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The dynamical core of the Aeolus 1.0 statistical–dynamical atmosphere model: validation and parameter optimization

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Supplementary Information

S1 Planetary Waves

S1.1 Calculation of planetary waves at tropospheric levels excluding EBL-level

At other tropospheric levels than the EBL, the components are calculated by

$$\langle u^*(z)\rangle = -\frac{1}{f\rho_0}\nabla_\phi\langle p_z^*\rangle \tag{S1}$$

$$\langle v^*(z)\rangle = \frac{1}{f\rho_0}\nabla_\lambda\langle p_z^*\rangle, \tag{S2}$$

The azonal component is computed assuming isothermal expansion of air parcels in planetary waves

$$\langle p_z^*\rangle = \langle p_{EBL}^*\rangle \exp[(z - z_{EBL})/H_0] + \frac{p_* g}{\Gamma R}\exp[-z/H_0]\left\{\ln\left[\frac{T(z)}{T(z_{EBL})}\right] - \ln\left[\overline{\frac{T(z)}{T(z_{EBL})}}\right]\right\} \tag{S3}$$

and

$$\langle p_{EBL}^*\rangle = \rho\left\{\overline{\langle u_{EBL}\rangle}\right\}\nabla_\phi\langle\psi_{EBL}^*\rangle + 2\left(\frac{\overline{\langle u_{EBL}\rangle}}{a\cos(\phi)} + \Omega\right)\sin(\phi)\langle\psi_{EBL}^*\rangle \tag{S4}$$

S1.2 Orographically induced stream function

For the waves excited by the orography, the stream function is calculated by

$$\beta\nabla_\lambda\langle\psi_{or,0,EBL}^*\rangle = -\frac{f}{H_0}\langle w_{or}\rangle + \frac{f^2}{g}\frac{\partial\langle u'v'\rangle^*}{\partial z} \tag{S5}$$

where f is the Coriolis parameter and $\beta = \nabla_\phi f$ and

$$w_{or} = \langle u\rangle\nabla_\phi h_{or} + \langle v\rangle\nabla_\lambda h_{or} + a_{std}(\langle u\rangle^2 + \langle v\rangle^2 + \langle u'^2\rangle + \langle v'^2\rangle)^{1/2} h_{std}. \qquad \text{(S6)}$$

The variable h_{or} describes the grid cell averaged orography height h_{std} the subgrid scale standard deviation of the height of mountains, and $a_{std}$is an additional tuning parameter.

The azonal component describes quasi-stationary planetary waves and is subdivided into a geostrophic and ageostrophic term:

$$u^* = u^*_{geos} + u^*_{ageos}$$

$$v^* = v^*_{geos} + v^*_{ageos}$$

S1.3 Zeroth order solution of the thermally induced waves of the barotropic vorticity equation at the EBL

We start from the z-projection of the baroclinic vorticity equation, which can be derived from the simplified Navier-Stokes-equation :

$$\bar{u}\frac{\partial}{\partial x}\left(\frac{\partial^2\langle\Psi^*_{EBL}\rangle}{\partial x^2} + \frac{\partial^2\langle\Psi^*_{EBL}\rangle}{\partial y^2}\right) + \beta\frac{\partial\langle\psi^*\rangle}{\partial x} = -\frac{\rho}{T_0}\frac{\partial\langle T^*_{EBL}\rangle}{\partial x}\frac{\partial\bar{\mathrm{p}}}{\partial y}\frac{1}{{\rho_0}^2} \qquad \text{(S7)}$$

with $= \frac{2\Omega}{a}\cos\phi$, and Ω is the earth's rotation angular velocity, a is the earth's radius and ϕ the latitude.

In Eq. (S7) $\langle\Psi^*_{EBL}\rangle$ is the stream function of the azonal large-scale component at the equivalent barotropic level z_{EBL}, x and y are the horizontal and vertical direction, T_0 is the constant reference temperature and $\langle T^*_{EBL}\rangle$ is the large-scale long-term azonal temperature at the EBL. The variable $\bar{u}$ is the zonal mean zonal wind velocity, ρ_0 stands for the density near surface and $\bar{\mathrm{p}}$ is the zonal mean pressure.

For the stream function of the azonal large-scale component of motion at the equivalent barotropic level z_{EBL}we use the ansatz

$$\langle\Psi^*_{EBL}\rangle = \langle\Psi^*_{0,EBL}\rangle + \epsilon\langle\Psi^*_{1,EBL}\rangle + \ldots$$

For the zeroth order approximation, we can neglect higher order derivations of Ψ:

$$\beta\frac{\partial\langle\Psi^*_{0,EBL}\rangle}{\partial x} = -\frac{1}{\rho\, T_0}\frac{\partial\langle T^*_{EBL}\rangle}{\partial x}\frac{\partial}{\partial y}\frac{\int_0^\infty \rho\langle[T(z)]\rangle\, dz}{H_0} \qquad \text{(S8)}$$

In eq. (S8), we replaced $\bar{\mathrm{p}} = \int_0^\infty R\rho\langle[T(z)]\rangle\, dz/H_0$ and $H_0 = RT_0/g$ and $\rho = \rho_0\exp(-z/H_0)$, R is the gas constant, ρ is the air density, T is the temperature , H_0 is the atmospheric scale height, and g the gravity acceleration . Per definition, one can replace the term with

$$\frac{\int_0^\infty p\,dz}{H_0} = \frac{gR}{RT_0} 2\int_0^{z_{EBL}} \rho\langle[T(z)]\rangle\,dz$$

Such that

$$\frac{\partial\langle\psi^*_{0,EBL}\rangle}{\partial x} = -\frac{agR}{2\Omega R T_0^2 \rho_0 \cos\phi} 2\frac{\partial}{\partial y}\int_0^{z_{EBL}} \rho\langle[T(z)]\rangle\,dz \frac{\partial\langle T^*_{EBL}\rangle}{\partial x}$$

With latter equation and $\frac{1}{\mathrm{a}}\nabla_\phi = \partial/\partial y$, we can then derive

$$\langle\psi^*_{0,EBL}\rangle = -\langle T^*_{EBL}\rangle \frac{g}{\Omega\rho_0 T_0^2 \cos\phi}\nabla_\phi \int_0^{z_{EBL}} \rho\langle[T(z)]\rangle\,dz$$

S2 Derivation of the zonal mean meridional wind velocity

The zonal mean meridional wind velocity $\overline{\langle v(z,\phi)\rangle}$ which also accounts for convective contribution is calculated by

$\overline{\langle v(z,\phi)\rangle} =$

$$\frac{d_1\cdot\left(-2\tan(\phi)\left(\overline{\langle u^*v^*\rangle}+\overline{\langle u'v'\rangle}\right)\right) + d_2\cdot\left(\frac{\partial}{\partial\phi}\left(\overline{\langle u^*v^*\rangle}+\overline{\langle u'v'\rangle}\right)\right) + d_3\cdot\left(\left(-\frac{dK_z}{z}+\frac{K_z}{H_0}\right)\frac{\partial\overline{\langle u\rangle}}{\partial z}a\right) + d_4\cdot(A)}{n_1 * (\tan(\phi)\overline{\langle u\rangle}) + n_2 * \left(-\frac{\partial\overline{\langle u\rangle}}{\partial\phi}\right) + n_3 * (2\Omega a\sin(\phi))} \quad \text{(S9)}$$

.

With $K_z = 0.005\,z$ and

$$A = \frac{\mathcal{L}\overline{\langle P_{co}\rangle}}{H_0}\frac{\overline{\langle u_{sf}\rangle}}{\Gamma_a - \Gamma_0 - \Gamma_1(T_a - T_0)\left(1 - a_q q_s^2\right) + \Gamma_2 n_c}$$

whereby the parameters are given in **Table 1**. We roughly approximate $\overline{\langle u_{sf}\rangle}$ by constant profile for this experiment

$$\overline{\langle u_{sf}\rangle} = \begin{cases} 2, & |\phi| > 40 \\ -2\cos\left(\phi\frac{\pi}{40°}\right), & \text{otherwise} \end{cases}. \quad \text{(S10)}$$

The additional calculation of $\overline{\langle u_{sf} \rangle}$ instead of the calculated surface zonal velocity is done to avoid instabilities. Instabilities can emerge due to the strong positive feedback between the meridional temperature and vertical wind velocity, which lead to high latent heat. In nature these would be damped out but due to fixed troposphere height, we parameterize it in the above described way.

For the derivation we start with the differential equation of the zonal wind component

$$\frac{du}{dt} = \frac{\tan\phi}{a} uv + fv - \frac{1}{\rho}\Delta_\lambda p + F_u \tag{S11}$$

Whereby a is the Earth radius, f is the Coriolis factor and F_u is the frictional force in u-direction. Multiplying the equation with ρ and using that $\rho\frac{du}{dt} = \frac{d(\rho u)}{dt} - u\frac{d\rho}{dt}, \frac{d(\rho u)}{dt} = \frac{\partial(\rho u)}{\partial t} + \boldsymbol{V}\cdot\boldsymbol{\Delta}\,(\rho u)$ and $\boldsymbol{V}\cdot\boldsymbol{\Delta}\,(\rho u) = \Delta\,(\rho u\boldsymbol{V}) - \,(\rho u)\boldsymbol{\Delta}\cdot \boldsymbol{V}$, we get

$$\frac{\partial(\rho u)}{\partial t} + \Delta\,(\rho u \boldsymbol{V}\,) - u\left(\frac{d\rho}{dt} + (\rho u)\boldsymbol{\Delta}\cdot\boldsymbol{V}\right) = \frac{\tan\phi}{a}\rho uv + f\rho v - \Delta_\lambda p + \rho F_u$$

With the continuity equation and using spherical coordinates, the equation simplifies to

$$\begin{aligned}\frac{\partial(\rho u)}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial(\rho u^2)}{\partial\lambda} + \frac{1}{a\cos\phi}\frac{\partial(\rho\cos\phi\; uv)}{\partial\phi} + \frac{\partial(\rho w u)}{\partial z} \\ = \frac{\tan\phi}{a}\rho uv + f\rho v - \frac{1}{a\cos\phi}\frac{\partial p}{\partial\lambda} + \rho F_u\end{aligned} \tag{S12}$$

We calculate the zonal average $(\overline{\dots})$, take into account that $\frac{\overline{\partial x}}{\partial\lambda} = 0$ and assume a vertical dependence of the density $\left(\rho = \rho_0(z)\right)$:

$$\frac{\overline{\partial(\rho_0 u)}}{\partial t} + \frac{1}{a}\frac{\overline{\partial(\rho_0 uv)}}{\partial\phi} + \frac{\overline{\partial(\rho_0 wu)}}{\partial z} = 2\frac{\tan\phi}{a}\rho\overline{uv} + f\rho\bar{v} + \rho_0\overline{F_u}$$

We split the wind variables into an synoptic scale waves, planetary waves and zonal mean wind ($u = \bar{u} + u^* + u'$). Under the assumption that $\bar{u}$ and v^* are independent, the result of the zonal mean over the azonal component is zero:

$$\begin{aligned}\overline{uv} &= \overline{\bar{u}\bar{v} + \bar{u}v^* + \bar{u}v' + u^*\bar{v} + u^*v^* + u^*v' + u'\bar{v} + u'v^* + u'v'} \\ &= \overline{\bar{u}\bar{v} + \bar{u}v' + u^*v^* + u'\bar{v} + u'v'}\end{aligned}$$

We average eq. (S12) over time and phase speed ($\langle\ldots\rangle$). Due to a "gap" in the three-dimensional (period-wavelength-phase velocity) spectrum of atmospheric processes (see, e.g., Fraedrich & Böttger 1978, Coumou et al. 2011), the synoptic-scale component in its interaction with the large-scale long-term component of the atmospheric fields on the time scales about 10-20 days and longer could be, to a first approximation, represented (described) in terms of its ensemble (statistical) characteristics (the second and higher-order moments), and not as the individual eddies (Saltzman, 1978). We can simplify the terms $\langle\bar{u}v'\rangle = \langle u'\bar{v}\rangle = 0$. In addition, it is $\overline{\bar{u}\bar{v}} = \bar{u}\bar{v}$ due to quasi stationarity of both terms. It is also $\langle\frac{dx}{dt}\rangle = 0$ and $\langle\bar{u}\bar{v}\rangle = \langle\bar{u}\rangle\langle\bar{v}\rangle$ since the oscillations of $\bar{u}$ and $\bar{v}$ are very small and independent of each other. By using the continuity equation $\frac{\rho_0}{a}\frac{\partial\langle\bar{v}\rangle}{\partial\phi} - \frac{\tan\phi}{a}\rho_0\bar{v} + \frac{\partial(\rho_0\langle\bar{w}\rangle)}{\partial z} = 0$, we can simplify eq. (S12) to

$$\frac{1}{a}\rho_0\langle\bar{v}\rangle\frac{\partial\langle\bar{u}\rangle}{\partial\phi} + \frac{\rho_0}{a}\frac{\partial\left(\overline{\langle v^*u^*\rangle} + \overline{\langle v'u'\rangle}\right)}{\partial\phi} + \rho_0\langle\bar{w}\rangle\frac{\partial\langle\bar{u}\rangle}{\partial z} + \frac{\partial\left(\rho_0\langle\overline{w^*u^*}\rangle + \langle\overline{w'u'}\rangle\right)}{\partial z} = \frac{\tan\phi}{a}\rho_0\langle\bar{u}\rangle\langle\bar{v}\rangle + 2\,\frac{\tan\phi}{a}\left(\overline{\langle v^*u^*\rangle} + \overline{\langle v'u'\rangle}\right) + f\rho\bar{v} + \rho_0\bar{F}_u \quad \text{(S13)}$$

With the assumption that $\rho_0 = e^{-z/H_0}$ and $\rho_0\bar{F}_u = \frac{\partial\,\bar{\tau}}{\partial z} = \frac{\partial}{\partial z}\left(\kappa\rho_0\frac{\partial\langle\bar{u}\rangle}{\partial z}\right) = \kappa\frac{\partial\rho_0}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial\kappa}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\kappa\frac{\partial^2\langle\bar{u}\rangle}{\partial z^2} = -\kappa\frac{\rho_0}{H_0}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial\kappa}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z}$, we obtain

$$\rho_0\langle\bar{v}\rangle\left(\frac{1}{a}\frac{\partial\langle\bar{u}\rangle}{\partial\phi} - \frac{\tan\phi}{a}\langle\bar{u}\rangle - f\right) = 2\,\frac{\tan\phi}{a}\left(\overline{\langle v^*u^*\rangle} + \overline{\langle v'u'\rangle}\right) - \frac{\rho_0}{a}\frac{\partial\left(\overline{\langle v^*u^*\rangle} + \overline{\langle v'u'\rangle}\right)}{\partial\phi} - \rho_0\langle\bar{w}\rangle\frac{\partial\langle\bar{u}\rangle}{\partial z} - \frac{\partial(\rho_0\langle\overline{w^*u^*}\rangle + \langle\overline{w'u'}\rangle)}{\partial z} - \kappa\frac{\rho_0}{H_0}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial\kappa}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z} \quad \text{(S14)}$$

The contribution to the vertical exchange of the atmospheric momentum from stationary eddies described in our case by zonally averaged $\langle\overline{w^*u^*}\rangle$ is shown negligibly small (Hantel and Hacker, 1978). Also, the scale analysis attests that $\langle\bar{w}\rangle\frac{\partial\langle u\rangle}{\partial z}$ are small (Petoukhov et al., 2003):

$$-\rho_0\langle\bar{w}\rangle\frac{\partial\langle\bar{u}\rangle}{\partial z} - \frac{\partial(\rho_0\langle\overline{w^*u^*}\rangle + \langle\overline{w'u'}\rangle)}{\partial z} - \kappa\frac{\rho_0}{H_0}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial\kappa}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z} \approx -\frac{\partial(\rho_0\langle\overline{w'u'}\rangle)}{\partial z} - \kappa\frac{\rho_0}{H_0}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial\kappa}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z}$$

Hence the eq. (S14) can be rewritten into

$$\rho_0\langle\bar{v}\rangle\left(\frac{1}{a}\frac{\partial\langle\bar{u}\rangle}{\partial\phi} - \frac{\tan\phi}{a}\langle\bar{u}\rangle - f\right) = 2\,\frac{\tan\phi}{a}\left(\overline{\langle v^*u^*\rangle} + \overline{\langle v'u'\rangle}\right) - \frac{\rho_0}{a}\frac{\partial\left(\overline{\langle v^*u^*\rangle} + \overline{\langle v'u'\rangle}\right)}{\partial\phi} - \rho_0\langle\bar{w}\rangle\frac{\partial\langle\bar{u}\rangle}{\partial z} - \frac{\partial(\rho_0\langle\overline{w^*u^*}\rangle + \langle\overline{w'u'}\rangle)}{\partial z} - \kappa\frac{\rho_0}{H_0}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial\kappa}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z} \quad \text{(S15)}$$

With $\overline{\langle u'w'\rangle} = -\kappa' \frac{\partial\langle\bar{u}\rangle}{\partial z}$, whereby κ' is the coefficient of large-scale turbulent exchange for the momentum due to transient synoptic eddies (Williams and Davies, 1965), we get

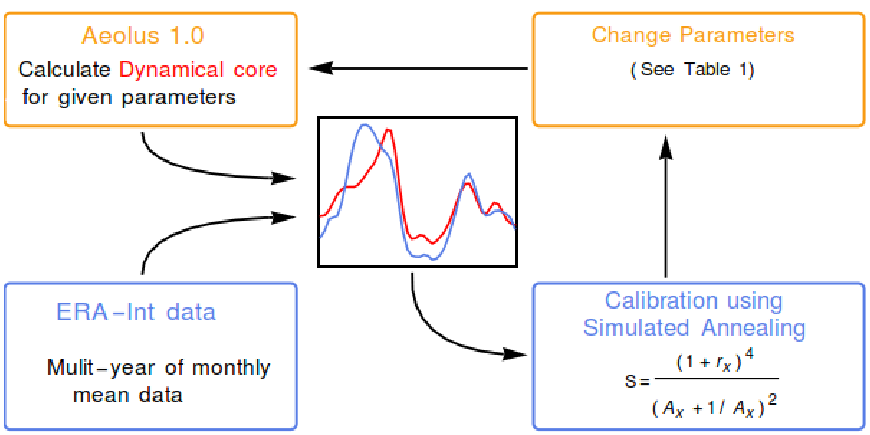
$$\rho_0\langle\bar{v}\rangle\left(\frac{1}{a}\frac{\partial\langle\bar{u}\rangle}{\partial\phi} - \frac{\tan\phi}{a}\langle\bar{u}\rangle - f\right) = 2\frac{\tan\phi}{a}\left(\overline{\langle v^*u^*\rangle} + \overline{\langle v'u'\rangle}\right) - \frac{\rho_0}{a}\frac{\partial\left(\overline{\langle v^*u^*\rangle} + \overline{\langle v'u'\rangle}\right)}{\partial\phi} - (\kappa+\kappa')\frac{\rho_0}{H_0}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial(\kappa+\kappa')}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z}$$

With $K_z = \kappa + \kappa'$ we can simplify the equation to

$$\overline{\langle v(z,\phi)\rangle} = \frac{-2\tan(\phi)\left(\overline{\langle u^*v^*\rangle} + \overline{\langle u'v'\rangle}\right) + \frac{\partial}{\partial\phi}\left(\overline{\langle u^*v^*\rangle} + \overline{\langle u'v'\rangle}\right) + \left(-\frac{dK_z}{dz} + \frac{K_z}{H_0}\right)\frac{\partial\overline{\langle u\rangle}}{\partial z}a}{\tan(\phi)\overline{\langle u\rangle} - \frac{\partial\overline{\langle u\rangle}}{\partial\phi} + 2\Omega a\sin(\phi)} \quad \text{(S16)}$$

The derived equation for the meridional velocity does not account for latent heat release associated with convective precipitation. To capture this additional term we include convective precipitation and finally introduce tuning parameters, which have values close to 1.

S3 Schematic plot of the optimization process



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