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An Application of LADM-Padé Approximation for the Analytical Solution of the SIR Infectious Disease Model

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ABSTRACT

This article is aimed at finding analytical solution to the dynamics of the epidemic model. The autonomous nonlinear differential equation was reduced to algebraic equation via Laplace transform method subject to the initial condition. The LADM was then employed to obtain the approximate analytical solution for the pertinent parameters of interest. In view to match the obtained solution as far as possible, the Padé approximant is applied to the partial sum of the obtained analytical solution to improve its convergence. Maple is used in the computation and the results are presented graphically and in tables. The result obtained agrees with literature and shows the adopted method is accurate, flexible, and reliable.

Keywords: *Epidemic model, Adomian decomposition, LaplaceAdomian decomposition, Padé Approximation, Runge-Kutta, Decomposition series, Laplace Transformation*

I. INTRODUCTION

From prehistoric times until now, the world has been ravaged by diverse kinds of epidemics ranging from malaria, Ebola, Dengue, HIV-AIDS, COVID-19, and others. These diseases have caused mankind problems in their wake with severe fatalities especially in the third world countries who neither have the wherewithal nor the technology to nip them in the bud. For example, the world health organization in their latest bulletin stressed if the scourging effects of malaria is not mitigated by increased vaccination and qualified personnel, an estimated half a billion infant mortality wouldn't be averted in Africa and North America. Equally, the report painted a gory picture of paucity of funds from third world countries and donor organization to tackle frontally polio myelitis which have been prevalent and been the major cause of child mortality in sub-Saharan Africa.

Infectious diseases have adverse and tremendous effect on human population. Millions of human beings either suffer or die from diverse infectious disease every year [1]. Among these diseases, the childhood diseases are the most common form of killer infectious diseases. They include measles, mumps, chicken pox, polio myelitis etc... to which every child under age five is born susceptible to and contracts [2]. These diseases spread faster among children because they are always in close contact either at school or during play. Therefore, containing these childhood diseases to protect children from contracting them by way of early vaccination been the viable and effective strategy became a front burner among health authorities especially in the third world countries which are most prone [3]. Equally, aside early vaccination against these diseases, mathematical models have also been used extensively to study the dynamics and spread of these epidemics and diseases. The models can quantitatively predict how the disease spread and the factors responsible for their progression. The result obtained from these models are useful in the implementation of strategy to curb the spread and development of these diseases.Kermack and McKendrick developed the classical SIR model for epidemic diseases in 1927. The framework for these model fits exactly how various diseases spread and affects humanity even after vaccination. In this model, the total population denoted N is subdivided into three class namely: Those susceptible to the disease denoted $N,$ the infected number of people, I and the removed number or people immune to the disease after treatment, R. This model has been successfully used to describe several epidemiological diseases. [4-10].

The Adomian decomposition method introduced by G. Adomian [11-13] has shown great potential in solving a wide variety of problems ranging from linear as well as nonlinear differential and partial differential equations. This method requires writing the unknown function as a decomposition series, the nonlinear terms as an Adomian polynomial and matching both sides to obtain a recursive algorithm where the rapidly convergent series solution is then obtained which is equivalent to the closed form or exact solution of the problem if it exists. It has been successfully applied to solve problems in many scientific fields such as plasma physics, fluid mechanics, solid state physics, chemical kinetics, population dynamics and engineering. Equally, most recently most authors adore its appeal and has applied it to the following areas: A Comparison between Adomian decomposition method and Taylor series method in the series solutions, Analytical solution of a time-fractional Navier-Stokes equation by Adomian decomposition, delay differential equations, systems of nonlinear equations, nonlinear integro-differential equations, nonlinear analytical techniques, convergence of nonlinear equations and Falkner-Skan equation for a Wedge. [14-21]. This method is advantageous over other methods in that it does not require linearization, perturbation, and discretization.

The hybrid Laplace transform, and the domain decomposition method introduced by Khuri [22-23] has equally be given considerable attention. This method is powerful and preferable to Adomian decomposition because it accelerate the rapid convergence of the series solution. [24] employed the Laplace Adomian decomposition method to solve linear and nonlinear systems of PDEs. Coupled systems of PDEs have been examined using LADM by [25], [26] explored the Duffing equation numerically using the combined Laplace Adomian decomposition method. [27] used LADM to examine the HIV model. The Newell-Whitehead-Segel have been investigated using LADM by [28]. Combined Laplace transform and Adomian decomposition method have been used to analyze systems of systems of ordinary differential equation [29]. [30-38] have also employed LADM to investigate the following problems viz: linear and nonlinear Volterra integral equations with weak kernel, nonlinear Volterraintegro-differential equations, nth order integro-differential equations, integro-differential equations, two-dimensional viscous fluid with shrinking sheet, numerical solution of logistics differential equations, convection diffusion-dissipation equation, nonlinear fractional differential equation and numerical solution of the crime deterrence model in society.
In this article, we use the Laplace Adomian decomposition method to seek analytical solution of the SIR epidemic model. The nonlinear differential equations are solved for the governing parameters for the problem. The article is composed as follows. The introduction and fundamentals of the Adomian decomposition methods are in sections 1& 2. The combination of Laplace and Adomian decomposition method is contained in section 3. Sections 4& 5 presents the Padé approximation and application of LADM to the model. Section 6 presents the graphical representation of the analytical solution and its comparison in tables. Finally, the conclusion in section 7.

II. BASICS OF THE ADOMIAN DECOMPOSITION METHOD

Consider a functional inhomogeneous differential equation of the form

$$Fy = g(x) \tag{1}$$

Where F is a nonlinear differential operator and y, g are the unknown function and the source term. Dividing the operator as the sum of $L + R + N$ as follows. Eq. (1) now become

$$Ly + Ry + Ny = g(x) \tag{2}$$

Where L is the highest order derivative that's invertible, R is a linear differential operator, N is a nonlinear term and g is the source term.
Rewriting Eq. (2) in the form

$Ly = g(x) - Ry - Ny$
(3)Applying L^{-1}on both sides of the equation **(3)** to obtain
$L^{-1}(Ly) = L^{-1}\big(g(x)\big) - L^{-1}(Ry) - L^{-1}(Ny)$ (4)
$y(x) = L^{-1}\big(g(x)\big) - L^{-1}(Ry) - L^{-1}(Ny)$
Eq. (4**)** can be written alternatively as
$y(x) = \phi - L^{-1}(Ry) - L^{-1}(Ny)$ (5)
Where ϕ represents the term arising from integrating the source term, g. That is, $[L^{-1}(g)]$ and from the given conditions.
Using the standard Adomian decomposition method the zeroth component is written as,
$y_0 = \phi$
And the recursive relation is given by
$y_{n+1} = -L^{-1}(Ry_n) - L^{-1}(Ny_n), \quad n \geq 0$ (6)
For $n = 0$
$y_1 = -L^{-1}(Ry_0) - L^{-1}(Ny_0)$
For $n = 1$
$y_2 = -L^{-1}(Ry_1) - L^{-1}(Ny_1)$
For $n = 2$
$y_3 = -L^{-1}(Ry_2) - L^{-1}(Ny_2)$
Now, writing the unknown the solution of the problem as a decomposition series of the form
$y(x) = \sum_{n=0}^{\infty} y_n(x)$ (7)
Here, the nonlinear term can be determined by an infinite series of Adomian polynomials
$Ny = \sum_{n=0}^{\infty} A_n$ (8)
where $A_n^1 s$ are obtained from the relation
$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N(\sum_{x=0}^{n} \lambda^i y_i)]_{t=0}, \qquad n = 0,1,2,3,\ldots$ (9)

III. FUNDAMENTALS OF THE LAPLACE ADOMIAN DECOMPOSITION METHOD

In this subsection, we discuss the use of the hybrid Laplace transformation and Adomian decomposition algorithm for the nonlinear autonomous first order differential equations governing the problem. For convenience, we consider a first order nonhomogeneous functional differential equation subject to initial condition of the form

$$L[u(x)] + R[u(x)] + N[u(x)] = g(x) \tag{10}$$

$$u(0) = f(x) \tag{11}$$

$$L[u(x)] = g(x) - R[u(x)] - N[u(x)] \tag{12}$$

Applying Laplace transform to both sides of Eq. (10), and using the differentiation property, we get

$$s\mathcal{L}\{u(x)\} - f(x) = \mathcal{L}\{g(x)\} - \mathcal{L}\{Ru(x)\} - \mathcal{L}\{Nu(x)\}$$

$$s\mathcal{L}\{u(x)\} = f(x) + \mathcal{L}\{g(x)\} - \mathcal{L}\{Ru(x)\} - \mathcal{L}\{Nu(x)\}$$

$$\mathcal{L}\{u(x)\} = \frac{f(x)}{s} + \frac{1}{s}\mathcal{L}\{g(x)\} - \frac{1}{s}\mathcal{L}\{Ru(x)\} - \frac{1}{s}\mathcal{L}\{Nu(x)\} \tag{13}$$

Applying the inverse Laplace transform to both sides of Eq. (13), we obtain

$$u(x) = \phi(x) - \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\{Ru(x)\} - \frac{1}{s}\mathcal{L}\{Nu(x)\}\right] \tag{14}$$

Where $\phi(x)$ is the term arising from the first three terms on the right-hand side of Eq. (14).

Next, we assume the solution of the problem as a decomposing series in the form

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \tag{15}$$

Similarly, the nonlinear terms are written in terms of the Adomian polynomials as

$$Nu(x) = \sum_{n=0}^{\infty} A_n \tag{16}$$

Where the $A_n^{'s}$ represents the Adomian polynomials defined in the form

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{\infty} \lambda^i y_i\right)\right]_{i=0}, n = 0,1,2,3 \tag{17}$$

Plugging Eqs. (15) and (16) into Eq. (17), we obtain

$$\sum_{n=0}^{\infty} u_n(x) = \phi(x) - \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\{R\sum_{n=0}^{\infty} u_n(x)\} - \frac{1}{s}\mathcal{L}\{N\sum_{n=0}^{\infty} A_n\}\right] \tag{18}$$

Matching both sides of Eq. (18), we obtain an iterative algorithm in the form

$$u_0(x) = \phi(x)$$
$$u_1(x) = -\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left\{R\sum_{n=0}^{\infty} u_0(x)\right\} - \frac{1}{s}\mathcal{L}\left\{N\sum_{n=0}^{\infty} A_0\right\}\right]$$
$$u_3(x) = -\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\{R\sum_{n=0}^{\infty} u_2(x)\} - \frac{1}{s}\mathcal{L}\{N\sum_{n=0}^{\infty} A_2\}\right] \tag{19}$$
$$\vdots$$
$$u_{n+1}(x) = -\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left\{R\sum_{n=0}^{\infty} u_n(x)\right\} - \frac{1}{s}\mathcal{L}\left\{N\sum_{n=0}^{\infty} A_n\right\}\right]$$

Then the solution of the differential equation is obtained as the sum of decomposed series in the form

$$u(x) \approx u_0(x) + u_1(x) + u_2(x) + \cdots \tag{20}$$

IV. PADÉ APPROXIMATION

In mathematics and other applied sciences, rational functions are functions which the degree of the denominator is either equal or greater than the degree of the numerator all expressed as polynomials. In seeking to write these functions in Taylor series form, difficulties often arise in the form of divergent, singularities and radius of convergence which blow up occur. To curtail these inherent difficulties, there was need for a new way of expressing rational functions.

The Padé approximant is a particular and classical type of rational approximation originally credited to George Frobenius who introduced the idea and study the features of the rational power approximation. Henri Padé around 1890 was the one who made significant contributions by expressing it as the quotient of two polynomials with varying degrees. It is superior to the Taylor series expansion in that it provides better approximation of the function than truncating the Taylor series especially where the Taylor series does not converge and when it contains poles. The approximation has been extensively applied to calculate time delay and in computer science. [39-43].

Now, given two polynomials, $P_L(x)$ and $Q_M(x)$ with highest degrees of N and M. Then the Pade approximant of function, $f(x)$in each closed interval $[a, b]$ denoted $[L/M]$ is the ratio of the polynomials in the form.

$$[L/M] = \frac{P_L(x)}{Q_M(x)} \tag{21}$$

The power series of the given function is given as

$$f(x) = \sum_{n=0}^{\infty} a_k x^k \tag{22}$$

To determine the coefficients of the polynomials $P_L(x)$ and $Q_M(x)$, we use the equation

$$f(x) = \frac{P_L(x)}{Q_M(x)} + O(x^{L+M+1}) \tag{23}$$

Using the standard normalization condition

$$Q_M(0) = 1.0 \tag{24}$$

Expressing the coefficients of the two polynomials explicitly in the form

$$\begin{cases} P_L(x) = p_0 + p_2 x + \cdots + p_L x^L \\ Q_M(x) = 1 + q_1 x + \cdots + q_M x^M \end{cases} \tag{25}$$

In view of Eq. (25), multiplying both sides of Eq. (23) by $Q_M(x)$ yield the system of equations and linearized the coefficient equation in the form.

$$\left.\begin{array}{l} q_m a_{L-M+1} + q_{M-1} a_{L-M+2} + \cdots + q_0 a_{L+1} = 0 \\ q_m a_{L-M+2} + q_{M-1} a_{L-M+3} + \cdots + q_0 a_{L+2} = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ q_M a_{L-M+2} + q_{M-1} a_{L-M+3} + \cdots + q_0 a_{L+2} = 0 \end{array}\right] \tag{26}$$

We set $q_0 = 1$, and if $j > 0$, then $a_i = 0$. The system in Eq. (26) reduced to the form

$$\begin{pmatrix} a_{L-M+1} & a_{L-M+2} \dots . a_{L+1} \\ a_{L-M+2} & a_{L-M+3} \dots\dots a_{L+2} \\ \dots\dots\dots\dots & \dots\dots\dots\dots \\ a_L & a_{L+1} \dots\dots\dots\dots a_{L+M} \end{pmatrix} \begin{pmatrix} q_m \\ q_{M-1} \\ \vdots \\ q_1 \end{pmatrix} = \begin{pmatrix} a_{L+1} \\ a_{L+2} \\ \vdots \\ a_{L+M} \end{pmatrix} \tag{27}$$

Solving the system in Eq. (27) produces the solution in the form

$$\left.\begin{array}{r} a_0 = p_0 \\ a_0 + a_0 q_1 + \cdots = p_1 \\ \dots\dots\dots\dots\dots\dots\dots \\ a_L + a_{L-1} q_1 + \cdots + a_0 q_1 = p_L \end{array}\right] \tag{28}$$

The Padé approximant of different orders such as $[2/2], [3/3], [4/4], [5/5]$ and $[6/6]$ are obtained using Maple 20.

V. APPLICATION OF LADM TO THE EPIDEMIC MODEL

We consider the epidemiological disease model as follows

$$\begin{bmatrix} \frac{dS}{dt} = -\beta SI \\ \frac{dI}{dt} = \beta SI - \mu I \\ \frac{dR}{dt} = \mu I \end{bmatrix} \tag{29}$$

Subject to the initial conditions

$$S(0) = 0.9,\ I(0) = 0.1, R(0) = 0 \tag{30}$$

Taking the Laplace transform of both sides of Eq. (29), we get

$$\begin{cases} \mathcal{L}\left\{\frac{dS}{dt}\right\} = \mathcal{L}\{-\beta SI\} \\ \mathcal{L}\left\{\frac{dI}{dt}\right\} = \mathcal{L}\{\beta SI - \mu I\} \\ \mathcal{L}\left\{\frac{dR}{dt}\right\} = \mathcal{L}\{\mu I\} \end{cases} \quad (31)$$

$$\begin{cases} \mathcal{L}\left\{\frac{dS}{dt}\right\} = -\beta\mathcal{L}\{SI\} \\ \mathcal{L}\left\{\frac{dI}{dt}\right\} = \beta\mathcal{L}\{SI\} - \mu\mathcal{L}\{I\} \\ \mathcal{L}\left\{\frac{dR}{dt}\right\} = \mu\mathcal{L}\{I\} \end{cases} \quad (32)$$

Applying the differentiation law of Laplace transforms, we obtain

$$w\mathcal{L}\{S\} - S(0) = -\beta\mathcal{L}\{SI\} \quad (33)$$

$$w\mathcal{L}\{I\} - I(0) = \beta\mathcal{L}\{SI\} - \mu\mathcal{L}\{I\} \quad (34)$$

$$w\mathcal{L}\{R\} - R(0) = \mu\mathcal{L}\{I\} \quad (35)$$

Using the initial condition to the above Eqs. (33) – (35), we get

$$\mathcal{L}\{S\} = \frac{0.9}{w} - \frac{\beta}{w}\mathcal{L}\{A\} \quad (36)$$

$$\mathcal{L}\{I\} = \frac{0.1}{w} + \frac{\beta}{w}\mathcal{L}\{A\} - \frac{\mu}{w}\mathcal{L}\{I\} \quad (37)$$

$$\mathcal{L}\{R\} = \frac{\mu}{w}\mathcal{L}\{I\} \quad (38)$$

Where $A = SI$

Next, we represent the pertinent parameters as an infinite series of the form

$$S = \sum_{n=0}^{\infty} S_n, \quad I = \sum_{n=0}^{\infty} I_n, \quad R = \sum_{n=0}^{\infty} R_n \quad (39)$$

Where the terms S_n, I_n and R_n are to be determined recursively.

Similarly, the nonlinear term is equally decomposed in the form

$$A = \sum_{n=0}^{\infty} A_n \quad (40)$$

Where A_n are called the Adomian polynomials. The first five polynomials are considered as follows

$$A = SI$$

$$A_0 = S_0 I_0$$

$$A_1 = S_0 I_1 + S_1 I_0$$

$$\begin{aligned} A_2 &= S_0 I_2 + S_1 I_1 + S_2 I_0 \\ A_3 &= S_0 I_3 + S_1 I_2 + S_2 I_1 + S_3 I_0 \end{aligned} \quad (41)$$

$$\begin{aligned} A_4 &= S_0 I_4 + S_1 I_3 + S_2 I_2 + S_3 I_1 + S_4 I_0 \\ A_5 &= S_0 I_5 + S_1 I_4 + S_2 I_3 + S_3 I_2 + S_4 I_1 + S_5 I_0 \\ A_6 &= S_0 I_6 + S_1 I_5 + S_2 I_4 + S_3 I_3 + S_4 I_2 + S_5 I_1 + S_6 I_0 \end{aligned}$$

Substituting Eqs (39) and (40) into Eqs (36) – (38) yield

$$\mathcal{L}\{\sum_{n=0}^{\infty} S_n\} = \frac{0.9}{w} - \frac{\beta}{w}\mathcal{L}\{\sum_{n=0}^{\infty} A_n\} \quad (42)$$

$$\mathcal{L}\{\sum_{n=0}^{\infty} I_n\} = \frac{0.1}{w} + \frac{\beta}{w}\mathcal{L}\{\sum_{n=0}^{\infty} A_n\} - \frac{\mu}{w}\mathcal{L}\{\sum_{n=0}^{\infty} I_n\} \tag{43}$$

$$\mathcal{L}\{\sum_{n=0}^{\infty} R_n\} = \frac{\mu}{w}\mathcal{L}\{\sum_{n=0}^{\infty} I_n\} \tag{44}$$

Matching the sides of Eqs. (42) – (44) yield the following iterative algorithm

$$\begin{aligned}
\mathcal{L}\{S_0\} &= \frac{0.9}{w} \\
\mathcal{L}\{S_1\} &= -\frac{\beta}{w}\mathcal{L}\{A_0\} \\
\mathcal{L}\{S_2\} &= -\frac{\beta}{w}\mathcal{L}\{A_1\} \\
\mathcal{L}\{S_3\} &= -\frac{\beta}{w}\mathcal{L}\{A_2\} \\
&\vdots \\
\mathcal{L}\{S_{n+1}\} &= -\frac{\beta}{w}\mathcal{L}\{A_n\}
\end{aligned} \tag{45}$$

$$\begin{aligned}
\mathcal{L}\{I_0\} &= \frac{0.1}{w} \\
\mathcal{L}\{S_1\} &= \frac{\beta}{w}\mathcal{L}\{A_0\} - \frac{\mu}{w}\mathcal{L}\{I_0\} \\
\mathcal{L}\{S_2\} &= \frac{\beta}{w}\mathcal{L}\{A_1\} - \frac{\mu}{w}\mathcal{L}\{I_1\} \\
\mathcal{L}\{S_3\} &= \frac{\beta}{w}\mathcal{L}\{A_2\} - \frac{\mu}{w}\mathcal{L}\{I_2\} \\
&\vdots \\
\mathcal{L}\{I_{n+1}\} &= \frac{\beta}{w}\mathcal{L}\{A_n\} - \frac{\mu}{w}\mathcal{L}\{I_n\}
\end{aligned} \tag{46}$$

$$\begin{aligned}
\mathcal{L}\{R_0\} &= 0 \\
\mathcal{L}\{R_1\} &= \frac{\mu}{w}\mathcal{L}\{I_0\} \\
\mathcal{L}\{R_2\} &= \frac{\mu}{w}\mathcal{L}\{I_1\} \\
\mathcal{L}\{R_3\} &= \frac{\mu}{w}\mathcal{L}\{I_2\} \\
&\vdots \\
\mathcal{L}\{R_{n+1}\} &= \frac{\mu}{w}\mathcal{L}\{I_n\}
\end{aligned} \tag{47}$$

Applying the inverse Laplace transform to the first Eqs. (45) – (47), we get

$$\mathcal{L}\{S_0\} = \frac{0.9}{w}, \mathcal{L}\{I_0\} = \frac{0.1}{w}, \mathcal{L}\{R_0\} = 0 \tag{48}$$

Substitution of the above values of S_0, I_0 and R_0 into the second and third Eqs (45) – (47), we get

$$\mathcal{L}\{S_1\} = -\frac{0.18}{w^3}, \mathcal{L}\{I_1\} = \frac{0.18}{w^3} - \frac{0.05}{w^2}, \mathcal{L}\{R_1\} = \frac{0.05}{w^2} \tag{49}$$

$$\begin{aligned}
\mathcal{L}\{S_2\} &= \frac{0.1}{w^3} - \frac{0.144}{w^5} \\
\mathcal{L}\{I_2\} &= \frac{0.144}{w^5} - \frac{0.02}{w^3} - \frac{0.018}{w^4} + \frac{0.001}{w^3} \\
\mathcal{L}\{R_2\} &= \frac{0.018}{w^4} - \frac{0.001}{w^3}
\end{aligned} \tag{50}$$

Substituting the Laplace transform of the quantities on the right-hand side of Eqs (44) – (47) and applying the inverse Laplace transform, we obtain the values
$S_2(t), I_2(t), R_2(t)$. Similarly, the other remaining terms $S_3(t), S_4(t) \dots S_n(t), I_3(t), I_4(t) \dots I_n(t)$ and $R_3(t), R_4(t) \dots R_n(t)$ can be recursively obtained.

VI. NUMERICAL APPLICATION

In this section, we apply the LADM to the epidemiological model. Taking $S(0) = 0.9, I(0) = 0.1$ and $R(0) = 0$ for the three parameters of interest. Setting $\beta = 2, \mu = 0.5$ and the first few calculations for $S(t), I(t)$ and $R(t)$ are calculated and presented below

$$S(t) = \frac{0.9}{w} - \frac{0.16}{w^3} - \frac{0.144}{w^5}$$
$$I(t) = \frac{0.1}{w} - \frac{0.01}{w^2} + \frac{0.161}{w^3} - \frac{0.018}{w^4} + \frac{0.144}{w^5} \quad (51)$$
$$R(t) = \frac{0.1}{w^2} - \frac{0.001}{w^3} + \frac{0.018}{w^4}$$

Applying the inverse Laplace transform on both sides of Eq. (51), we obtain the analytical solutions for the governing parameters.

$$S(t) = 0.9 - 0.08t^2 - 0.006t^4$$
$$I(t) = 0.1 - 0.01t + 0.0805t^2 - 0.003t^3 + 0.0006t^4 \quad (52)$$
$$R(t) = 0.1t - 0.0005t + 0.003t^2$$

Using symbolic computational software Maple 20, we calculate the $[5/5]$ Pade approximant of the infinite series of Eq. (52) which gives the following rational approximations to the solution.

$$S_{Pade}(t) = \frac{0.91234 - 1.45 \times 10^{-16}t - 0.080101t^2 - 3.69 \times 10^{-15}t^3 - 0.00112345t^4 + 5.7 \times 10^{-16}t^5}{1 - 1.6 \times 10^{-16}t - 1.2 \times 10^{-15}t^2 - 4.1 \times 10^{-15}t^3 - 7.3 \times 10^{-17}t^4 + 3.07 \times 10^{-16}t^5}$$

$$I_{Pade}(t) = \frac{0.1 - 0.00981t + 0.080t^2 - 0.0021782t^3 + 0.00059392226t^4 + 0.00000120892t^5}{1 + 0.020t - 2.32 \times 10^{-13}t^2 - 2.045 \times 10^{-14}t^3 + 8.42 \times 10^{-16}t^4 + 4.3 \times 10^{-17}t^5}$$

$$R_{Pade}(t) = \frac{0.1t - 0.0005t^2 + 0.00297t^3 + 3.014 \times 10^{-7}t^4 - 8.910731504 \times 10^{-7}t^5}{1 + 0.0059t - 0.0297t^2 - 5.174 \times 10^{-15}t^3 + 9.68 \times 10^{-17}t^4 + 1.559 \times 10^{-16}t^5}$$

Table 1: Numerical comparison for Susceptible using six iterates

t	LADM	LADM-Padé	4^{th} Order R-K
0	0.900000	1.0000	1.0000
0.2	0.896790	0.896790	0.896800
0.4	0.88705	0.887046	0.887045
0.6	0.870422	0.870350	0.870421
0.8	0.846342	0.846344	0.8463450
1.0	0.814000	0.814000	0.814000
1.2	0.772358	0.772357	0.772350

Table 2: Numerical comparison for Infected using six iterates

t	LADM	LADM-Padé	4^{th} Order R-K
0	0.10000	0.10000	0.10000
0.2	0.101197	0.1011641	0.101198
0.4	0.108703	0.108704	0.108705
0.6	0.122410	0.122410	0.122410
0.8	0.142230	0.142240	0.142235
1.0	0.16810	0.168100	0.168910
1.2	0.19998	0.19990	0.202882

Table 3: Numerical comparison for Removed using six iterates

t	LADM	LADM-Padé	4^{th} Order R-K
0	0.0000	0.0000	0.0000
0.2	0.020004	0.0200023	0.0200040
0.4	0.040112	0.04112	0.040038
0.6	0.604680	0.603870	0.604680
0.8	0.081216	0.081215	0.0812160
1.0	0.102500	0.102501	0.102500
1.2	0.124464	0.1244640	0.1244604

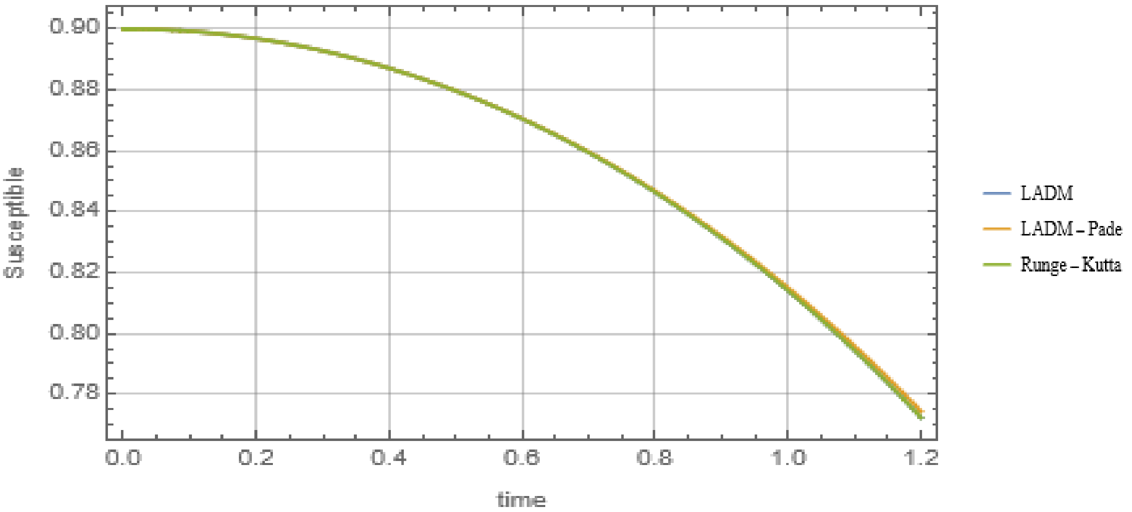

Figure 1. Comparison of LADM solution of Susceptible for six iterates

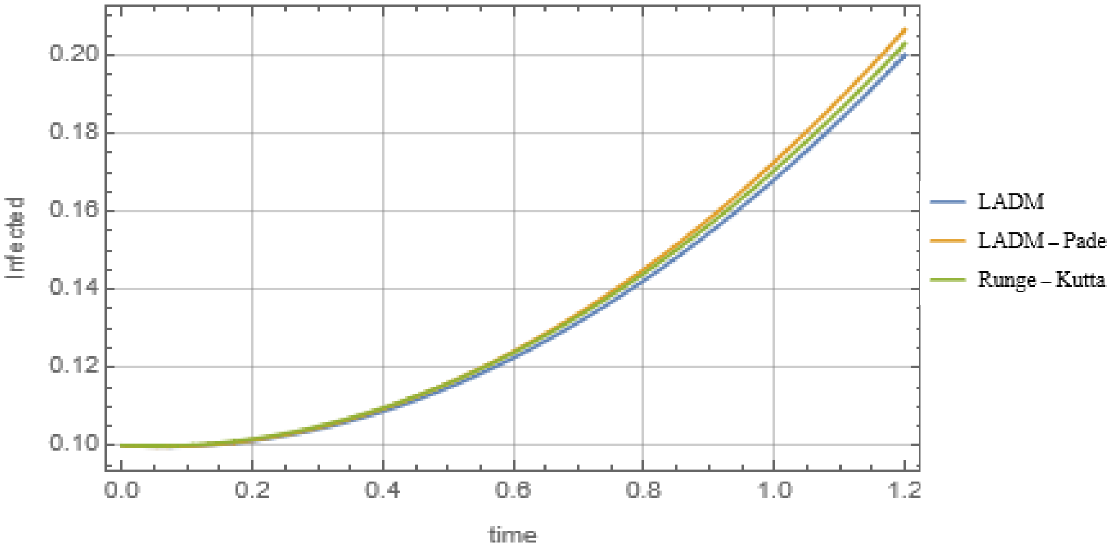

Figure 2. Comparison of LADM solution of Infected for six iterates

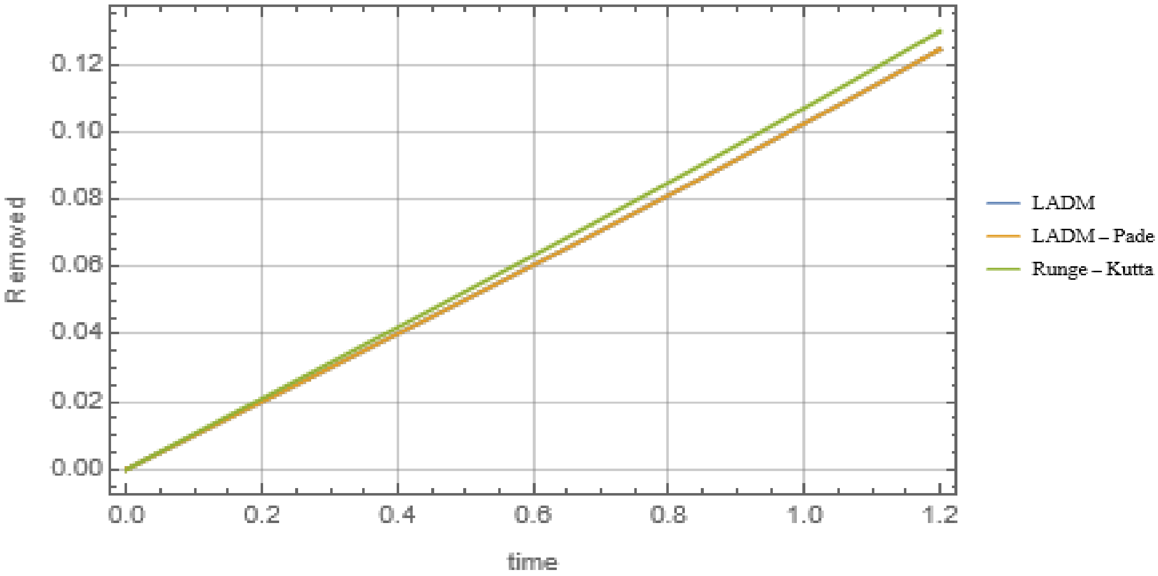

Figure 3. Comparison of LADM solution of Removed for six iterates

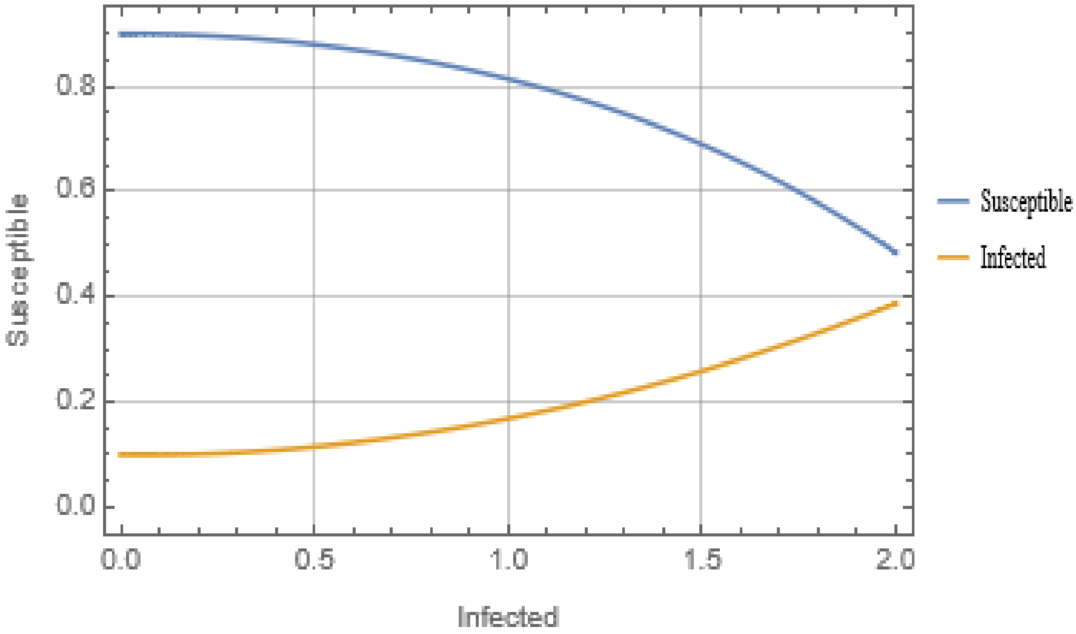

Figure 4. Comparison of Susceptible against Infected for six iterates

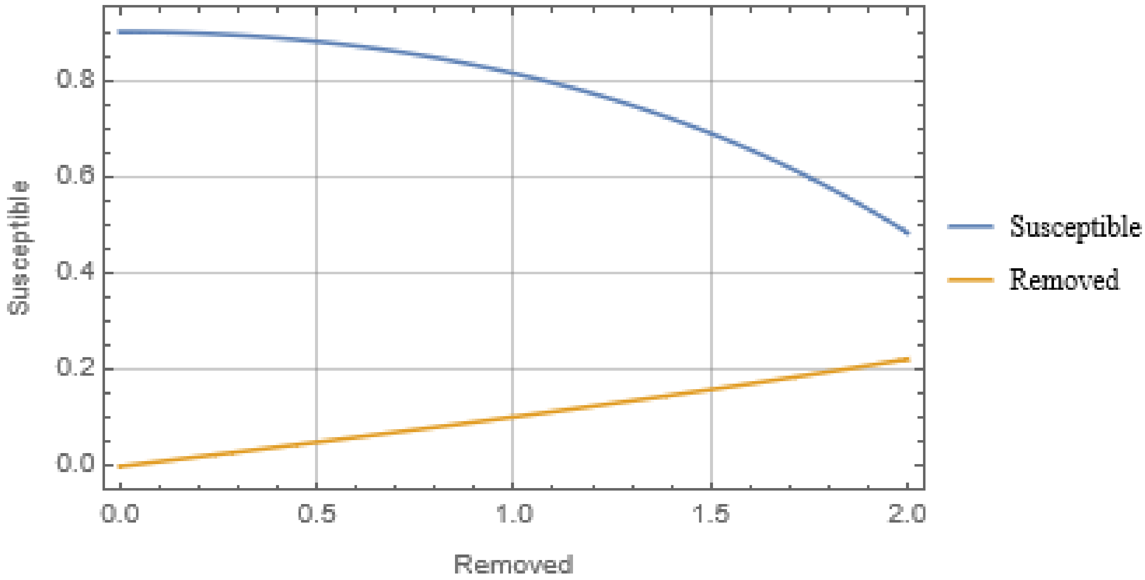

Figure 5. Comparison of Susceptible against Removed for six iterates

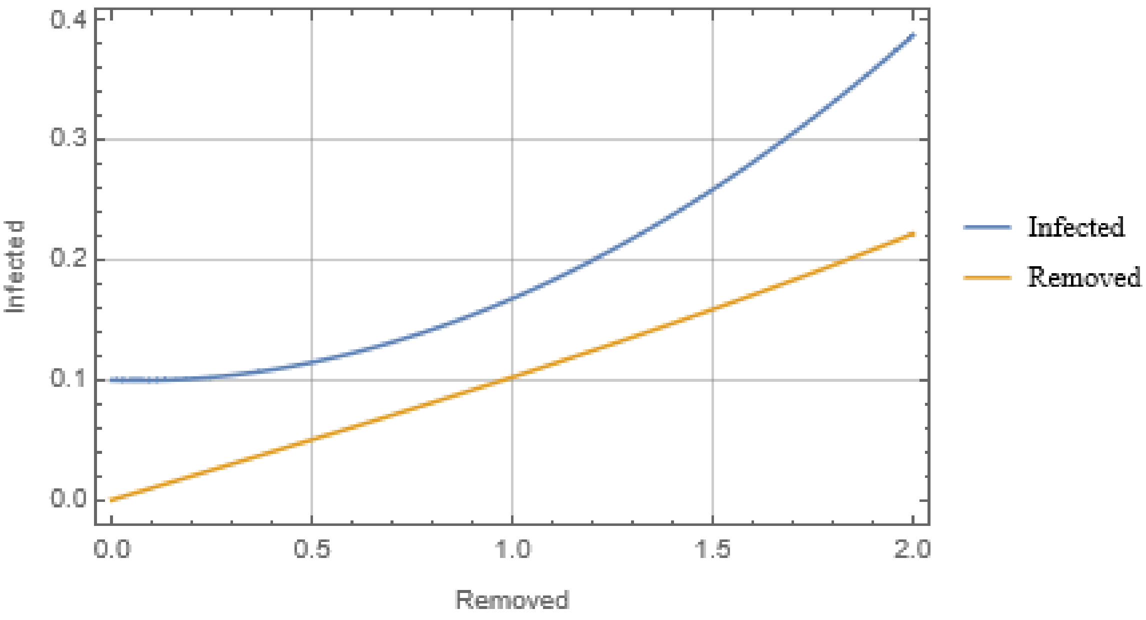

Figure 6. Comparison of Infected against Removed for six iterates

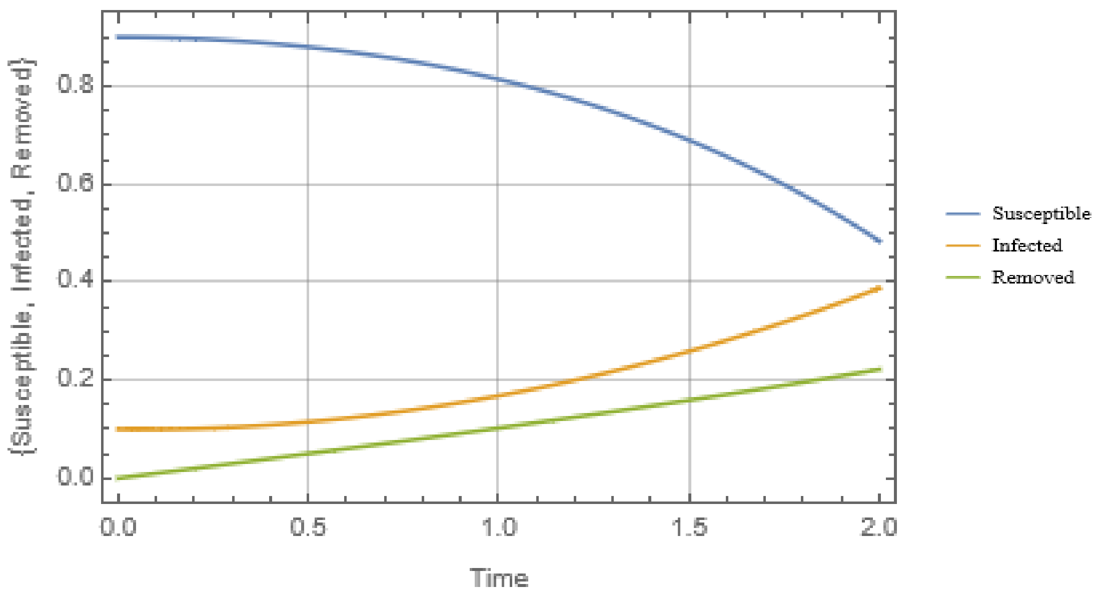

Figure 7. Comparison of Susceptible, Infected and Removed against Time

VII. CONCLUSION

In this research article, we proposed the Laplace Adomian decomposition method and the Padé approximant to find analytical solution to the SIR epidemic model. The governing nonlinear autonomous differential equations comprising the parameters of interest were solved using the LADM. The obtained results were approximated using the Padé approximant to validate the earlier result and improve upon it. Comparison is then made between the results with the fourth order Runge-Kutta method. The solution obtained agreed with literature, simple, efficient, and applicable for a large time interval.

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