Hybridizing sequential and variational data assimilation for robust high-resolution hydrologic forecasting

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Supplemental information

Table 1. Parameter assignments and error metrics for all the configurations of Experiment 1 (Blue River), Scenario 1 (November 16th, 1996). Configuration 0 corresponds to the default model. All other errors are computed using the OPTIMISTS ensemble mean.

Config.	Δt	Objs.	n	p_{samp}	$k_{F\text{-}class}$	Assimilation period			Forecast period		
						NSE_{t2}	NSE_{t1}	MARE	NSE_{t2}	NSE_{t1}	MARE
0	-- No data assimilation --					0.801	0.613	26.2%	0.553	0.404	16.5%
1	1 w	1	200	0.4	no	0.968	0.861	8.6%	-1.109	-0.496	45.4%
2	1 w	1	200	0.4	yes	0.980	0.879	9.6%	-1.519	-0.749	55.5%
3	1 w	1	200	1.0	no	0.899	0.746	15.3%	-1.455	-0.519	43.9%
4	1 w	1	200	1.0	yes	0.960	0.828	11.4%	-0.465	-0.348	42.7%
5	2 w	1	200	0.4	no	0.922	0.798	12.3%	-1.287	-0.564	47.6%
6	2 w	1	200	0.4	yes	0.964	0.823	13.3%	-0.353	-0.343	44.1%
7	2 w	1	200	1.0	no	0.877	0.721	16.9%	-1.252	-0.453	42.0%
8	2 w	1	200	1.0	yes	0.929	0.770	15.9%	-0.502	-0.271	38.5%
9	1 w	1	500	0.4	no	0.952	0.823	13.6%	-1.947	-1.003	67.2%
10	1 w	1	500	0.4	yes	0.969	0.851	8.7%	-0.509	-0.405	45.4%
11	1 w	1	500	1.0	no	0.896	0.749	14.7%	-1.552	-0.591	46.9%
12	1 w	1	500	1.0	yes	0.970	0.846	10.9%	-1.349	-0.538	45.3%
13	2 w	1	500	0.4	no	0.955	0.831	10.6%	-1.616	-0.500	42.0%
14	2 w	1	500	0.4	yes	0.969	0.843	12.5%	-0.488	-0.360	44.7%
15	2 w	1	500	1.0	no	0.884	0.733	16.0%	-1.314	-0.458	42.0%
16	2 w	1	500	1.0	yes	0.937	0.778	16.1%	-0.718	-0.356	41.1%
17	1 w	2	200	0.4	no	0.927	0.784	12.7%	-1.697	-0.664	49.3%
18	1 w	2	200	0.4	yes	0.946	0.813	10.2%	-2.005	-0.786	54.0%
19	1 w	2	200	1.0	no	0.873	0.720	17.5%	-1.256	-0.520	45.3%
20	1 w	2	200	1.0	yes	0.930	0.778	13.3%	-1.141	-0.419	40.9%
21	2 w	2	200	0.4	no	0.914	0.763	13.8%	-1.179	-0.403	40.2%
22	2 w	2	200	0.4	yes	0.889	0.731	15.7%	-1.138	-0.439	42.2%
23	2 w	2	200	1.0	no	0.846	0.692	18.7%	-1.189	-0.446	42.2%
24	2 w	2	200	1.0	yes	0.875	0.718	16.7%	-1.244	-0.473	43.3%
25	1 w	2	500	0.4	no	0.951	0.816	12.6%	-1.482	-0.850	61.4%
26	1 w	2	500	0.4	yes	0.960	0.825	11.3%	0.015	-0.080	33.9%
27	1 w	2	500	1.0	no	0.866	0.714	17.3%	-1.434	-0.554	45.9%
28	1 w	2	500	1.0	yes	0.913	0.765	14.7%	-1.603	-0.554	44.8%
29	2 w	2	500	0.4	no	0.890	0.733	16.8%	-1.258	-0.492	44.0%
30	2 w	2	500	0.4	yes	0.905	0.747	15.5%	-1.130	-0.466	43.8%
31	2 w	2	500	1.0	no	0.860	0.703	17.7%	-1.117	-0.424	41.6%
32	2 w	2	500	1.0	yes	0.881	0.723	16.4%	-1.150	-0.449	42.7%

Table 2. Parameter assignments and error metrics for all the configurations of Experiment 1 (Blue River), Scenario 2 (April 8th, 1997). Configuration 0 corresponds to the default model. All other errors are computed using the OPTIMISTS ensemble mean.

Config.	Δt	Objs.	n	p_{samp}	$k_{F\text{-class}}$	Assimilation period			Forecast period		
						NSE_{t2}	NSE_{t1}	MARE	NSE_{t2}	NSE_{t1}	MARE
0	-- No data assimilation --					-0.249	-0.641	122.5%	0.498	0.108	106.8%
1	1 w	1	200	0.4	no	0.402	-0.16	80.7%	0.479	0.129	99.7%
2	1 w	1	200	0.4	yes	0.505	-0.023	70.9%	-0.083	-0.061	110.7%
3	1 w	1	200	1	no	0.285	-0.285	91.6%	0.435	-0.003	111.5%
4	1 w	1	200	1	yes	0.283	-0.161	81.0%	0.555	0.179	79.9%
5	2 w	1	200	0.4	no	0.387	-0.177	82.5%	0.580	0.144	92.3%
6	2 w	1	200	0.4	yes	0.307	-0.126	78.0%	0.629	0.207	82.2%
7	2 w	1	200	1	no	0.324	-0.245	88.1%	0.512	0.067	103.1%
8	2 w	1	200	1	yes	0.287	-0.142	79.5%	0.552	0.157	83.7%
9	1 w	1	500	0.4	no	0.473	-0.078	75.2%	0.592	0.215	90.0%
10	1 w	1	500	0.4	yes	0.636	0.373	37.4%	0.371	0.119	81.4%
11	1 w	1	500	1	no	0.323	-0.258	90.1%	0.488	0.052	107.2%
12	1 w	1	500	1	yes	0.279	-0.123	78.2%	0.508	0.187	74.0%
13	2 w	1	500	0.4	no	0.453	-0.087	79.9%	0.246	-0.043	113.9%
14	2 w	1	500	0.4	yes	0.503	0.026	66.9%	0.600	0.213	83.4%
15	2 w	1	500	1	no	0.343	-0.231	87.6%	0.497	0.054	102.8%
16	2 w	1	500	1	yes	0.284	-0.088	74.4%	0.573	0.224	72.9%
17	1 w	2	200	0.4	no	-0.381	-0.69	124.0%	0.427	-0.007	113.3%
18	1 w	2	200	0.4	yes	0.39	-0.148	79.9%	0.579	0.242	70.1%
19	1 w	2	200	1	no	-0.435	-0.703	124.8%	0.413	-0.022	114.7%
20	1 w	2	200	1	yes	0.566	0.125	56.8%	0.542	0.404	41.1%
21	2 w	2	200	0.4	no	-0.76	-0.823	133.7%	0.425	0.002	114.9%
22	2 w	2	200	0.4	yes	0.311	-0.162	80.6%	0.592	0.170	85.4%
23	2 w	2	200	1	no	-0.677	-0.781	131.1%	0.428	0.000	114.0%
24	2 w	2	200	1	yes	0.284	-0.263	89.6%	0.517	0.073	100.8%
25	1 w	2	500	0.4	no	-0.116	-0.528	110.9%	0.504	0.072	99.5%
26	1 w	2	500	0.4	yes	0.502	0.089	59.4%	0.463	0.365	43.8%
27	1 w	2	500	1	no	-0.324	-0.652	121.1%	0.449	0.017	112.2%
28	1 w	2	500	1	yes	0.467	0.067	60.4%	0.458	0.383	39.4%
29	2 w	2	500	0.4	no	-0.174	-0.572	113.9%	0.460	0.038	110.5%
30	2 w	2	500	0.4	yes	0.326	-0.203	84.6%	0.571	0.135	94.0%
31	2 w	2	500	1	no	-0.457	-0.701	124.3%	0.445	0.014	112.1%
32	2 w	2	500	1	yes	0.304	-0.209	85.0%	0.538	0.101	95.1%

Table 3. Parameter assignments and error metrics for all the configurations of Experiment 1 (Blue River), Scenario 2 (February 24th, 1998). Configuration 0 corresponds to the default model. All other errors are computed using the OPTIMISTS ensemble mean.

Config.	Δt	Objs.	n	p_{samp}	$k_{F\text{-class}}$	Assimilation period			Forecast period		
						NSE_{t2}	NSE_{t1}	MARE	NSE_{t2}	NSE_{t1}	MARE
0	-- No data assimilation --					0.608	0.174	141.7%	0.758	0.480	101.7%
1	1 w	1	200	0.4	no	0.689	0.387	92.2%	0.864	0.609	81.6%
2	1 w	1	200	0.4	yes	0.608	0.361	96.0%	0.885	0.662	73.4%
3	1 w	1	200	1	no	0.66	0.365	98.0%	0.871	0.594	85.2%
4	1 w	1	200	1	yes	0.586	0.376	89.5%	0.824	0.619	71.3%
5	2 w	1	200	0.4	no	0.648	0.346	100.8%	0.866	0.603	72.8%
6	2 w	1	200	0.4	yes	0.57	0.329	99.4%	0.884	0.635	72.1%
7	2 w	1	200	1	no	0.606	0.329	101.8%	0.886	0.630	74.1%
8	2 w	1	200	1	yes	0.539	0.299	104.8%	0.887	0.637	73.5%
9	1 w	1	500	0.4	no	0.74	0.437	85.5%	0.858	0.594	75.4%
10	1 w	1	500	0.4	yes	0.691	0.489	62.9%	0.865	0.645	70.9%
11	1 w	1	500	1	no	0.656	0.375	98.3%	0.874	0.603	85.6%
12	1 w	1	500	1	yes	0.611	0.373	92.1%	0.861	0.645	72.0%
13	2 w	1	500	0.4	no	0.623	0.379	91.9%	0.894	0.636	72.5%
14	2 w	1	500	0.4	yes	0.682	0.381	94.2%	0.875	0.630	66.2%
15	2 w	1	500	1	no	0.613	0.339	100.3%	0.892	0.632	72.5%
16	2 w	1	500	1	yes	0.532	0.301	104.1%	0.889	0.643	72.9%
17	1 w	2	200	0.4	no	0.631	0.233	122.2%	0.849	0.560	90.0%
18	1 w	2	200	0.4	yes	0.69	0.339	105.8%	0.813	0.525	94.0%
19	1 w	2	200	1	no	0.667	0.284	114.6%	0.853	0.566	91.2%
20	1 w	2	200	1	yes	0.67	0.351	98.9%	0.874	0.603	81.9%
21	2 w	2	200	0.4	no	0.588	0.1	144.7%	0.823	0.555	80.3%
22	2 w	2	200	0.4	yes	0.586	0.297	106.7%	0.886	0.632	73.4%
23	2 w	2	200	1	no	0.602	0.106	145.1%	0.814	0.545	82.0%
24	2 w	2	200	1	yes	0.571	0.289	108.7%	0.880	0.619	75.2%
25	1 w	2	500	0.4	no	0.623	0.258	120.8%	0.852	0.571	87.0%
26	1 w	2	500	0.4	yes	0.705	0.369	98.5%	0.852	0.560	93.3%
27	1 w	2	500	1	no	0.637	0.269	117.6%	0.851	0.565	88.6%
28	1 w	2	500	1	yes	0.675	0.347	101.3%	0.858	0.576	88.5%
29	2 w	2	500	0.4	no	0.634	0.198	127.5%	0.880	0.613	74.8%
30	2 w	2	500	0.4	yes	0.56	0.308	103.3%	0.884	0.629	75.5%
31	2 w	2	500	1	no	0.61	0.133	140.7%	0.822	0.550	82.1%
32	2 w	2	500	1	yes	0.54	0.298	105.1%	0.886	0.634	73.7%

Table 4. Parameter assignments and error metrics for all the configurations of Experiment 2 (Indiantown Run), Scenario 1 (July 26th, 2009). Configuration 0 corresponds to the default model. All other errors are computed using the OPTIMISTS ensemble mean.

Config.	Δt	Objs.	n	p_{samp}	w_{root}	Assimilation period			Forecast period		
						NSE_{t2}	NSE_{t1}	MARE	NSE_{t2}	NSE_{t1}	MARE
0	-- No data assimilation --					0.478	-0.015	24.9%	0.022	-0.003	54.3%
1	1 h	2	100	0.25	60%	-9E+04	-1E+02	2.8E+01	0.067	-0.759	197.1%
2	1 h	2	100	0.25	95%	-2E+03	-3E+01	7.4E+00	0.365	0.193	73.5%
3	1 h	2	100	1.00	60%	-1E+01	-3.324	98.3%	0.279	-0.360	143.9%
4	1 h	2	100	1.00	95%	-1.455	-0.862	44.9%	0.346	-0.099	107.0%
5	1 h	2	200	0.25	60%	-2E+05	-1E+02	3.4E+01	0.075	-0.769	196.1%
6	1 h	2	200	0.25	95%	-3E+03	-3E+01	6.8E+00	0.357	0.275	62.2%
7	1 h	2	200	1.00	60%	-1E+01	-3.900	111.6%	0.284	-0.330	143.5%
8	1 h	2	200	1.00	95%	-2.320	-1.192	54.1%	0.341	-0.182	120.1%
9	2 w	2	100	0.25	60%	0.471	0.208	18.1%	0.317	0.381	43.2%
10	2 w	2	100	0.25	95%	0.099	-0.001	23.2%	0.320	0.375	44.3%
11	2 w	2	100	1.00	60%	0.571	0.371	13.3%	0.292	0.393	38.3%
12	2 w	2	100	1.00	95%	0.572	0.353	14.1%	0.323	0.373	45.7%
13	2 w	2	200	0.25	60%	0.531	0.295	15.5%	0.327	0.367	47.0%
14	2 w	2	200	0.25	95%	0.520	0.305	15.4%	0.312	0.380	42.6%
15	2 w	2	200	1.00	60%	0.360	0.211	17.5%	0.322	0.338	50.1%
16	2 w	2	200	1.00	95%	0.427	0.175	18.7%	0.332	0.357	49.0%
17	1 h	1	100	0.25	60%	-5E+02	-3E+01	7.0E+00	-0.429	-1.338	2.8E+00
18	1 h	1	100	0.25	95%	-5E+01	-7.655	196.9%	0.094	-0.736	193.1%
19	1 h	1	100	1.00	60%	-1E+01	-3.201	96.0%	0.314	-0.269	132.4%
20	1 h	1	100	1.00	95%	-9.818	-3.016	90.8%	0.290	-0.354	142.2%
21	1 h	1	200	0.25	60%	-2E+05	-6E+02	1.4E+02	-5E+01	-7.699	1.4E+01
22	1 h	1	200	0.25	95%	-1E+05	-4E+02	9.7E+01	-5E+01	-7.337	1.4E+01
23	1 h	1	200	1.00	60%	-1E+01	-3.965	113.2%	0.254	-0.447	153.8%
24	1 h	1	200	1.00	95%	-1E+01	-3.601	103.3%	0.283	-0.374	144.9%
25	2 w	1	100	0.25	60%	0.453	0.071	21.8%	0.320	0.373	44.2%
26	2 w	1	100	0.25	95%	0.279	-0.053	24.6%	0.329	0.368	46.3%
27	2 w	1	100	1.00	60%	0.500	0.177	18.9%	0.331	0.369	46.3%
28	2 w	1	100	1.00	95%	0.358	0.041	22.2%	0.330	0.364	47.4%
29	2 w	1	200	0.25	60%	0.494	0.177	18.8%	0.324	0.366	46.3%
30	2 w	1	200	0.25	95%	0.362	0.036	22.3%	0.329	0.362	47.4%
31	2 w	1	200	1.00	60%	0.452	0.158	19.2%	0.329	0.362	46.7%
32	2 w	1	200	1.00	95%	0.480	0.155	19.4%	0.333	0.362	47.2%

Table 5. Parameter assignments and error metrics for all the configurations of Experiment 2 (Indiantown Run), Scenario 2 (August 26th, 2009). Configuration 0 corresponds to the default model. All other errors are computed using the OPTIMISTS ensemble mean.

Config.	Δt	Objs.	n	p_{samp}	w_{root}	Assimilation period			Forecast period		
						NSE$_{t2}$	NSE$_{t1}$	MARE	NSE$_{t2}$	NSE$_{t1}$	MARE
0	-- No data assimilation --					-0.926	-0.963	68.0%	-0.428	-0.733	58.3%
1	1 h	2	100	0.25	60%	-8E+02	-2E+01	8.5E+00	-5.171	-2.290	104.3%
2	1 h	2	100	0.25	95%	-2E+02	-7.769	3.9E+00	-0.581	-0.517	46.2%
3	1 h	2	100	1.00	60%	0.672	0.570	16.1%	-1.459	-1.065	66.7%
4	1 h	2	100	1.00	95%	0.637	0.643	11.4%	-0.971	-0.799	57.0%
5	1 h	2	200	0.25	60%	-5E+03	-3E+01	1.0E+01	-1E+01	-3.291	134.0%
6	1 h	2	200	0.25	95%	-3E+02	-9.816	5.0E+00	-1.156	-0.797	54.9%
7	1 h	2	200	1.00	60%	0.708	0.518	19.2%	-1.493	-1.082	67.2%
8	1 h	2	200	1.00	95%	0.657	0.625	12.9%	-1.259	-0.966	63.0%
9	2 w	2	100	0.25	60%	-0.607	-0.154	35.9%	0.322	0.118	25.8%
10	2 w	2	100	0.25	95%	0.041	0.013	29.1%	0.248	0.070	27.1%
11	2 w	2	100	1.00	60%	0.122	0.096	25.6%	0.048	-0.067	31.1%
12	2 w	2	100	1.00	95%	-0.113	-0.164	35.1%	0.451	0.212	23.2%
13	2 w	2	200	0.25	60%	0.186	0.161	23.5%	0.051	-0.089	32.1%
14	2 w	2	200	0.25	95%	0.118	0.088	26.0%	0.132	-0.010	29.4%
15	2 w	2	200	1.00	60%	-0.019	-0.052	30.9%	0.255	0.077	26.9%
16	2 w	2	200	1.00	95%	0.022	-0.030	30.3%	0.279	0.085	26.8%
17	1 h	1	100	0.25	60%	-3.015	-2.226	138.7%	-2E+01	-4.216	164.2%
18	1 h	1	100	0.25	95%	-0.028	-0.550	61.0%	-7.080	-2.694	115.6%
19	1 h	1	100	1.00	60%	0.539	0.542	14.8%	-1.544	-1.083	66.8%
20	1 h	1	100	1.00	95%	0.565	0.442	21.7%	-2.307	-1.433	78.4%
21	1 h	1	200	0.25	60%	-5E+03	-1E+02	4.6E+01	-6E+02	-2E+01	8.5E+00
22	1 h	1	200	0.25	95%	-1E+04	-1E+02	4.9E+01	-7E+02	-2E+01	9.4E+00
23	1 h	1	200	1.00	60%	0.538	0.548	16.1%	-2.240	-1.396	77.8%
24	1 h	1	200	1.00	95%	0.551	0.594	12.7%	-2.089	-1.318	74.2%
25	2 w	1	100	0.25	60%	0.272	0.414	15.6%	-0.859	-0.577	46.7%
26	2 w	1	100	0.25	95%	0.307	0.422	14.7%	-0.779	-0.494	43.5%
27	2 w	1	100	1.00	60%	0.230	0.300	18.2%	-0.501	-0.368	39.8%
28	2 w	1	100	1.00	95%	0.220	0.268	19.4%	-0.475	-0.359	39.6%
29	2 w	1	200	0.25	60%	0.296	0.460	14.6%	-0.757	-0.527	45.2%
30	2 w	1	200	0.25	95%	0.377	0.485	13.8%	-0.916	-0.564	45.7%
31	2 w	1	200	1.00	60%	0.207	0.285	18.8%	-0.493	-0.348	39.0%
32	2 w	1	200	1.00	95%	0.202	0.278	18.9%	-0.525	-0.382	40.3%

Table 6. Parameter assignments and error metrics for all the configurations of Experiment 3 (Indiantown Run), Scenario 1 (July 26th, 2009). Configuration 0 corresponds to the default model. All other errors are computed using the OPTIMISTS ensemble mean.

Config.	Δt	p_{samp}	g	Assimilation period			Forecast period		
				$NSE_{\ell 2}$	$NSE_{\ell 1}$	MARE	$NSE_{\ell 2}$	$NSE_{\ell 1}$	MARE
0	-- No data assimilation --			0.478	-0.015	24.9%	0.022	-0.003	54.3%
1	1 h	0.4	0.5	-1E+05	-2E+02	5.2E+01	0.359	0.286	60.6%
2	1 h	0.4	1.0	-5E+03	-3E+01	8.3E+00	0.366	0.271	63.1%
3	1 h	1.0	0.5	-3.682	-1.729	64.1%	0.358	-0.069	107.8%
4	1 h	1.0	1.0	-2.038	-1.146	50.9%	0.371	0.031	94.3%
5	6 h	0.4	0.5	-6E+02	-1E+01	2.9E+00	0.354	0.307	57.1%
6	6 h	0.4	1.0	-9E+02	-1E+01	317.4%	0.357	0.291	59.9%
7	6 h	1.0	0.5	-2.107	-1.230	52.0%	0.357	0.086	84.0%
8	6 h	1.0	1.0	-1.431	-0.889	43.0%	0.361	0.292	60.1%
9	1 d	0.4	0.5	-2E+02	-4.030	119.4%	0.351	0.312	56.1%
10	1 d	0.4	1.0	-2E+03	-6.205	171.3%	0.350	0.340	52.3%
11	1 d	1.0	0.5	-1.406	-0.891	42.8%	0.353	0.325	55.0%
12	1 d	1.0	1.0	-1.440	-0.893	42.9%	0.352	0.315	56.6%
13	3.5 d	0.4	0.5	-3E+03	-7.748	2.1E+00	0.337	0.311	54.2%
14	3.5 d	0.4	1.0	-6E+01	-1.096	49.0%	0.328	0.337	50.2%
15	3.5 d	1.0	0.5	0.224	0.067	22.1%	0.331	0.306	55.4%
16	3.5 d	1.0	1.0	0.058	-0.095	25.6%	0.333	0.360	48.8%
17	2 w	0.4	0.5	-8E+01	-1.527	59.6%	0.339	0.326	54.1%
18	2 w	0.4	1.0	0.133	-0.190	28.1%	0.333	0.357	49.0%
19	2 w	1.0	0.5	0.399	0.242	16.3%	0.301	0.394	40.2%
20	2 w	1.0	1.0	0.560	0.340	14.4%	0.308	0.389	41.3%
21	4 w	0.4	0.5	-0.743	-0.735	42.0%	0.355	0.112	79.6%
22	4 w	0.4	1.0	-1.469	-1.161	52.2%	0.358	0.047	87.4%
23	4 w	1.0	0.5	-2.873	-1.763	66.4%	0.355	0.033	90.8%
24	4 w	1.0	1.0	-0.577	-0.622	39.2%	0.351	0.159	74.2%

Table 7. Parameter assignments and error metrics for all the configurations of Experiment 3 (Indiantown Run), Scenario 2 (August 26th, 2009). Configuration 0 corresponds to the default model. All other errors are computed using the OPTIMISTS ensemble mean.

Config.	Δt	p_{samp}	g	Assimilation period			Forecast period		
				$NSE_{\ell2}$	$NSE_{\ell1}$	MARE	$NSE_{\ell2}$	$NSE_{\ell1}$	MARE
0	-- No data assimilation --			-0.926	-0.963	68.0%	-0.428	-0.733	58.3%
1	1 h	0.4	0.5	-1E+03	-1E+01	6.2E+00	-0.133	-0.204	37.1%
2	1 h	0.4	1.0	-6E+02	-8.948	4.4E+00	-1.516	-0.967	60.3%
3	1 h	1.0	0.5	0.604	0.686	8.0%	-0.352	-0.459	46.3%
4	1 h	1.0	1.0	0.664	0.596	14.0%	-1.591	-1.133	68.3%
5	6 h	0.4	0.5	-1E+02	-4.849	2.7E+00	-1.270	-0.769	52.7%
6	6 h	0.4	1.0	-2E+01	-0.391	57.4%	-0.371	-0.393	42.1%
7	6 h	1.0	0.5	0.529	0.634	8.6%	-1.098	-0.679	49.6%
8	6 h	1.0	1.0	0.494	0.606	9.2%	-0.388	-0.460	45.5%
9	1 d	0.4	0.5	-7.399	0.020	39.8%	-0.830	-0.563	46.2%
10	1 d	0.4	1.0	0.475	0.444	14.3%	-0.213	-0.286	38.6%
11	1 d	1.0	0.5	0.398	0.432	14.7%	-1.096	-0.717	51.5%
12	1 d	1.0	1.0	0.453	0.512	11.9%	-0.656	-0.501	44.9%
13	3.5 d	0.4	0.5	-1.056	0.047	27.5%	-0.706	-0.465	42.7%
14	3.5 d	0.4	1.0	0.296	0.366	15.6%	-0.690	-0.437	41.6%
15	3.5 d	1.0	0.5	0.028	0.076	24.9%	-0.454	-0.402	41.9%
16	3.5 d	1.0	1.0	0.122	0.149	22.7%	-0.522	-0.376	40.0%
17	2 w	0.4	0.5	-2E+01	-0.396	50.0%	0.094	-0.044	30.5%
18	2 w	0.4	1.0	-0.002	-0.062	31.7%	0.358	0.140	25.3%
19	2 w	1.0	0.5	-0.157	-0.171	35.0%	0.340	0.108	26.4%
20	2 w	1.0	1.0	0.032	-0.024	30.2%	0.301	0.109	26.0%
21	4 w	0.4	0.5	0.379	0.491	11.4%	-1.398	-0.877	56.6%
22	4 w	0.4	1.0	0.330	0.424	13.4%	-1.281	-0.834	55.4%
23	4 w	1.0	0.5	0.308	0.397	14.1%	-0.907	-0.653	49.8%
24	4 w	1.0	1.0	0.327	0.425	13.2%	-1.159	-0.771	53.4%

Table 8. ANOVA table for Experiment 1 for the improvements in NSE$_{\ell 2}$ with respect to the default model. DF: degrees of freedom; Adj SS: adjusted sum of squares; Adj MS: adjusted mean sum of squares; Sims.: n; Samples: p_{samp}; F-kernels: $k_{\text{F−class}}$.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	33	69.586	2.109	28.910	0.000
Scenario	2	66.861	33.431	458.420	0.000
Linear	5	0.914	0.183	2.510	0.039
Objs.	1	0.070	0.070	0.960	0.330
Sims.	1	0.002	0.002	0.030	0.865
TimeStep	1	0.240	0.240	3.300	0.074
Samples	1	0.002	0.002	0.030	0.872
F-kernels	1	0.599	0.599	8.220	0.006
2-Way Interactions	10	0.622	0.062	0.850	0.581
Objs.*Sims.	1	0.096	0.096	1.320	0.255
Objs.*TimeStep	1	0.041	0.041	0.560	0.457
Objs.*Samples	1	0.021	0.021	0.290	0.595
Objs.*F-kernels	1	0.118	0.118	1.610	0.209
Sims.*TimeStep	1	0.049	0.049	0.670	0.416
Sims.*Samples	1	0.153	0.153	2.090	0.153
Sims.*F-kernels	1	0.087	0.087	1.190	0.279
TimeStep*Samples	1	0.005	0.005	0.060	0.805
TimeStep*F-kernels	1	0.032	0.032	0.430	0.513
Samples*F-kernels	1	0.022	0.022	0.300	0.587
3-Way Interactions	10	0.848	0.085	1.160	0.332
Objs.*Sims.*TimeStep	1	0.002	0.002	0.030	0.860
Objs.*Sims.*Samples	1	0.027	0.027	0.370	0.543
Objs.*Sims.*F-kernels	1	0.002	0.002	0.030	0.870
Objs.*TimeStep*Samples	1	0.008	0.008	0.120	0.735
Objs.*TimeStep*F-kernels	1	0.095	0.095	1.300	0.259
Objs.*Samples*F-kernels	1	0.022	0.022	0.300	0.588
Sims.*TimeStep*Samples	1	0.299	0.299	4.090	0.047
Sims.*TimeStep*F-kernels	1	0.051	0.051	0.700	0.406
Sims.*Samples*F-kernels	1	0.329	0.329	4.510	0.038
TimeStep*Samples*F-kernels	1	0.014	0.014	0.190	0.668
4-Way Interactions	5	0.340	0.068	0.930	0.467
Objs.*Sims.*TimeStep*Samples	1	0.010	0.010	0.140	0.714
Objs.*Sims.*TimeStep*F-kernels	1	0.001	0.001	0.010	0.924
Objs.*Sims.*Samples*F-kernels	1	0.027	0.027	0.360	0.548
Objs.*TimeStep*Samples*F-kernels	1	0.086	0.086	1.180	0.282
Sims.*TimeStep*Samples*F-kernels	1	0.217	0.217	2.970	0.090
5-Way Interaction	1	0.001	0.001	0.020	0.903
Objs.*Sims.*TimeStep*Samples*F-kernels	1	0.001	0.001	0.020	0.903
Error	62	4.521	0.073		
Total	95	74.107			

Table 9. ANOVA table for Experiment 1 for the improvements in $\text{NSE}_{\ell 1}$ with respect to the default model. DF: degrees of freedom; Adj SS: adjusted sum of squares; Adj MS: adjusted mean sum of squares; Sims.: n; Samples: p_{samp}; F-kernels: $k_{\text{F}-\text{class}}$.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	33	20.943	0.635	47.830	0.000
Scenario	2	20.338	10.169	766.350	0.000
Linear	5	0.288	0.058	4.340	0.002
Objs.	1	0.004	0.004	0.320	0.576
Sims.	1	0.002	0.002	0.130	0.720
TimeStep	1	0.034	0.034	2.530	0.117
Samples	1	0.014	0.014	1.090	0.301
F-kernels	1	0.234	0.234	17.630	0.000
2-Way Interactions	10	0.101	0.010	0.760	0.666
Objs.*Sims.	1	0.008	0.008	0.590	0.446
Objs.*TimeStep	1	0.022	0.022	1.670	0.200
Objs.*Samples	1	0.003	0.003	0.250	0.620
Objs.*F-kernels	1	0.005	0.005	0.350	0.557
Sims.*TimeStep	1	0.004	0.004	0.270	0.607
Sims.*Samples	1	0.009	0.009	0.690	0.411
Sims.*F-kernels	1	0.027	0.027	2.020	0.160
TimeStep*Samples	1	0.014	0.014	1.030	0.315
TimeStep*F-kernels	1	0.010	0.010	0.740	0.393
Samples*F-kernels	1	0.000	0.000	0.000	0.984
3-Way Interactions	10	0.151	0.015	1.140	0.348
Objs.*Sims.*TimeStep	1	0.003	0.003	0.200	0.657
Objs.*Sims.*Samples	1	0.004	0.004	0.300	0.585
Objs.*Sims.*F-kernels	1	0.000	0.000	0.020	0.898
Objs.*TimeStep*Samples	1	0.002	0.002	0.120	0.728
Objs.*TimeStep*F-kernels	1	0.040	0.040	3.030	0.086
Objs.*Samples*F-kernels	1	0.004	0.004	0.270	0.604
Sims.*TimeStep*Samples	1	0.021	0.021	1.600	0.211
Sims.*TimeStep*F-kernels	1	0.025	0.025	1.850	0.178
Sims.*Samples*F-kernels	1	0.051	0.051	3.840	0.055
TimeStep*Samples*F-kernels	1	0.002	0.002	0.170	0.683
4-Way Interactions	5	0.065	0.013	0.980	0.434
Objs.*Sims.*TimeStep*Samples	1	0.007	0.007	0.500	0.480
Objs.*Sims.*TimeStep*F-kernels	1	0.000	0.000	0.010	0.931
Objs.*Sims.*Samples*F-kernels	1	0.000	0.000	0.010	0.922
Objs.*TimeStep*Samples*F-kernels	1	0.005	0.005	0.400	0.532
Sims.*TimeStep*Samples*F-kernels	1	0.053	0.053	4.010	0.050
5-Way Interaction	1	0.001	0.001	0.040	0.845
Objs.*Sims.*TimeStep*Samples*F-kernels	1	0.001	0.001	0.040	0.845
Error	62	0.823	0.013		
Total	95	21.766			

Table 10. ANOVA table for Experiment 1 for the improvements in MARE with respect to the default model. DF: degrees of freedom; Adj SS: adjusted sum of squares; Adj MS: adjusted mean sum of squares; Sims.: n; Samples: p_{samp}; F-kernels: $k_{F-class}$.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	33	5.556	0.168	9.240	0.000
Scenario	2	4.935	2.468	135.470	0.000
Linear	5	0.383	0.077	4.210	0.002
Objs.	1	0.009	0.009	0.500	0.484
Sims.	1	0.008	0.008	0.470	0.497
TimeStep	1	0.003	0.003	0.140	0.708
Samples	1	0.006	0.006	0.320	0.575
F-kernels	1	0.357	0.357	19.630	0.000
2-Way Interactions	10	0.136	0.014	0.740	0.680
Objs.*Sims.	1	0.000	0.000	0.010	0.922
Objs.*TimeStep	1	0.030	0.030	1.630	0.207
Objs.*Samples	1	0.000	0.000	0.020	0.902
Objs.*F-kernels	1	0.022	0.022	1.230	0.272
Sims.*TimeStep	1	0.007	0.007	0.410	0.527
Sims.*Samples	1	0.003	0.003	0.140	0.708
Sims.*F-kernels	1	0.009	0.009	0.480	0.489
TimeStep*Samples	1	0.010	0.010	0.540	0.464
TimeStep*F-kernels	1	0.039	0.039	2.150	0.147
Samples*F-kernels	1	0.015	0.015	0.840	0.363
3-Way Interactions	10	0.085	0.008	0.470	0.906
Objs.*Sims.*TimeStep	1	0.000	0.000	0.000	0.981
Objs.*Sims.*Samples	1	0.001	0.001	0.050	0.824
Objs.*Sims.*F-kernels	1	0.004	0.004	0.210	0.647
Objs.*TimeStep*Samples	1	0.004	0.004	0.240	0.627
Objs.*TimeStep*F-kernels	1	0.028	0.028	1.540	0.219
Objs.*Samples*F-kernels	1	0.001	0.001	0.070	0.785
Sims.*TimeStep*Samples	1	0.017	0.017	0.930	0.339
Sims.*TimeStep*F-kernels	1	0.004	0.004	0.250	0.622
Sims.*Samples*F-kernels	1	0.009	0.009	0.470	0.495
TimeStep*Samples*F-kernels	1	0.016	0.016	0.890	0.348
4-Way Interactions	5	0.015	0.003	0.160	0.975
Objs.*Sims.*TimeStep*Samples	1	0.000	0.000	0.020	0.892
Objs.*Sims.*TimeStep*F-kernels	1	0.000	0.000	0.020	0.886
Objs.*Sims.*Samples*F-kernels	1	0.001	0.001	0.040	0.839
Objs.*TimeStep*Samples*F-kernels	1	0.000	0.000	0.010	0.936
Sims.*TimeStep*Samples*F-kernels	1	0.013	0.013	0.730	0.397
5-Way Interaction	1	0.002	0.002	0.120	0.728
Objs.*Sims.*TimeStep*Samples*F-kernels	1	0.002	0.002	0.120	0.728
Error	62	1.129	0.018		
Total	95	6.685			

Table 11. ANOVA table for Experiment 2 for the improvements in NSE$_{\ell 2}$ with respect to the default model. DF: degrees of freedom; Adj SS: adjusted sum of squares; Adj MS: adjusted mean sum of squares; Candidates: n; Roots: w_{root}.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	32	65.769	2.055	3.450	0.000
Scenario	1	11.325	11.325	19.020	0.000
Linear	5	40.151	8.030	13.490	0.000
Time_step	1	25.442	25.442	42.730	0.000
Objectives	1	7.834	7.834	13.160	0.001
Candidates	1	0.866	0.866	1.450	0.237
Optimization	1	5.274	5.274	8.860	0.006
Roots	1	0.735	0.735	1.230	0.275
2-Way Interactions	10	9.786	0.979	1.640	0.140
Time_step*Objectives	1	1.096	1.096	1.840	0.185
Time_step*Candidates	1	0.697	0.697	1.170	0.288
Time_step*Optimization	1	3.586	3.586	6.020	0.020
Time_step*Roots	1	0.575	0.575	0.970	0.333
Objectives*Candidates	1	0.395	0.395	0.660	0.421
Objectives*Optimization	1	1.635	1.635	2.750	0.108
Objectives*Roots	1	0.801	0.801	1.350	0.255
Candidates*Optimization	1	0.522	0.522	0.880	0.356
Candidates*Roots	1	0.015	0.015	0.020	0.876
Optimization*Roots	1	0.464	0.464	0.780	0.384
3-Way Interactions	10	3.644	0.364	0.610	0.792
Time_step*Objectives*Candidates	1	0.475	0.475	0.800	0.379
Time_step*Objectives*Optimization	1	1.025	1.025	1.720	0.199
Time_step*Objectives*Roots	1	0.581	0.581	0.980	0.331
Time_step*Candidates*Optimization	1	0.372	0.372	0.620	0.435
Time_step*Candidates*Roots	1	0.000	0.000	0.000	0.980
Time_step*Optimization*Roots	1	0.664	0.664	1.120	0.299
Objectives*Candidates*Optimization	1	0.296	0.296	0.500	0.486
Objectives*Candidates*Roots	1	0.025	0.025	0.040	0.839
Objectives*Optimization*Roots	1	0.172	0.172	0.290	0.595
Candidates*Optimization*Roots	1	0.034	0.034	0.060	0.813
4-Way Interactions	5	0.861	0.172	0.290	0.915
Time_step*Objectives*Candidates*Optimization	1	0.437	0.437	0.730	0.398
Time_step*Objectives*Candidates*Roots	1	0.028	0.028	0.050	0.831
Time_step*Objectives*Optimization*Roots	1	0.267	0.267	0.450	0.508
Time_step*Candidates*Optimization*Roots	1	0.073	0.073	0.120	0.728
Objectives*Candidates*Optimization*Roots	1	0.056	0.056	0.090	0.761
5-Way Interaction	1	0.003	0.003	0.010	0.944
Time_step*Objectives*Candidates*Optimization*Roots	1	0.003	0.003	0.010	0.944
Error	31	18.457	0.595		
Total	63	84.226			

Table 12. ANOVA table for Experiment 2 for the improvements in NSE$_{\ell 1}$ with respect to the default model. DF: degrees of freedom; Adj SS: adjusted sum of squares; Adj MS: adjusted mean sum of squares; Candidates: n; Roots: w_{root}.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	32	39.898	1.247	13.330	0.000
Scenario	1	0.105	0.105	1.120	0.298
Linear	5	30.924	6.185	66.140	0.000
Time_step	1	23.271	23.271	248.860	0.000
Objectives	1	3.714	3.714	39.720	0.000
Candidates	1	0.208	0.208	2.220	0.146
Optimization	1	2.923	2.923	31.260	0.000
Roots	1	0.807	0.807	8.630	0.006
2-Way Interactions	10	6.188	0.619	6.620	0.000
Time_step*Objectives	1	0.836	0.836	8.930	0.005
Time_step*Candidates	1	0.128	0.128	1.360	0.252
Time_step*Optimization	1	2.229	2.229	23.840	0.000
Time_step*Roots	1	0.665	0.665	7.110	0.012
Objectives*Candidates	1	0.022	0.022	0.230	0.635
Objectives*Optimization	1	1.077	1.077	11.520	0.002
Objectives*Roots	1	0.651	0.651	6.960	0.013
Candidates*Optimization	1	0.079	0.079	0.850	0.364
Candidates*Roots	1	0.002	0.002	0.020	0.900
Optimization*Roots	1	0.500	0.500	5.350	0.028
3-Way Interactions	10	2.244	0.224	2.400	0.030
Time_step*Objectives*Candidates	1	0.050	0.050	0.530	0.470
Time_step*Objectives*Optimization	1	0.825	0.825	8.820	0.006
Time_step*Objectives*Roots	1	0.530	0.530	5.660	0.024
Time_step*Candidates*Optimization	1	0.046	0.046	0.500	0.487
Time_step*Candidates*Roots	1	0.001	0.001	0.010	0.942
Time_step*Optimization*Roots	1	0.566	0.566	6.050	0.020
Objectives*Candidates*Optimization	1	0.014	0.014	0.150	0.703
Objectives*Candidates*Roots	1	0.000	0.000	0.000	0.963
Objectives*Optimization*Roots	1	0.211	0.211	2.250	0.143
Candidates*Optimization*Roots	1	0.003	0.003	0.030	0.867
4-Way Interactions	5	0.412	0.082	0.880	0.506
Time_step*Objectives*Candidates*Optimization	1	0.030	0.030	0.320	0.576
Time_step*Objectives*Candidates*Roots	1	0.000	0.000	0.000	0.992
Time_step*Objectives*Optimization*Roots	1	0.290	0.290	3.100	0.088
Time_step*Candidates*Optimization*Roots	1	0.016	0.016	0.170	0.685
Objectives*Candidates*Optimization*Roots	1	0.076	0.076	0.810	0.374
5-Way Interaction	1	0.026	0.026	0.280	0.603
Time_step*Objectives*Candidates*Optimization*Roots	1	0.026	0.026	0.280	0.603
Error	31	2.899	0.094		
Total	63	42.797			

Table 13. ANOVA table for Experiment 2 for the improvements in MARE with respect to the default model. DF: degrees of freedom; Adj SS: adjusted sum of squares; Adj MS: adjusted mean sum of squares; Candidates: n; Roots: w_{root}.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	32	27.446	0.858	16.570	0.000
Scenario	1	2.191	2.191	42.320	0.000
Linear	5	18.024	3.605	69.630	0.000
Time_step	1	13.747	13.747	265.540	0.000
Objectives	1	1.943	1.943	37.530	0.000
Candidates	1	0.254	0.254	4.910	0.034
Optimization	1	1.567	1.567	30.270	0.000
Roots	1	0.513	0.513	9.910	0.004
2-Way Interactions	10	5.046	0.505	9.750	0.000
Time_step*Objectives	1	1.148	1.148	22.180	0.000
Time_step*Candidates	1	0.186	0.186	3.600	0.067
Time_step*Optimization	1	1.448	1.448	27.970	0.000
Time_step*Roots	1	0.502	0.502	9.700	0.004
Objectives*Candidates	1	0.108	0.108	2.090	0.159
Objectives*Optimization	1	1.001	1.001	19.320	0.000
Objectives*Roots	1	0.223	0.223	4.300	0.047
Candidates*Optimization	1	0.118	0.118	2.270	0.142
Candidates*Roots	1	0.005	0.005	0.100	0.756
Optimization*Roots	1	0.308	0.308	5.950	0.021
3-Way Interactions	10	1.880	0.188	3.630	0.003
Time_step*Objectives*Candidates	1	0.145	0.145	2.800	0.105
Time_step*Objectives*Optimization	1	0.910	0.910	17.580	0.000
Time_step*Objectives*Roots	1	0.201	0.201	3.880	0.058
Time_step*Candidates*Optimization	1	0.117	0.117	2.260	0.143
Time_step*Candidates*Roots	1	0.008	0.008	0.150	0.703
Time_step*Optimization*Roots	1	0.294	0.294	5.690	0.023
Objectives*Candidates*Optimization	1	0.112	0.112	2.170	0.151
Objectives*Candidates*Roots	1	0.016	0.016	0.310	0.580
Objectives*Optimization*Roots	1	0.064	0.064	1.240	0.274
Candidates*Optimization*Roots	1	0.012	0.012	0.230	0.638
4-Way Interactions	5	0.253	0.051	0.980	0.448
Time_step*Objectives*Candidates*Optimization	1	0.100	0.100	1.930	0.175
Time_step*Objectives*Candidates*Roots	1	0.009	0.009	0.170	0.686
Time_step*Objectives*Optimization*Roots	1	0.063	0.063	1.220	0.278
Time_step*Candidates*Optimization*Roots	1	0.016	0.016	0.300	0.586
Objectives*Candidates*Optimization*Roots	1	0.066	0.066	1.270	0.269
5-Way Interaction	1	0.053	0.053	1.020	0.321
Time_step*Objectives*Candidates*Optimization*Roots	1	0.053	0.053	1.020	0.321
Error	31	1.605	0.052		
Total	63	29.051			

Table 14. ANOVA table for Experiment 3 for the improvements in NSE$_{\ell 2}$ with respect to the default model. DF: degrees of freedom; Adj SS: adjusted sum of squares; Adj MS: adjusted mean sum of squares; Samples: p_{samp}.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	24	7.368	0.307	1.710	0.102
Scenario	1	3.551	3.551	19.770	0.000
Linear	7	2.313	0.330	1.840	0.128
Time_step	5	2.310	0.462	2.570	0.055
Samples	1	0.002	0.002	0.010	0.915
Greed	1	0.000	0.000	0.000	0.973
2-Way Interactions	11	1.483	0.135	0.750	0.682
Time_step*Samples	5	0.144	0.029	0.160	0.975
Time_step*Greed	5	1.321	0.264	1.470	0.238
Samples*Greed	1	0.019	0.019	0.110	0.748
3-Way Interactions	5	0.021	0.004	0.020	1.000
Time_step*Samples*Greed	5	0.021	0.004	0.020	1.000
Error	23	4.132	0.180		
Total	47	11.500			

5   **Table 15. ANOVA table for Experiment 3 for the improvements in NSE$_{\ell 1}$ with respect to the default model. DF: degrees of freedom; Adj SS: adjusted sum of squares; Adj MS: adjusted mean sum of squares; Samples: p_{samp}.**

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	24	2.158	0.090	2.290	0.026
Scenario	1	0.000	0.000	0.000	0.971
Linear	7	1.644	0.235	5.980	0.000
Time_step	5	1.625	0.325	8.270	0.000
Samples	1	0.017	0.017	0.440	0.514
Greed	1	0.002	0.002	0.040	0.843
2-Way Interactions	11	0.501	0.046	1.160	0.366
Time_step*Samples	5	0.156	0.031	0.790	0.564
Time_step*Greed	5	0.344	0.069	1.750	0.163
Samples*Greed	1	0.000	0.000	0.000	0.999
3-Way Interactions	5	0.013	0.003	0.070	0.997
Time_step*Samples*Greed	5	0.013	0.003	0.070	0.997
Error	23	0.904	0.039		
Total	47	3.062			

Table 16. ANOVA table for Experiment 3 for the improvements in MARE with respect to the default model. DF: degrees of freedom; Adj SS: adjusted sum of squares; Adj MS: adjusted mean sum of squares; Samples: p_{samp}.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	24	1.438	0.060	6.780	0.000
Scenario	1	0.643	0.643	72.820	0.000
Linear	7	0.630	0.090	10.190	0.000
Time_step	5	0.605	0.121	13.700	0.000
Samples	1	0.021	0.021	2.320	0.141
Greed	1	0.005	0.005	0.530	0.473
2-Way Interactions	11	0.152	0.014	1.570	0.175
Time_step*Samples	5	0.115	0.023	2.610	0.052
Time_step*Greed	5	0.033	0.007	0.760	0.590
Samples*Greed	1	0.004	0.004	0.430	0.519
3-Way Interactions	5	0.012	0.002	0.270	0.925
Time_step*Samples*Greed	5	0.012	0.002	0.270	0.925
Error	23	0.203	0.009		
Total	47	1.642			