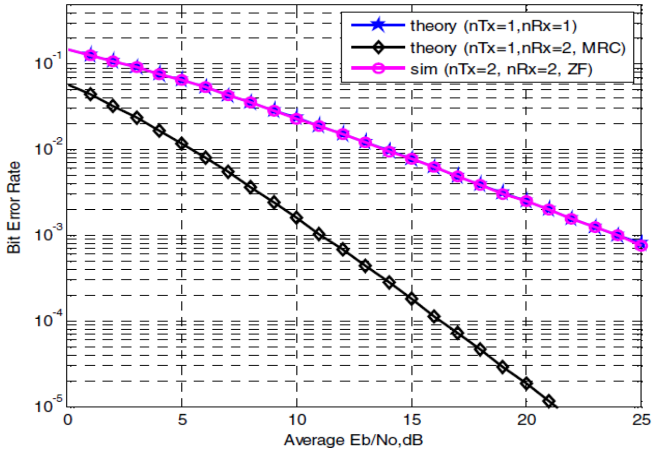

Fig. 4 : BER plot for 2x2 MIMO channel with ZF equalizer (BPSK modulation in Rayleigh channel)

Hence we might not be able to solve for the two unknown transmitted symbols even if we have two received symbols. In case of 3x3 MIMO shown in Fig. 5 has some discontinuity due to interference effect.

BER for BPSK modulation with 3x3 MIMO and ZF equalizer (Rayleigh channel)

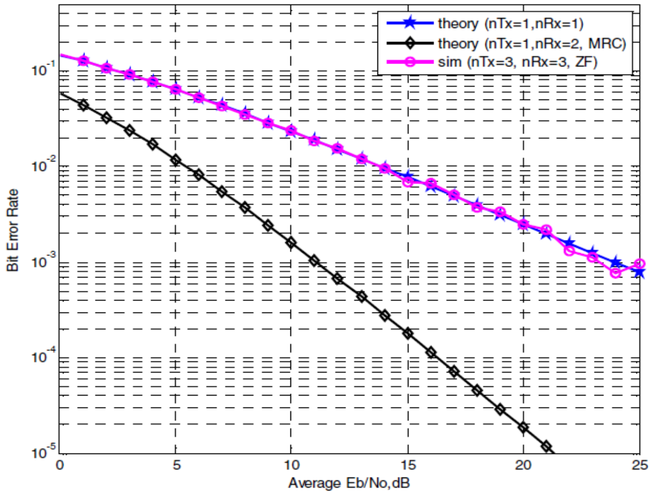

Fig. 5 : BER plot for 3x3 MIMO channel with ZF equalizer(BPSK modulation in Rayleigh channel)

IV. CONCLUSIONS

This paper presents a simulation study on the performance analysis of ZF equalizer based MIMO receiver. The simulation result shows the BER characteristics for the ZF equalizer. From the simulation result we can summarize that, ZF equalization in addition of noise gets boosted up and thus spoils the overall signal to noise ratio. Hence it is considered good to a receiver under noise free conditions. The multiple antennas are used to increase data rates through multiplexing or to improve performance through diversity. This technique offers higher capacity to wireless systems and the capacity increases linearly with the number of antennas and link range without additional bandwidth and power requirements.

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