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TRANSPORT CORRECTION OF HIGHER ORDER AND REMOVAL SYSTEMS

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T R A N S P O R T C O R R E C T I O N O F H I G H E R O R D E R A N D R E M O V A L S Y S T E M S

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ABSTRACT

In this paper the transport corrections of higher order are presented. The results are applicated on removal systems. The comparison of the removal diffusion and removal P1 systems based on the transport equations of higher order with the NRN method and Spinney's method is presented.

1.0 Introduction

The angular dependence of the kinetic equation brings a lot of complications into calculations, i.e. into the economy and the practicability of calculations.

For neutrons with low energies (where the anisotropy of scattering is low as well) the scattering is considered to be divided into two parts, i.e. isotropic scattering and scattering with maximum anisotropy /3/.

This process gives a transport approximation of the kinetic equation the main advantage of which consists in the possibility to perform the calculation for the isotropic case with effective scatter data.

The papers /1/, /4/ endeavours to present a logical extension on the transport correction of higher order. This demands the application of the SN, DSN and PN methods, especially for calculations in shielding physics, where for higher energies the anisotropy of scattering increases.

A similar problem occurs also for gamma ray transport, where the anisotropy of scattering is especially high. For this case with regard to the anisotropy of diffusion is suitable to observe the classical access depending on the division of the radiation into the direct passing and scattered radiation.

The same problem takes place for neutrons using the removal method with removal cross-sections dependent on energy. Neutrons are to be divided into "scattered" and "non-scattered" parts. The "non-scattered" part of neutrons includes the purely non-scattered neutrons and neutrons scattered directly forward with small scatter angles.

The removal cross-section characterizes the attenuation of the original beam. There exist several definitions of the removal cross-section and several procedures of calculation /2/.

In this paper we shall deduce the removal cross-section on the basis of the transport correction of higher order. This cross-section will be compared with the mostly used definitions of the removal cross-section, i.e. the Spiney's model and the semiempiric NRN method.

2. General Part

2.1. Formulation of the Problem

Let us discusse the Boltzmann kinetic equation for neutron fields

$$\vec{\Omega}\nabla\varphi(\vec{r},\vec{\Omega},E)+\Sigma(\vec{r},E)\,\varphi(\vec{r},\vec{\Omega},E)=\int_{E}^{\infty}dE'\int_{4\pi}d\vec{\Omega}'\,w(\vec{r},\mu_0,E'\to E)\,\varphi(\vec{r},\vec{\Omega}',E')+S(\vec{r},\vec{\Omega},E) \tag{2.1}$$

$\vec{v}'=v'\vec{\Omega}'$ ... is the neutron velocity vector before scattering

$\vec{v}=v\vec{\Omega}$ ... is the neutron velosity vector after scattering

$\varphi(\vec{r},\vec{\Omega},E)$ ... is the differential neutron flux, where

$$\int_{4\pi}d\vec{\Omega}\,\varphi(\vec{r},\vec{\Omega},E)=\phi(\vec{r},E)$$

$\phi(\vec{r},E)$ ... is the neutron flux

$S(\vec{r},\vec{\Omega},E)$ ... is the external source of neutrons

$\Sigma(\vec{r},E)$ ... is the macroscopic total cross-section for neutron interactions

$$\mu_0 = \vec{\Omega}\vec{\Omega}' = (\vec{i}\sin\vartheta\cos\psi + \vec{j}\sin\vartheta\sin\psi + \vec{k}\cos\vartheta)\cdot$$

$$\cdot(\vec{i}\sin\vartheta'\cos\psi' + \vec{j}\sin\vartheta'\sin\psi' + \vec{k}\cos\vartheta') =$$

$$= \cos\vartheta\cos\vartheta' + \sqrt{1-\cos^2\vartheta}\cdot\sqrt{1-\cos^2\vartheta'}\cos(\psi-\psi') =$$

$$= \mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\psi-\psi'),$$

$$w(\vec{r},\mu_0,E'\to E) = \sum_{g,h} \nu_{gh}(E')N_g(\vec{r})\sigma_{A_h,g}(E')\,p_{gh}(\mu_0,E'\to E),$$

where "g" signifies the g'th sort of nuclei, "h" signifies the h'th type of neutron interaction the consequence of which is the emission of secondary neutrons.

$\nu_{gh}(E')$ is the neutron profit per scatter interaction of the h'th type with a nucleus of the g'th sort, i.e.

(n,f)	fission	$\nu_{gf} = \nu_{gh}(E')$
(n,2n)	....	$\nu_{g,(n,2n)} = 2$
(n,n)	elastic scattering	$\nu_{g,el} = 1$
(n,n')	inelastic scattering	$\nu_{g,in} = 1$

Only the interactions (n,n), (n,n') take place in shielding physics calculations. Therefore

$$w(\vec{r},\mu_0,E'\to E) = \sum_{g,h} N_g(\vec{r})\sigma_{s_h,g}(E')\,p_{gh}(\mu_0,E'\to E) = \sum_{g,h}\Sigma_{s_h,g}(\vec{r},E')\,p_{gh}(\mu_0,E'\to E) =$$

$$= \sum_{g,h}\Sigma_{s_h,g}(\vec{r},\mu_0,E'\to E) = \Sigma_s(\vec{r},\mu_0,E'\to E),$$

$\Sigma_s(\vec{r},\mu_0,E'\to E)$ ... is the differential macroscopic cross-section

$p_{gh}(\mu_0,E'\to E)$ ... is the indicatrix of scattering, where

$$\int_0^\infty dE \int_{4\pi} d\vec{\Omega}\, p_{gh}(\mu_0,E'\to E) = 1 \; . \tag{2.2}$$

For the next we shall use the following definitions and formulae:

The Legendrian polynomials

$$P_n(\mu) = P_n(\cos\vartheta) = \frac{1}{2^n n!}\,\frac{d^n}{d\cos\vartheta^n}(\cos^2\vartheta - 1)^n \; , \quad n = 0,1,2,\cdots .$$

The Legendrian functions

$$P_n^m(\mu) = P_n^m(\cos\vartheta) = \sin^m\vartheta\,\frac{d^m}{d\cos\vartheta^m}P_n(\cos\vartheta), \quad n = 0,1,2,\cdots;\; m = -n,\cdots,n$$

The spherical harmonic functions

$$P_n^m(\vec{\Omega}) = (-1)^m\sqrt{\frac{(n-m)!}{(n+m)!}}\,e^{im\psi}P_n^m(\cos\vartheta) \; ,$$

$$P_n^{m*}(\vec{\Omega}) = (-1)^m\sqrt{\frac{(n-m)!}{(n+m)!}}\,e^{-im\psi}P_n^m(\cos\vartheta) \; , \qquad n = 0,1,2,\cdots;\; m = -n,\cdots,n$$

The addition theorem

$$P_n(\mu_0) = P_n(\mu)P_n(\mu') + 2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}P_n^m(\mu)P_n^m(\mu')\cos m(\psi-\psi') =$$

$$= \sum_{m=-n}^{n} P_n^m(\vec{\Omega})P_n^{m*}(\vec{\Omega}') \; .$$

$$\int_{-1}^{1} d\mu\, P_n(\mu) P_\ell(\mu) = \frac{2}{2n+1}\,\delta_{n\ell} \;,$$

$$\int_{4\pi} d\vec{\Omega}\, P_n^m(\vec{\Omega}) P_\ell^{\kappa *}(\vec{\Omega}) = \frac{4\pi}{2n+1}\,\delta_{n\ell}\,\delta_{m\kappa} \;.$$

In tables we can usually find the scatter matrices with elements $\sigma_{gh}(E' \to E)$. Therefore we shall write for each "g" and "h"

$$\sigma(E')\,p(\mu_0, E' \to E) = \sigma(E' \to E)\,p(\mu_0) \;,$$

$$p(\mu_0, E' \to E) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi}\, p_\ell(E' \to E) P_\ell(\mu_0) \;,$$

$$p_\ell(E' \to E) = 2\pi \int_{-1}^{1} d\mu_0\, p(\mu_0, E' \to E) P_\ell(\mu_0) \;,$$

$$p_0(E' \to E) = 2\pi \int_{-1}^{1} d\mu_0\, p(\mu_0, E' \to E) \;.$$

Looking at (2.2) we have

$$\int_0^\infty dE \int_0^{2\pi} d\psi \int_{-1}^{1} d\mu\, p(\mu_0, E' \to E) = \int_0^\infty dE \int_0^{2\pi} d\psi \int_{-1}^{1} d\mu \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi}\, p_\ell(E' \to E) P_\ell(\mu_0) =$$

$$= \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} \int_0^\infty dE\, p_\ell(E' \to E) \int_0^{2\pi} d\psi \int_{-1}^{1} d\mu\, P_\ell(\mu) P_\ell(\mu') =$$

$$= \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} \int_0^\infty dE\, p_\ell(E' \to E) P_\ell(\mu') \int_{-1}^{1} d\mu\, P_\ell(\mu) P_0(\mu) =$$

$$= \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} \int_0^\infty dE\, p_\ell(E' \to E) P_\ell(\mu') \frac{2}{2\ell+1}\,\delta_{\ell 0} = \int_0^\infty dE\, p_0(E' \to E) = 1 \;,$$

$$\int_{-1}^{1} d\mu_0 \int_0^{\infty} dE \; p(\mu_0, E' \to E) = \frac{1}{2\pi}$$

$$\int_0^{\infty} dE \; p_0(E' \to E) = 1$$

Let us write:

$$\sigma(E' \to E) = \sigma(E') \, p_0(E' \to E) \, ,$$

$$\sigma(E') \, p(\mu_0, E' \to E) = \sigma(E') \, p_0(E' \to E) \, p(\mu_0) \, ,$$

$$p(\mu_0, E' \to E) = p_0(E' \to E) \, p(\mu_0) \, ,$$

$$\int_0^{\infty} dE \int_{-1}^{1} d\mu_0 \, p(\mu_0, E' \to E) = \int_0^{\infty} dE \; p_0(E' \to E) \int_{-1}^{1} d\mu_0 \, p(\mu_0) = \frac{1}{2\pi} \, ,$$

$$\int_{-1}^{1} d\mu_0 \, p(\mu_0) = \frac{1}{2\pi \int_0^{\infty} dE \; p_0(E' \to E)} = \frac{1}{2\pi} \, .$$

Now we can write:

$$w(\vec{r}, \mu_0, E' \to E) = \sum_{g,h} N_g(\vec{r}) \, \sigma_{s_h, g}(E' \to E) \, p_{gh}(\mu_0) =$$

$$= \sum_{g,h} \Sigma_{s_h, g}(\vec{r}, E' \to E) \, p_{gh}(\mu_0) = \Sigma_s(\vec{r}, \mu_0, E' \to E) \, .$$

Further we can write:

$$w(\vec{r}, \mu_0, E' \to E) = \Sigma_s(\vec{r}, \mu_0, E' \to E) = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} \Sigma_{s,l}(\vec{r}, E' \to E) \, P_l(\mu_0) =$$

$$= \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} \sum_{g,h} N_g(\vec{r}) \, \sigma_{s_h, g}(E' \to E) \, p_{gh, l} \, P_l(\mu_0)$$

where $P_\ell(\mu_0) = \sum_{K=-\ell}^{\ell} P_\ell^K(\vec{\Omega}) P^{K*}(\vec{\Omega})$

It is evident that $w_\ell(\vec{r}, E' \to E) = \Sigma_{s,\ell}(\vec{r}, E' \to E)$ (2.3)

2.2 Removal Method

Let us divide the differential flux φ in (2.1) into the "non-scattered" part φ_r and the "scattered" part φ_c :

$$\varphi(\vec{r}, \vec{\Omega}, E) = \varphi_r(\vec{r}, \vec{\Omega}, E) + \varphi_c(\vec{r}, \vec{\Omega}, E)$$

$\varphi_r(\vec{r}, \vec{\Omega}, E)$ ... is the flux of those neutrons, which did not experience any interaction since their penetration into the discussed region.

The equation (1) may be rewritten

$$\vec{\Omega}\nabla(\varphi_r + \varphi_c) + \Sigma(\varphi_r + \varphi_c) = \int_0^\infty dE' \int_{4\pi} d\vec{\Omega}' \, w(\vec{r}, \mu_0, E' \to E)(\varphi_r + \varphi_c) + S \tag{2.4}$$

Let us divide the equation (2.4) into two equations for φ_r and φ_c :

$$\vec{\Omega}\nabla\varphi_r(\vec{r}, \vec{\Omega}, E) + \Sigma(\vec{r}, E)\varphi_r(\vec{r}, \vec{\Omega}, E) = S(\vec{r}, \vec{\Omega}, E) \tag{2.5}$$

$$\vec{\Omega}\nabla\varphi_c(\vec{r}, \vec{\Omega}, E) + \Sigma(\vec{r}, E)\varphi_c(\vec{r}, \vec{\Omega}, E) = \int_0^\infty dE' \int_{4\pi} d\vec{\Omega}' \, w(\vec{r}, \mu_0, E' \to E)\varphi_c(\vec{r}, \vec{\Omega}', E') +$$
$$+ \int_0^\infty dE' \int_{4\pi} d\vec{\Omega}' \, w(\vec{r}, \mu_0, E' \to E)\varphi_r(\vec{r}, \vec{\Omega}', E') . \tag{2.6}$$

can be written

$$w(\vec{r},\mu_0,E'\to E)=\sum_{\ell=0}^{\infty}\frac{2\ell+1}{4\pi}w_\ell(\vec{r},E'\to E)P_\ell(\mu_0)=$$

$$=\sum_{\ell=0}^{\infty}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}w_\ell(\vec{r},E'\to E)P_\ell^K(\vec{\Omega})P_\ell^{K*}(\vec{\Omega}')=$$

$$=\sum_{\ell=0}^{L}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}w_\ell(\vec{r},E'\to E)P_\ell^K(\vec{\Omega})P_\ell^{K*}(\vec{\Omega}')+$$

$$+\sum_{\ell=L+1}^{\infty}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}w_\ell(\vec{r},E'\to E)P_\ell^K(\vec{\Omega})P_\ell^{K*}(\vec{\Omega}')\,. \qquad (2.7)$$

[illegible] The Transport Correction of Higher Order

Let us make the following assumptions:

I) $w_\ell \le w_{L+1}$ for $\ell > L$. (2.8)

According to I) let us make the following assumption:

II) $w_\ell(\vec{r},E'\to E)=w_{L+1}(\vec{r},E'\to E)$ for $\ell > L$. (2.9)

For $\ell > L$ may be written

$$\sum_{\ell=L+1}^{\infty}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}w_\ell(\vec{r},E'\to E)P_\ell^K(\vec{\Omega})P_\ell^{K*}(\vec{\Omega}')=$$

$$=w_{L+1}(\vec{r},E'\to E)\sum_{\ell=L+1}^{\infty}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}P_\ell^K(\vec{\Omega})P_\ell^{K*}(\vec{\Omega}')=$$

$$=w_{L+1}(\vec{r},E'\to E)\sum_{\ell=0}^{\infty}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}P_\ell^K(\vec{\Omega})P_\ell^{K*}(\vec{\Omega}')-$$

$$-w_{L+1}(\vec{r},E'\to E)\sum_{\ell=0}^{L}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}P_\ell(\vec{\Omega})P_\ell^{K*}(\vec{\Omega}')\,.$$

Now we shall discuss the δ-function $\delta(\mu_0-1)$:

$$\delta(\mu_0-1)=\sum_{\ell=0}^{\infty} c_\ell P_\ell(\mu_0),$$

$$\int_{-1}^{1} d\mu_0 P_K(\mu_0)\delta(\mu_0-1)=\sum_{\ell=0}^{\infty} c_\ell \int_{-1}^{1} d\mu_0 P_\ell(\mu_0)P_K(\mu_0)=\frac{2}{2K+1}c_K ,$$

$$c_\ell=\frac{2\ell+1}{2}\int_{-1}^{1} d\mu_0 P_\ell(\mu_0)\,\delta(\mu_0-1)=\frac{2\ell+1}{2}\int_{-\infty}^{+\infty} d\mu_0 P_\ell(\mu_0)\delta(\mu_0-1)=$$

$$=\frac{2\ell+1}{2}P_\ell(1).$$

From the integral relation

$$P_\ell(\mu_0)=\frac{1}{\pi}\int_0^\pi d\psi\left(\mu_0+\sqrt{\mu_0^2-1}\cos\psi\right)^\ell$$

follows $P_\ell(1)=1$ for $\ell=0,1,2,\cdots$.

Therefore

$$c_\ell=\frac{2\ell+1}{2},$$

$$\delta(\mu_0-1)=\sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}P_\ell(\mu_0),$$

$$\sum_{\ell=0}^{\infty}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}P_\ell^K(\vec{\Omega})P_\ell^{K*}(\vec{\Omega}')=\frac{1}{2\pi}\sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}P_\ell(\mu_0)=\frac{1}{2\pi}\delta(\mu_0-1).$$

From above follows (with regard to (2.9)):

$$\sum_{\ell=L+1}^{\infty}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}w_\ell(\vec r,E'\to E)P_\ell^K(\vec\Omega)P_\ell^{K*}(\vec\Omega')=$$

$$=w_{L+1}(\vec r,E'\to E)\frac{\delta(\mu_0-1)}{2\pi}-$$

$$-w_{L+1}(\vec r,E'\to E)\sum_{\ell=0}^{L}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}P_\ell^K(\vec\Omega)P_\ell^{K*}(\vec\Omega')\;. \tag{2.10}$$

Using (2.7) and (2.10) we may write

$$w(\vec r,\mu_0,E'\to E)=\sum_{\ell=0}^{L}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}w_\ell^*(\vec r,E'\to E)P_\ell^K(\vec\Omega)P_\ell^{K*}(\vec\Omega')+$$

$$+w_{L+1}(\vec r,E'\to E)\frac{\delta(\mu_0-1)}{2\pi}, \tag{2.11}$$

where $w_\ell^*(\vec r,E'\to E)=w_\ell(\vec r,E'\to E)-w_{L+1}(\vec r,E'\to E)$

for $\ell\le L$. (2.12)

Further may be written for the equation (2.5)

$$\int_E^{\infty}dE'\int_{4\pi}d\vec\Omega'\,w(\vec r,\mu_0,E'\to E)\left[\varphi_r(\vec r,\vec\Omega',E')+\varphi_c(\vec r,\vec\Omega',E')\right]=$$

$$=\int_E^{\infty}dE'\int_{4\pi}d\vec\Omega'\sum_{\ell=0}^{L}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}w_\ell^*(\vec r,E'\to E)P_\ell^K(\vec\Omega)P_\ell^{K*}(\vec\Omega')[\varphi_r+\varphi_c]+$$

$$+\int_E^{\infty}dE'\,w_{L+1}(\vec r,E'\to E)\int_{4\pi}d\vec\Omega'\,[\varphi_r+\varphi_c]\frac{\delta(\mu_0-1)}{2\pi}, \tag{2.13}$$

where $\mu_0=\vec\Omega\vec\Omega'$.

Let us calculate the last integral on the right side of (2.13), following the definition of the δ-function

$$\delta(\mu_0-1)=\begin{cases}\infty & \text{for } \mu_0=1\\ 0 & \text{for } \mu_0\neq 1\end{cases}.$$

To find out the integrals $\int\limits_{4\pi} d\vec{\Omega}'\,\delta(\mu_0-1)$, $\int\limits_{4\pi} d\vec{\Omega}'\,\varphi(\vec{\Omega}')\,\delta(\mu_0-1)$, we shall use the Fourier series for $\delta(\mu_0-1)$

$$\delta(\mu_0-1)=\sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}P_\ell(\mu_0)$$

Then we can write

$$\begin{aligned}\int\limits_{4\pi} d\vec{\Omega}'\,\delta(\mu_0-1)&=\sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}\int\limits_{4\pi} d\vec{\Omega}'\,P_\ell(\mu_0)=\\ &=\sum_{\ell=0}^{\infty}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{2}P_\ell^K(\vec{\Omega})\int\limits_{4\pi} d\vec{\Omega}'\,P_\ell^{K*}(\vec{\Omega}')=\\ &\sum_{\ell=0}^{\infty}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{2}P_\ell^K(\vec{\Omega})\,\frac{4\pi}{2\ell+1}\,\delta_{\ell 0}\,\delta_{K0}=2\pi,\end{aligned} \tag{2.14}$$

$$\begin{aligned}\int\limits_{4\pi} d\vec{\Omega}'\,\varphi(\vec{\Omega}')\,\delta(\mu_0-1)&=\sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}\int\limits_{4\pi} d\vec{\Omega}'\,\varphi(\vec{\Omega}')\,P(\mu_0)=\\ &=\sum_{\ell=0}^{\infty}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{2}P_\ell^K(\vec{\Omega})\int\limits_{4\pi} d\vec{\Omega}'\,\varphi(\vec{\Omega}')\,P_\ell^{K*}(\vec{\Omega}')=\\ &=2\pi\sum_{\ell=0}^{\infty}\sum_{K=-\ell}^{\ell}\frac{2\ell+1}{4\pi}\varphi_{\ell K}\,P_\ell^K(\vec{\Omega})=2\pi\,\varphi(\vec{\Omega}).\end{aligned} \tag{2.15}$$

Now we shall write the integral $\int\limits_{4\pi} d\vec{\Omega}'\,\varphi(\mu')\,\delta(\mu_0-1)$ for plane geometry:

$$\int\limits_{4\pi} d\vec{\Omega}'\,\varphi(\mu')\,\delta(\mu_0-1)=\sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}\int\limits_{4\pi} d\vec{\Omega}'\,\varphi(\mu')\,P_\ell(\mu_0)=$$

$$=\sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}\int_0^{2\pi} d\Psi'\int_{-1}^{1} d\mu'\,\varphi(\mu')\Big[P_\ell(\mu)P_\ell(\mu')+$$

$$+2\sum_{K=1}^{\ell}\frac{(\ell-K)!}{(\ell+K)!}P_\ell^K(\mu)P_\ell^K(\mu')\cos K(\Psi-\Psi')\Big]=$$

$$=\sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}P_\ell(\mu)\,2\pi\int_{-1}^{1} d\mu'\,\varphi(\mu')P_\ell(\mu')=2\pi\sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}\varphi_\ell\,P_\ell(\mu)=$$

$$=2\pi\,\varphi(\mu)\,. \tag{2.16}$$

Remark: During the calculation of the integrals the following relations had been used:

$$P_0^0(\vec{\Omega})=P_0^{0*}(\vec{\Omega})=1\,,\quad \int_0^{2\pi} d\Psi'\cos K(\Psi-\Psi')=0\,.$$

Now we shall write:

$$\int\limits_{4\pi} d\vec{\Omega}'\big[\varphi_r(\vec{r},\vec{\Omega}',E')+\varphi_c(\vec{r},\vec{\Omega}',E')\big]\delta(\mu_0-1)=2\pi\big[\varphi_r(\vec{r},\vec{\Omega},E')+\varphi_c(\vec{r},\vec{\Omega},E')\big],$$

$$\int_E^{\infty} dE'\,w_{L+1}(\vec{r},E'\to E)\int\limits_{4\pi} d\vec{\Omega}'\big[\varphi_r(\vec{r},\vec{\Omega}',E')+\varphi_c(\vec{r},\vec{\Omega}',E')\big]\frac{\delta(\mu_0-1)}{2\pi}=$$

$$=\int_E^{\infty} dE'\,w_{L+1}(\vec{r},E'\to E)\big[\varphi_r(\vec{r},\vec{\Omega},E')+\varphi_c(\vec{r},\vec{\Omega},E')\big]\,.$$

The expression (2.13) may be written in the form

$$\int_E^\infty dE' \int_{4\pi} d\vec{\Omega}'\, w(\vec{r},\mu_0,E'\to E)\left[\varphi_r(\vec{r},\vec{\Omega}',E') + \varphi_c(\vec{r},\vec{\Omega}',E')\right] =$$

$$= \int_E^\infty dE'\, w_{L+1}(\vec{r},E'\to E)\left[\varphi_r(\vec{r},\vec{\Omega},E') + \varphi_c(\vec{r},\vec{\Omega},E')\right] +$$

$$+ \int_E^\infty dE' \int_{4\pi} d\vec{\Omega}' \sum_{l=0}^{L} \sum_{k=-l}^{l} \frac{2l+1}{4\pi} w_l^*(\vec{r},E'\to E) P_l^k(\vec{\Omega}) P_l^{k*}(\vec{\Omega}')\left[\varphi_r(\vec{r},\vec{\Omega}',E') + \varphi_c(\vec{r},\vec{\Omega}',E')\right].$$

The kinetic equation (2.6) for φ_c takes the form

$$\vec{\Omega}\nabla\varphi_c(\vec{r},\vec{\Omega},E) + \Sigma(\vec{r},E)\varphi_c(\vec{r},\vec{\Omega},E) = \int_E^\infty dE'\, w_{L+1}(\vec{r},E'\to E)\left[\varphi_r(\vec{r},\vec{\Omega},E') + \varphi_c(\vec{r},\vec{\Omega},E')\right] +$$

$$+ \int_E^\infty dE' \int_{4\pi} d\vec{\Omega}' \sum_{l=0}^{L} \sum_{k=-l}^{l} \frac{2l+1}{4\pi} w_l^*(\vec{r},E'\to E) P_l^k(\vec{\Omega}) P_l^{k*}(\vec{\Omega}')\left[\varphi_r(\vec{r},\vec{\Omega}',E') + \varphi_c(\vec{r},\vec{\Omega}',E')\right]. \quad (2.17)$$

Let us add the equations (2.5) and (2.17):

$$\vec{\Omega}\nabla\left[\varphi_r(\vec{r},\vec{\Omega},E) + \varphi_c(\vec{r},\vec{\Omega},E)\right] + \Sigma(\vec{r},E)\left[\varphi_r(\vec{r},\vec{\Omega},E) + \varphi_c(\vec{r},\vec{\Omega},E)\right] =$$

$$= \int_E^\infty dE' \int_{4\pi} d\vec{\Omega}' \sum_{l=0}^{L} \sum_{k=-l}^{l} \frac{2l+1}{4\pi} w_l^*(\vec{r},E'\to E) P_l^k(\vec{\Omega}) P_l^{k*}(\vec{\Omega}')\left[\varphi_r(\vec{r},\vec{\Omega}',E') + \varphi_c(\vec{r},\vec{\Omega}',E')\right] +$$

$$+ \int_E^\infty dE'\, w_{L+1}(\vec{r},E'\to E)\left[\varphi_r(\vec{r},\vec{\Omega},E') + \varphi_c(\vec{r},\vec{\Omega},E')\right] + S(\vec{r},\vec{\Omega},E). \quad (2.18)$$

For the plane geometry the (2.18) takes the form

$$\mu\frac{d}{dz}\left[\varphi_r(z,\mu,E) + \varphi_c(z,\mu,E)\right] + \Sigma(z,E)\left[\varphi_r(z,\mu,E) + \varphi_c(z,\mu,E)\right] =$$

$$= \int_E^\infty dE' \int_0^{2\pi} d\psi' \int_{-1}^{1} d\mu' \sum_{l=0}^{L} \frac{2l+1}{4\pi} w_l^*(z,E'\to E)\Big\{P_l(\mu)P_l(\mu') +$$

$$+ 2\sum_{k=1}^{l} \frac{(l-k)!}{(l+k)!} P_l^k(\mu) P_l^k(\mu')\cos k(\psi-\psi')\Big\}\left[\varphi_r(z,\mu',E') + \varphi_c(z,\mu',E')\right] +$$

$$+ \int_E^\infty dE'\, w_{L+1}(z,E'\to E)\left[\varphi_r(z,\mu,E') + \varphi_c(z,\mu,E')\right] + S(z,\mu,E). \quad (2.19)$$

After integration according to Ψ the equation (2.19) can be written as :

$$\mu \frac{d}{dz}\left[\varphi_r(z,\mu,E)+\varphi_c(z,\mu,E)\right]+\Sigma(z,E)\left[\varphi_r(z,\mu,E)+\varphi_c(z,\mu,E)\right]=$$

$$=\int_E^\infty dE'\int_{-1}^{1}d\mu'\sum_{\ell=0}^{L}\frac{2\ell+1}{2}w_\ell^*(z,E'\to E)\left[\varphi_r(z,\mu',E')+\varphi_c(z,\mu',E')\right]P_\ell(\mu)P_\ell(\mu')+$$

$$+\int_E^\infty dE'\, w_{L+1}(z,E'\to E)\left[\varphi_r(z,\mu,E')+\varphi_c(z,\mu,E')\right]+S(z,\mu,E). \qquad (2.20)$$

Let us replace the φ_r and φ_c by the Fourier series

$$\varphi_r(z,\mu',E')=\sum_{n=0}^{\infty}\frac{2n+1}{4\pi}\varphi_{r,n}(z,E')P_n(\mu'),$$

$$\varphi_{r,n}(z,E')=2\pi\int_{-1}^{1}d\mu'\,\varphi_r(z,\mu',E')P_n(\mu'),$$

$$\varphi_c(z,\mu',E')=\sum_{n=0}^{\infty}\frac{2n+1}{4\pi}\varphi_{c,n}(z,E')P_n(\mu'),$$

$$\varphi_{c,n}(z,E')=2\pi\int_{-1}^{1}d\mu'\,\varphi_c(z,\mu',E')P_n(\mu').$$

The scatter integral in (2.20) takes the form

$$\int_E^\infty dE'\int_{-1}^{1}d\mu'\sum_{\ell=0}^{L}\frac{2\ell+1}{2}w_\ell^*(z,E'\to E)P_\ell(\mu)P_\ell(\mu')\sum_{n=0}^{\infty}\frac{2n+1}{4\pi}\left[\varphi_{r,n}(z,E')+\varphi_{c,n}(z,E')\right]P_n(\mu')=$$

$$=\int_E^\infty dE'\sum_{\ell=0}^{L}\frac{2\ell+1}{2}w_\ell^*(z,E'\to E)P_\ell(\mu)\sum_{n=0}^{\infty}\frac{2n+1}{4\pi}\left[\varphi_{r,n}(z,E')+\varphi_{c,n}(z,E')\right]\int_{-1}^{1}d\mu'\,P_n(\mu')P_\ell(\mu')=$$

$$=\int_E^\infty dE'\sum_{\ell=0}^{L}\frac{2\ell+1}{2}w_\ell^*(z,E'\to E)P_\ell(\mu)\sum_{n=0}^{\infty}\frac{2n+1}{4\pi}\left[\varphi_{r,n}(z,E')+\varphi_{c,n}(z,E')\right]\frac{2}{2n+1}\delta_{n\ell}=$$

$$=\int_E^\infty dE'\sum_{\ell=0}^{L}\frac{2\ell+1}{4\pi}w_\ell^*(z,E'\to E)\left[\varphi_{r,\ell}(z,E')+\varphi_{c,\ell}(z,E')\right]P_\ell(\mu).$$

Let us put it into (2.20):

$$\mu \frac{d}{dz}[\varphi_r(z,\mu,E)+\varphi_c(z,\mu,E)]+\Sigma(z,E)[\varphi_r(z,\mu,E)+\varphi_c(z,\mu,E)]=$$

$$=\int_E^\infty dE' \sum_{\ell=0}^{L} \frac{2\ell+1}{4\pi} w_\ell^*(z,E'\to E)[\varphi_{r,\ell}(z,E')+\varphi_{c,\ell}(z,E')]P_\ell(\mu)+$$

$$+\int_E^\infty dE' w_{L+1}(z,E'\to E)[\varphi_r(z,\mu,E')+\varphi_c(z,\mu,E')]+S(z,\mu,E). \qquad (2.21)$$

2.4. Multi-Group Equations

The equation (2.21) may be written in the multi-group form

$$\mu \frac{d}{dz}[\varphi_r^j(z,\mu)+\varphi_c^j(z,\mu)]+\Sigma^j(z)[\varphi_r^j(z,\mu)+\varphi_c^j(z,\mu)]=$$

$$=\sum_{i=1}^{j} \sum_{\ell=0}^{L} \frac{2\ell+1}{4\pi} w_\ell^{i\to j *}(z)[\varphi_{r,\ell}^j(z)+\varphi_{c,i}^{i}(z)]P_\ell(\mu)+$$

$$+\sum_{i=1}^{j} w_{L+1}^{i\to j}(z)[\varphi_r^i(z,\mu)+\varphi_c^i(z,\mu)]+S^j(z,\mu),$$

where $$\varphi^i(z,\mu)=\int_{E_i}^{E_{i-1}} dE\,\varphi(z,\mu,E). \qquad (2.22)$$

The equation (2.22) may be rewritten as

$$\mu \frac{d}{dz}[\varphi_r^j(z,\mu)+\varphi_c^j(z,\mu)]+[\Sigma^j(z)-w_{L+1}^{j\to j}(z)][\varphi_r^j(z,\mu)+\varphi_c^j(z,\mu)]=$$

$$=\sum_{i=1}^{j} \sum_{\ell=0}^{L} \frac{2\ell+1}{4\pi} w_\ell^{i\to j *}(z)[\varphi_{r,\ell}^i(z)+\varphi_{c,\ell}^i(z)]P_\ell(\mu)+$$

$$+\sum_{i=1}^{j-1} w_{L+1}^{i\to j}(z)[\varphi_r^i(z,\mu)+\varphi_c^i(z,\mu)]+S^j(z,\mu). \qquad (2.23)$$

Let us divide the last equation into two equations for φ_r^j and φ_c^j as previously.

$$\mu \frac{d}{dz}\varphi_r^j(z,\mu)+[\Sigma^j(z)-w_{L+1}^{j\to j}(z)]\varphi_r^j(z,\mu)=S^j(z,\mu), \qquad (2.24)$$

$$\mu \frac{d}{dz}\varphi_c^j(z,\mu)+\left[\Sigma^j(z)-\overset{j\to j}{w}_{L+1}(z)\right]\varphi_c^j(z,\mu)=\sum_{i=1}^{j-1}\overset{i\to j}{w}_{L+1}(z)\left[\varphi_r^i(z,\mu)+\varphi_c^i(z,\mu)\right]+$$

$$+\sum_{i=1}^{j}\sum_{l=0}^{L}\frac{2l+1}{4\pi}\overset{i\to j}{w_l^*}(z)\left[\varphi_{r,l}^i(z)+\varphi_{c,l}^i(z)\right]P_l(\mu). \qquad (2.25)$$

Now let us use for the previous discussion a different procedure.

The equation (2.1) for plane geometry takes the form

$$\mu \frac{d}{dz}\varphi(z,\mu,E)+\Sigma(z,E)\varphi(z,\mu,E)=\int_E^\infty dE'\int_0^{2\pi}d\psi'\int_{-1}^{1}d\mu'\, w(z,\mu_0,E'\to E)\varphi(z,\mu',E')+$$

$$+S(z,\mu,E). \qquad (2.26)$$

In the multigroup form we get

$$\mu\frac{d}{dz}\varphi^j(z,\mu)+\Sigma^j(z)\varphi^j(z,\mu)=\sum_{i=1}^{j}\int_0^{2\pi}d\psi'\int_{-1}^{1}d\mu'\,\overset{i\to j}{w}(\mu_0,z)\varphi^i(z,\mu')+S^j(z,\mu). \qquad (2.27)$$

Let us write

$$\overset{i\to j}{w}(z,\mu_0)=\sum_{l=0}^{\infty}\frac{2l+1}{4\pi}\overset{i\to j}{w_l}(z)P_l(\mu_0)=$$

$$=\sum_{l=0}^{\infty}\frac{2l+1}{4\pi}\overset{i\to j}{w_l}(z)\left[P_l(\mu)P_l(\mu')+2\sum_{k=1}^{l}\frac{(l-k)!}{(l+k)!}P_l^k(\mu)P_l^k(\mu')\cos k(\psi-\psi')\right],$$

$$\overset{i\to j}{w_l}(z)=2\pi\int_{-1}^{1}d\mu_0\,\overset{i\to j}{w}(z,\mu_0)P_l(\mu_0),$$

$$\varphi^j(z,\mu)=\sum_{n=0}^{\infty}\frac{2n+1}{4\pi}\varphi_n^j(z)P_n(\mu),$$

$$\varphi_n^j(z)=2\pi\int_{-1}^{1}d\mu\,\varphi^j(z,\mu)P_n(\mu).$$

The scatter integral in (2.27) may be written as

$$\sum_{i=1}^{j}\int_{0}^{2\pi} d\varphi' \int_{-1}^{1} d\mu' \, w^{i\to j}(z,\mu_0)\,\varphi^{i}(z,\mu') =$$

$$=\sum_{i=1}^{j}\int_{0}^{2\pi} d\varphi' \int_{-1}^{1} d\mu' \sum_{l=0}^{\infty}\frac{2l+1}{4\pi}\, w_l^{i\to j}(z)P_l(\mu_0)\sum_{n=0}^{\infty}\frac{2n+1}{4\pi}\,\varphi_n^{i}(z)P_n(\mu') =$$

$$=\sum_{i=1}^{j}\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\frac{2l+1}{4\pi}\,\frac{2n+1}{4\pi}\, w_l^{i\to j}(z)\varphi_n^{i}(z)\Big[P_l(\mu)\int_{-1}^{1}d\mu' P_l(\mu')P_n(\mu')\int_{0}^{2\pi}d\varphi' +$$

$$+2\sum_{k=1}^{l}\frac{(l-k)!}{(l+k)!}P_l^k(\mu)\int_{-1}^{1}d\mu' P_l^k(\mu')P_n^k(\mu')\int_{0}^{2\pi}d\varphi'\cos k(\varphi-\varphi')\Big] =$$

$$=\sum_{i=1}^{j}\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\frac{2l+1}{4\pi}\,\frac{2n+1}{4\pi}\, w_l^{i\to j}(z)P_l(\mu)\frac{4\pi}{2n+1}\,\delta_{nl}\,\varphi_n^{i}(z) =$$

$$=\sum_{i=1}^{j}\sum_{l=0}^{\infty}\frac{2l+1}{4\pi}\, w_l^{i\to j}(z)\varphi_l^{j}(z)P_l(\mu).$$

Now we shall use the assumptions (2.8) and (2.9).

$$\sum_{i=1}^{j}\sum_{l=0}^{\infty}\frac{2l+1}{4\pi}\, w_l^{i\to j}(z)\,\varphi_l^{i}(z)P_l(\mu) =$$

$$=\sum_{i=1}^{j}\Big[\sum_{l=0}^{L}\frac{2l+1}{4\pi}\, w_l^{i\to j}(z)\varphi_l^{i}(z)P_l(\mu)+\sum_{l=L+1}^{\infty}\frac{2l+1}{4\pi}\, w_l^{i\to j}(z)\varphi_l^{i}(z)P_l(\mu)\Big] =$$

$$=\sum_{i=1}^{j}\Big[\sum_{l=0}^{L}\frac{2l+1}{4\pi}\, w_l^{i\to j}(z)\,\varphi_l^{i}(z)P_l(\mu) + w_{L+1}^{i\to j}(z)\sum_{l=0}^{\infty}\frac{2l+1}{4\pi}\,\varphi_l^{j}(z)P_l(\mu) -$$

$$- w_{L+1}^{i\to j}(z)\sum_{l=0}^{L}\frac{2l+1}{4\pi}\,\varphi_l^{i}(z)P_l(\mu)\Big] =$$

$$=\sum_{i=1}^{j}\Big\{\sum_{l=0}^{L}\frac{2l+1}{4\pi}\Big[w_L^{i\to j}(z)-w_{L+1}^{i\to j}(z)\Big]\varphi_l^{i}(z)P_l(\mu)+w_{L+1}^{i\to j}(z)\,\varphi^{i}(z,\mu)\Big\}.$$

Let us put the final expression to (2.27):

$$\mu\frac{d}{dz}\varphi^{j}(z,\mu)+\Sigma^{j}(z)\varphi^{j}(z,\mu)=\sum_{i=0}^{j}\sum_{\ell=0}^{L}\frac{2\ell+1}{4\pi}\left[w_{\ell}^{i\to j}(z)-w_{L+1}^{i\to j}(z)\right]\varphi_{\ell}^{i}(z)P_{\ell}(\mu)$$
$$+\sum_{i=1}^{j}w_{L+1}^{i\to j}(z)\varphi^{i}(z,\mu)+S^{j}(z,\mu). \tag{2.28}$$

Further we get

$$\mu\frac{d}{dz}\varphi^{j}(z,\mu)+\left[\Sigma^{j}(z)-w_{L+1}^{j\to j}(z)\right]\varphi^{j}(z,\mu)=\sum_{i=1}^{j-1}w_{\ell}^{i\to j}(z)\varphi^{i}(z,\mu)+$$
$$+\sum_{i=1}^{j}\sum_{\ell=0}^{L}\frac{2\ell+1}{4\pi}\left[w_{\ell}^{i\to j}(z)-w_{L+1}^{i\to j}(z)\right]\varphi_{\ell}^{i}(z)P_{\ell}(\mu)+S^{j}(z,\mu). \tag{2.29}$$

The equation (2.29) may be divided as:

$$\mu\frac{d}{dz}\varphi_{r}^{j}(z,\mu)+\left[\Sigma^{j}(z)-w_{L+1}^{j\to j}(z)\right]\varphi_{r}^{j}(z,\mu)=S^{j}(z,\mu), \tag{2.30}$$

$$\mu\frac{d}{dz}\varphi_{c}^{j}(z,\mu)+\left[\Sigma^{j}(z)-w_{L+1}^{j\to j}(z)\right]\varphi_{c}^{j}(z,\mu)=\sum_{i=1}^{j-1}w_{L+1}^{i\to j}(z)\left[\varphi_{r}^{i}(z,\mu)+\varphi_{c}^{i}(z,\mu)\right]+$$
$$+\sum_{i=1}^{j}\sum_{\ell=0}^{L}\frac{2\ell+1}{4\pi}w_{\ell}^{*\,i\to j}(z)\left[\varphi_{r,\ell}^{i}(z)+\varphi_{c,\ell}^{i}(z)\right]P_{\ell}(\mu), \tag{2.31}$$

where $w_{\ell}^{*\,i\to j}(z)=w_{\ell}^{i\to j}(z)-w_{L+1}^{i\to j}(z)$, $\ell=0,1,2,\dots,L$.

So the (2.30) and (2.31) may be written in the form(for shielding calculations)

$$\mu\frac{d}{dz}\varphi_{r}^{j}(z,\mu)+\left[\Sigma^{j}(z)-\Sigma_{s,L+1}^{j\to j}(z)\right]\varphi_{r}^{j}(z,\mu)=S^{j}(z,\mu), \tag{2.32}$$

$$\mu\frac{d}{dz}\varphi_{c}^{j}(z,\mu)+\left[\Sigma^{j}(z)-\Sigma_{s,L+1}^{j\to j}(z)\right]\varphi_{c}^{j}(z,\mu)=\sum_{i=1}^{j-1}\Sigma_{s,L+1}^{i\to j}(z)\left[\varphi_{r}^{i}(z,\mu)+\varphi_{c}^{i}(z,\mu)\right]$$
$$+\sum_{i=1}^{j}\sum_{\ell=0}^{L}\frac{2\ell+1}{4\pi}\Sigma_{s,\ell}^{i\to j}(z)\left[\varphi_{r,\ell}^{i}(z)+\varphi_{c,\ell}^{i}(z)\right]P_{\ell}(\mu), \tag{2.33}$$

where $\quad w(z,\mu_0,E'\to E)=\Sigma_s(z,\mu_0,E'\to E)=\Sigma_{s,el}+\Sigma_{s,in}$,

$$\overset{i\to j}{\Sigma^{*}_{s,\ell}}(z)=\overset{i\to j}{\Sigma_{s,\ell}}(z)-\overset{i\to j}{\Sigma_{s,L+1}}(z),\quad \ell=0,1,2,\dots,L.$$

In /4/ the equation (2.29) has the form (2.34):
Let us add and subtract to (2.29) the expression

$$\sum_{i=1}^{j-1}\overset{i\to j}{w_{L+1}}(z)\varphi^{j}(z,\mu)\ :$$

Then we get

$$\mu\frac{d}{dz}\varphi^{j}(z,\mu)+\Big[\Sigma^{j}(z)-\sum_{i=1}^{j}\overset{i\to j}{w_{L+1}}(z)\Big]\varphi^{j}(z,\mu)=$$

$$=\sum_{i=1}^{j}\sum_{\ell=0}^{L}\frac{2\ell+1}{4\pi}\overset{i\to j}{w^{*}_{\ell}}(z)\varphi^{i}_{\ell}(z)P_{\ell}(\mu)+$$

$$+\sum_{i=1}^{j-1}\overset{i\to j}{w_{L+1}}(z)\Big[\varphi^{i}(z,\mu)-\varphi^{j}(z,\mu)\Big]+S^{j}(z,\mu).\qquad(2.34)$$

Dividing (2.34) into two equation for φ_r and φ_c we get (2.32) and (2.33), where

$$\overset{i\to j}{\Sigma_{s,\ell}}(z)=\sum_{g,k}N_g(z)\overset{i\to j}{\sigma_{s_k,g}}P_{gk,\ell}\ .$$

2.5. Solution of φ_c Using PN - Approximation

Let us write equation (2.33) in the PN-approximation:

$$\varphi^{j}_{c}(z,\mu)=\sum_{n=0}^{\infty}\frac{2n+1}{4\pi}\varphi^{j}_{c,n}(z)P_n(\mu),$$

$$\varphi^{j}_{c,n}(z)=2\pi\int_{-1}^{1}d\mu\,\varphi^{j}_{c}(z,\mu)P_n(\mu),\qquad(2.36)$$

$$\Sigma^{j*}(z) = \Sigma^{j}(z) - \Sigma^{i\to j}_{s,L+1}(z),$$

$$\sum_{n=0}^{\infty} \frac{2n+1}{4\pi} \frac{d}{dz} \varphi^{j}_{c,n}(z) \mu P_n(\mu) + \Sigma^{j*}(z) \sum_{n=0}^{\infty} \frac{2n+1}{4\pi} \varphi^{j}_{c,n}(z) P_n(\mu) =$$
$$= \sum_{i=1}^{j} \sum_{l=0}^{L} \frac{2l+1}{4\pi} \Sigma^{*\,i\to j}_{s,l}(z) \left[\varphi^{i}_{r,l}(z) + \varphi^{i}_{c,l}(z) \right] P_l(\mu) +$$
$$+ \sum_{i=1}^{j-1} \Sigma^{i\to j}_{s,l+1}(z) \sum_{n=0}^{\infty} \frac{2n+1}{4\pi} \left[\varphi^{i}_{r,n}(z) + \varphi^{i}_{c,n}(z) \right] P_n(\mu) .$$

Let us multiply the last expression by $P_m(\mu)$ and integrate according to μ for $\mu \in \langle -1, 1 \rangle$:

$$\sum_{n=0}^{\infty} (2n+1) \frac{d}{dz} \varphi^{j}_{c,n}(z) \int_{-1}^{1} d\mu\, \mu P_n(\mu) P_m(\mu) + \Sigma^{j*}(z) \sum_{n=0}^{\infty} (2n+1) \varphi^{j}_{c,n}(z) \int_{-1}^{1} d\mu\, P_n(\mu) P_m(\mu) =$$
$$= \sum_{i=1}^{j} \sum_{l=0}^{L} (2l+1) \Sigma^{*\,i\to j}_{s,l}(z) \left[\varphi^{i}_{r,l}(z) + \varphi^{i}_{c,l}(z) \right] \int_{-1}^{1} d\mu\, P_l(\mu) P_m(\mu) +$$
$$+ \sum_{i=1}^{j-1} \Sigma^{i\to j}_{s,L+1}(z) \sum_{n=0}^{\infty} (2n+1) \left[\varphi^{i}_{r,n}(z) + \varphi^{i}_{c,n}(z) \right] \int_{-1}^{1} d\mu\, P_n(\mu) P_m(\mu) .$$

(2.37)

It applies

$$\mu P_n(\mu) = \frac{n}{2n+1} P_{n-1}(\mu) + \frac{n}{2n+1} P_{n+1}(\mu)$$

$$\sum_{n=0}^{\infty}(2n+1)\frac{d}{dz}\varphi^{j}_{c,n}(z)\int_{-1}^{1}d\mu\,\mu P_n(\mu)P_m(\mu)=\sum_{n=0}^{\infty}n\frac{d}{dz}\varphi^{j}_{c,n}(z)\int_{-1}^{1}d\mu\,P_{n-1}(\mu)P_m(\mu)\ldots$$

$$+\sum_{n=0}^{\infty}(n+1)\frac{d}{dz}\varphi^{j}_{c,n}(z)\int_{-1}^{1}d\mu\,P_{n+1}(\mu)P_m(\mu)=\sum_{n=1}^{\infty}n\frac{d}{dz}\varphi^{j}_{c,n}(z)\int_{-1}^{1}d\mu\,P_n(\mu)P_m(\ldots)\ldots$$

$$\sum_{n=1}^{\infty}n\frac{d}{dz}\varphi^{j}_{c,n-1}(z)\int_{-1}^{1}d\mu\,P_n(\mu)P_m(\mu)=$$

$$2\sum_{n=0}^{\infty}\left[\frac{n+1}{2n+1}\frac{d}{dz}\varphi^{j}_{c,n+1}(z)+\frac{n}{2n+1}\frac{d}{dz}\varphi^{j}_{c,n-1}(z)\right]\delta_{nm}=$$

$$2\left[\frac{m+1}{2m+1}\frac{d}{dz}\varphi^{j}_{c,m+1}(z)+\frac{m}{2m+1}\frac{d}{dz}\varphi^{j}_{c,m-1}(z)\right].$$

Now the equation (2.37) turns into an infinite system of equations for $m = 0, 1, 2, \ldots$:

$$\frac{m+1}{2m+1}\frac{d}{dz}\varphi^{j}_{c,m+1}(z)+\frac{m}{2m+1}\frac{d}{dz}\varphi^{j}_{c,m-1}(z)+\Sigma^{j*}(z)\varphi^{j}_{c,m}(z)=$$

$$=\sum_{i=1}^{j}\sum_{\ell=0}^{L}\Sigma^{i\to j}_{s,\ell}{}^{*}(z)\left[\varphi^{i}_{r,\ell}(z)+\varphi^{i}_{c,\ell}(z)\right]\delta_{\ell m}+$$

$$+\sum_{i=1}^{j-1}\Sigma^{i\to j}_{s,L+1}(z)\left[\varphi^{i}_{r,m}(z)+\varphi^{i}_{c,m}(z)\right]. \qquad (2.38)$$

After some arrangement the equations (2.38) turn into an infinite system of equations for $n = 0, 1, 2, \ldots$:

$$\frac{n+1}{2n+1}\frac{d}{dz}\varphi^{j}_{c,n+1}(z)+\frac{n}{2n+1}\frac{d}{dz}\varphi^{j}_{c,n-1}(z)+\Sigma^{j*}(z)\varphi^{j}_{c,n}(z)=$$

$$=\sum_{i=1}^{j}\Sigma^{i\to j}_{s,n}{}^{*}(z)\left[\varphi^{i}_{r,n}(z)+\varphi^{j}_{c,n}(z)\right]\delta_{L}+\sum_{i=1}^{j-1}\Sigma^{i\to j}_{s,L+1}(z)\left[\varphi^{i}_{r,n}(z)+\varphi^{i}_{c,n}(z)\right], \qquad (2.39)$$

$$\delta_L = \begin{cases} 1 & \text{for } n \leq L \\ 0 & \text{for } n > L. \end{cases}$$

In the PN - approximation we shall restrain ourselves for the first N + 1 equations of (2.39) putting $\varphi^j_{c,N+1} \sim 0$.
For L ≤ N (look at (2.8), (2.9)) we get in the PN - approximation

$$\frac{n+1}{2n+1}\frac{d}{dz}\varphi^j_{c,n+1}(z) + \frac{n}{2n+1}\frac{d}{dz}\varphi^j_{c,n-1}(z) + \left[\Sigma^j(z) - \Sigma^{j\to j}_{s,L+1}(z)\right]\varphi^j_{c,n}(z) =$$

$$= \sum_{i=1}^{j-1} \Sigma^{i\to j}_{s,n}\left[\varphi^i_{r,n}(z) + \varphi^i_{c,n}(z)\right]\delta_L + \left[\Sigma^{j\to j}_{s,n}(z) - \Sigma^{j\to j}_{s,L+1}(z)\right]\left[\varphi^j_{r,n}(z) + \varphi^j_{c,n}(z)\right]\delta_L +$$

$$+ \sum_{i=1}^{j-1} \Sigma^{i\to j}_{s,L+1}(z)\left[\varphi^i_{r,n}(z) + \varphi^i_{c,n}(z)\right](1 - \delta_L),$$

$$n = 0, 1, 2, \dots, N; \quad \varphi^j_{c,N+1}(z) = 0; \quad L \leq N. \tag{2.40}$$

After some arrangements we obtain

$$\frac{n+1}{2n+1}\frac{d}{dz}\varphi^j_{c,n+1}(z) + \frac{n}{2n+1}\frac{d}{dz}\varphi^j_{c,n-1}(z) +$$

$$+ \left[\Sigma^j(z) - \Sigma^{j\to j}_{s,n}(z)\delta_L - \Sigma^{j\to j}_{s,L+1}(z)(1 - \delta_L)\right]\varphi^j_{c,n}(z) =$$

$$= \sum_{i=1}^{j-1} \Sigma^{i\to j}_{s,n}(z)\left[\varphi^i_{r,n}(z) + \varphi^i_{c,n}(z)\right]\delta_L + \left[\Sigma^{j\to j}_{s,n}(z) - \Sigma^{j\to j}_{s,L+1}(z)\right]\delta_L\,\varphi^j_{r,n}(z) +$$

$$+ \sum_{i=1}^{j-1} \Sigma^{i\to j}_{s,L+1}(z)(1 - \delta_L)\left[\varphi^i_{r,n}(z) + \varphi^i_{c,n}(z)\right], \tag{2.41}$$

$$n = 0, 1, 2, \dots, N; \quad \varphi^j_{c,N+1}(z) = 0; \quad L \leq N; \quad \delta_{L \geq n} = 1, \quad \delta_{L < n} = 0.$$

2.6. Removal-Diffusion and Removal - P1 Systems Based on the Transport Theory

For the removal - P1 system we get from (2.41) for N = 1, L = 1

$$\mu \frac{d}{dz}\varphi_r^j(z,\mu) + \left[\Sigma^j(z) - \Sigma_{s,2}^{j\to j}(z)\right]\varphi_r^j(z,\mu) = S^j(z,\mu), \tag{2.42}$$

$$\frac{d}{dz}\varphi_{c,1}^j(z) + \left[\Sigma^j(z) - \Sigma_{s,0}^{j\to j}(z)\right]\varphi_{c,0}^j(z) = \sum_{i=1}^{j-1}\Sigma_{s,0}^{i\to j}(z)\left[\varphi_{r,0}^i(z) + \varphi_{c,0}^i(z)\right] + \left[\Sigma_{s,0}^{j\to j}(z) - \Sigma_{s,2}^{j\to j}(z)\right]\varphi_{r,0}^j(z), \tag{2.43}$$

$$\frac{1}{3}\frac{d}{dz}\varphi_{c,0}^j + \left[\Sigma^j(z) - \Sigma_{s,1}^{j\to j}(z)\right]\varphi_{c,1}^j(z) = \sum_{i=1}^{j-1}\Sigma_{s,1}^{i\to j}(z)\left[\varphi_{r,1}^i(z) + \varphi_{c,1}^i(z)\right] + \left[\Sigma_{s,1}^{j\to j}(z) - \Sigma_{s,2}^{j\to j}(z)\right]\varphi_{r,1}^j(z), \tag{2.44}$$

where

$$\varphi_{r,0}^j(z) = 2\pi\int_{-1}^{1} d\mu\, \varphi_r^j(z,\mu),$$

$$\varphi_{r,1}^j(z) = 2\pi\int_{-1}^{1} d\mu\, \mu\, \varphi_r^j(z,\mu).$$

Similarly for the removal - diffusion system putting $Q_1(z) \sim 0$, where $Q_1(z)$ is the second Fourier coefficient of the source-expansion, we get for N = 1, L = 0:

$$\mu \frac{d}{dz}\varphi_r^j(z,\mu) + \left[\Sigma^j(z) - \Sigma_{s,1}^{j\to j}(z)\right]\varphi_r^j(z,\mu) = S^j(z,\mu) \tag{2.45}$$

$$\frac{d}{dz}\varphi_{c,1}^j(z) + \left[\Sigma^j(z) - \Sigma_{s,0}^{j\to j}(z)\right]\varphi_{c,0}^j(z) = \sum_{i=1}^{j-1}\Sigma_{s,0}^{i\to j}(z)\left[\varphi_{r,0}^i(z) + \varphi_{c,0}^i\right] + \left[\Sigma_{s,0}^{j\to j}(z) - \Sigma_{s,1}^{j\to j}(z)\right]\varphi_{r,0}^j(z), \tag{2.46}$$

$$\frac{1}{3}\frac{d}{dz}\varphi_{c,0}^{j}+\left[\Sigma^{j}(z)-\Sigma_{s,1}^{j\to j}(z)\right]\varphi_{c,1}^{j}(z)=0 \quad . \tag{2.47}$$

To determine the sources it is advisable to use a finer division of removal groups. Then we may write:

$$\mu\frac{d}{dz}\varphi_{r}^{\alpha}(z,\mu)+\Sigma_{rem}^{\alpha}(z)\varphi_{r}^{\alpha}(z,\mu)=S^{\alpha}(z,\mu), \tag{2.48}$$

where for the removal-P1 approximation:

$$\Sigma_{rem}^{\alpha}(z)=\tilde{\Sigma}^{\alpha}(z)-\tilde{\tilde{\Sigma}}_{s,2}^{\alpha\to\alpha}(z). \tag{2.49}$$

(The wave-line above Σ refers to the maner of the veraging of data. Look at chapter 3).

For the removal-diffusion approximation is

$$\Sigma_{rem}^{\alpha}(z)=\tilde{\Sigma}^{\alpha}(z)-\tilde{\tilde{\Sigma}}_{s,1}^{\alpha\to\alpha}(z). \tag{2.50}$$

Marking $\Sigma_{n}^{j}(z)=\Sigma^{j}(z)-\Sigma_{s,n}^{j\to j}(z)$ we get for φ_c in the P1 - approximation

$$\frac{d}{dz}\varphi_{c,1}^{j}(z)+\Sigma_{0}^{j}\varphi_{c,0}^{j}(z)=\sum_{i=1}^{j-1}\Sigma_{s,0}^{i\to j}(z)\varphi_{c,c}^{i}(z)+\sum_{\alpha=1}^{\beta(j)-1}\tilde{\tilde{\Sigma}}_{s,0}^{\alpha\to j}(z)\varphi_{r,0}^{\alpha}(z)+$$
$$+\sum_{\alpha=\beta(j)}^{\gamma(j)}\left[\tilde{\tilde{\Sigma}}_{s,0}^{\alpha\to j}(z)-\tilde{\tilde{\Sigma}}_{s,2}^{\alpha\to j}(z)\right]\varphi_{r,0}^{\alpha}(z), \tag{2.51}$$

$$\frac{1}{3}\frac{d}{dz}\varphi_{c,0}^{j}(z)+\Sigma_{1}^{j}(z)\varphi_{c,1}^{j}(z)=\sum_{i=1}^{j-1}\Sigma_{s,1}^{i\to j}(z)\varphi_{c,1}^{i}(z)+$$
$$+\sum_{\alpha=1}^{\beta(j)-1}\tilde{\tilde{\Sigma}}_{s,1}^{\alpha\to j}(z)\varphi_{r,1}^{\alpha}(z)+\sum_{\alpha=\beta(j)}^{\gamma(j)}\left[\tilde{\tilde{\Sigma}}_{s,1}^{\alpha\to j}(z)-\tilde{\tilde{\Sigma}}_{s,2}^{\alpha\to j}(z)\right]\varphi_{r,1}^{\alpha}(z), \tag{2.52}$$

where $\beta(j)$ is the ranking number of the group of the fine division the upper limit of which is the upper limit of the j'th group. Similarly $\gamma(j)$ is the ranking number of the group of the fine division the lower limit of which is the lower limit of the j'th group.

For φ_c in the removal-diffusion approximation the following equations take place

$$\frac{d}{dz}\varphi_{c,1}^{j}(z)+\Sigma_0^j(z)\varphi_{c,0}^{j}(z)=\sum_{i=1}^{j-1}\Sigma_{s,0}^{i\to j}(z)\varphi_{c,0}^{i}(z)+$$

$$+\sum_{\alpha=1}^{\beta(j)-1}\tilde{\tilde{\Sigma}}_{s,0}^{\alpha\to j}(z)\varphi_{r,0}^{\alpha}(z)+\sum_{\alpha=\beta(j)}^{\gamma(j)}\left[\tilde{\tilde{\Sigma}}_{s,0}^{\alpha\to j}(z)-\tilde{\tilde{\Sigma}}_{s,1}^{\alpha\to j}(z)\right]\varphi_{r,0}^{\alpha}(z), \tag{2.53}$$

$$\frac{1}{3}\frac{d}{dz}\varphi_{c,0}^{j}(z)+\Sigma_1^j\varphi_{c,1}^{j}(z)=0. \tag{2.54}$$

3. Energy Dependence of Removal Cross-Sections and Manner of Their Averaging

In the previous chapter we have found out the removal cross-sections.

For the removal diffusion systems we use

$$\Sigma_{rem}(E)=\Sigma(E)-\Sigma_{s,1}(E), \tag{3.1}$$

where $\Sigma_{s,1}(E)=\overline{\mu(E)\Sigma_s(E)}=\sum_{g,h}N_g\sigma_{s_h,g}(E)\mu_{s_h,g}(E)$.

Supposing the isotropy of the inelastic scattering (i.e. $\mu_{in}=0$) we can write

$$\Sigma_{s,1}(E) = \overline{\mu_{el}(E)\Sigma_{el}(E)},$$

and the expression (3.1) consents with the Spiney's definition of the removal cross-section used in the RASH and MAC-RAD models.

For the removal - P1 systems we have got

$$\Sigma_{rem}(E) = \Sigma(E) - \Sigma_{s,2}(E). \qquad (3.2)$$

In the following paragraph we shall compare the expression (3.2) with the semiempiric NRN-method in which the removal cross-section is defined as

$$\Sigma_{rem}(E) = \Sigma(E) - 2\pi\int_{-1}^{1} d\mu\, w(\mu)\Sigma_{el}(E_\mu), \qquad (3.3)$$

where

$$w(\mu) = \begin{cases} 0 & \text{for } \mu < \mu_T \\ 1 & \text{for } \mu > \mu_T \end{cases}$$

On the basis of the comparison with experiments it has been recommended

$$\mu_T = \begin{cases} 0.45 & \text{for } A = 1 \\ 0.6 & \text{for } A > 1 \end{cases},$$

where μ_T is the cosine of the scatter angle in the centre-of-mass system. It is important to observe that in the NRN-method is another important arrangement.

In the solution of the diffusion equation the diffusion coefficient determined with corrections /2/, /3/ is being used :

$$\Sigma_{tr}(E)=\Sigma(E)\left[1-\bar{\mu}(E)\right]-\frac{5}{4}\Sigma_{b,i}(E)+\frac{9}{5}\Sigma_{b,i}(E)\bar{\mu}(E), \qquad (3.4)$$

$\Sigma_{b,i}(E)$ is the cross-section for all absorbing reactions and for those scatter reactions, which contribute to the removal of neutrons below the bottom E_i of the i'th group, where the averiging of the diffusion coefficient takes place.

To determine the group-cross-sections the averiging is to be executed using two ways, i.e. for the "non-scattered" and for the scattered neutrons. For gamma ray these problems are discussed in /6/, for neutrons this formula is used:

$$\tilde{\Sigma}^{\alpha}=\frac{\int_{E_\alpha}^{E_{\alpha-1}} dE\,\Sigma(E)N(E)}{\int_{E_\alpha}^{E_{\alpha-1}} dE\,N(E)}, \qquad (3.5)$$

where N(E) is the spectrum of the source.

Similarly for the scatter-matrix with the assumption $\phi^r_\ell(r,E')\approx N(E')$ we have

$$\tilde{\Sigma}^{\alpha\to j}_{s,\ell}=\frac{\int_{E_j}^{E_{j-1}} dE\int_{E_\alpha}^{E_{\alpha-1}} dE'\,\Sigma_{s,\ell}(E'\to E)N(E')}{\int_{E_\alpha}^{E_{\alpha-1}} dE'\,N(E')}, \qquad (3.6)$$

where "g" is the index of the scatter neutron-group, "h" is the index of the removal group.

For neutrons N(E) is the fission spectrum.

Following /7/ we use

$$N(E) = \alpha e^{-\beta E} \sinh \sqrt{\gamma E} \tag{3.7}$$

where α is a normalization constant

$$\alpha = \frac{2}{\int_0^\infty dE \left(\frac{e^{-\sqrt{\gamma E}}}{e^{-\beta E}} - \frac{1}{e^{\sqrt{\gamma E} + \beta E}} \right)} \tag{3.8}$$

For β and γ the following values are recommended:

	β	γ
U 233	1.05	2.3
U 235	1.036	2.29
Pu 239	1.0	2.2
Pu 241	1.0	2.0

With regard to /8/ for U 235 - fission the following values are recommended:

$\beta = 1.013$ $\gamma = 1.932$.

Watt suggests $\beta = 1$ $\gamma = \sqrt{2}$ $\alpha = 0.484$

For the scattered neutrons to add the influence of the slowing-down spectrum and its deformation caused by the resonances /5/ is necessary.

Comparison of Removal Methods.

A detailed comparison of various removal approximations are given in /2/. We suspect it is necessary to complete these comparisons by some other facts. Furthermore it is advisable to compare the proposed foundations for the arrangement of one-dimensional and two-dimensional removal codes REM-DIF and REM-P1 based on the transport corrections /10/, /11/ with typical removal codes RASH, MAC and NRN.

The main differences may be searched using the following way:

a) The determination of the removal cross-section.
b) The coupling of removal groups and scattered neutron - groups.
c) The slowing down model.
d) The division of groups.
e) The method of the data averaging (with regard to the resonance structure).
f) The consideration of anisotropy of scattering in the scattered neutron-groups.

The RASH, MAC and REM-DIF codes use the same definition of the removal cross-section corresponding to the Spiney's model.

The anisotropy of scattering in the scattered neutron - groups is used only in the REM-P1 code. In this code the removal scatter angle conserving neutrons in the "non - scattered" beam is picked out by taking the Fourier coefficient of the indicatrix for n = 2 into consideration. Therefore the removal angle is smaller in comparison with the Spiney's model in which the angle is picked out using the Fourier coefficient of the indicatrix for n = 1.

In the above described methods the removal cross-section is a function of energy. In the NRN method it was determined as a quantity independent on energy by using experimental methods.

In the coupling between the removal groups and the scattered neutron-groups for the RASH and MAC codes is a limitation in the scatter matrices.

Furthermore in the RASH and MAC codes the neutron conservation law for the transfer of neutrons from the removal groups into the scattered neutron-groups is not fulfilled. The general transfer from the removal groups into the scattered neutron-groups and between the scattered neutron-groups are not permitted in the RASH and MAC codes. They are permitted in the NRN, REM-DIF and REM-P1 codes.

We consider the including of the influence of the resonance structure on the group-data averaging is the most important question. In the mean time none of the used codes (i.e. NRN, MAC, RASH) includes the influence of the resonances.

This may influence the experimental determination of the removal angle in the NRN code. The codes REM-DIF and REM-P1 suppose the group-data everaging, influenced by the resonances /5/. In the NRN code the diffusion coefficient is corrected /3/, /4/.

Let us return to the definition of the removal cross - section:

$$\Sigma_{rem}(E) = \Sigma(E) - f(E)\,\Sigma_{s}(E)\,, \tag{4.1}$$

where $\Sigma_{s}(E)$ is the elastic scattering cross-section, f is the fraction of elastic collision which deliberate scattering into the directions of the "non-scattered" neutrons. Following Spiney

$$f = w_1 = \mu_L \quad . \tag{4.2}$$

According to the REM-P1 method $f = w_2$ (i.e. the third Fourier coefficient of the indicatrix in the L-system).

The NRN method gives the definition:

$$f_{NRN}(E) = \frac{\int_0^{\Theta_{rem}} d\Psi_c \, \sigma(E,\Psi_c) \sin\Psi_c}{\int_0^{\pi} d\Psi_c \, \sigma(E,\Psi_c) \sin\Psi_c} , \tag{4.3}$$

where $\sigma(\Theta)$ is the cross-section for elastic scattering into unit solid angle at a direction Θ with the incident neutron direction in the centre of mass system, Θ_{rem} is the scatter angle above which a collision is considered to be a removal.

The experiments /2/ have given $\mu_{rem} = \cos\Theta_{rem} = 0.6$ which gives $\Theta_{rem}^{NRN} \simeq 53°$ for materials with $A > 1$ and $\mu_{rem} = 0.45$ which gives $\Theta_{rem}^{NRN} \simeq 63°$ for hydrogen ($A = 1$).

Therefore it is convenient to compare the quantity f or the angle Θ_{rem} corresponding to the removal codes mentioned above. For simplicity we shall use an approsimate expression deliberating the anisotropy in the centre of mass system /3/

$$\bar{\mu}_c = 0.07 A^{2/3} E < 1 \tag{4.4}$$

The formula (4.4) gives only a rough approximation useful for low energies $\sim$ 100 KeV. Nevertheless it is used for higher energies and small A (look at /3/). Therefore we are able to use (4.4) for comparison only for small A, so that the expression (4.4) does not lose its mathematical meaning for energies up to 2 MeV.

For E in [MeV] is $\sigma(\psi_c) = \frac{\sigma_s}{4\pi}\left[1 + 3\bar{\mu}_c \cos\psi_c\right]$ (4.5)

and $$f_{NRN} = \frac{\int_0^\infty dE\, n(E) \int_{\mu_{rem}}^1 d\mu\, (1 + 3\bar{\mu}_c \mu)}{\int_0^\infty dE\, n(E) \int_{-1}^1 d\mu\, (1 + 3\bar{\mu}_c \mu)} . \qquad (4.6)$$

Here n(E) is the fission spectrum, f is the average value independent on energy. With regard to the linear dependence of $\bar{\mu}_c$ on energy it is possible to use the average value of energy of the fission spectrum, i.e. 2 MeV. Therefore

$$f_{NRN}(A, \mu_{rem}) = \frac{\int_{\mu_{rem}}^1 d\mu\, (1 + 0.42 A^{2/3} \mu)}{\int_{-1}^1 d\mu\, (1 + 0.42 A^{2/3} \mu)} =$$

$$= \frac{1}{2}\left[(1 - \mu_{rem}) + 0.21 A^{2/3} (1 - \mu_{rem}^2)\right]. \qquad (4.7)$$

Further we have

$$w_1(A, E) = \bar{\mu}_L = \frac{2}{3A} + 0.07 A^{2/3} E \left(1 - \frac{3}{5A^2}\right) \qquad (4.8)$$

To the expressions above there corresponds in the centre of mass system the angle

$$\Theta_T^{dif}(A, E) = \arccos\left\{\frac{1}{2\beta}\left[\sqrt{1 + 4\beta(1 + \beta - 2w_1)} - 1\right]\right\}, \qquad (4.9)$$

where $\beta = 0.21 A^{2/3}$.

Analogously to paragraph 7 we can write

$$w_2(A,E) = 2\pi\int_{-1}^{1} d\mu \frac{1}{4\pi}(1+3\bar{\mu}_c\mu)P_2[\cos\Psi_L(\mu)],$$

$$\cos\Psi_L(\mu) = \frac{A\cos\Psi_c + 1}{\sqrt{A^2+2A\cos\Psi_c+1}} = \frac{A\mu+1}{\sqrt{A^2+2A\mu+1}},$$

$$w_2(A,E) = \frac{1}{2}\int_{-1}^{1} d\mu P_2\left(\frac{A\mu+1}{\sqrt{A^2+2A\mu+1}}\right) + \frac{3}{2}\mu_c\int_{-1}^{1} d\mu\, \mu P_2\left(\frac{A\mu+1}{\sqrt{A^2+2A\mu+1}}\right) =$$

$$= \frac{1}{8}\left[5-3A^2-\frac{3(A^2-1)^2}{2A}\ln\frac{A-1}{A+1}\right] + 0.1575A^{2/3}E\left\{\frac{A}{3}+\frac{1}{A}\left[1+\frac{A^2+1}{2}\left(\frac{A^2+1}{2}-2\right)\right]+\right.$$

$$\left.+\frac{1}{A^2}\frac{A^2+1}{2}\left[1+\frac{A^2+1}{2}\left(\frac{A^2+1}{2}-2\right)\right]\ln\frac{A-1}{A+1}\right. .$$

(4.10)

For $A=1$ the terms containing logarithm turn to zero and therefore

$$w_2(A,E) = \frac{1}{4} + 0.0525E . \qquad (4.11)$$

For $A \geq 2$

$$w_2(A,E) = 3\sum_{i=1}^{\infty}\frac{1}{(2i-1)(2i+1)(2i+3)}\frac{1}{A^{2i}} -$$

$$- C(A,E)\sum_{i=0}^{\infty}\frac{(i+1)}{(2i-1)(2i+1)(2i+3)(2i+5)}\frac{1}{A^{2i+1}}, \qquad (4.12)$$

where

$$C(A,E) = 18\bar{\mu}_c = 1.26A^{2/3}E,$$

The corresponding angle in the centre of mass system is

$$\Theta_{rem}^{p1}(A,E) \simeq \arccos\left\{\frac{1}{2\beta}\left[\sqrt{1+4\beta(1+\beta-w_2)}-1\right]\right\} \qquad (4.13)$$

where $\beta = 0.21 A^{1/3}$

The following table presents the comparison of the quantitie f_{NRN}, w_1 and w_2 as functions of A for energy E = 2 MeV.

A :	1	2	10
f_{NRN}	.359	.307	.510
w_1	.723	.522	.716
w_2	.355	.175	.08
$\alpha^{rem}_{T,NRN}$	0.45	0.6	0.6
Θ^{rem}_{NRN}	63°	53°	53°
$\alpha^{rem}_{T,dif}$	-.243	0.27	0.394
Θ^{rem}_{dif}	104°	$74^{\circ}\ 20'$	$66^{\circ}\ 45'$
$\alpha^{rem}_{T,P1}$	.45	0.78	0.938
Θ^{rem}_{P1}	$.63^{\circ}$	$51^{\circ}\ 40'$	20°

In spite of the rough approximation of the angular dependence of scattering (look at 4.4) from the table above is evident the REM-P1 determines the "removal angle" in a good consent with the experimentally determined angle in the NRN method for small A. For larger A the formula (4.4) loses its mathematical meaning.

5. Conclusion

The proposed method of determination of the removal cross-section based on the transport correction of higher order enables us to determine the removal cross-section comparatively simply. For the REM-P1 system we have got

$$\Sigma_{rem}(E) = \Sigma(E) - \Sigma_{s,2}(E).$$

Further the received scheme for the REM-P1 system expands the idea of the removal method in the way of selection of the removal angle as a function of energy and in a better consideration of anisotropy. Furthermore the selection of the removal angle conserving neutrons in the "non-scattered" beam is in good conformity for small A (look at paragraph 4.) in both the NRN and REM-P1 approximations.

Receiving a sufficient set of experimental informations the removal cross-sections may be later corrected in the manner of the NRN approximation.

The complication of the calculations in REM-DIF and REM-P1 approximations is practically the same.

The removal cross-section used in the REM-DIF approximation determined by using the transport approximation corresponds to Spiney's model

$$\Sigma_{rem}(E) = \Sigma(E) - \Sigma_{s,1}(E) = \Sigma(E) - \bar{\mu}_L(E)\Sigma_s(E).$$

We suppose that the use of the group cross-section influenced by resonances is important. Only in this case it is possible to execute correct comparisons of the removal angle with experiments.

In /11/ the procedure REM-P1 is described. The general transfers of neutrons both between the scattered neutron -

groups and from the removal groups into the scattered neutron-groups are taken into consideration. Therefore this procedure is useful for all known removal systems.

A next possible arrangement of the REM-P1 approximation consists in the consideration of transfers of neutrons between the removal groups as this corresponds to the loss of energy caused by the elastic scattering into an angle less than the determined removal angle.

The difficulties in the calculations of gamma ray distributions by using the build-up factors provoke an idea to aply the REM-P1 approximation for gamma ray field problems, too.

Finally it is necessary to be reflected on the matter of the practicability of the transport corrections of higher order, i.e. objectively on the fulfilling of the basic assumptions (look at 2.8, 2.9).

From the results (paragraph 7) it follows the assumption 2.8 $w_{L+1} > w_{L+p}$ for $p \geq 2$ is not fulfilled. This fact, and therefore the applicability of /1/ and /4/, should be taken into consideration.

The transport corrections may be safely used up to $n = L+1 \leq 2$. For higher order approximations the determinatio of the scattering cross-section in the form of a series may lead to the well hnown difficulties (i.e. for instance in the SN, DSN and discrete ordinates approximations the negative values of fluxes may be received).

Further the detailed comparisons of the NRN and REM-P1 approximations for concrete cases are presented.

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7. Enclosure: Determination of Fourier Coefficients of the Scatter Indicatrix for Isotropic Scattering in Centre-Of-Mass System

From the neutron conservation law we obtain /9/

$$2\pi \sigma_{el}(\Psi_L) \sin\Psi_L \, d\Psi_L = 2\pi \sigma_{el}(\Psi_c) \sin\Psi_c \, d\Psi_c$$

where Ψ_L is the scatter angle in the laboratory system,

Ψ_c is the scatter angle in the centre-of-mass system.

Supposing the scattering to be isotropic in the centre-of-mass system, we have got

$$\sigma_{el}(\Psi_c) = \frac{1}{4\pi} \sigma_{el}$$

Considering

$$\Sigma_s = N\sigma_{el} \, , \quad \Sigma_{s,n} = \Sigma_s w_n \, ,$$

where $$\Sigma_{s,n} = 2\pi \int_{-1}^{1} d\cos\Psi_L \, \Sigma_s(\Psi_L) P_n(\cos\Psi_L)$$

and $$d\cos\Psi_L = -\sin\Psi_L \, d\Psi_L \, .$$

Using the expression

$$\cos\Psi_L = \frac{A\cos\Psi_c + 1}{\sqrt{A^2 + 2A\cos\Psi_c + 1}}$$

we get

$$w_n(A) = \frac{1}{2} \int_{-1}^{1} d\cos\Psi_c \, P_n\left(\frac{A\cos\Psi_c + 1}{\sqrt{A^2 + 2A\cos\Psi_c + 1}}\right)$$

Expressing

$$\mathcal{P}_n(\xi) = \sum_{K=0}^{[n/2]} \frac{(-1)^K (2n-2K)!}{2^n K!(n-K)!(n-2K)!} \xi^{n-2K} = \sum_{K=0}^{[n/2]} C_{n,K} \xi^{n-2K},$$

$$\mathcal{P}_n\left(\frac{Ax+1}{\sqrt{A^2+2Ax+1}}\right) = \sum_{K=0}^{[n/2]} C_{n,K} \left(\frac{Ax+1}{\sqrt{A^2+2Ax+1}}\right)^{n-2K}, \tag{7.1}$$

we get

$$w_n(A) = \sum_{K=0}^{[n/2]} \frac{C_{n,K}}{2} \int_{-1}^{1} dx \left(\frac{Ax+1}{\sqrt{A^2+2Ax+1}}\right)^{n-2K} \tag{7.2}$$

After the integration and some arrangements we obtain a general formula for all $n = 0, 1, 2, \ldots\ldots$:

$$w_n(A) = \sum_{K=0}^{[n/2]} \Bigg\{ \sum_{\substack{r=0 \\ r \neq \frac{n}{2}-K+1}}^{n-2K} \beta^*_{n,K,r} \frac{1}{A} \left[(A+1)^{n-2K-r+2} (A-1)^r - (A+1)^r (A-1)^{n-2K-r+2} \right] -$$

$$- \delta_{n0} \delta_{r,\frac{n}{2}-K+1} \beta_{n,K,\frac{n}{2}-K+1} \frac{1}{A} (A+1)^{\frac{n}{2}-K+1} (A-1)^{\frac{n}{2}-K+1} \ln \frac{A-1}{A+1} \Bigg\} \tag{7.3}$$

$$\delta_{n0} = \begin{cases} 1 & \text{for } n \neq 0 \\ 0 & \text{for } n = 0 \end{cases}$$

$$\delta_{r,\frac{n}{2}-K+1} = \begin{cases} 1 & \text{for } r = \frac{n}{2}-K+1 \\ 0 & \text{for } r \neq \frac{n}{2}-K+1 \end{cases}$$

$$\beta^*_{n,k,r} = \frac{\beta_{n,k,r}}{n-2k-2r+2} \quad , \; n \geq 1 ,$$

$$\beta_{n,k,r} = \frac{(-1)^{k+r}(2n-2k)!}{2^{2n-2k+1}\,k!\,(n-k)!\,r!\,(n-2k-r)!} \; , \; n \geq 1 ,$$

$$\beta_{n,k,\frac{n}{2}-k+1} = \frac{(-1)^{\frac{n}{2}+1}(2n-2k)!}{2^{2n-2k+1}\,k!\,(n-k)!\,(\frac{n}{2}-k+1)!\,(\frac{n}{2}-k-1)!} \; , \; n \geq 2 \; .$$

Let us write the expression $w_n(A)$ for several values of "n" by using the formula (7.3):

$$w_0(A) = 1 ,$$

$$w_1(A) = \frac{2}{3A} ,$$

$$w_2(A) = \frac{1}{8}\left[5-3A^2-\frac{3(A^2-1)^2}{2A}\ln\frac{A-1}{A+1}\right] ,$$

$$w_3(A) = 0 .$$

It is necessary to discusse the following questions:

A) The seattering in hydrogen ($A = 1$).

B) The limit for $A \to \infty$

C) For which values of "n" is $w_n(A) = 0$.

Let us write

$$w_n(1) = \frac{1}{2}\int_{-1}^{1} d\cos\Psi_c\, P_n\left(\frac{1+\cos\Psi_c}{\sqrt{2+2\cos\Psi_c}}\right) = \frac{1}{2}\int_{-1}^{1} dx\, P_n\left(\frac{1+x}{\sqrt{2+2x}}\right) =$$

$$= \frac{1}{2}\sum_{K=0}^{[n/2]} C_{n,K}\int_{-1}^{1} dx\left(\frac{x+1}{\sqrt{2x+2}}\right)^{n-2K} =$$

$$= \frac{1}{2}\sum_{K=0}^{[n/2]} C_{n,K}\int_{-1}^{1} dx\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)^{n-2K} =$$

$$= \sum_{K=0}^{[n/2]} \frac{C_{n,K}}{2^{\frac{n}{2}-K+1}}\int_{-1}^{1} dx\,(x+1)^{\frac{n}{2}-K} .$$

Finally we get

$$w_n(1) = \sum_{K=0}^{[n/2]} \frac{(-1)^K (2n-2K)!}{2^{n-1} K!(n-K)!(n-2K)!(2n-2K+2)} =$$

$$= \sum_{K=0}^{[n/2]} \frac{(-1)^K (2n-2K)!(2n-2K+1)}{2^{n-1} K!(n-K)!(2n-2K+2)!} = \sum_{K=0}^{[n/2]} \frac{2^{n-2K+2}}{n-2K+2}\beta_{n,K,0} . \tag{7.4}$$

Further

$$w_n(+\infty) = \lim_{A\to+\infty} w_n(A) = \frac{1}{2}\lim_{A\to+\infty}\int_{-1}^{1} dx\, P_n\left(\frac{Ax+1}{\sqrt{A^2+2Ax+1}}\right) =$$

$$= \frac{1}{2}\int_{-1}^{1} dx \lim_{A\to+\infty} P_n\left(\frac{x+\frac{1}{A}}{\sqrt{1+\frac{2x}{A}+\frac{1}{A^2}}}\right) = \frac{1}{2}\int_{-1}^{1} dx\, P_n(x) =$$

$$= \frac{1}{2}\int_{-1}^{1} dx\, P_n(x) P_0(x) = \frac{1}{2}\, 2\,\delta_{n0} = \delta_{n0} = \begin{cases} 1 & \dots\ n=0 \\ 0 & \dots\ n\neq 0 \end{cases} . \tag{7.5}$$

The formulae (7.3) and (7.4) are expressively unstable for numerical calculations. Therefore to determine the integral

$w_n(A)$ we have used another way which is suitable for numerical calculations. For $A > 1$ is valid

$$w_n(A) = \frac{1}{2}\int_{-1}^{1} dx\, P_n\left(\frac{Ax+1}{\sqrt{A^2+2Ax+1}}\right) .$$

Let us use the substitution

$$y = \frac{Ax+1}{\sqrt{A^2+2Ax+1}} , \qquad \begin{array}{l} x = -1 \cdots y = -1 , \\ x = 1 \cdots y = 1 , \end{array}$$

$$x_{1,2} = \frac{1}{A}\left[-1+y^2 \pm y\sqrt{y^2-1+A^2}\right] .$$

With regard to the limits of the integral we are to take the root with $+$:

$$x = \frac{1}{A}\left[y^2-1 + y\sqrt{y^2-1+A^2}\right] ,$$

$$dx = \frac{1}{A}\left[2y + \frac{2y^2-1+A^2}{\sqrt{y^2-1+A^2}}\right] dy .$$

Let us mark

$$c^2 = A^2-1 > 0 \qquad \text{for } A > 1 .$$

Expanding the square root into a series we receive after some arrangements for $A > \sqrt{2}$

$$dx = \frac{2}{A} y\, dy + \frac{c}{A}\left[1 + \frac{3}{2}\left(\frac{y}{c}\right)^2 - \frac{5}{8}\left(\frac{y}{c}\right)^4 + \frac{21}{48}\left(\frac{y}{c}\right)^6 - \frac{135}{384}\left(\frac{y}{c}\right) + \cdots\right] dy .$$

Defining $(-1)!! = 1,\ (-3)!! = -1$, we can write

$$dx = \frac{2}{A} y\, dy + \frac{C}{A} \sum_{K=0}^{\infty} (-1)^{K+1} \frac{(2K-3)!!}{2^K K!} (2K+1) \left(\frac{y}{C}\right)^{2K} dy =$$

$$= \frac{2}{A} y\, dy + \frac{C}{A} \sum_{K=0}^{\infty} \alpha_K\, y^{2K}\, dy \;,$$

$$w_n(A) = \frac{1}{A} \int_{-1}^{1} dy\; y P_n(y) + \frac{C}{2A} \int_{-1}^{1} dy \sum_{K=0}^{\infty} \alpha_K\, y^{2K} P_n(y) \;,$$

$$w_1(A) = \frac{1}{A} \int_{-1}^{1} dy\, y^2 = \frac{2}{3A} \;, \tag{7.6}$$

$$w_{2\ell+1}(A) = 0 \qquad \text{for } \ell = 1, 2, 3, \cdots . \tag{7.7}$$

For even $n > 0$ the following is valid:

$$w_n(A) = \frac{C}{2A} \int_{-1}^{1} dy\, P_n(y) \sum_{K=0}^{\infty} \alpha_K\, y^{2K} = \frac{C}{2A} \sum_{K=\frac{n}{2}}^{\infty} \alpha_K \int_{-1}^{1} dy\; y^{2K} P_n(y) \;,$$

$$\alpha_K = (-1)^{K+1} \frac{(2K-3)!!}{2^K K!} (2K+1) \frac{1}{C^{2K}} \;,$$

$$y^{2K} = \sum_{\ell=0}^{K} \beta_{\ell,K}\, P_{2\ell}(y) \;,$$

$$\beta_{0,K} = \frac{1}{2K+1} \;, \quad \beta_{\ell,K} = (4\ell+1) \frac{2K(2K-2)\cdots(2K-2\ell+2)}{(2K+1)(2K+3)\cdots(2K+2\ell+1)} \;.$$

As

$$\int_{-1}^{1} dy\, y^{2K} P_n(y) = \sum_{\ell=0}^{K} \beta_{\ell,K} \int_{-1}^{1} dy\, P_n(y) P_{2\ell}(y) = \frac{2}{2n+1} \beta_{\frac{n}{2},K} \;,$$

where

$$\beta_{\frac{n}{2},K} = (2n+1)\,\frac{2K(2K-2)\cdots(2K-n+2)}{(2K+1)(2K+3)\cdots(2K+n+1)},$$

we get

$$w_n(A) = \frac{c}{A}\sum_{K=\frac{n}{2}}^{\infty}\frac{1}{2n+1}\alpha_K\,\beta_{\frac{n}{2},K} = \sqrt{1-\frac{1}{A^2}}\sum_{K=\frac{n}{2}}^{\infty}\frac{1}{2n+1}\alpha_K\,\beta_{\frac{n}{2},K}\ .$$

So we get for all $A>1$

$$w_{2\ell}(A) = \sqrt{1-\frac{1}{A^2}}\sum_{K=\ell}^{\infty}(-1)^{K+1}\frac{(2K-3)!!}{2^{K-\ell}(K-\ell)!(2K+3)(2K+5)\cdots(2K+2\ell+1)}\,\frac{1}{(A^2-1)^K} \qquad (7.8)$$

where $\ell\geq 1$ and $(-3)!!=-1$, $(-1)!!=1$ is defined.

For $w_0(A)$ we get directly $w_0(A)=1$ which may be determined also from (7.8) taking into consideration

$$\frac{1}{(2K+3)(2K+5)\cdots(2K+2\ell+1)} = \frac{(2K+1)!!}{(2K+2\ell+1)!!}\ .$$

For $\ell=0$

$$\frac{(2K+1)!!}{(2K+2\ell+1)!!} = 1.$$

Then

$$\omega_0(A) = \sqrt{1-\frac{1}{A^2}}\sum_{K=0}^{\infty}(-1)^{K+1}\frac{(2K-3)!!}{2^K K!}\,\frac{1}{(A^2-1)^K}\ .$$

Let us write for real α :

$$(1+x)^{\alpha} = \sum_{K=0}^{\infty}\frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-K+1)}{1\cdot 2\cdot 3\cdots K}\,x^K\ , \quad |x|<1\ .$$

Therefore

$$(1+x)^{\frac{1}{2}} = \sum_{K=0}^{\infty} (-1)^{K+1} \frac{(2K-3)!!}{2^K K!} x^K \quad , \quad |x| < 1 .$$

Let us put

$$x = \frac{1}{A^2-1} = \frac{1}{C^2} .$$

Then

$$(1+x)^{\frac{1}{2}} = \frac{A}{\sqrt{A^2-1}} ,$$

$$w_0(A) = \sqrt{1-\frac{1}{A^2}}\ \frac{A}{\sqrt{A^2-1}} = 1 . \tag{7.9}$$

For $A=1$ we get

$$w_n(1) = \frac{1}{2}\int_{-1}^{1} dx\, P_n\left(\frac{x+1}{\sqrt{2}\sqrt{x+1}}\right) = \frac{1}{2}\int_{-1}^{1} dx\, P_n\left(\sqrt{\frac{x+1}{2}}\right) =$$

$$= 2\int_0^1 dy\, y P_n(y) = 2\int_0^1 dy\, P_n(y) P_1(y) .$$

Following J.M.Ryshik, J.S.Gradstein: "Tables of Series, Products and Integrals" we get

$$\int_0^1 dx\, P_n(x) P_m(x) = \begin{cases} \dfrac{1}{2n+1} & n=m \\[2ex] 0 & \begin{array}{l} n \neq m \\ \text{even}(n-m) \end{array} \\[2ex] \dfrac{(-1)^{\frac{1}{2}(n+m+1)}\ n!\,m!}{2^{n+m-1}(n-m)(n+m+1)\left[\left(\frac{n}{2}\right)!\left(\frac{m-1}{2}\right)!\right]^2} & \begin{array}{l} n \neq m \\ \text{odd}(n-m) \end{array} \end{cases}$$

From that for even "n" and n = 1

$$\int_0^1 dx\, P_n(x) P_1(x) = \frac{(-1)^{\frac{n+2}{2}}\, n!}{2^n (n-1)(n+2)\left[\left(\frac{n}{2}\right)!\right]^2} = \frac{(-1)^{\frac{n}{2}+1} (n-3)!!}{2^{\frac{n}{2}+1}\left(\frac{n}{2}+1\right)!},$$

$$w_n(1) = 2\int_0^1 dx\, P_n(x) P_1(x) = \frac{(-1)^{\frac{n}{2}+1} (n-3)!!}{2^{\frac{n}{2}}\left(\frac{n}{2}+1\right)!}. \qquad (7.10)$$

The behaviour of $w_n(A)$ was determined using calculations for n = 0, 1, 2, 3, ..., 14 and for A = 1, 2, 5, 10, 50, 100, 200. The reviev of the results is given in the table T.1.

The exactness of the determination of the scatter matrix using a finite Fourier series for $\mu = -1, 0, 1$ may be judged from the table T.2 for A = 1, 2, 5, 10, 50, 100, 200 and from the pictures 1, 2, 3 and 4.

The partial sums of the scatter indicatrix were found using the formula

$$S_N(\mu) = \sum_{i=0}^{N} \frac{2i+1}{2} w_i P_i(\mu), \qquad (7.11)$$

$$P_i(1) = 1, \qquad P_i(-1) = (-1)^i,$$

$$P_{2i}(0) = (-1)^i \frac{1\cdot 3\cdot 5\cdots(2i-1)}{2\cdot 4\cdot 6\cdots 2i},$$

$$P_{2i+1}(0) = 0.$$

We can see the limitation of the Fourier series of the scatter indicatrix appears to be strong for small A for which the damping of the oscillations is very low. For large [illegible] the [illegible] laboratory system and centre-of-mass [illegible] and [illegible] the results limit to the asymptotic [illegible].

[illegible] of the partial sums of the scatter indicatrix [illegible] and the characteristics of its [illegible] demonstrate-

a) [illegible] validity of the transport corrections of higher order is limited,

b) the [illegible] of the expansion of the scatter indicatrix [illegible] a series with a finite number of terms for the [illegible] of numerical calculations (like the SN, DSN and the discrete ordinates methods) may disagreeably influence [illegible] of the differential flux.

[illegible] it is advisable to express the scatter indicatrix on the basis of the scatter law in a [illegible] form for discrete values of $\vec{\Omega}$ and $\vec{\Omega}'$.

Табл. [illegible]

$w_n(A)$

A	n = 1	2	4	6
1	0,666667	0,25	-0,0416667	0,0156250
2	0,333333	.519541/-01	-.115050/-02	.478627/-04
5	0,133333	.804634/-02	-.259641/-04	.155354/-06
10	0,066667	.200287/-02	-.159601/-05	.235451/-08
50	0.013333	.800046/-04	-.254024/-08	.149244/-12
100	0,006667	.200003/-04	-.158739/-09	.233124/-14
200	0,003333	.500002/-05	-.992077/-11	.364228/-16

Table 1

$w_n(A)$

A	n = 8	10	12	14
1	−0.00781250	.0045572916	−.0029296875	.0020141601
2	−.240642/−05	.132674/−06	−.772445/−08	.466191/−09
5	−.111791/−08	.880026/−11	−.730595/−13	.628229/−15
10	−.417481/−11	.809579/−14	−.165540/−16	.350563/−19
50	−.105369/−16	.813528/−21	−.662271/−25	.558346/−29
100	−.411414/−19	.793995/−24	−.161569/−28	.340487/−33
200	−.160691/−21	.775271/−27	−.394382/−32	.207771/−37

$w_0(A)=1, \; w_{2\ell+1}(A)=0, \; \ell=1,2,3,\cdots.$

The partial sums $S_N(\mu)=\sum_{i=0}^{N}\frac{2i+1}{2}w_i(A)P_i(\mu)$

for $\mu=-1,0,1$:

Table 2

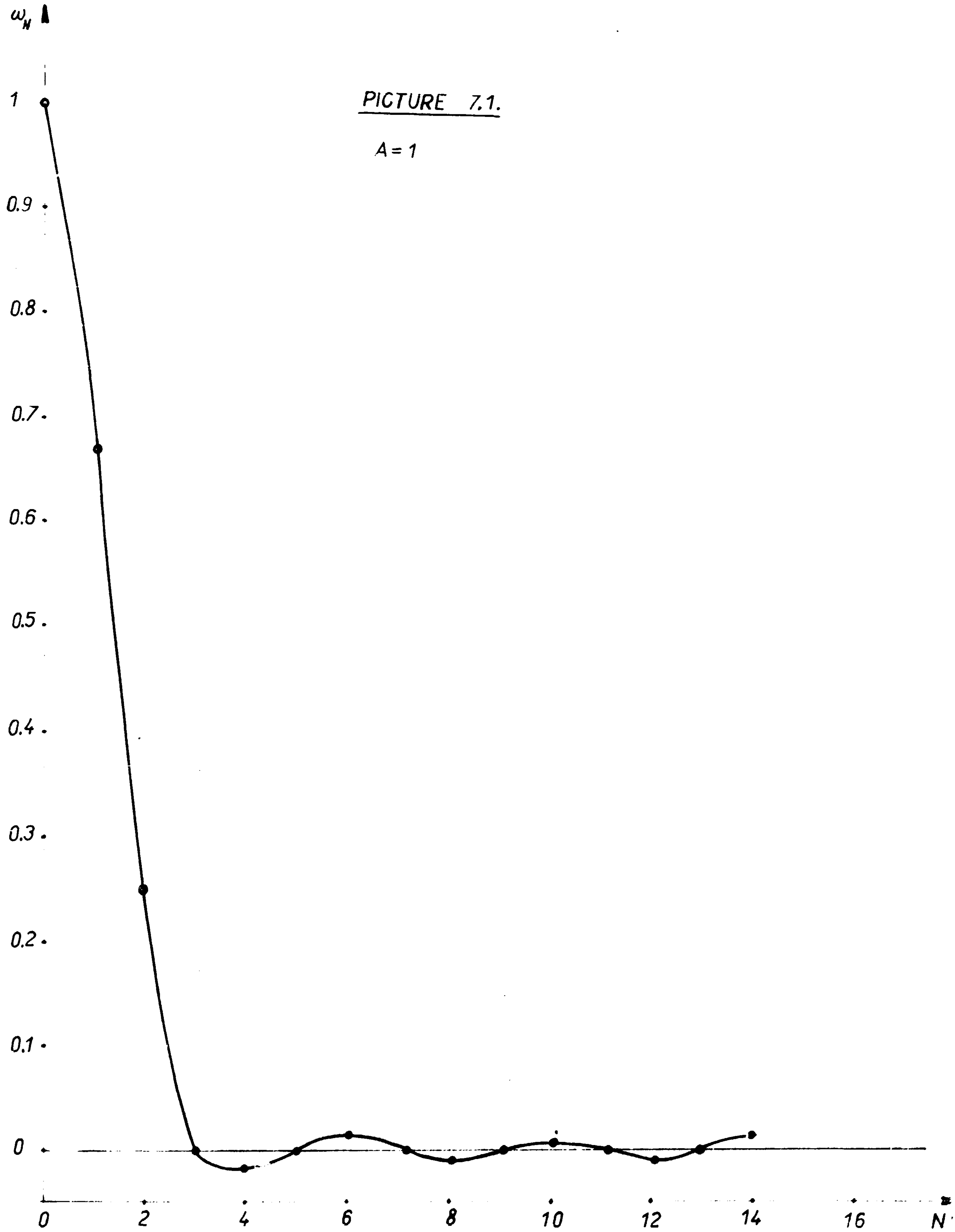
A	1			2		
n	$S_N(-1)$	$S_N(0)$	$S_N(1)$	$S_N(-1)$	$S_N(0)$	$S_N(1)$
0	.500000	.500000	.500000	.500000	.500000	.500000
1	-.500001	.500000	1.50000	.000000	.500000	.999999
2	.124999	.734375	2.12500	.129886	.548707	1.12988
3	.124999	.734375	2.12500	.129886	.548707	1.12988
4	-.0625007	.683105	1.93750	.124709	.547291	1.12471
5	.0625007	.683105	1.93750	.124709	.547291	1.12471
6	.0390618	.706017	2.03906	.125020	.547361	1.12502
7	.0390618	.706017	2.03906	.125020	.547361	1.12502

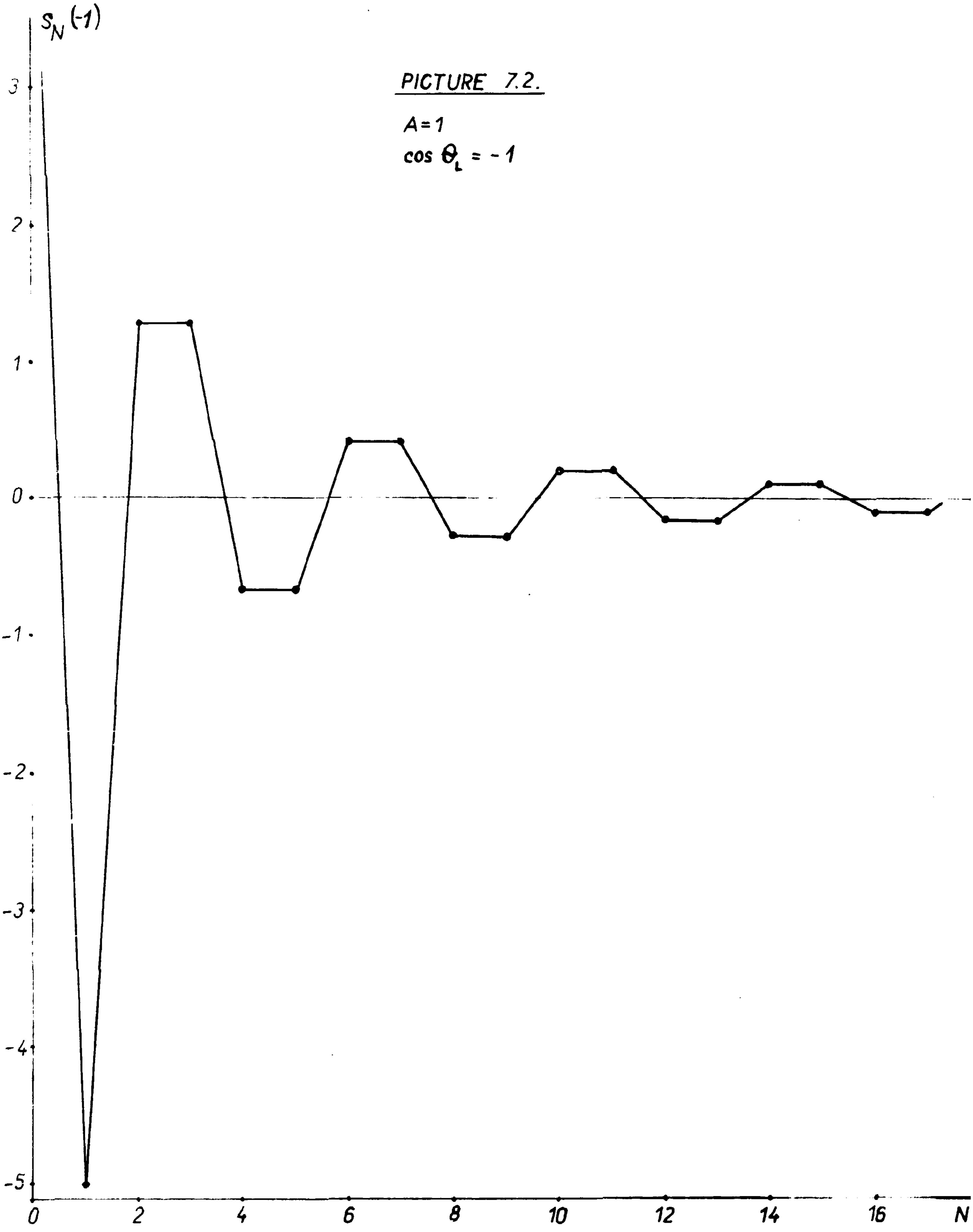
A	1			2		
n	$S_N(-1)$	$S_N(0)$	$S_N(1)$	$S_N(-1)$	$S_N(0)$	$S_N(1)$
8	-.0273444	.692976	1.97266	.124999	.547357	1.12500
9	-.0273444	.692976	1.97266	.124999	.547357	1.12500
10	.0205071	.701407	2.02051	.125001	.547358	1.12500
11	.0205071	.701407	2.02051	.125001	.547358	1.12500
12	-.0161140	.695504	1.98389	.125000	.547358	1.12500
13	-.0161140	.695504	1.98389	.125000	.547358	1.12500
14	-.130913	.699869	2.01309	.125000	.547358	1.12500
15	.130913	.699869	2.01309	.125000	.547358	1.12500

A	5			10		
n	$S_N(1)$	$S_N(0)$	$S_N(-1)$	$S_N(1)$	$S_N(0)$	$S_N(-1)$
0	.500000	.500000	.500000	.500000	.500000	.500000
1	.699999	.500000	.300001	.600000	.500000	.400000
2	.720115	.507543	.320116	.605007	.501878	.405007
3	.720115	.507543	.320116	.605007	.501878	.405007
4	.719999	.507511	.320000	.605000	.501876	.405000
5	.719999	.507511	.320000	.605000	.501876	.405000
6	.720000	.507512	.320001	.605000	.501876	.405000
7	.720000	.507512	.320001	.605000	.501876	.405000
8	.720000	.507512	.320001	.605000	.501876	.405000

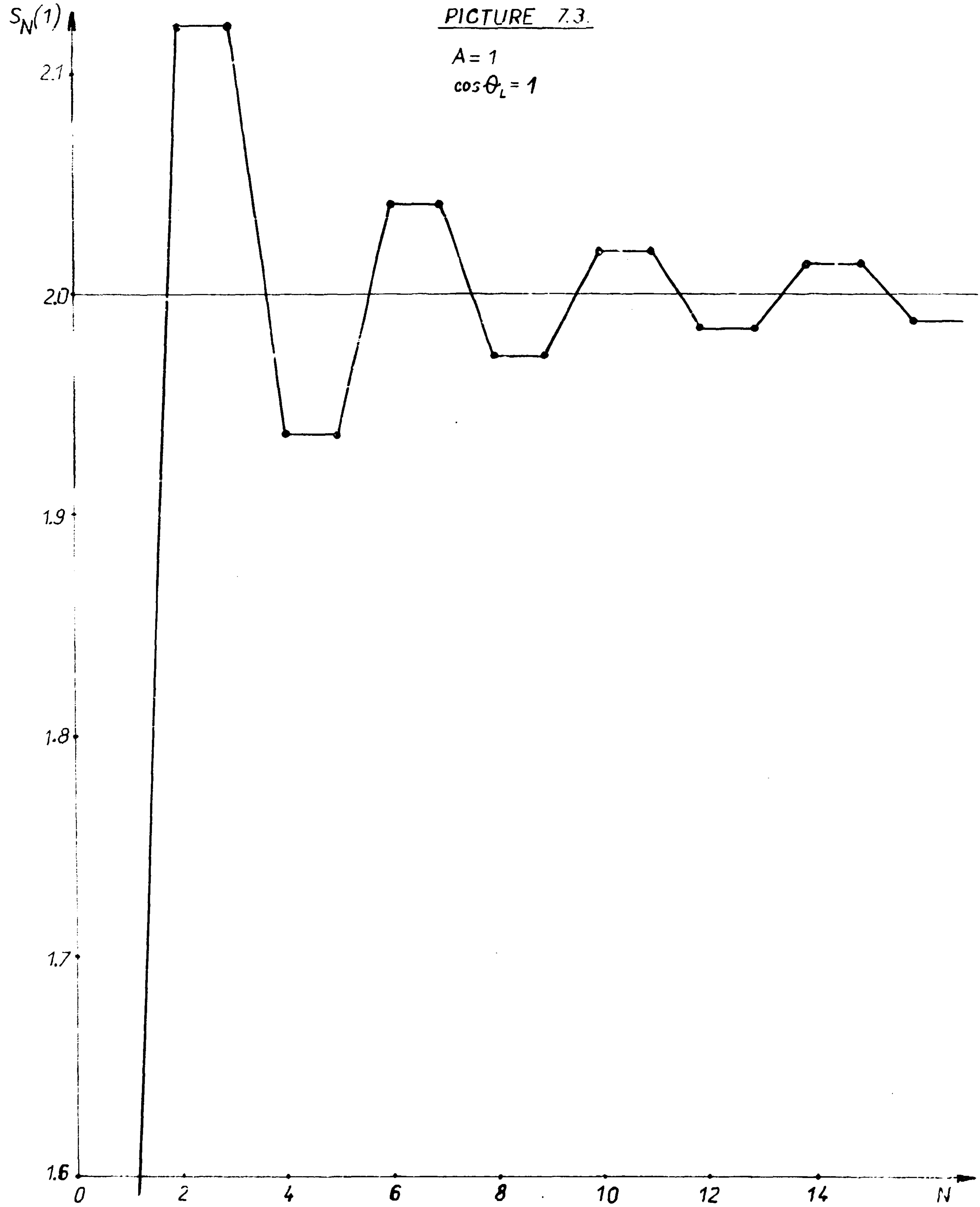
A	50			100		
n	$S_N(1)$	$S_N(0)$	$S_N(-1)$	$S_N(1)$	$S_N(0)$	$S_N(-1)$
0	.500000	.500000	.500000	.500000	.500000	.500000
1	.520000	.500000	.480000	.510000	.500000	.490000
2	.520200	.500075	.480200	.510050	.500019	.490050
3	.520200	.500075	.480200	.510050	.500019	.490050
4	.520200	.500075	.480200	.510050	.500019	.480050

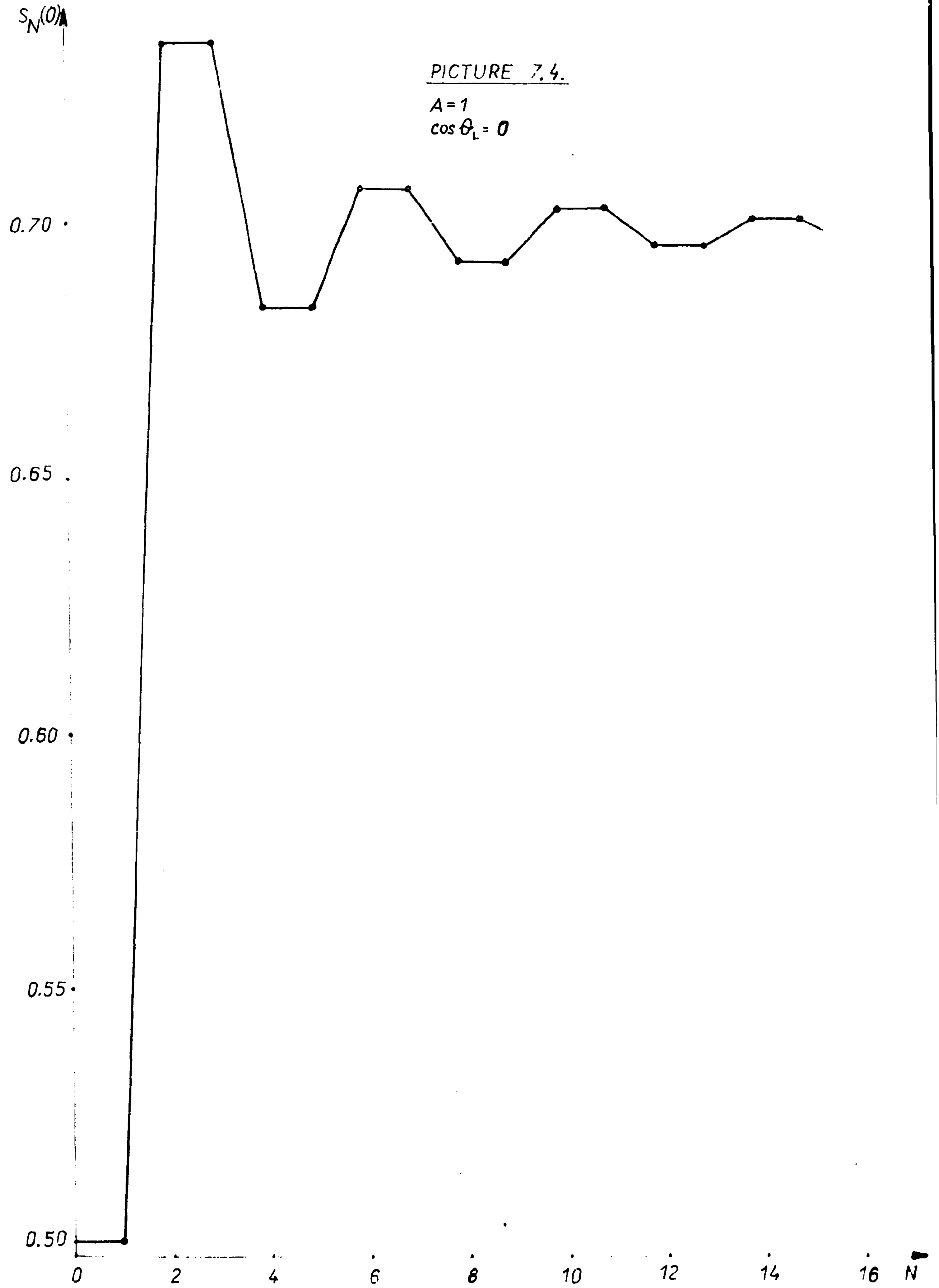
A	200		
n	$S_N(1)$	$S_N(0)$	$S_N(-1)$
0	.500000	.500000	.500000
1	.505000	.500000	.495000
2	.505012	.500005	.495013
3	.505012	.500005	.495013
4	.505012	.500005	.495013


PICTURE 7.1.
A = 1
ω_N
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
16
N



$s_N(-1)$
PICTURE 7.2.
$A=1$
$\cos\theta_L = -1$
3
2
1
0
-1
-2
-3
-4
-5
0
2
4
6
8
10
12
14
16
N


PICTURE 7.3.
A = 1
$\cos\theta_L = 1$
$S_N(1)$
2.1
2.0
1.9
1.8
1.7
1.6
0
2
4
6
8
10
12
14
N



$S_N(0)$
PICTURE 7.4.
$A = 1$
$\cos\theta_L = 0$
0.70
0.65
0.60
0.55
0.50
0
2
4
6
8
10
12
14
16
N