



1 **The potential uses of tracer cycles for groundwater dating in** 2 **heterogeneous aquifers**

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9 **Abstract**

10 The use of the annual cycles of stable isotopes to estimate the parameters of transit time distribution functions has been
11 recently criticised by Kirchner (2016). The author shows that the mean residence time of heterogeneous catchments
12 calculated from the damping of the amplitude of the input signal are very often over-estimates, sometimes by large factors.
13 We show here that the overestimation depends on the relative time scales of the cycle's frequency and the mean transit time
14 and that tracer cycles can still be used, at least for groundwater systems sustained by baseflow.

15 Firstly it appears that an exponential model is a good approximation for the transit time distribution of a heterogeneous
16 groundwatershed if the sub groundwatersheds' transit time distributions are themselves exponential and their mean transit
17 times are in the same range or slightly higher than the period of the tracer cycle.

18 Secondly, we suggest that tracer cycles can still be used as secondary data to test whether the degree of heterogeneity of the
19 subsurface is small enough to warrant approximating it by a homogeneous medium.

20 Lastly, we develop a model predicting the amplitude of groundwater temperature from the annual air temperature cycle, and
21 show that even though temperature is not a conservative tracer, it can be useful for groundwater dating. The potential use of
22 the temperature cycle is illustrated in the case-study of a sandstone aquifer drained by contact springs.

23 **1. Introduction**

24 Of the different methods that were developed to estimate groundwater transit time from environmental isotopes, fitting a
25 model to the damping of the amplitude of the atmospheric input (Maloszewski et al., 1983) is probably the most simple. It
26 only requires short times series of observations at the outlet (half a cycle at worst, i.e. half a year for the annual signal of
27 stable isotopes), and the fitting of a sine function. Until now, its main recognized drawback was the limited window of mean
28 transit time (MTT) accessible, as the function describing the damping of the amplitude with increasing MTT reaches the
29 quantification error of stable isotope measurements (about 1‰ for deuterium and 0.1‰ for ¹⁸O) after only a few years.

30 The use of tracer cycles to estimate the parameters of the transit time distribution (TTD) function has been recently
31 reexamined by Kirchner (2016) for heterogeneous catchments. Using a simple model combining the transit time distribution
32 (TTD) of two theoretical sub-catchments characterised by a MTT diverging by at least a factor of two and calculating the
33 MTT from the resulting amplitude of the output tracer signal, he showed that the strong non-linearity of the relationship
34 between amplitude and MTT was liable to cause a systematic underestimation of the “true” MTT. These results contrast
35 with those of Luther and Haitjema (1998) who after conducting a series of numerical experiments using groundwater flow
36 models and spatially variable hydraulic conductivities, recharge and porosities, concluded that the exponential function



37 “provides a good approximation to the residence time distributions for watersheds with heterogeneity that is not significant
38 and distinct”.

39 Since all catchments are to some degree heterogeneous, Kirchner's result seem to spell the end of the method altogether for
40 dating purposes. In this contribution we wish to build upon his excellent work and show that there still is room for
41 differentiation, at least for relatively slowly-responsive groundwater systems. To that effect, we adopt Kirchner's
42 methodology, which we slightly modify to shift the whole perspective from estimating the MTT to a more general
43 assessment of TTD estimation (in the vein of the paper by Luther and Haitjema).

44 This is done in order to study the effect of heterogeneity (in the sense of Kirchner's toy model) on the *shape* of the TTD, and
45 consequently on the robustness of the estimation of the MTT from tracer cycles. To this aim, we fit an exponential model
46 directly to the tracer output calculated from the transit time distribution (TTD) obtained from the sum of the sub-catchments'
47 TTDs in order to compare the (synthetic) real TTD with its exponential approximation. This comparison is done for
48 different degrees of heterogeneities and MTTs of different magnitudes relative to the period of the cycle (one year).

49 This analysis then leads to our suggestion of using tracer cycles as secondary data with which to test the “homogeneity
50 hypothesis”

51 We also introduce temperature as an alternative tracer for groundwater dating, and show that although it is not conservative,
52 its large seasonal amplitude and smooth annual cycle advantageously compare to stable isotopes and allow an extension of
53 the dating window to a mean residence time of about ten years in favourable cases. We illustrate the second and third points
54 with a case-study of a sandstone aquifer drained by contact springs.

55 **2. Modification of Kirchner's thought experiment**

56 Three remarks are of the order to begin with. Firstly, we assume that the tracer mean transit time is in first approximation
57 equal to the water mean transit time. Secondly, the mean transit time in itself is of limited interest to us, although useful
58 estimates can be gained from it, such as the total volume of water in storage or the average saturated porosity (Maloszewski
59 et al., 2002). But as a “characteristic measure” of a catchment's reaction time, it suffers from the same drawback as the
60 average value of any set of measurements in that it does not convey any information about the distribution of the transit time
61 around that mean value, which depends on the chosen TTD model. In our view, the mean transit time is simply a fitting
62 parameter of the TTD, and as such, much less appropriate than the *shape* of the TTD to study the effect of “heterogeneity”
63 on a catchment's response time. Thirdly, we restrict our analysis to groundwater systems in which the discharge in the outlet
64 is sustained exclusively by baseflow.

65 In order to study the effect of subcatchments' heterogeneities on the shape of a groundwater's TTD, we modify
66 Kirchner's methodology as follows. Firstly, we increase number of subcatchments to study a possible convergence of the
67 total TTD towards a stable distribution. Secondly, we do not enforce a factor 2 of difference between the sub-catchments'
68 MTTs (which Kirchner presents without further arguments as “truly heterogeneous catchments”). Instead, we investigate
69 the effect of the magnitude of subcatchments' MTTs *relative* to the period of the tracer cycle of one year.

70 The methodology is as follows:

- 71 1. The number “n” of subcatchments is set
- 72 2. An interval I for the subcatchments' MTTs is defined ($I \in \mathbb{R}_{>0}$).
- 73 3. The MTT for each subcatchment is defined by drawing a random number from the interval I assuming a uniform
74 probability distribution.



- 75 4. the TTDs of the subcatchments are added up and scaled to unity, yielding the TTD of the entire groundwatershed
 76 (TTD_{composite})
 77 5. The input tracer cycle is convolved with the TTD_{composite} obtained, yielding an output signal characterized by a
 78 given amplitude damping.
 79 6. A theoretical exponential function is fitted to the output signal obtained from the TTD_{composite}, yielding a TTD_{theory}.
 80 This step simulates the real situation in tracer studies where the parameter of the TTD_{theory} is estimated inversely
 81 from the input and output signal.

82 The procedure is repeated for different subcatchments sizes n and for the different intervals I .

83 The function chosen for the TTD_{theory} is the exponential model, which was shown by Luther and Haitjema (1998) and
 84 Etcheverry (2001) to describe exactly the distribution of transit times in a homogeneous semi-confined aquifer for which the
 85 product nH/R (porosity times aquifer thickness divided by annual recharge) is piecewise constant. Another important result
 86 of Luther and Haitjema's numerical simulations is that mild heterogeneous hydraulic conductivities fields or unconfined
 87 aquifer with gently sloping groundwater tables lead to TTDs nearly indistinguishable from an exponential function as well.
 88 Following previous groundwater dating studies performed for the aquifer presented in the case study, a piston-flow
 89 component is added to the transfer function to simulate the transit time through the unsaturated zone (Farlin et al., 2013b).

90 3. Groundwater dating using the annual temperature cycles

91 To our knowledge, all publications using tracer cycles to estimate the parameters of a TTD are based on the seasonal signal
 92 of stable isotopes. One of the problems with that signal is its lack of smoothness. Depending on the year and the location,
 93 the isotopic seasonal variations in rainfall in some cases hardly qualify as sinusoidal. On the other opposite, air temperature
 94 and consequently soil temperature follow much more closely the seasonal pattern of the near-surface air temperature. In
 95 case of the soil temperature, soil thermal inertia imparts both a lag and a damping of the amplitude of the air temperature
 96 with depth which can be modeled by a sinusoidal function (Hillel, 1998)

$$97 \quad T(z, t) = T_a + A_0 e^{-z/d} \sin \left[\frac{2\pi(t - t_0)}{365} - \frac{z}{d} - \frac{\pi}{2} \right] \quad (1)$$

98 Where $T(z, t)$ =soil temperature [°C] at depth z [m] and time t [d], T_a =average annual soil temperature [°C], A_0 =annual
 99 amplitude of the surface soil temperature [°C], d =damping depth [m] and t_0 =time lag from January 1 to the occurrence of
 100 the minimum temperature in a year [d]

101 The damping depth is given by

$$102 \quad d = \sqrt{\frac{2D}{\omega}} \quad (2)$$

103 With D =thermal diffusivity of the soil [$\text{m}^2 \cdot \text{s}^{-1}$] and $\omega=2\pi/365$ [d^{-1}]

104 Assuming sufficient time for infiltrating water to come to thermal equilibrium with the soil, equation 1 can be used to
 105 predict soil water temperature as a function of depth and of the thermal properties of the soil.

106 After leaving the soil compartment, the additional damping of the temperature signal that occurs during transport in the
 107 aquifer can be modeled by the following convolution integral (Maloszewski and Zuber, 1982)

$$108 \quad T_{out}(t) = \int_0^t T_{in}(t) g(t - \tau) \exp(-\lambda\tau) d\tau \quad (3)$$



109 Where $T_{\text{out}}(t)$ =water temperature in the outlet at time t [$^{\circ}\text{C}$], $T_{\text{in}}(t)$ =water temperature at the groundwater table at time t [$^{\circ}\text{C}$],
110 $g=\text{TTD}_{\text{theory}}$, and λ =loss constant [d^{-1}]. λ usually simulates decay for radioactive tracers such as tritium or degradation for
111 non-conservative tracers. For temperature, we assume long-term equilibrium with the surrounding rock, and set $\lambda=0$.
112 The exponential piston-flow TTD is given by

$$113 \quad g(t) = \frac{\eta}{\nu} \exp\left(-\frac{\eta t}{\nu} + \eta - 1\right) \text{ for } t \geq \nu(1 - \eta^{-1}) \quad (4)$$

$$114 \quad g(t) = 0 \text{ otherwise}$$

115 With $\nu=\text{MTT}$ [d] and η =total volume divided by the exponential flow volume [-]

116 Thus, the temperature signal in water at the outlet of the groundwatershed can be modeled from the annual cycle of air
117 temperature transformed in series first by its transfer first through the soil (equation 1) and then through the aquifer
118 (equation 3). Equation 3 can also be used with another tracer, in which case $T_{\text{out}}(t)$ is the tracer concentration in the outlet. In
119 any case, $T_{\text{out}}(t)$ and $T_{\text{in}}(t)$ are known, and ν and η are the two free parameters that must be estimated by curve fitting. The
120 complete soil-aquifer model developed here consists of 4 free parameters: the soil depth z , the mean soil thermal diffusivity
121 D , the MTT ν and the piston-flow parameter η .

122 4. Field sites

123 The fractured-rock aquifer known as the Luxembourg Sandstone is the main groundwater resource of the country of
124 Luxembourg. The aquifer, unconfined in its northern part, is drained by numerous contact springs emerging where its
125 impervious basis intersects the land surface. More details about the aquifer can be found in Farlin et al. (2013a).

126 Water temperature and stable isotopes were measured weekly in 11 contact springs draining the Luxembourg Sandstone
127 aquifer between May 2013 and July 2014. Additionally, between 1 and 5 tritium measurements were also available
128 depending on the spring, covering the period 2008-2014. The atmospheric input functions for tritium and ^{18}O was created by
129 extending backwards the monthly measurements of the station Trier available from 1978 to the present with the monthly
130 measurements of the station Vienna using linear regressions. The input time series were further weighted by the ratio of
131 summer to winter infiltration coefficient (Grabczak et al., 1984) using the stable isotope measurements.

132 The TTD of each spring was parameterized jointly using tritium and ^{18}O measurements. Only parameters that lead to output
133 signals within the analytical error of both tracers were retained, yielding a range of estimated MTTs for each spring. The
134 stable isotope signal could not be used for sine fitting as it was either flat, or showed an interannual trend depending on the
135 spring.

136 The mean annual air temperature in the region is 10°C with an annual amplitude also equal to 10°C . The minimum air
137 temperature is usually reached at the end of January, so t_0 is set to 30 days. Soil thermal diffusivity depends on mineralogy,
138 water content and bulk density. For a bulk density of 1.5 Mg.m^{-3} , measured D vary between to $0.2 \cdot 10^{-6}$ and $10^{-5} \text{ m}^2.\text{s}^{-1}$
139 (deVries, 1963;Farouki, 1986). Furthermore, the depth of the soil also influences the damping of the temperature signal.
140 Soil depth in the catchments is variable and can reach 2 meters (Farlin et al., 2013a). Since the mean value of D and z
141 cannot be estimated separately for each groundwatershed, calculations were made using minimum and maximum known
142 values, yielding bounding curves in the plot of temperature amplitude versus MTT.



143 5. Results

144 Three different intervals were used for the TTD estimations. The interval adopted by Kirchner (I_3 : 0.1-20 years), as well as
145 two sub-intervals (I_1 : 0.1-1 year and I_2 : 5-20 years). Figure 1 shows the result of the comparison of the $TTD_{\text{composite}}$ with the
146 TTD_{theory} for five different realisations of the $TTD_{\text{composite}}$ for the three different MTT ranges. Results are insensitive to the
147 number of subcatchments and are shown exemplarily for $n=10$. The realisations shown are a small subset of all the
148 simulations performed chosen to illustrate the different TTD shapes obtained.

149 Both for short MTTs (I_1) and the entire MTT range adopted by Kirchner (I_3), the obtained TTDs display a significant
150 departure from an exponential function. All simulated TTDs are more strongly curved than the exponential distribution. For
151 MTTs shorter than the period of the annual cycle (I_1), the younger fraction dominates in the output signal which causes the
152 exponential function to follow the slope of that fraction. When MTTs are drawn from the entire range of 30 days to 20 years
153 (I_3), the slope of the fitted exponential function often lies between the two segments of the $TTD_{\text{composite}}$ in order to balance
154 out the respective influence of the “young” and “old” fractions. As a consequence, the young water contribution is
155 overestimated, while the opposite holds for the old water contribution. The situation is different for the medium MTT range
156 (I_2). In that case, the entire TTD_{theory} obtained inversely from fitting follows closely the $TTD_{\text{composite}}$ over the entire interval
157 of transit times of 0 to 20 years.

158 A plot of the measured amplitude versus the calculated MTT together with the theoretical curves obtained for different
159 parameterisations of the soil compartment is presented on Figure 2. Seven out of eleven springs fall within the interval
160 spanned by the theoretical curves (eight out of eleven taking the error bars into account). For the last three springs, the
161 measured amplitude of water temperature is higher than expected from the calculated MTTs. Two of these springs probably
162 drain a more superficial aquifer than the others, and thus may display a TTD more curved than the exponential model of the
163 type shown for the interval I_3 on Figure 1. Figure 2 also illustrates the fact that for a shallow soil and an amplitude of the
164 input temperature signal of 10°C, MTTs of up to 20 years can be estimated before the measurement error of $\pm 0.1^\circ\text{C}$ is
165 reached.

166 6. Discussion

167 Our re-exploration of Kirchner’s model has shown that (i) as long as the range of subcatchments’ MTTs is in the same order
168 of magnitude as the period of the cycle, and supposing all TTDs are exponential, the total TTD can still be approximated by
169 an exponential distribution and consequently that (ii) although using tracer cycles to estimate the parameter of a TTD must
170 be used with caution and acknowledging the heterogeneity problem, the method still appears sufficiently robust for
171 particular applications such as groundwater studies as long as the quickflow contribution to discharge is negligible
172 compared to baseflow.

173 The first point confirms the findings of Luther and Haitjema concerning heterogeneities that are “not significant and
174 distinct” using Kirchner’s simple toy model. As Kirchner puts it, “it will be generally difficult or impossible to characterize
175 the system’s heterogeneity”. We can only agree, and wish to add that this shortcoming becomes much less glaring if a
176 catchment’s heterogeneity is simulated using a groundwater flow model as was done by Luther and Haitjema (1998), since
177 the variables’ physical parameters (porosity, hydraulic conductivity and recharge rate) can be compared to real-world
178 measurements. This compares favourably to the type of guesswork necessary for a model based on combining
179 subcatchments TTDs, where the “likely range of variation in subcatchments MTTs” (Kirchner, 2016) is difficult to estimate,
180 and which Kirchner supposes to go from “fractions of a year to perhaps several years” without substantiating his claim.



181 The second point introduces a degree of differentiation to the results obtained by Kirchner. Our analysis shows that the
182 decisive point is not so much the difference between sub-catchments MTTs, but rather its magnitude relative to the time
183 scale of the cycle used for dating. Even when individual MTTs differ by a factor of up to 4, as long as they are drawn from
184 an interval slightly higher than the period of a cycle (in most cases one year), the total TTD follows closely an exponential
185 function if the individual TTDs are themselves exponential. This is not surprising considering the strong smoothing effect
186 exercised by the TTD on the input function.

187 We also present a groundwater dating model based on the annual temperature cycle by combining a function describing soil
188 temperature as a function of soil depth and the traditional convolution integral of Maloszewski and Zuber (1982).
189 Temperature is not a conservative tracer, and as such suffers from a number of drawbacks. Firstly, Eq. (1) is strictly valid
190 using soil surface temperature, and not air temperature as we have done. Wu and Nofziger (1999) however have shown that
191 this approximation leads to a systematic underestimation of soil temperature for bare soils, which can be corrected simply
192 by adding 2 degrees to the prediction. Since our analysis is based solely on the amplitude, and not on absolute temperature,
193 such a correction is unnecessary. Secondly, Eq. (1) predicts soil temperature, and not water temperature. Thus we had to
194 assume thermal equilibrium between the soil and the infiltrating water. This approximation of course should not hold if fast
195 preferential flow is the main infiltration mechanism. However, since we calculate water temperature at the soil-bedrock
196 interface, and we expect the density of preferential flow pathways to decrease with depth, the model assumption appears
197 reasonable in the absence of actual field measurements of water temperature at such depth. Thirdly, the natural geothermal
198 gradient imparts an increase of subsurface temperature of 1°C per 20 to 40 m (Anderson, 2005). As for the difference
199 between air temperature and soil surface temperature, an increasing heat content is not problematic as long as it is constant
200 in time, since only the amplitude of the temperature signal is used in the analysis.

201 On the other hand, the advantages of the temperature cycle should not be overlooked. Temperature measurements in
202 groundwater or spring water are often readily available, measurement is cheap and can be done automatically using field
203 probes, and one cycle measured at the outlet is enough to estimate the MTT parameter of the TTD. Furthermore, both input
204 and output signals are much smoother than the often noisy stable isotope cycles.

205 The case study we present constitutes a first test of our theoretical analysis. The groundwatersheds of the springs sampled
206 appears in eight out of eleven cases to be sufficiently homogeneous to allow approximating their TTDs with an exponential
207 function. Furthermore, the case study illustrates how tracer cycles can be used to verify whether the aquifer is locally
208 sufficiently homogeneous for such an approximation. The fact that three springs fall outside of the envelope of Figure 2
209 could indicate that the underlying exponential TTD fails to represent the true TTD of the three groundwatersheds. As we
210 have seen, this situation can be observed in the presence of strong local heterogeneities leading to a TTD more strongly
211 curved than an exponential curve. In such a case the estimated MTT is meaningless, since it parameterises a function that
212 underestimates the old water component and often overestimates the young water component as well.

213 In conclusion, we recognize two uses for tracer cycles:

- 214 • In subsurface systems that react relatively slowly to precipitation events, the cycle can be used for dating purposes.
215 Water temperature in particular can be used in that way when a more complex approach based on costlier tracers
216 cannot be adopted.
- 217 • In dating studies making use of tracers such as tritium or CFC measurements, the additional MTT estimates
218 obtained from the amplitude damping can be compared to those calculated from the environmental tracer. A
219 disagreement could indicate that the chosen TTD is inappropriate, possibly because of strong and distinct local



220 heterogeneities within the groundwatershed.

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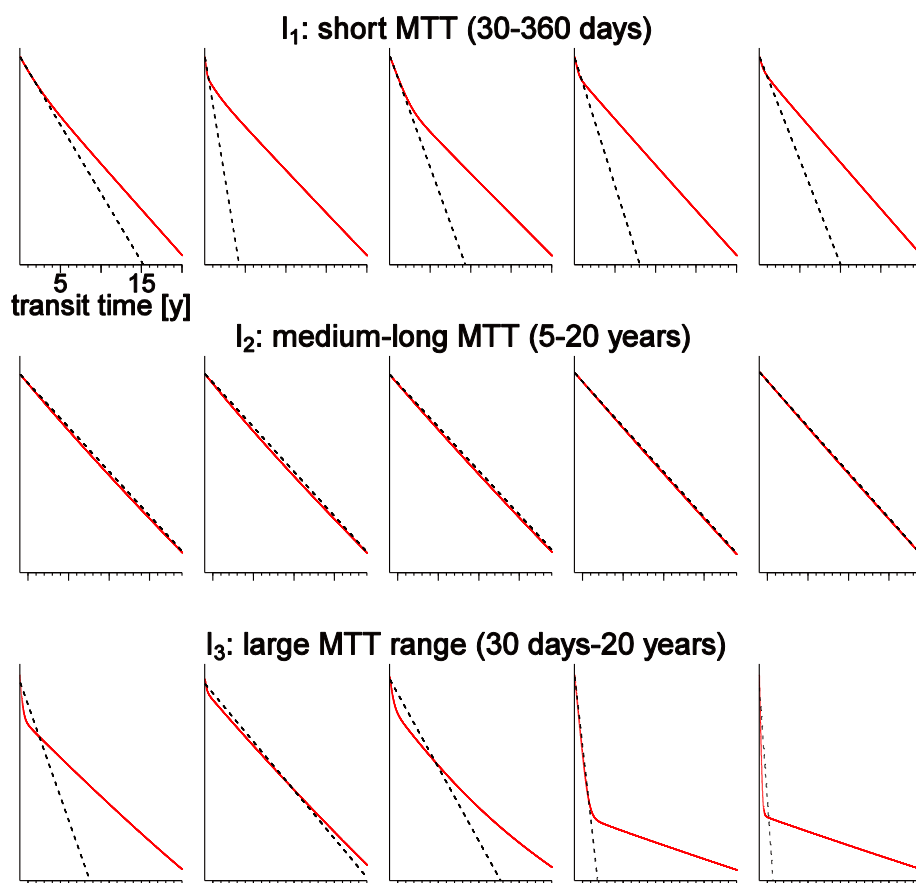


Figure 1: Comparison of the TTD_{theory} (black dotted line) and $TTD_{composite}$ (plain red line) for different MTT ranges (exponential y-axis)

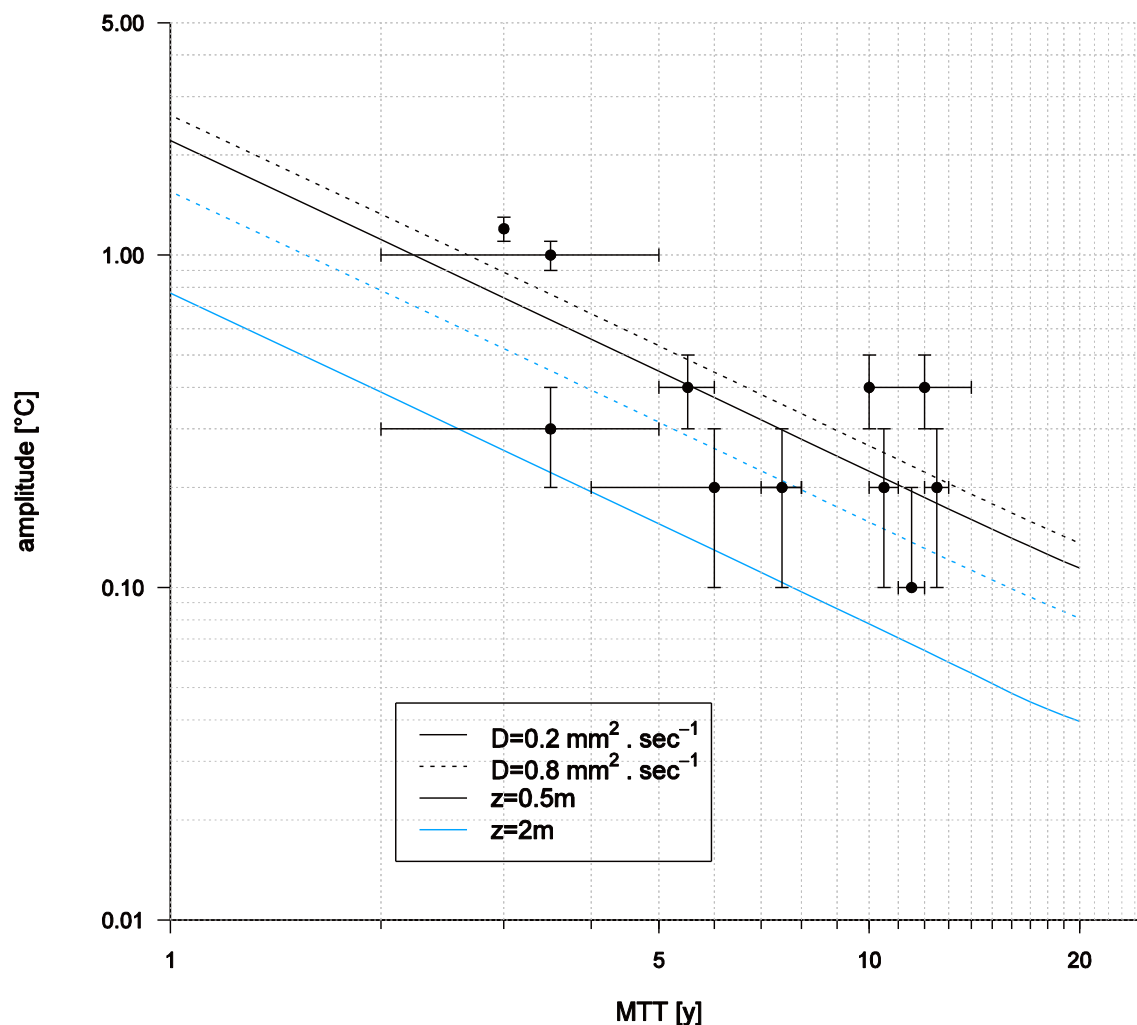


Figure 2: Amplitude damping of the temperature signal as function of the MTT and for different values of D and z , and observed relationships for the sampled springs. The horizontal error bars mark the range of MTTs estimated using tritium and ^{18}O measurements, the vertical error bars correspond to the instrumental error of the temperature probe ($0.1\text{ }^{\circ}\text{C}$).

222 **Author contribution**

223 J. Farlin performed the field work and the calculations and wrote the manuscript. P. Małoszewski added elements to the
224 draft.

225 **Competing interest**

226 The authors declare that they have no conflict of interest.