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Some Features of $(3, \bar{3}) \oplus (\bar{3}, 3)$ Breaking of Chiral Symmetry(*)

In this lecture I will illustrate some general features of lagrangian theories of strong interactions where the lagrangian can be meaningfully separated into two terms: a term $\mathcal{L}_0$ symmetrical under the chiral $SU(3) \otimes SU(3)$ group, and a breaking term $\mathcal{L}_B$ which is assumed to transform, under the same group, according to the representation $(3, \bar{3}) \oplus (\bar{3}, 3)$. This is the kind of theory one abstracts from simple quark models (in these models $\mathcal{L}_B$ representing just a quark-mass term) or from more elaborate σ-models (where $\mathcal{L}_B$ is related to fundamental scalar fields). As we shall see, the theory displays a remarkable degree of symmetry and allows some general features of hadron spectrum to be interpreted in a very simple way.

In the following I will restrict myself to rather general considerations, without going much into a detailed connection of the parameters appearing in the theory with physically measurable

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quantities (such as masses, decay amplitudes, etc.). These aspects are fully covered in the lectures by R. Dashen at this School, where also a comparison with other schemes, different from the one discussed here, is given.

Let me conclude this introduction apologizing for not presenting any particularly new material, except for some speculation on scale invariance discussed in the end.

This lecture has a rather pedagogical character, and I hope can provide some useful introduction to the wide literature appeared in the last few years on the subject.

The relevance of chiral SU(3) ⊗ SU(3) was pointed out by M. Gell-Mann since 1962[1], in the frame of what has afterward been called the free quark model. So, let me start by considering a free, massive quark lagrangian:

$$\mathcal{L} = i\,\overline{\psi}_\alpha \not\partial \psi_\alpha + \overline{\psi}_\alpha \epsilon_{\alpha\beta} \psi_\beta \equiv i\,\overline{\psi} \not\partial \psi + \overline{\psi} \epsilon \psi \qquad (1)$$

ϵ is a 3x3 real diagonal matrix, representing quark masses, and α and β run from 1 to 3.

It is useful to introduce the left and right handed quark fields:

$$\psi_{L\alpha} = a_+ \psi_\alpha \quad ; \quad \psi_{R\alpha} = a_- \psi_\alpha \quad ; \quad a_\pm = \frac{1 + \gamma_5}{2}$$

If the quarks were massless, so that $\mathcal{L}$ would reduce to the kinetic energy term, $\mathcal{L}$ would be fully symmetrical under the set of transformations:

$$\psi_{L\alpha} \rightarrow U_{\alpha\beta}\, \psi_{L\beta} \quad ; \quad \psi_{R\alpha} = V_{\alpha\beta}\, \psi_{R\beta} \qquad (2)$$

U and V being 3x3 unitary, unimodular matrices.

This set of transformations, each of which is evidently characterized by the pair of matrices (U, V), is the chiral $SU(3) \otimes SU(3)$ group.

Actually the massless version of Eq. (1) is symmetrical under the transformations Eq. (2) even when U and V are unitary but not unimodular, this corresponding to symmetry under $U(3) \otimes U(3) = U(1) \otimes U(1) \otimes SU(3) \otimes SU(3)$.

One of the two U(1) groups can be identified with quark number conservation (corresponding in the real world to baryon number conservation) and the other one with the quark helicity conservation.

The relevance to the real world of this latter symmetry is, at the moment, more dubious than anything I will say in this lecture, so that I will forget about it in the following[(2)]. Also, I will not mention quark number conservation anymore, it being implicit in all I will say. $SU(3) \otimes SU(3)$ symmetry brings with it sixteen conserved currents, which can be written as:

$$L^{\mu}_{\alpha\beta}(x) = \bar{\psi}_{L\beta}(x)\,\gamma_{\mu}\,\psi_{L\alpha}(x)$$

$$R^{\mu}_{\alpha\beta}(x) = \bar{\psi}_{R\beta}(x)\,\gamma_{\mu}\,\psi_{R\alpha}(x) \qquad (3)$$

or, in terms of the more familiar vector and axial vector currents,

$$V^{i}_{\mu} = \mathrm{Tr}\,(L^{\mu} + R^{\mu})\,\frac{\lambda_i}{2} = \bar{\psi}\,\gamma_{\mu}\,\frac{\lambda_i}{2}\,\psi$$

$$A^{i}_{\mu} = \mathrm{Tr}\,(L^{\mu} - R^{\mu})\,\frac{\lambda_i}{2} = \bar{\psi}\,\gamma_{\mu}\,\gamma_5\,\frac{\lambda_i}{2}\,\psi \qquad (4)$$

λ_i being the eight Gell-Mann matrices.

In terms of quark fields we also construct the operators:

$$M_{\alpha\beta}(x) = \overline{\psi}_{R\beta}(x)\,\psi_{L\alpha}(x)$$
$$(M^{+})_{\alpha\beta}(x) = \overline{\psi}_{L\beta}(x)\,\psi_{R\alpha}(x) \tag{5}$$

which again are connected to the familiar scalar and pseudoscalar densities[1] according to:

$$u_i = \mathrm{Tr}\,(M + M^{+})\,\lambda_i = \overline{\psi}\,\lambda_i\,\psi$$
$$v_i = \mathrm{Tr}\,\frac{M - M^{+}}{i}\,\lambda_i = i\,\overline{\psi}\,\lambda_i\,\gamma_5\,\psi \tag{6}$$

here i goes from 0 to 8.

The matrices M and M^+ have simple transformation properties under the transformations of the quark fields given by Eq. (2):

$$M \xrightarrow[(U,V)]{} U\,M\,V^{+}$$
$$M^{+} \xrightarrow[(U,V)]{} V\,M^{+}\,U^{+} \tag{7}$$

which reveal that M transforms according to the $(3, \overline{3})$ representation, and M^+ according to the $(\overline{3}, 3)$. We will also consider infinitesimal transformations, both left and right-handed:

$$(U, V) = (1 + i\,\alpha^{i}\,\lambda_i\,,\ 1)$$
$$\delta_L\,M = i\,\alpha^{i}\,\lambda_i\,M, \qquad \delta_L\,M^{+} = -\,i\,\alpha^{i}\,M^{+}\,\lambda_i \tag{8}$$

$$(U, V) = (1, 1 + i\beta^{i}\lambda_i) \qquad (9)$$

$$\delta_R M = -i\beta^{i} M \lambda_i \quad ; \quad \delta_R M^{+} = i\beta^{i}\lambda_i M^{+}$$

We conclude these preliminaries by giving the transformation properties of the currents, which can be derived from Eq. (3):

$$L_\mu \xrightarrow[(U,V)]{} U L_\mu U^{+} \qquad (10)$$

$$R_\mu \xrightarrow[(U,V)]{} V R_\mu V^{+} \qquad (11)$$

Eqs. (10) and (11) indicate that L^μ transforms as the (8, 1) and R^μ as the (1, 8) representations.

We now introduce e.m. and weak interactions in the lagrangian eq. (1), writing

$$\mathcal{L} = i\bar{\psi}\,\partial\!\!\!/\,\psi + \bar{\psi}\epsilon\psi + g\left[W^\mu\,\bar{\psi}\gamma_\mu(1+\gamma_5)\lambda_w\psi + h.c.\right] +$$

$$+ e A^\mu\,\bar{\psi}\gamma_\mu\lambda_Q\psi = i(\bar{\psi}_L\,\partial\!\!\!/\,\psi_L + \bar{\psi}_R\,\partial\!\!\!/\,\psi_R) +$$

$$+ (\bar{\psi}_R\epsilon\psi_L + \bar{\psi}_L\epsilon\psi_R) + 2g\left[W^\mu\,\bar{\psi}_L\gamma_\mu\lambda_w\psi_L + h.c.\right] +$$

$$+ e A^\mu(\bar{\psi}_L\gamma_\mu\lambda_Q\psi_L + \bar{\psi}_R\gamma_\mu\lambda_Q\psi_R) =$$

$$= \mathcal{L}_0 + \mathrm{Tr}(M\epsilon + M^{+}\epsilon) + 2g\left[W^\mu\,\mathrm{Tr}(L_\mu\lambda_w) + h.c.\right] +$$

$$+ e A^\mu\,\mathrm{Tr}(L_\mu + R_\mu)\lambda_Q$$

We fix the charge spectrum by requiring

$$\lambda_Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} \qquad (13)$$

λ_w is then determined(by the requirements of having charge +1 and of generating with λ_w^+ an SU(2) group[3]) to have the form:

$$\lambda_w = \begin{pmatrix} 0 & \cos\theta & \sin\theta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (14)$$

We see that the lagrangian Eq. (12) can be thus splitted into:

i) a symmetric term $\mathcal{L}_0$;

ii) an $SU(3) \otimes SU(3)$ breaking term $\mathcal{L}_B$, transforming as $(3, \overline{3}) \oplus (\overline{3}, 3)$;

iii) e.m. and weak terms, transforming according to $(8, 1) \oplus (1, 8)$ and given in terms of the $SU(3) \otimes SU(3)$ currents via the matrices λ_Q and λ_w.

θ is (with some qualification to be given later) the Cabibbo angle[4].

Properties i), ii) and iii) are those which we want to abstract from quark model, and constitute the basis of our considerations.

However the breaking term $\mathcal{L}_B$ as given by Eq. (12) is not the most general $(3, \overline{3}) \oplus (\overline{3}, 3)$ element, compatible with hermiticity of $\mathcal{L}_B$. This is rather given by

$$\mathcal{L}_B = \mathrm{Tr}\,(M\epsilon^+ + M^+\epsilon) \qquad (15)$$

ϵ being any 3x3 matrix. If ϵ is real and diagonal we get back Eq. (12).

In conclusion, let us write our Lagrangian as:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_B(\epsilon) + \mathcal{L}_{e.m.}(\lambda_Q) + \mathcal{L}_w(\lambda_w) \qquad (16)$$

λ_Q and λ_w are given by eqs. (13) and (14), $\mathcal{L}_B$ is given by eq. (15), ϵ being any matrix consistent with change conservation:

$$\epsilon = \begin{pmatrix} \epsilon_1 & 0 & 0 \\ 0 & a & b \\ 0 & c & d \end{pmatrix} \qquad \text{a, b, c, d = arbitrary complex numbers} \qquad (17)$$

In the free quark model $\mathcal{L}_0$ and $\mathcal{L}_B$ are trivial, but it is easy to construct more complicated models in which $\mathcal{L}_0$ and $\mathcal{L}_B$ both contain non trivial interactions, like the σ-model or the gluon model.

Our first problem will be that of determining what symmetries can $\mathcal{L}_B$ retain or violate[5].

The first observation is that, if ϵ has the general form eq. (17) $\mathcal{L}_B$ seems to violate strangeness and parity. Why? Let us rewrite Eq. (15) as

$$\mathcal{L}_B = \mathrm{Tr}\left(\frac{M+M^+}{2}\right)(\epsilon + \epsilon^+) + \mathrm{Tr}\left(\frac{M-M^+}{2i}\right)\left(\frac{\epsilon-\epsilon^+}{i}\right)$$

comparing with Eq. (16), we see that $\mathcal{L}_B$ contains terms of the form $(\bar{n}\lambda)$ and $i(\bar{n}\gamma_5\lambda)$ i.e. both parity (P) and strangeness (S) violating. Actually there have been attempts to connect at least parts of the S-violating non leptonic decay amplitudes to an "effective" lagrangian transforming as a piece of a $(3,\bar{3}) \oplus (\bar{3},3)$. What we will show now is that, in the scheme represented by Eq. (16), these pieces are illusory because:

a) $\mathcal{L}_B$ can violate P only if it violates also CP;

b) $\mathcal{L}_B$ is always strangeness conserving.

If $\mathcal{L}_B$ contains other pieces besides the $(3,\bar{3}) \oplus (\bar{3},3)$ term, then b) does not hold any more, whereas a) is still true. Since the main part of the observed non leptonic decays is CP-conserving, then a $(3,\bar{3})$ can represent at most P-conserving, $\Delta S = 1$ non leptonic amplitudes, provided $\mathcal{L}_B$ contains more terms than in our scheme (e.g. an $(8.1 \oplus (1.8)$ piece). Elementary considerations, which I will not report here[6], show that any 3x3 matrix ϵ can always be written as:

$$\epsilon = U \ \epsilon_D \ V^+ \ e^{i\frac{\varphi}{3}} \tag{18}$$

U and V being suitable unitary and unimodular matrices, ϵ_D a real diagonal matrix and φ some real phase (Eq. (18) holds in general for nxn matrices, with the substitution $\frac{1}{3}\varphi \to \frac{1}{n}\varphi$).

Suppose now to apply to the basic fields in $\mathcal{L}$ (whatever they are) the $SU(3) \otimes SU(3)$ transformation (U, V). Then:

$$\mathcal{L}_0 \to \mathcal{L}_0$$

$$\mathcal{L}_B \to Tr(MV^+ \epsilon^+ U + M^+ U^+ \epsilon V) = Tr(M \epsilon_D e^{-i\frac{\varphi}{3}} + M^+ \epsilon \ e^{+i\frac{\varphi}{3}})$$

$$\mathcal{L}_{e.m.} \to e A^\mu \ Tr(L_\mu U^+ \lambda_Q U + R_\mu \ V^+ \lambda_Q V)$$

$$\mathcal{L}_w \to 2g \left[W^\mu \ Tr(L_\mu U^+ \lambda_w U) + h.c. \right]$$

Since $\left[\epsilon, \lambda_Q\right] = 0$, we can furthermore choose U, and V such that they commute with λ_Q, so that $\mathcal{L}_{e.m.}$ remains unchanged.

Then we see that the lagrangian eq.(16) is equivalent to:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_B\,(\epsilon_D\, e^{i\frac{\varphi}{3}}) + \mathcal{L}_{e.m.}(\lambda_Q) + \mathcal{L}_w\,(U^+ \lambda_w U)$$

where

a) ϵ is replaced by a diagonal matrix, so that it has no more S-violating terms;

b) $\lambda_w \to U^+ \lambda_w U$, this corresponding just to a redefinition of the Cabibbo angle.

In fact, since $\left[U, \lambda_Q\right] = 0$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{pmatrix}$$

$$U^+ \lambda_w\; U = \begin{pmatrix} 0 & \cos(\theta+\varphi) & \sin(\theta+\varphi) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

This disposes of S-violation.

As for parity, we observe that the now $\mathcal{L}_B$, if $\varphi \neq 0$, contains both terms like $\bar{p}p$ and terms like $i\,\bar{p}\,\gamma_5\,p$ which have not only opposite P, but also opposite CP (having the same C).

In conclusion, the best Eq.(16) can do is to provide us with CP and P violation in $\Delta S = 0$ channels (i.e. for example in nuclear levels).

This restricts φ to be extremely small, and from now on we will assume that ϵ is such that $\varphi = 0$. Actually it is very simple to see if a given ϵ will give a CP-conserving theory. From Eq. (18)

$$\det \epsilon = e^{i\varphi} \det \epsilon_D$$

i.e. if $\det \epsilon$ is real $\varphi = 0$ and viceversa. Two comments, are in order.

First observe that if $\mathcal{L}_0$ were invariant under $U(3) \otimes U(3)$ we could put the factor $e^{i\frac{\varphi}{3}}$ into U or V^+, and in this case we could always eliminate CP-violation just as we have done for S-violation.

This means that in free quark models (or even in the gluon model) one can never get a CP violation out of Eq. (16). The second comment is this. If ϵ_D is proportional to the unit matrix, we can make an additional transformation (U_θ, U_θ) of the basic fields such that

$$U_\theta^+ \lambda_w U_\theta = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

i.e. in absence of SU(3) breaking, we can always rotate away the Cabibbo angle eliminating S-violation also from weak interactions. This points to a deep connection between the actual value of θ ($\theta \sim 0.22$) and the breaking of SU(3) and possibly of $SU(3) \otimes SU(3)$, and to an interplay of weak and strong interactions, which must cooperate somehow to produce the bizarre angle observed in nature.

This idea has been pursued by various authors[6-8] even with encouraging results, but a real breakthrough has not yet been achieved.

The value of θ remains still as one of the most challenging problems for theorists.

From now we will understand ϵ as to be of the form:

$$\epsilon = \begin{pmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix} \qquad (19)$$

Using Eq. (6), $\mathcal{L}_B$ can be written alternatively, as, (see e.g. ref.(9)):

$$\mathcal{L}_B = \alpha_0 u_0 + \alpha_8 u_8 + \alpha_3 u_3$$

$\alpha_{0,8,3}$ being related to the ϵ_i' s according to:

$$\alpha_0 = \frac{1}{\sqrt{6}} (\epsilon_1 + \epsilon_2 + \epsilon_3)$$

$$\alpha_8 = \frac{1}{2\sqrt{3}} (\epsilon_1 + \epsilon_2 - 2\epsilon_3) \qquad (20)$$

$$\alpha_3 = \frac{1}{2} (\epsilon_1 - \epsilon_2)$$

Observe that we have left open the possibility that $\mathcal{L}_B$ contains some I-spin violating term.

Before connecting $\mathcal{L}_B$ to experiments (e.g. ratios of ϵ_i or α_i to meson spectrum) one has to be sure that the parametrization Eq. (19) is unique, and that there are no ambiguities.

Actually this not so[10]. The requirement that ϵ is diagonal does not fix uniquely the ϵ_i's. We can still perform $SU(3) \otimes SU(3)$

transformations which: 1) exchange two ϵ_i's (actually only ϵ_2 and ϵ_3, if we want to keep the spectrum of λ_Q fixed), 2) flip the sign of any two of them. Any of these transformations do not change $\mathcal{L}_{e.m.}$ or $\mathcal{L}_w$ (apart from trivial redefinitions) but drastically change the pattern of the parameters ϵ_i or α_i so that it does not make sense at the present level to attach, say, to α_3 the meaning of an I-spin violation. In quark language, these transformations correspond to exchange n and λ quarks and/or change the intrinsic parities of any two quarks.

We will fix this ambiguity in the following[11] and will see that its elimination is obtained only when one takes into account the fact that ϵ_D by itself <u>does not</u> give a complete description of the symmetry breaking.

From now on let us neglect $\mathcal{L}_{e.m.}$ and $\mathcal{L}_w$, and restrict to $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_B$.

We introduce a very important quantity, which is the vacuum expectation value (VEV) of the fields contained in M(x)[12,5]:

$$\langle 0|M(x)|0\rangle = \langle 0|M(0)|0\rangle = \eta$$

η is, analogously to ϵ, a 3x3 matrix.

If our theory were exactly invariant under SU(3) $\otimes$ SU(3), η would vanish. To see this, recall that it can be shown that all symmetries of the vacuum are symmetries of the world[13], so that if $\mathcal{U}(U, V)$ is the Hilbert space operator corresponding to the element (U, V) of SU(3) $\otimes$ SU(3), then $\mathcal{U}(U, V)|0\rangle = |0\rangle$ implies:

$$\begin{aligned}\eta &= \langle 0|M|0\rangle = \langle 0|\mathcal{U}(U, V)\, M\, \mathcal{U}^{\dagger}(U, V)|0\rangle = \\ &= U\langle 0|M|0\rangle V^{+} = U\,\eta\, V^{+}\end{aligned} \qquad (21)$$

for any (U, V). This can be satisfied only if $\eta = 0$.

When $\mathcal{L}_B \neq 0$ we have then $\eta \neq 0$. However, in certain models it happens that η does not vanish even in the limit $\mathcal{L}_B = 0$. This situation is usually referred to as "spontaneous breaking" of SU(3) $\otimes$ SU(3) and is the one I will discuss here. It corresponds to the presence of stable solutions for the vacuum which display a lower degree of symmetry than the lagrangian. In fact in the limit $\mathcal{L} = \mathcal{L}_0$ the lagrangian is symmetric under the full SU(3) $\otimes$ SU(3) whereas the vacuum is left invariant only by those transformations such that Eq. (21) holds. This phenomenon in turn is connected with the appearance of massless spin zero bosons (Goldstone bosons), one for each generator of SU(3) $\otimes$ SU(3) which does not leave invariant η [14,9].

Before touching upon the argument of Goldstone bosons, however, let me consider in some detail the symmetry structure of η.

To visualize the situation, let us consider first a classical example, that of a ferromagnet.

Consider a system of spins in an infinite volume. In absence of external fields, the system is described by a rotationally invariant Hamiltonian $\mathcal{H}_0$. Call $|0\rangle$ the ground state of $\mathcal{H}_0$.

Usually the magnetization of the ground state

$$\langle 0 | \vec{m} | 0 \rangle$$

vanishes. This is certainly so if $|0\rangle$ itself is rotationally invariant.

However, in the case of a ferromagnet $\mathcal{H}_0$ is such that the stable ground state has a non vanishing magnetization $\vec{m} = \langle 0 | \vec{m} | 0 \rangle$, so that $|0\rangle$ is not symmetrical under the full rotation group, but only under rotations around $\vec{m}$. Of course, we can orientate $\vec{m}$ in whatever direction we want, and ground states with different orientations of $\vec{m}$ are degenerate with respect to energy.

These states do not lie in the same Hilbert space, but rather each of them corresponds to mathematically inequivalent though physically identical theories.

Suppose we introduce now a weak external magnetic field $\vec{H}$. The Hamiltonian is changed into:

$$\mathcal{H} = \mathcal{H}_0 + \vec{m} \cdot \vec{H}$$

and out of all the infinite number of degenerate ground states one is selected as the lowest energy state, this being that one in which $\vec{m}$ is parallel to $\vec{H}$. How are $\vec{m}$ and $\vec{H}$ connected? For weak fields $|\vec{m}|$ depends little upon $\vec{H}$, and is mainly determined by $\mathcal{H}_0$. However, no matter how weak is the external "breaking" $\vec{H}$, there is always a strong correlation in that the stable ground state has to have $\vec{m}$ parallel to $\vec{H}$.

Let us now go back to our case. Here η is the analog of $\vec{m}$ and in the limit $\epsilon = 0$ identical theories correspond to matrices related by $SU(3) \otimes SU(3)$ transformations:

$$\eta \rightarrow U \eta V^+$$

Using Eq. (18) then we can always choose a frame where η is diagonal so that we have always S-conservation. However, when $\epsilon \neq 0$ the question arises whether η is diagonal in the same frame where ϵ is such, i.e. whether ϵ and η are in some way constrained to be "parallel" as in the ferromagnet analogy. This is actually the case. We insert Eqs. (8), (9) and (15) in the divergence formulae:

$$\partial^{\mu} L^{i}_{\mu} = \frac{\delta \mathcal{L}}{\delta \alpha_i} = \frac{\delta \mathcal{L}_B}{\delta \alpha_i}$$

$$\partial^{\mu} R^{i}_{\mu} = \frac{\delta \mathcal{L}}{\delta \beta_i} = \frac{\delta \mathcal{L}_B}{\delta \beta_i}$$

and we obtain (ϵ is real and diagonal):

$$\partial^{\mu} L^{i}_{\mu} = i \, \mathrm{Tr} \, (\lambda^{i} M - M^{+} \lambda^{i}) \, \epsilon$$

$$\partial^{\mu} R^{i}_{\mu} = -i \, \mathrm{Tr} \, (M \lambda^{i} - \lambda^{i} M^{+}) \, \epsilon \qquad (22)$$

The VEV of Eq. (22) must vanish by translation invariance and this gives us a set of relations between ϵ and η . It is then a simple matter of algebra to show that these equations imply precisely that ϵ and η can be simultaneously diagonalized by an SU(3) $\otimes$ SU(3) rotation.

It remains the possibility that ϵ and η may not be relatively real. This would give rise to a kind of "spontaneous breakdown of P and CP"[(15)] which has the same features and disadvantages as the one previously discussed. I will not insist therefore on this, and assume that η and ϵ can be both brought into a real diagonal form. This form does in general suffer from the ambiguities we mentioned above, but now we are in the position to fix them.

We have already mentioned that η must possess all the symmetries of the vacuum, i.e. of the real world. Now the particle spectrum clearly displays a very good I-spin symmetry and a more approximate SU(3) symmetry. This implies that it must be possible to choose an SU(3) $\otimes$ SU(3) frame such that η takes the form:

$$\eta = \begin{pmatrix} \eta_1 & 0 & 0 \\ 0 & \eta_2 & 0 \\ 0 & 0 & \eta_3 \end{pmatrix}$$

with

$$\eta_1 \sim \eta_2 \qquad \text{up to I-spin corrections}$$

$$\eta_{1,2} \sim \eta_3 \qquad \text{up to SU(3) corrections}$$

Note that this frame is now uniquely defined. We are not allowed any more to exchange η_2 with η_3 or to change sign to any two η_i's, as this would spoil our approximate equalities[11]. In this frame then the diagonal elements of both η and ϵ have a physical meaning and can be compared to physical quantities. Also, this frame defines the physical Cabibbo angle.

Moreover, if the symmetries of η are also symmetries of the vacuum (i.e. if η gives a complete description of symmetry breaking in the vacuum) then if we want chiral symmetry to be much more badly broken in particle spectrum then SU(2) or SU(3) (as it is indicated by the absence of parity doublets) we must also require

$$|\eta_i| >> |\eta_i - \eta_j| \qquad \text{for any i and j}$$

This statement, however, is more model dependent than the others, and we are unable to prove it in general. It is in fact verified in the σ-model and we shall assume its validity.

The ferromagnet analogy also clearly indicates what is the connection between η and ϵ. In the limit $\epsilon \to 0$ we expect η to tend to a finite value which will display all those symmetries which are not spontaneously broken, i.e. realized with particle multiplets.

SU(2) and SU(3) symmetries appear indeed to be of such type, so that we expect:

$$\lim_{\epsilon \to 0} \eta = \eta_0 \propto \mathbb{1} \tag{23}$$

η_0 is determined by $\mathcal{L}_0$ only, and we have an octet of pseudoscalar massless bosons. When $\epsilon \neq 0$, we expect

$$\eta = \eta_0 + O(\epsilon) \tag{24}$$

If $\mathcal{L}_B$ can be considered in some sense a small perturbation, η will depend very little upon ϵ and be mainly determined by $\mathcal{L}_0$. In this scheme, it may be meaningful to apply perturbation theory in $\mathcal{L}_B$, starting from the spontaneously broken solution η_0, as discussed by Dashen[9].

Let me now very briefly discuss pseudoscalar meson mass formulae[12, 16], as an illustration of the arguments presented above. I will restrict to π and K masses, and, for the sake of brevity, will not give any derivation, but simply quote from ref. (2). Then (neglecting I-spin violations, i.e. putting $\epsilon_1 = \epsilon_2$, $\eta_1 = \eta_2$) we have:

$$m_\pi^2 = - Z_\pi^2 \frac{\epsilon_1}{\eta_1} \tag{25}$$

$$m_K^2 = - Z_K^2 \frac{\epsilon_1 + \epsilon_3}{\eta_1 + \eta_3} \tag{26}$$

$$m_\pi^2 F_\pi = - Z \epsilon_1 Z_\pi \tag{27}$$

$$m_K^2 F_K = - (\epsilon_1 + \epsilon_3) Z_K \tag{28}$$

where we have defined:

$$Z_\pi = \langle \pi | v^\pi | 0 \rangle \quad , \quad Z_K = \langle K | v^K | 0 \rangle$$

$$\langle \pi | \partial^\mu A_\mu | 0 \rangle = m_\pi^2 \; F_\pi \quad , \quad \langle K | \partial^\mu A_\mu^K | 0 \rangle = m_K^2 F_K$$

Eqs.(27) and (28) can be re-written as:

$$Z_\pi F_\pi = 2\,\eta_1 \quad , \quad Z_K F_K = \eta_1 + \eta_3 \tag{29}$$

These equations clearly display the Goldstone phenomenon: if η remains finite when $\epsilon \to 0$, then by Eqs.(25), (26) and (29) both m_π and m_K vanish in this limit. Also, if we assume Eq.(24) to hold, then

$$Z_\pi = Z_K + O(\epsilon) \; , \qquad F_\pi = F_K + O(\epsilon)$$

so that, to lowest order in ϵ:

$$\frac{m_\pi^2}{m_K^2} = \frac{2\,\epsilon_1}{\epsilon_1 + \epsilon_3}$$

which indicates that $\epsilon_1 \ll \epsilon_3$ (this corresponds in Dashen's notations to $C \sim -\sqrt{2}$).

Finally, these equations show that the vacuum breaking η is connected to F_π and F_K (which in fact display an approximate SU(3) symmetry) whereas ϵ is related to the meson masses, the smallness of pion mass indicating that $\mathcal{L}_B$ is, to a good approximation, $SU(2) \otimes SU(2)$ invariant.

A complete analysis of the relations linking ϵ and η to the observed p-s meson masses, and to the decay coupling constants F_π and F_K is beyond the scope of this lecture and I will not elaborate on this any further (see e.g. ref.(2), (9), (16) and also ref.(17)).

Rather I will conclude by presenting some further speculations on the symmetry structure of $\mathcal{L}$, which have recently received some attention.

If we go back to eq. (1), we see that the quark kinetic energy term is not only invariant under SU(3) ⊗ SU(3), but also under <u>scale transformations</u>, as discussed in Prof. Callan's lectures[18]. It is then interesting to see what happens if we conjecture the same to be true for the symmetric lagrangian $\mathcal{L}_0$ appearing in Eq. (12), i.e. assume that scale invariance is broken only by the same term which breaks SU(3) ⊗ SU(3) (apart from c-numbers)[19,20].

It is very easy to see qualitatively what is going on in this case. What happens is that, if in the limit $\epsilon \to 0$, η stays finite, we will have, in addition to SU(3) ⊗ SU(3) breaking, a spontaneous breaking of scale invariance. Correspondingly, a scalar Goldstone boson (called "dilation") appears. If in the same limit Eq. (23) holds, the dilaton is coupled mainly to the SU(3) singlet scalar density u_0. When $\epsilon \neq 0$, the $(\text{mass})^2$ of the dilaton is of order ϵ (i.e. $\sim m_K^2$) and, as a consequence, we will have a systematic enhancement[20,21] of the matrix elements of u_0 with respect to the matrix elements of u_8.

Another consequence of this assumption is that, since[18]

$$\theta^{\mu}_{\mu} = -(4-d)\mathcal{L}_B$$

θ^{ν}_{μ} being the "improved" energy-momentum tensor[22] and d the dimension of $\mathcal{L}_B$, the mass of any hadron A is proportional to the matrix element of $\mathcal{L}_B$:

$$M_A = -(4-d) \langle A|\, \mathcal{L}_B \,|A\rangle \qquad (30)$$

Eq. (30) may look rather strange since we expect in general M_A to be approximately SU(3) invariant, whereas we have seen that $\mathcal{L}_B$ is far from being so. However, the enhancement of $< A|u_0|A >$ with respect to $<A|u_8|A>$ that we have just mentioned compensates for the unsymmetrical nature of $\mathcal{L}_B$, and makes M_A to have a large SU(3) singlet part.

It is very unclear at present if these considerations are of any value especially in view of the anomalies found to appear (in perturbation theory) in the Ward identities derived from approximate scale invariance[23]. It is however interesting that they are qualitatively supported by recent calculations of the σ-term in π-N scattering, by Cheng and Dashen[24] and by others[25]. The large value for σ there found is qualitatively in agreement with the nucleon mass[20,25], as given by Eq. (30) with d = 3 (as in the quark model). It also gives evidence for the presence of an enhancement of $< N|u_0|N >$ with respect to $< N|u_8|N >$ of the right order of magnitude.

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Riassunto - Alcuni aspetti della violazione della simmetria chirale con termini della rappresentazione $(3, \bar{3}) \oplus (\bar{3}, 3)$.

Si discutono alcuni aspetti generali delle teorie in cui la simmetria chirale è violata da un termine che si trasforma secondo la rappresentazione $(3, \bar{3}) \oplus (\bar{3}, 3)$. In particolare vengono illustrate le connessioni tra la rottura del vuoto e la rottura della lagrangiana e il modo in cui si eliminano possibili ambiguità nelle relazioni che legano i parametri della rottura a quantità osservate. Infine vengono illustrate alcune congetture avanzate recentemente sulla rottura della simmetria di scala.

Abstract - Some Features of $(3, \bar{3}) \oplus (\bar{3}, 3)$ Breaking of Chiral Symmetry.

In this lecture some general properties of the $(3, \bar{3}) \oplus (\bar{3}, 3)$ breaking of chiral $SU(3) \otimes SU(3)$ are discussed. These include the relations between vacuum breaking and explicit breaking and the elimination of possible ambiguities in the connection of the breaking parameters to experimentally observed quantities. Some recently advanced speculations on the breaking of scale invariance are illustrated.

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