



# EDGE DETECTION IMAGE OPERATORS IN HEXAGONAL PIXEL FRAMEWORK

**Prathibha Varghese**

Research Scholar, Noorul Islam Centre for Higher Education, Kumaracoil, Tamil Nadu, India

**Dr. G. Arockia Selva Saroja**

Associate Professor, Department Electronics and Communication Engineering, Noorul Islam Centre for Higher Education, Kumaracoil, Tamil Nadu, India

## ABSTRACT

*With the advancement in computer vision and image processing field and due to the high computing power of graphic devices, it is high time to think of an alternate sampling grid called hexagonal grid which exactly resembles our rods and cones in the photoreceptors of human retina(eyes). Hexagonal grid structure framework provides higher angular resolution, equidistant property and consistent connectivity. As the angular resolution is more in hexagonal grid, curved structures can be well represented in hexagonal grid. As there is no hexagonal capturing and display device, software simulation methods are used for hexagonal image processing before proceeding the edge detection in hexagonal framework. Edge detection algorithm is the cornerstone for many pre-processing task in image processing field This paper covers different edge detection algorithms like robert, sobel, prewitt, log and canny in hexagonal and rectangular grid and the performance is assessed based on the design metrics like PSNR (peak signal to noise ratio) and MSE (mean square error). For all edge detection algorithms, results shows that hexagonal structural framework outperforms rectangular grid framework.*

**Key words:** Hexagonal image processing, Edge detection algorithms, Resampling, Spiral addressing architecture.

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## 1. INTRODUCTION

Edge detection and image enhancement are the two areas to which computer vision concepts expands and will be in need for increasing demand to know the structure of the irregular curves contained in that image. So it's of great importance to come out with different methodologies to identify the structure of an image.so the first step in the process of computer vision is the

edge detection. Most of the algorithm's efficiency is limited to the curved structural features. The promising solution to overcome this difficulty is to represent the images using an alternate sampling grid called hexagonal grid framework. Hexagonal sampling scheme provides good angular resolution, consistent connectivity, equidistant property, fixed neighbourhood connectivity and with reduced aliasing effect etc [1]. Hexagonal pixels are more suitable for human vision as they replicate the structure of rods and cones in our human eyes [2]. This structure provides less sensitivity in the diagonal direction than compared to the horizontal direction. Hence, in these papers we present different frame work for processing hexagonal sampled images and shows different key aspects: conversion from square to hexagonal sampled images, detecting edges using different edge detection algorithms in both grids. More importantly, we would like to prove that hexagonal sampled images is a better sampling grid in the edge detection image processing field. In order to avoid repetition and convenience we use the terms hexagonal sampled image and square sampled images as hexagonal image and square image.

In this paper we begin with a research objective followed by a review of some hexagonal image processing algorithms in section 3. Section 4 deals with the features of different edge detection operators. Section 5 is the detailed review of the methodology implementation used in our work. Section 6 deals with performance evaluation and section 5 is the conclusion.

## 2. RESEARCH OBJECTIVE

Numerous different edge detection and hexagonal image processing algorithms have been studied and investigated in the past years by researchers in this area. But combination of these methods have not discussed in the forefronts because of the unavailability of hardware to capture and display the hexagonal images. To alleviate this problem, images in rectangular grid is converted into hexagonal grid using software coding. Briefly, the objective is as follows:

- To create an hexagonally sampled image processing system, that involves the acquisition, resampling and processing of rectangular to hexagonally sampled images on the hexagonal grid of images.
- Hexagonal processed images are used in various edge detection algorithms, such as Robert, sobel, log, prewitt and canny.
- Performance comparison of these hexagonal and rectangular grid images on edge detection algorithms

## 3. HEXAGONAL IMAGE REPRESENTATION RESAMPLING TECHNIQUES

Hexagonal image processing has predominantly many advantages but it is not used in practical world owing to the lack of imaging sensors (CCD) and display systems. But fortunately there are different ways to simulate hexagonal images by resampling from the original square grid. The most commonly used 3 resampling techniques are

- Mimic Hexagonal pixels using square pixels
- Psuedo Hexagonal Pixel
- Mimic Hexagonal structure.

### 3.1 Hexagonal image processing (HIP) framework and Algorithms

Let  $f(x_1, x_2)$  represents an image,  $f_s(x_1, x_2)$  represents the sampled image and  $h(x)$  be the sampling kernel. The image sampling can be represented by

$$f_s(x_1, x_2) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} f(k_1, k_2)h(x_1 - k_1, x_2 - k_2)$$

Generally available sampled image in square lattice is  $f_{ss}(x)$ . Here, we are generating  $f_{sh}(x)$  with a hexagonal sampling lattice from  $f_{ss}(x)$  which is called the image resampling [3] as shown in Fig 1.

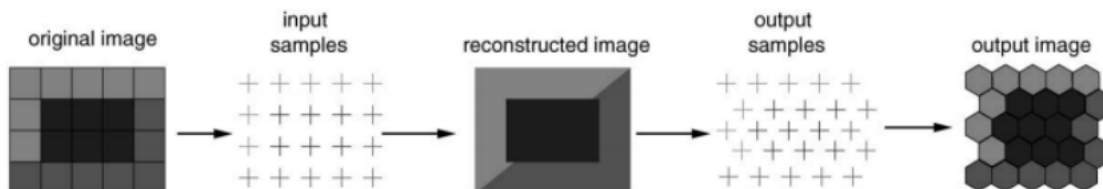


Figure 1 Hexagonal grid resampling

### 3.2 Mimic hexagonal pixels from square pixels

Horn [4] has defined how realistic hexagonal data can be collected on alternate sampling lines in horizontal direction by delaying the sampling by half a pixel width (see Fig. 2). The pixel type is square in his scheme. In other words, the sampling intervals are identical horizontally and vertically. This approach simplifies the hardware design by setting equal sample intervals horizontally and vertically. However the property of the hexagonal pixel equidistance is not preserved. If we denote a horizontal and vertical distance of 1 unit between the two neighbours, the distance of each neighbour in a diagonal direction will be shown  $\frac{\sqrt{3}}{2}$  as in Fig.2.

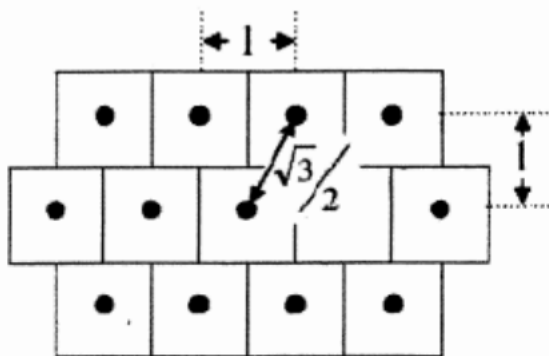


Figure 2 Half-pixel shift method

### 3.3 Psuedo hexagonal pixel structure

Lattice or grid represents different tiling or tessellation schemes. Let  $B= \{b_1, b_2\}$  be the basis vectors of the plane. The set  $B$  define the lattice as shown in equation 1.

$$L_B = \{n_1b_1 + n_2b_2 : n_i \in Z, i = 1, 2\}$$

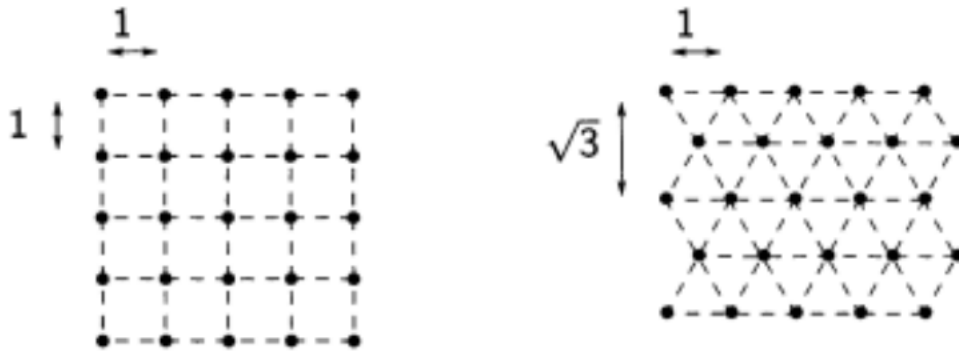
Different basis vectors are there for the generation different lattice. For example:

$B_s = \{(1,0), (0,1)\}$  will give square lattice. The predominant grids for generating hexagonal ones are

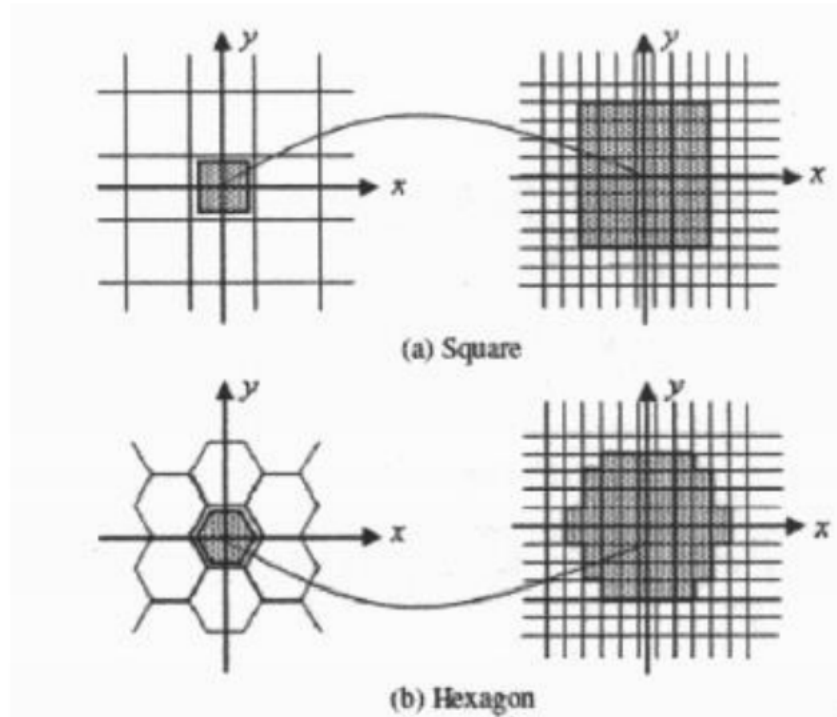
$$\mathbf{B}_{H1} = \left\{ (1, 0), \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right) \right\}$$

$$\mathbf{B}_{H2} = \left\{ \left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right), (0, 1) \right\}$$

By examining the relationship between  $\mathbf{B}_{H1}$  and  $\mathbf{B}_{H2}$ , only  $b_2$  is different, that means horizontal spacing is same as shown in Fig 3.



**Figure 3** Representation of relative spacing between square and hexagonal lattice



**Figure 4** Hyper pixel simulated

Using the best grid vectors discussed above, Middleton and Sivaswamy [5, 6] proposed a framework for simulated hexagonal pixel for practical hexagonal-image processing, where a process known as image re-sampling is employed to generate a hexagonally sampled image as shown in Fig 4., Hexagonal sampled images generated will have a resolution of 60 u 60 pels. Yabushita in [7] also proposed a similar pseudo hexagonal structure (hexelement), which is also composed of small square pixels with a aspect ratio is 12:14.

### 3.3 Mimic hexagonal structure

He [8] introduced mimic hexagonal structure consisting of four square pixels and its grey value is calculated using the average of the four square pixels as shown in Fig.5.

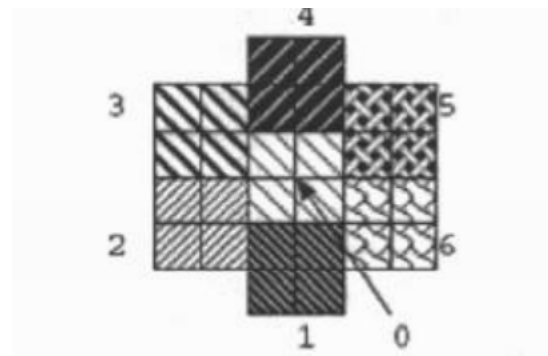


Figure 5 A cluster of 7 mic hexagons

This scheme preserves the six neighbour-hood property but it does not follow the equidistant property. This method provides less resolution as the average of four pixels is taken as the grey value.

### 4. Edge detection operators

Edge detection is an effective method for the improvement and detection, feature elimination, and detection of images to identify sudden changes in the light of sharp changes of image luminosity and discontinuities.[9] As the identification of the edges is an active field of study, detailed image analysis is possible. In the grey level image, for example, point, line, and edges, there are three types of discontinuities. All three types of image discontinuities can be defined through spatial masks [10]. Edge detection segment addresses the most frequently used discontinuities in image processing. Roberts, sobel, pre-witt, log and canny edge detection are the most popular edge detection operators in these category as shown in Fig.6. The edge detection detects variations with differential operators in gradients of grey levels. It is broken down into two major categories [11,12,13,14,15,16]:

- **Gradient** – Sobel operator, Prewitt operator, Robert operator-computes first order derivative images on a digital image
- **Gaussian** – Canny Edge Detector, Laplacian of Gaussian dependent operator-calculates second-order derivatives on a digital image

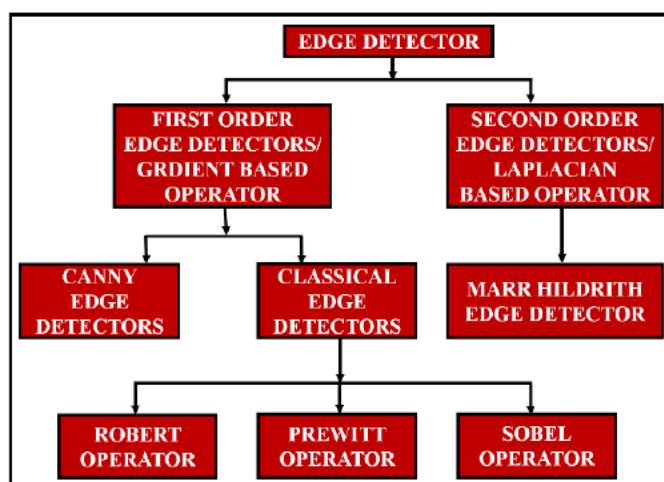


Figure 6 Types of edge detector

### 4.1 Sobel edge operator

It's a discerning distinguishing operator. For the detection of the edge of the image, the gradient approximation is calculated. The Sobel operator normally or the appropriate gradient vector is generated in the pixels of the image. In calculating derivative approximations of vertical or horizontal derivatives, it uses two kernels or masks 3 x 3 combined with the image data.

$$M_x = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix} M_y = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

### 4.2 Prewitt operator

This operator is almost the same as the operator of Sobel. There are also vertical and horizontal corners of the image. It is one of the best ways to detect the focus and magnitude of an image. Using kernels or masks.

$$M_x = \begin{pmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix} M_y = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

### 4.3 Robert operator

In this gradient-based operator, the number of squares is determined with discrete distinguishing between pixels in an image diagonally adjacent. The approximation of the gradient is then performed. You can use the following 2 x 2 kernels or masks.

$$M_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} M_y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

### 4.4 Laplacian of Gaussian (LoG) or Marr Hildrith operator

The Laplacian operator uses the second image derivative. It is a Gaussian operator. This works fine when the transition to the grey stage seems to be abrupt. It works on the zero-crossing rule, i.e. if the derivative of the second order crosses zero, the max. level corresponds to that particular location. The location is named on the rim. In this case the Gaussian operator eliminates the noise, and the Laplacian operator defines the sharp edges.

In the following formula the Gaussian function is defined:

$$G(x, y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{X^2 + Y^2}{2\sigma^2}\right)$$

Where  $\sigma$

It's the normal deviation.

And the operator from LoG is determined

$$LoG = \frac{\partial^2}{\partial x^2} G(x, y) + \frac{\partial^2}{\partial y^2} G(x, y) = \frac{x^2 + y^2 + 2\sigma^2}{\sigma^4} \exp\left(-\frac{X^2 + Y^2}{2\sigma^2}\right)$$

### 4.5 Canny operator

It is an edge detection operator based in Gaussian. This operator has no sensitivity to noise. It extracts picture features without altering the purpose or affecting them. The Canny edge detector offers an advanced algorithm derived from the Gaussian operator's previous work,

Laplacian. The most famous method is used for optimal border detection. It senses the edges in three ways:

- Error rate reduction.
- Edge points shall be precisely located.
- Only one edge detection can occur.

### 5. METHODOLOGY IMPLEMENTATION

Edge detection is a predominant pre-processing technique used in the image processing tasks. To study the edge detection algorithm in hexagonal grid hexagonal image processing (HIP) framework is utilised as shown in block diagram Fig 7. Following steps are used in the edge detection based on hexagonal grid:

- Step 1: Image is re-sampled to hexagonal domain using different software methods.
- Step 2: Hexagonal image enhancement using Gabor filter.
- Step3: Passing the images through various edge detection operators
- Step 4: Performance comparison of images in both hexagonal and rectangular grid.
- Step 5: Calculation of PSNR (Peak-signal to Noise ratio) and MSE (Mean square error).

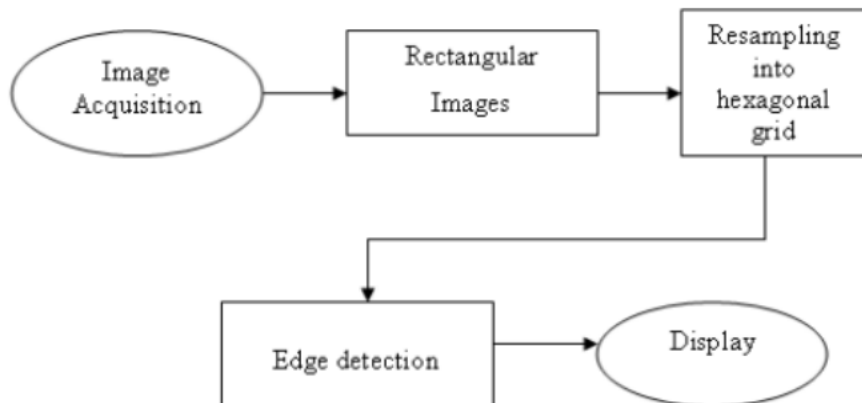


Figure 7 Block diagram of hexagonal based edge detection

#### 5.1 JK Algorithm

Using JK algorithm [17], each hexagonal pixel is made from 56 square pixel as shown in Fig 8.

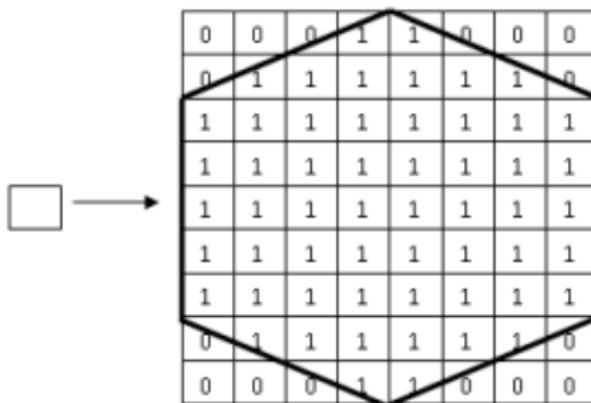


Figure 8 Psuedo hexagonal pixel from square pixel.

For developing a hexagonal pixel from square pixel, 9X8 matrix is considered, ie, total of 72 pixels. Out of these 72 pixels, pixels marked '0' are discarded. Only pixels marked '1' are used in the hexagonal pixel generation. This algorithm can be extended for colour images also by superimposing three channels.

### 5.2 Alternate Pixel Suppression Method (Resampling Method)

Pramod Sankar et.al [18], proposed alternate pixel suppression method in which hexagonal grid can be simulated from rectangular grid. The sub sampling equation is given as :

$$\begin{aligned}
 & pixel\_val_{hex(i,j)} \\
 &= \begin{cases} pixel\_val(2 * i, 2 * j), & \text{if } i \text{ is even} \\ pixel\_val(2 * i, 2 * j + 1), & \text{if } i \text{ is odd} \end{cases}
 \end{aligned}$$

In these method, alternate pixels in the rows and columns are suppressed in the rectangular grid. During processing stage, sub sampled image suppressed image cannot be considered for computation. The hexagonal pixel grid is as shown in Fig 9.



Figure 9 a) Rectangular grid b) Hexagonal grid

### 5.3 Half Pixel Shift Method (Resampling Method)

In order to obtain hexagonal grids from the standard rectangular grid, Senthil Periasamy proposed a method of a half-pixel shift described in [19]. In a half-pixel shift method, the midpoint between two adjacent pixels is to be identified for any odd line by means of a single linear technique.

Mid-point value = (left\_value + right\_value) / 2.

$$P^{new}(x, 2y) = P^{old}(x, 2y)$$

$$P^{new}(x, 2y + 1) = (P^{old}(x, 2y + 1) + P^{old}(x + 1, 2y + 1)) / 2$$

Fig.10 gives us a hexagonal map of a square or rectangular grid using half axis symmetry. In this method, mid values are kept, discarding left and right values.

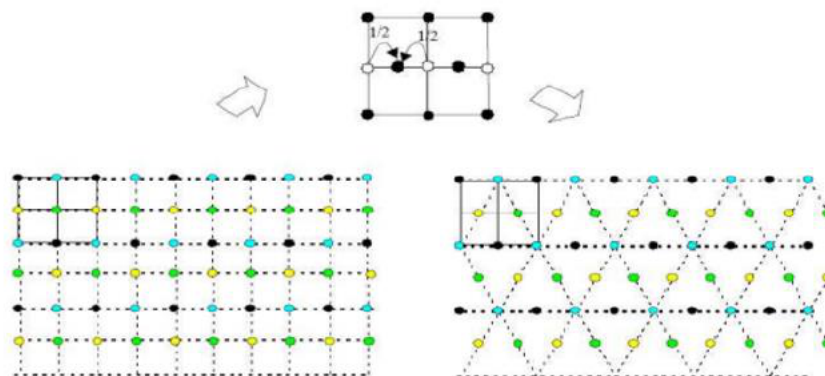


Figure 10 Half pixel hexagonal grid



## 6. PERFORMANCE EVALUATION

In this research work, various edge detection operators like prewit, sobel, log, Robert and canny are compared on the real rectangular grid structure with the hexagonal grid structure; 8 bit grey-level image of cameraman with size 256X256 is chosen as the sample image for the edge detection algorithms.

In order to compare the computational efficiency, a comparison is made on the basis of MSE (Mean square error) and PSNR (Peak signal to noise ratio)

### 6.1 Mean Square Error (MSE)

The average difference in pixel in the picture is indicated by MSE. An elevated MSE indicates a greater contrast.

$$MSE = \frac{1}{N} \sum_i \sum_j (X_{ij} - V_{ij})^2$$

Where N is the image size, the image is X and the initial image is V.

### 6.2 Peak Signal-to-Noise Ratio (PSNR)

$$PSNR = 10 \cdot \text{LOG}_{10} \left( \frac{n * 255^2}{\sum_i \sum_j (X_{ij} - v_{ij})^2} \right)$$

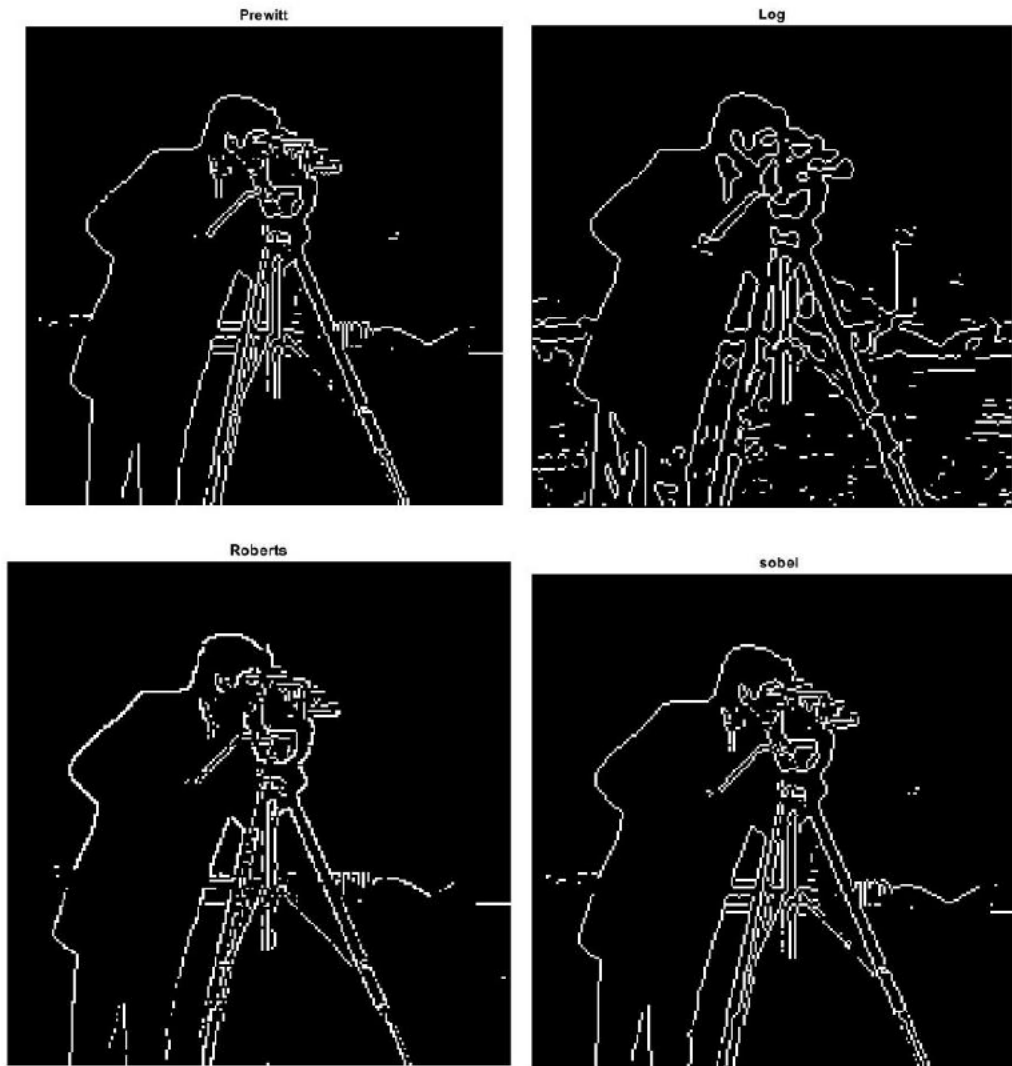
The greater the PSNR, the better the quality of the compressed or restored image. PSNR is used for the quantitative comparison [20]. MSE is inversely proportional to PSNR..

## 7. RESULTS AND DISCUSSIONS

Classical standard edge detection operators such as Robert, sobel, prewit, log and canny are highly sensitive to noise and very simple to apply.

In the rectangular domain, grid structure edge detection operators shows discontinuous edges. In the hexagonal domain, Edge detection operators on hexagonal domain shows good visual clarity and less MSE and high PSNR. It is due to the fact the hexagon geometry possesses three axis geometry at  $0^\circ, 60^\circ, 120^\circ$ . This geometry also possesses good angular resolution and as all pixels are equidistant. The results of various edge detection operators on rectangular domain and hexagonal domain are shown in Fig.11,12 and 13. performance comparison based on mse and psnr are also shown in Table 1.





**Figure 11** Rectangular grid -edge detection operators a)original cameraman image b)canny c)prewit d)log e) Robert f)sobel



Edge Detection Image Operators in Hexagonal Pixel Framework

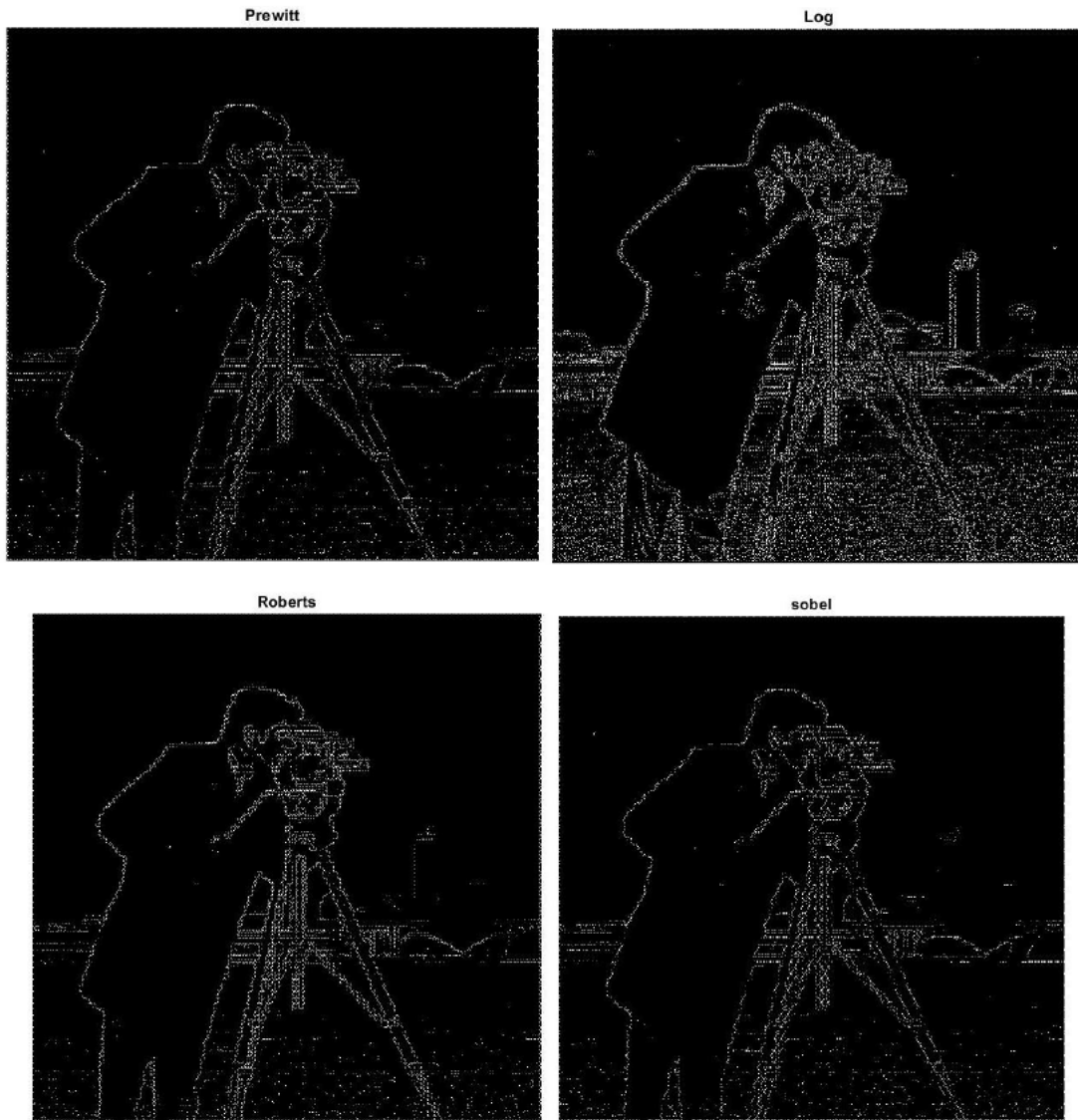
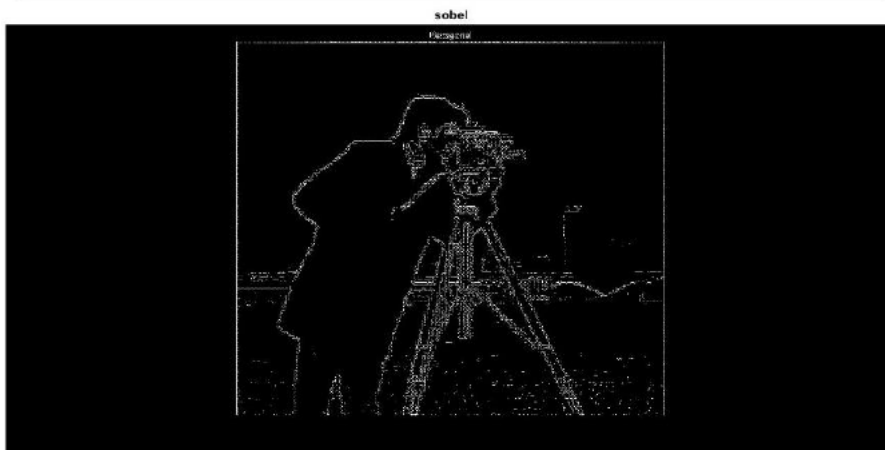
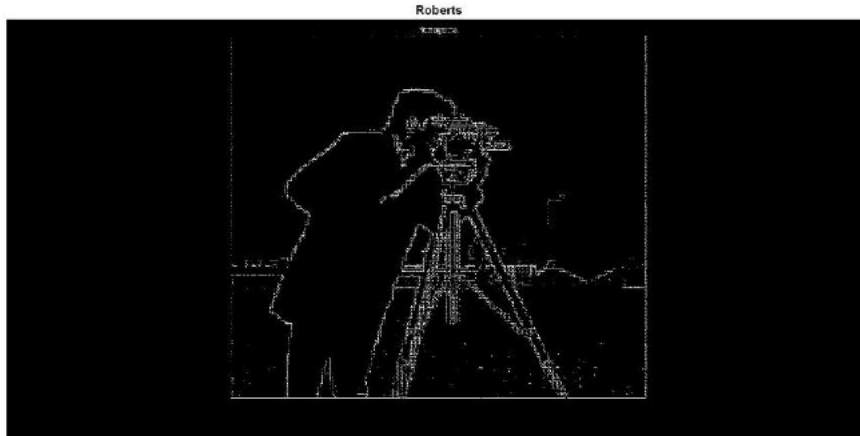
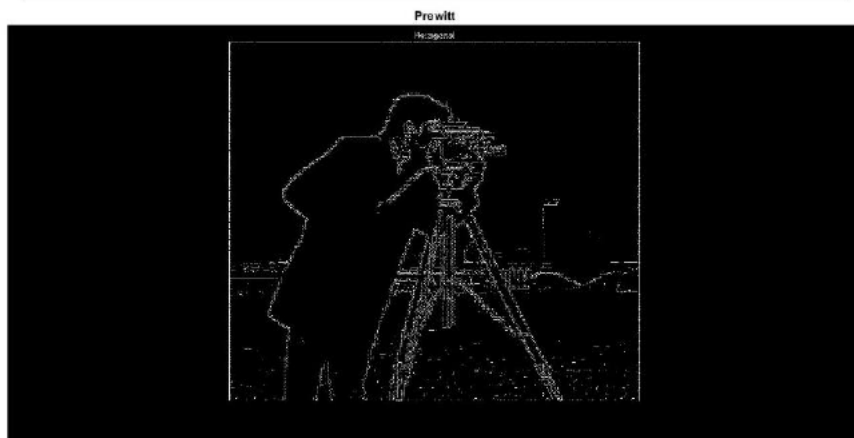
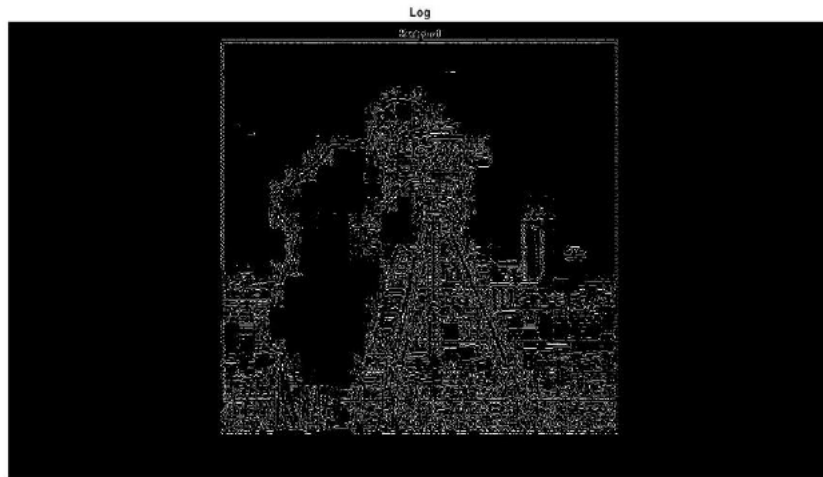
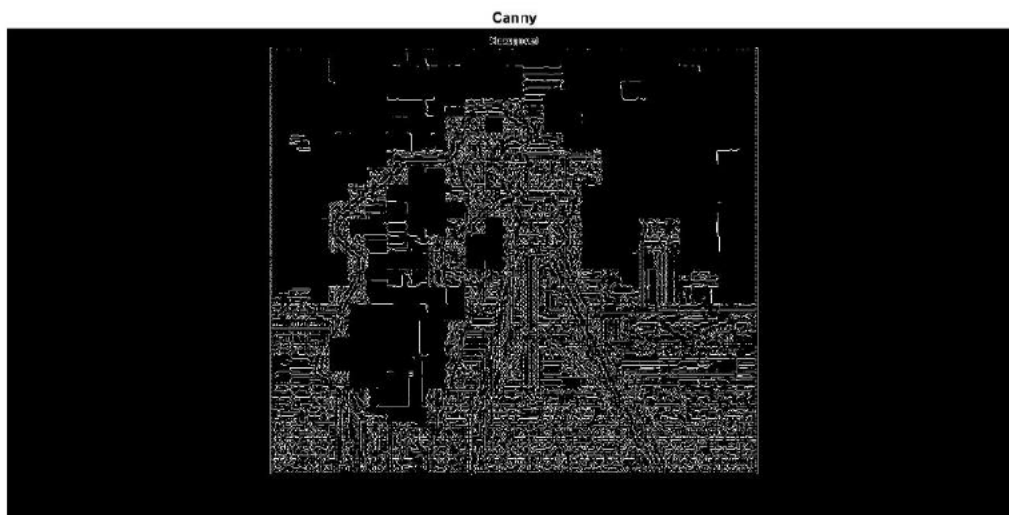


Figure 12 RESAMPLING method a) Hex cameraman image b)canny c)prewit d)log e)Robert f)sobel







**Figure 13** JK ALGORITHMN method a) Hex cameraman Image b) log c) prewit d) robert e) sobel f) canny

**Table 1** performance comparison of various edge detection operators in rectangular lattice and hexagonal lattice.

Methodology	Input image	Edge detection operator	Mse	Psnr
Jk algorithmn	Cameraman.jpg	Robert	32.71	33
		Sobel	32.71	32.99
		Prewit	32.88	33.03
		Log	32.76	32.95
		Canny	32.68	33.02
Resampling method	Cameraman.jpg	Robert	32.87	33.03
		Sobel	32.87	33.03
		Prewit	32.71	33.02
		Log	32.69	32.89
		Canny	32.66	33.03
Rectangular grid	Cameraman.jpg	Robert	33.27	33
		Sobel	32.93	32.99
		Prewit	32.88	32.98
		Log	33.24	32.95
		Canny	32.79	32.94

### 8.CONCLUSION

In this paper, various edge detection operators design procedure in hexagonal and rectangular grid structure is compared. As there is no hardware to capture hexagonal images, various software approach is used for generating pseudo hexagonal images via, JK algorithm and resampling and resizing of rectangular images. MSE and PSNR values which are used for comparison show that edge detection operator on hexagonal domain are more superior than rectangular domain based on both the objective and subjective quality. The only limitation of our work is lack of hexagonal lattice display and capture device hardware. A comparison of

both grids revealed that the proposed hexagonal method has been made more reliable, more definite and simpler overall.

## REFERENCES

- [1] Middleton, L., & Sivaswamy, J. (2001). Edge detection in a hexagonal-image processing framework. *Image and Vision Computing*, 19(14), 1071–1081. [https://doi.org/10.1016/S0262-8856\(01\)00067-1](https://doi.org/10.1016/S0262-8856(01)00067-1).
- [2] R.M.Gray,P.C.Cosman,Digital images and human vision,MITPress,Cambridge,MA,1993,chapter4,pg-35-52.
- [3] G.Wolberg,Digital Image Wrapping,IEEE Computer Society Press, Silver Spring,MD,1990.
- [4] Horn, B.K.P., Robot Vision. 1986: MIT Press, Cambridge, MA & McGraw-Hill, New York, NY.
- [5] Middleton, L. and J. Sivaswamy, Edge detection in a hexagonal-image processing framework. *Image and Vision Computing*, 2001. 19(14): p. 1071-108 1.
- [6] Middleton, L. and J. Sivaswamy, Framework for practical hexagonal-image processing. *Journal of Electronic Imaging*, 2002. 11(1): p. 104-114.
- [7] Yabushita, A. and K. Ogawa, Image reconstruction with a hexagonal grid. *Nuclear Science Symposium Conference Record*, 2002 IEEE, 2002. 3: p. 1500-1503 vol.3.
- [8] He, X., 2-D Object Recognition with Spiral Architecture. 1999, University of Technology, Sydney.
- [9] Mostafa, K., & Her, I, “An edge detection method for hexagonal images”, *International Journal of Image Processing*, Vol. 10, No. 4, (2016), 161-173
- [10] He, X., Wei, D., Lam, K. M., Li, J., Wang, L., Jia, W., & Wu, Q, “Canny edge detection using bilateral filter on real hexagonal structure”, In *International Conference on Advanced Concepts for Intelligent Vision Systems*, (2010) pp. 233-244
- [11] Rashmi, Mukesh Kumar, and Rohini Saxena, ALGORITHM AND TECHNIQUE ON VARIOUS EDGE DETECTION: A SURVEY, *Signal & Image Processing: An International Journal (SIPIJ)* Vol.4, No.3, June 2013.
- [12] R .Gonzalez and R. Woods, *Digital Image Processing*, Addison Wesley, 1992, pp 414 - 428.
- [13] S. Sridhar, Oxford university publication. , *Digital Image Processing*.
- [14] Shamik Tiwari, Danpat Rai & co.(P) LTD. “Digital Image processing”
- [15] J. F. Canny. “A computational approach to edge detection”. *IEEE Trans. Pattern Anal. Machine Intell.*, vol.PAMI-8, no. 6, pp. 679-697, 1986 *Journal of Image Processing (IJIP)*, Volume (3) : Issue (1)
- [16] James Clerk Maxwell,1868 DIGITAL IMAGE PROCESSING Mathematical and Computational Methods
- [17] Jeevan K. M. and S. Krishnakumar, An Algorithm for the Simulation of Pseudo Hexagonal Image Structure Using MATLAB, *I.J. Image, Graphics and Signal Processing*, 2016, 6, 57-63 Published Online June 2016 in MECS (<http://www.mecs-press.org/>) DOI: 10.5815/ijigsp.2016.06.073.
- [18] K. PramodSankar, T. Sanjay and E. G. Rajan, “Hexagonal Pixel Grid Modeling and Processing of Digital Images Using CLAP Algorithms”, *International Conference on Systemics, Cybernetics and Informatics*, pp.327-336, 2004
- [19] Periaswamy, Senthil. "Detection of microcalcifications in mamograms using hexagonal wavelets." Master's thesis, University of South Carolina, 1996
- [20] Kumar, M., & Saxena, R, “Algorithm and technique on various edge detection: A survey”, *Signal & Image Processing*, Vol. 4, No. 3, (2013), 65.