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ISABELLE PROJECT

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MASTER

INTERSECTING STORAGE ACCELERATOR NOTES

VACUUM CHAMBER EDDY CURRENTS

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In order to reduce the power losses due to the beam induced rf currents in the vacuum chamber of ISABELLE, it seems necessary to furnish the chamber with some kind of copper wall, either a thin film of copper, a few rf skin depths thick, about 0.1 mm, on a stainless steel tube, or a tube made completely of copper, perhaps about 1 mm thick.[1] In any case, since the resistivity of this copper[2] at 4.5°K will be about 100 times lower than that normally encountered at room temperature it seems worthwhile to estimate the effects of eddy currents induced by the slowly rising magnetic fields during the acceleration phase of the magnet cycle.

We will consider a very thin walled, exactly circular chamber of radius a whose surface resistivity, ρ (ohms per square) will be equal to the volume resistivity of copper divided by the wall thickness. We shall consider only the time that the magnetic fields are rising at a constant rate. When these fields reach their final values and become constant, the eddy effects will decay to zero. This transient will depend upon the detailed manner in which the impressed field changes from a ramp to a flat top.

If end effects are ignored, the impressed fields of the dipole bending magnets and quadrupoles (rising to full value in time T) and their corresponding magnetic vector potentials are written in cylindrical coordinates

1). Dipole

$$B'_r = (B'/T)\, t \sin\theta = \frac{1}{r}\frac{\partial A'_z}{\partial \theta}$$

$$B'_\theta = (B'/T)\, t \cos\theta = -\frac{\partial A'_z}{\partial r}$$

and $$A'_z = -r\,(B'/T)\, t \cos\theta$$

2) Quadrupole

$$B'_r = (G'/T)\ t\ r\ \sin 2\theta = \frac{1}{r}\ \frac{\partial A'_2}{\partial \theta}$$

$$B'_\theta = (G'/T)\ t\ r\ \cos 2\theta = -\frac{\partial A'_z}{\partial r}$$

and $$A'_z = \frac{-r^2}{2}\ (G'/T)\ t\ \cos 2\theta$$

We can represent each of these fields as a single circular harmonic

$$A_z' = \frac{-r^n}{n}\ A'_n\ \cos n\theta \qquad (1)$$

where $n = 1$ and $A'_n = (B/T)t$, for a dipole and where $n = 2$ and $A_n' = (G/T)t$ for a quadrupole.

Now these changing fields will induce currents in the thin tube which will induce new fields which are represented by the vector potential A_z. These quantities are related at the surface of the tube[3]

$$\mathcal{E} = \frac{dN}{dt} = \frac{-d}{dt}\ (A'_z + A_z)\Big|_{r=a} = \frac{\rho}{a d\theta}\ i\ a d\theta \qquad (2)$$

A'_z is increasing at a constant rate and hence the induced current and A_z will not vary with time. Therefore from (1) and (2)

$$i = \frac{-1}{\rho}\ \frac{d}{dt}\ A'_z\Big|_{r=a} = C_n\ \cos n\theta \qquad (3)$$

where: $$C_n = \frac{1}{\rho}\ \frac{a^n}{n}\ \frac{d}{dt}\left(A'_n\right)\ \cos n\theta$$

Smyth[4] shows that this current distribution produces in turn, a vector potential:

$$A_z = \frac{\mu a}{2}\frac{1}{n}\left(\frac{r}{a}\right)^n C_n \cos n\theta \ ; \qquad r<a \qquad (4)$$

Combining (3) and (4)

$$A_z = \frac{1}{2}\frac{\mu a}{\rho n^2} r^n \frac{dA'_n}{dt} \cos n\theta$$

Thus: (1) for the dipole $n = 1$ and $A'_n = (B'/T)\ t$

$$B_r = \frac{1}{r}\frac{\partial A_z}{\partial \theta} = -\frac{\mu a}{2\rho}(B'/T)\sin\theta$$

$$B_\theta = -\frac{\partial A_z}{\partial r} = -\frac{\mu a}{2\rho}(B'/T)\cos\theta$$

i.e. $B_{induced} = \left(-\frac{1}{2}\ \frac{\mu a}{\rho T}\right)$x Maximum impressed field

from (4)

$$i_{(\theta)} = \frac{a}{\rho}\ \frac{B'}{T}\cos\theta$$

(2) for the quadrupole $n = 2$ and $A'_n = (G/T)\ t$.

$$B_r = \frac{1}{r}\frac{\partial A_z}{\partial \theta} = \frac{-\mu a}{4\rho} r\,(G'/T)\sin 2\theta$$

$$B_\theta = \frac{\partial A_z}{\partial r} = \frac{-\mu a}{4\rho} r\,(G'/T)\cos 2\theta$$

i.e. $G_{induced} = \left(\frac{\mu a}{4\rho T}\right)$x Maximum gradient.

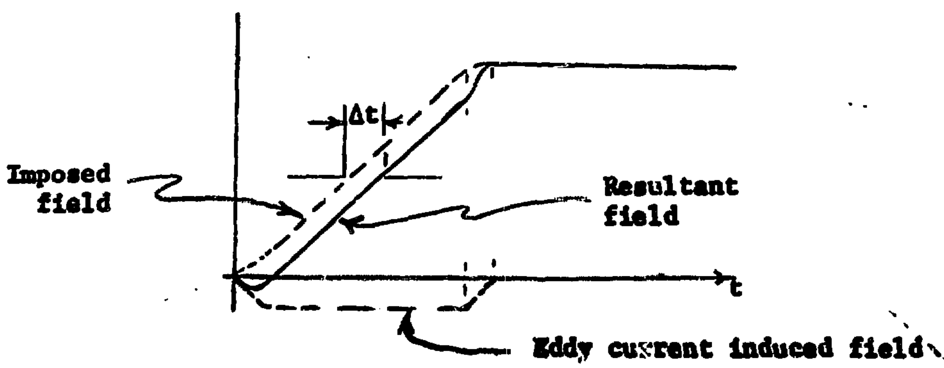
$$i(\theta) = \frac{a^2}{2\rho}\ (G'/T)\ \cos 2\theta$$

If we assume as an example, a tube of OFHC copper with a wall 1 mm thick with a volume resistivity of 1.6×10^{-10} ohm-meters at 4.3^{o}K then:

μ = $4\pi \times 10^{-7}$ henry/meter
ρ = 1.6×10^{-7} ohms/square surface resistance
a = 2.5×10^{-2} meters radius
B' = 4 Teslas max = 40 kgauss
G' = 100 Teslas/meter = 10 kgauss/cm
T = 120 seconds for each field to rise linearly to maximum value

Then: B $\approx$ 32.7 gauss induced by eddy currents in the dipole
G $\approx$ 4.09 gauss/cm induced by eddy currents in the quadrupole.

These induced fields are constant as long as the rate of rise of the magnet is constant and decrease the instantaneous field of the magnet by that amount but do not change its shape. Another way of considering this effect as seen by the sketch is that the eddy currents cause a constant delay in the build-up of field. About 90 ms for the dipole field and 48 ms for the quadrupole field.



There is a transient condition during the time the magnet current changes from a ramp to flat top whose exact nature can not be determined until the exact magnet program is known. However, from equations (1) and (2)

$$\frac{d}{dt} \quad (A'_n + A_n) = \frac{\rho 2n}{\mu a}$$

and hence if $\frac{d}{dt}$ (A'_n) should instantaneous become zero, the induced field would decay as

$$A_n = A_{no}\, e^{\frac{-2n\rho}{\mu a} t} .$$

Corresponding to a time constant of about 9.8 x 10^{-2} seconds for the dipole or 4.9 x 10^{-2} seconds for a quadrupole. In ISABELLE the quadrupoles and dipoles will be energized by different power supplies, no insurmountable difficulties in tracking the two sets of magnets should occur.

Eddy Current Heating

We have seen that the induced current in the tube is

$$i_{(\theta)} = \frac{a}{\rho} \; (B'/T) \cos \theta \quad \text{for dipoles}$$

$$i_{(\theta)} = \frac{a^2}{2\rho} \; (G'/T) \cos 2\,\theta \text{ for quadrupoles.}$$

In either case the power dissipated per meter length by eddy currents is

$$P = a \int_0^{2\pi} \rho i^2 \, d\theta$$

$$= \frac{\pi a^3}{\rho} \quad (B/T)^2 = 0.34 \text{ watts/meter (dipole)}$$

$$= \frac{\pi}{4} \; \frac{a^5}{\rho} \quad (G/T)^2 = 0.033 \text{ watts/meter (quadrupole).}$$

In the dipole case this loss per meter approaches the losses due to beam induced currents. They exist however only during the 120 seconds rise time and only in that fraction of the vacuum chamber passing through dipole magnets. It might be a very valid reason however to reduce the thickness of the copper wall.

All of the above estimates were made for a 1 mm thick cold copper vacuum chamber and are reduced as the thickness is reduced. The effects of the second wall which forms the vacuum chambers cooling channels and the inside of the dewar can be estimated in a similar way. Since the resistivity of stainless steel at these temperatures is about 3×10^3 times that of OFHC copper, the effects are small. It should be noted that the residual resistivity of the copper we have used here is much higher than that quoted for very pure copper. Under the assumptions we have made: uniform cylindrical tube; negligible end effects; and, in the case of the quadrupole, the tube axis coinciding with the magnet axis; the eddy currents do not introduce additional field harmonics.

References:

1) J.W. Bittner, P. Grand, ISABELLE Vacuum Chamber, CRISP 72-7.

2) A.F. Clark, G.E. Childs and G.H. Wallace "Electrical Resistivity of Some Engineering Alloys at Low Temperatures" Cryogenics 10, 4 August 1970.

3) W.R. Smythe, Static and Dynamic Electricity, 3rd Edition, p 400, McGraw Hill.

4) Ibid, p 299.

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