# "B U C K"

## A FORTRAN-IV CODE FOR THE COMPUTATION OF THE VOLUME AND GEOMETRICAL BUCKLING TO BE USED FOR THE DETERMINATION OF THE "SAFE CRITICAL" CONCENTRATIONS OF FISSILE MATERIALS CONTAINED IN VESSELS WHICH ARE NOT GEOMETRICALLY SAFE

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## ABSTRACT

The determination of the safe concentration of Fissile Material contained in vessels which are not geometrically safe, is a time-consuming problem which arises frequently in Nuclear Reprocessing Plants.

This programme was therefore elaborated in order to speed up the computations involved. It outputs the volume and the geometrical buckling of the solution contained in any standard-shaped vessel, for as many as 30 different values of the level.

It is written in Fortran-IV and is being run on a PDP8-I, equipped with an 8K core memory.


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# CONTENTS

## 1. INTRODUCTION

This programme originated from the extensive computations which the authors of this paper performed in the course of their systematic determination of the safe concentrations of fissile materials contained in vessels which are not geometrically safe.

The so-called "safe fissile material concentration limit" for a vessel is the concentration that will not exceed an appropriate fraction (safety factor) of the minimum critical mass in the layer that could be formed if the vessel were full and a uniform fissile material precipitation were to occur.

## 2. THE COMPUTATIONAL PROCEDURE

The method, for determining the concentration limit, runs as follows :

2.1. The volume and the geometrical buckling are calculated as a function of the level of the solution contained in the vessel.

2.2. For each value of the level, the criticality condition is expressed by equating the material buckling to the geometrical buckling ($B_m^2 = B_g^2$). The corresponding critical concentration of fissile material is then derived from the available data in the literature (e.g. Criticality Handbook by Carter, Kiel and Ridgway - Ref. 7.2.).

2.3. The critical mass corresponding to each solution level is obtained by multiplying the critical concentration by the volume of the solution.

2.4. The critical mass is plotted versus the solution level and the minimum critical mass is then given by the minimum of the curve thus obtained.

2.5. The safe critical concentration for the full vessel is calculated by dividing the minimum critical mass by the total volume and by applying an appropriate safety factor to the result.

The first step of the above procedure is by far the most tedious and time-consuming. Moreover, the determination of the safe concentration of solutions contained in vessels which are not geometrically safe is a problem which arises every now and again in a reprocessing plant. It was therefore felt that a programme specially designed to calculate the volume and the geometrical buckling automatically, would be very useful.

## 3. EXPLANATORY COMMENTS (cf. Appendix 1)

3.1. The successive values of the level are defined automatically and build an arithmetic series whose interval can be chosen arbitrarily (input data HU) according to the size of the vessel.

3.2. The programme has been written so as to output the volume and the geometrical buckling for a maximum of 30 different values of the level.

3.3. The vessels are grouped according to their general configuration, into three main categories which are identified by the input data ID.

ID = 1 : vertically mounted cylindrical vessels (Fig. 1, 2)
= 2 : parallelepipedic tanks (Fig. 3)
= 3 : horizontally mounted cylindrical vessels (Fig. 4)

3.4. In each particular case, the correct extrapolation distance ($\lambda$) can be introduced into the programme by the input data EXT.

3.5. To our knowledge, the smallest eigenvalue (geometrical buckling) of the wave equation :

$$\mathrm{div.grad}\ \varphi(\overline{\tau}) + B_g^2\ \varphi(\overline{\tau}) = 0$$

has been expressed analytically in the following cases only :

rectangular parallelepiped, finite vertical cylinder, sphere and infinite slab of finite thickness.

In order to take into account such geometries as round- or conical-bottomed vertical cylinders (Fig. 1 and 2), pyramidal-bottomed rectangular parallelepipeds (Fig. 3) and round-ended horizontal cylinders (Fig. 4), the following computational procedure has been developed. This method consists of approximating spherical sections by volumetrically equivalent vertical cylinders, and pyramids (as well as sections of horizontal cylinders) by volumetrically equivalent rectangualr slabs.

In other words, for each value of the level, the programme calculates the corresponding volume in the vessel under consideration. By equating this value to that of a hypothetical volume having the same height as the original geometry, "BUCK" then computes the equivalent radius or widths, as the case may be.

For example, in the case of a spherical section,

$$V = \frac{\pi h^2}{3}\left[3 R_S - h\right] = \pi h r_{eq}^2$$

where $R_S$ = radius of the sphere

$h$ = height of the level

$r_{eq}$ = equivalent radius of the hypothetical cylinder.

Hence $r_{eq} = \sqrt{\frac{V}{\pi h}}$

and $B_g^2 = \left(\frac{2,405}{r_{eq}+\lambda}\right)^2 + \left(\frac{\pi}{h+2\lambda}\right)^2$

In the case of a pyramid, we would have :

$V = a\ b\ h/3 = a'\ b'\ h$

with $\frac{a'}{a} = \frac{b'}{b}$

Hence, $b' = \sqrt{\frac{V}{h} \cdot \frac{b}{a}}$

$a' = \frac{a}{b} b'$

and $B_g^2 = \left(\frac{\pi}{a'+2\lambda}\right)^2 + \left(\frac{\pi}{b'+2\lambda}\right)^2 + \left(\frac{\pi}{h+2\lambda}\right)^2$

If we now consider a round-ended horizontal cylinder,

$$V = L.S = a.L.h$$

with $$L = X + h \frac{R_s - \sqrt{R_s^2 - R_c^2}}{R_c}$$

$$S = 2\,R_c^2 \arctan \frac{\sqrt{R_c^2 - (R_c - h)^2}}{R_c - h} - (R_c - h)\sqrt{R_c^2 - (R_c - h)^2}$$

where $X$ = length of the cylinder generatrix

$R_c$ = radius of the cylinder

$R_S$ = radius of the spherical ends

$h$ = height of the level

Hence $$a = \frac{S}{h}$$

and $$B_g^2 = \left(\frac{\pi}{h + 2\lambda}\right)^2 + \left(\frac{\pi}{L + 2\lambda}\right)^2 + \left(\frac{\pi}{a + 2\lambda}\right)^2$$

It can be shown that in each case the area of the surface of the volumetrically equivalent solid is smaller than that of the original figure. As a result of this approximation, the relative neutron leakage probability is therefore decreased and the computed values of the geometrical buckling and of the critical masses and concentrations, are conservative.

3.6. It might be mentioned at this point that we originally envisaged to include into the programme the second step of the general computational procedure (§ 2.2), in order to obtain also the critical concentrations of the fissile material.

The analytical expression of a curve, giving the critical plutonium concentration as a function of the material Buckling (Fig. 5), has therefore been derived by the usual curve-fitting methods.

However, this curve is only valid for Pu-$H_2O$ mixtures containing 0 % of Pu-240 and in order to achieve any reasonable degree of generality, a great number of similar curves would have to be expressed analytically.

In view of the enormous amount of work which this task would represent and considering the ease with which a one-to-one relationship can be determined by graphical means, we have decided to exclude this refinement from the present programme.

## 4. CONCLUSIONS

Extensive trial runs have been made, covering all the geometrical configurations considered. It can therefore be said that the programme whose listing is appended to the present report, operates satisfactorily.

The computer we have used is a Digital Equipment Corp., series PDP 8 I with an 8K core memory.

## 5. NUMERICAL EXAMPLE

Appendix 2 gives the print-out of a typical case, i.e. a vertical cylinder with a conical bottom containing a plutonium solution likely to precipitate. The first line recapitulates the input data which read from left to right :

the identification number (1), the number of different levels at which the volume and the geometrical buckling are to be calculated (30), the value of the level increment (2 cm), the radius of the cylinder (75 cm), the radius of the sphere (0), the height of the cone (50 cm), the lenght of the cylinder generatrix (0), the length of the sides of the parallelepiped (0), the height of the pyramid (0) and the extrapolation distance (4,5 cm).

It will be noticed that whenever a parameter does not apply to the case under consideration, it is set equal to zero.

The corresponding values of the plutonium concentration are looked up on (Fig. 6) and the successive values of the plutonium weight are then calculated and plotted as a function of the level (Fig. 7). The minimum of this curve gives a critical mass of 0.73 kg, which after multiplication by a safety factor of 0.75, gives us a safe critical mass of 0.54 kg.

## 6. ACKNOWLEDGEMENT

The authors wish to express their appreciation of the good will and competence which Mr G. Van Genechten has constantly displayed during the compiling, the debugging and the numerous trial runs of this programme.

## 7. REFERENCES

7.1. Plutonium Handbook, Vol. 2, Chap. 27

Nuclear Safety and Criticality of Plutonium

E.D. Clayton and W.A. Reardon

7.2. Criticality Handbook, Vol. 1

Carter, Kiel and Ridgway

7.3. The Elements of Nuclear Reactor Theory

S. Glasstone and M. Edlund

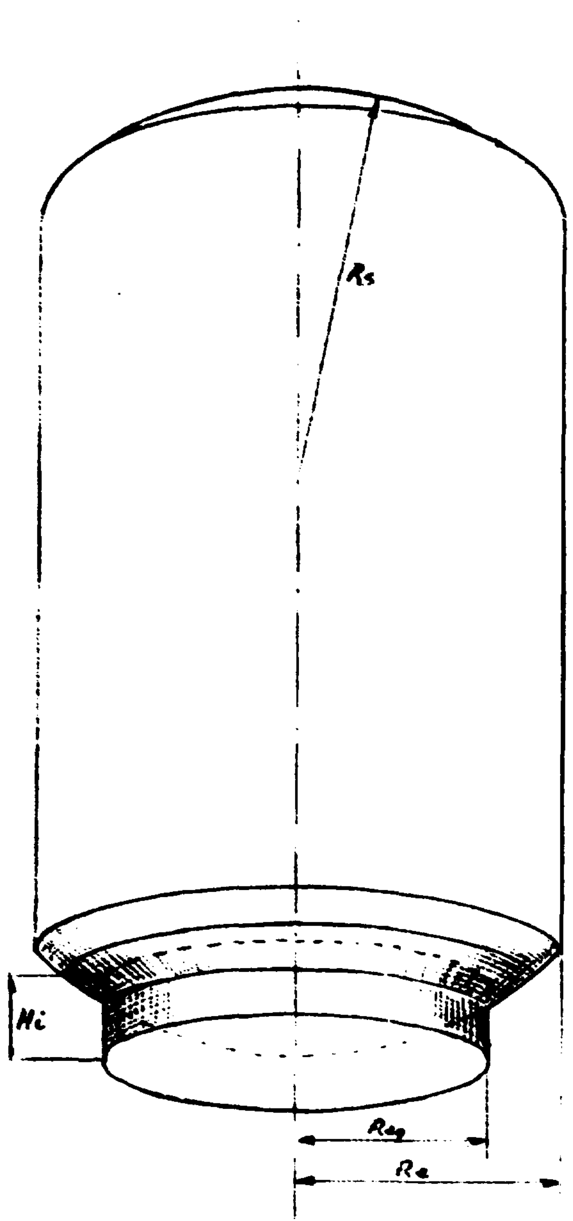



Fig. 1

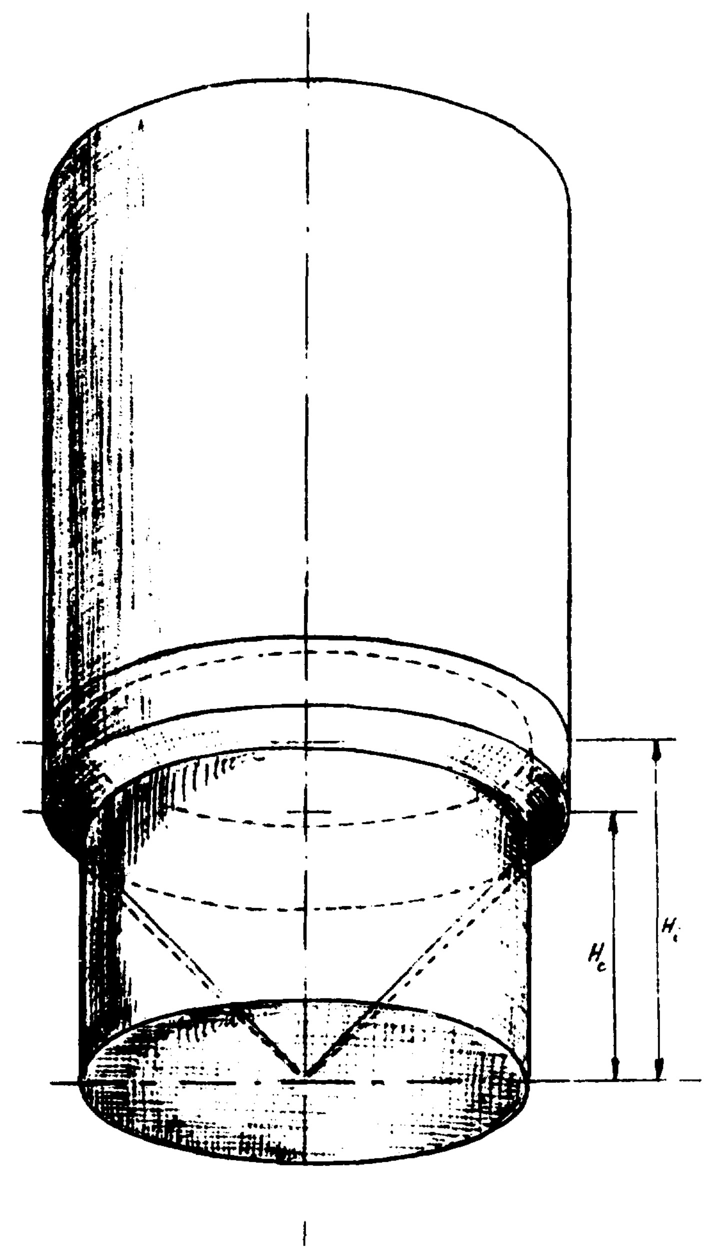



Fig. 2

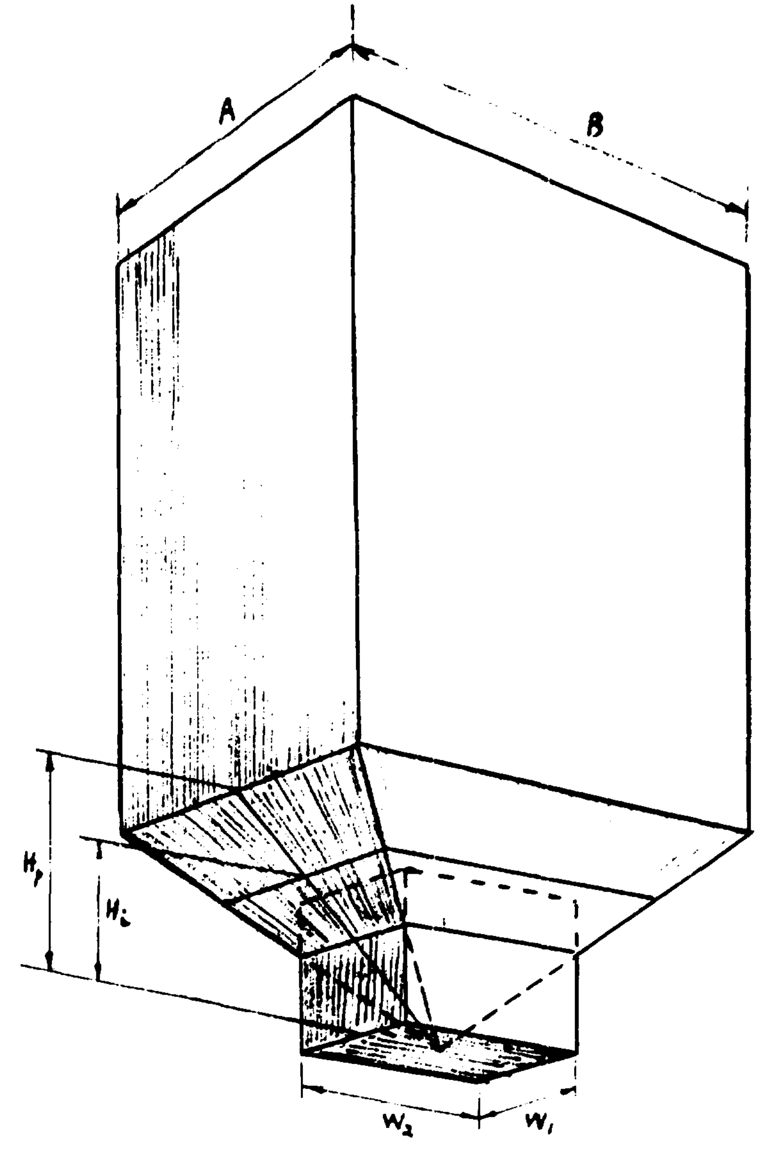



Fig. 3

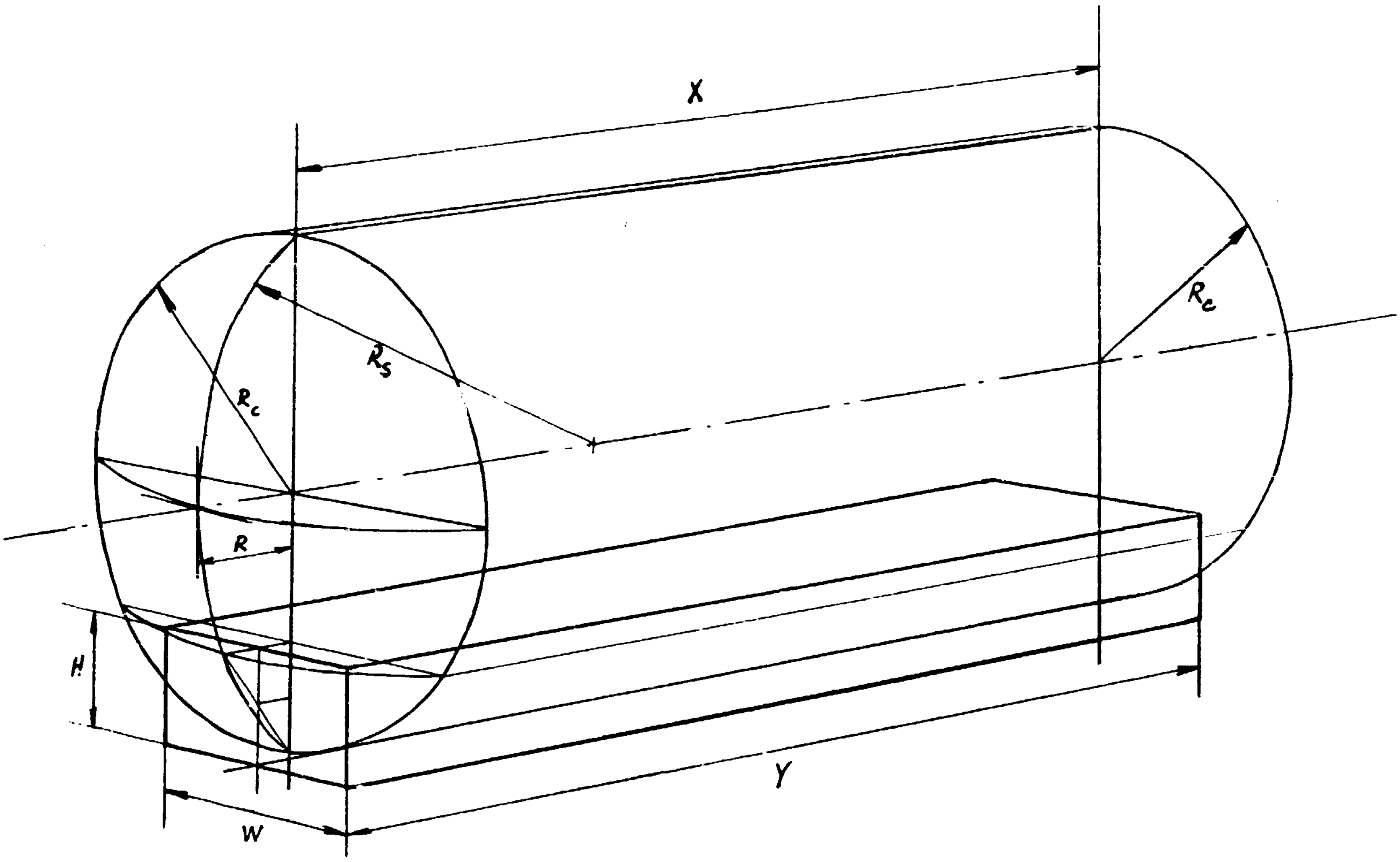



Fig. 4

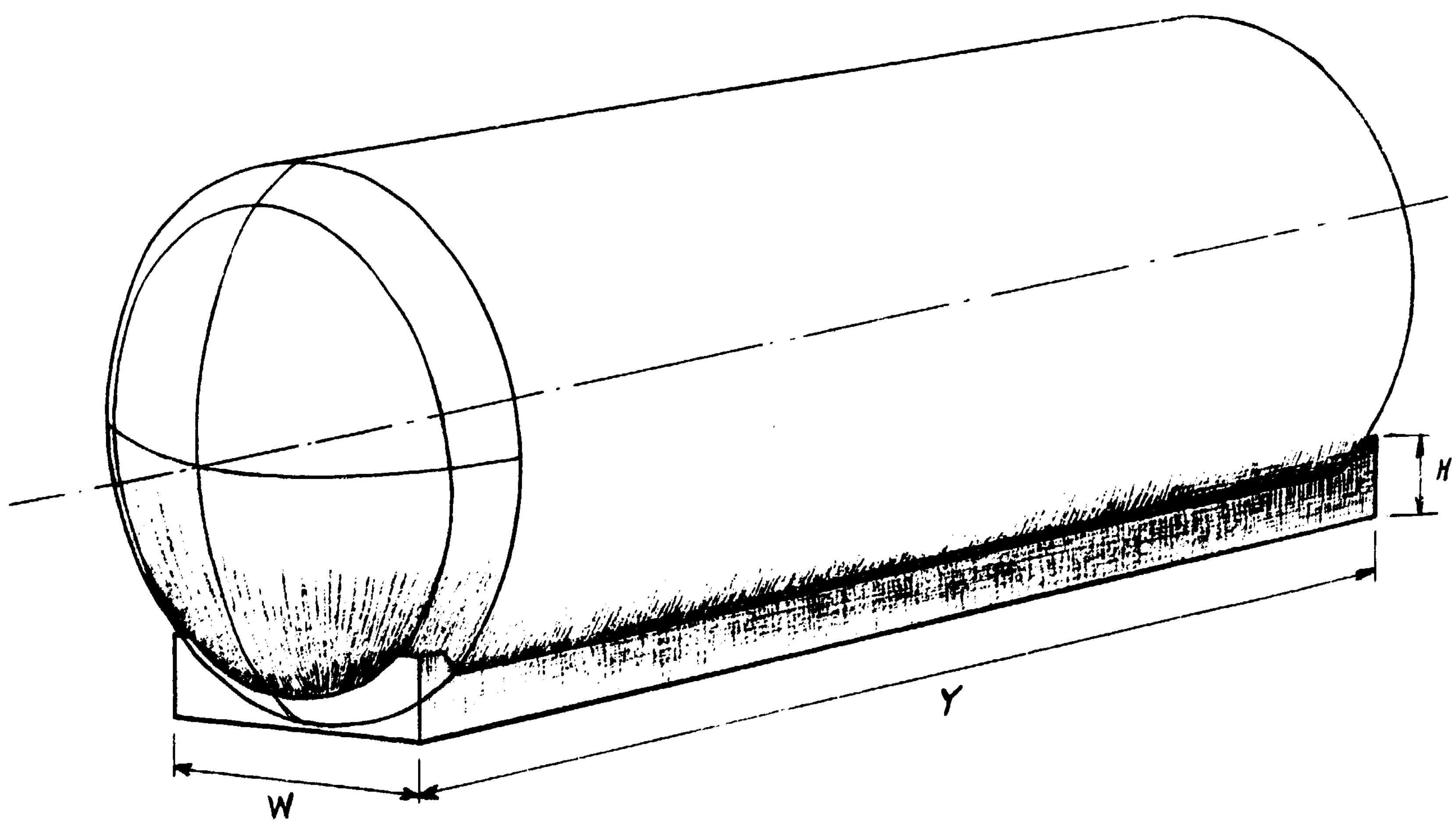
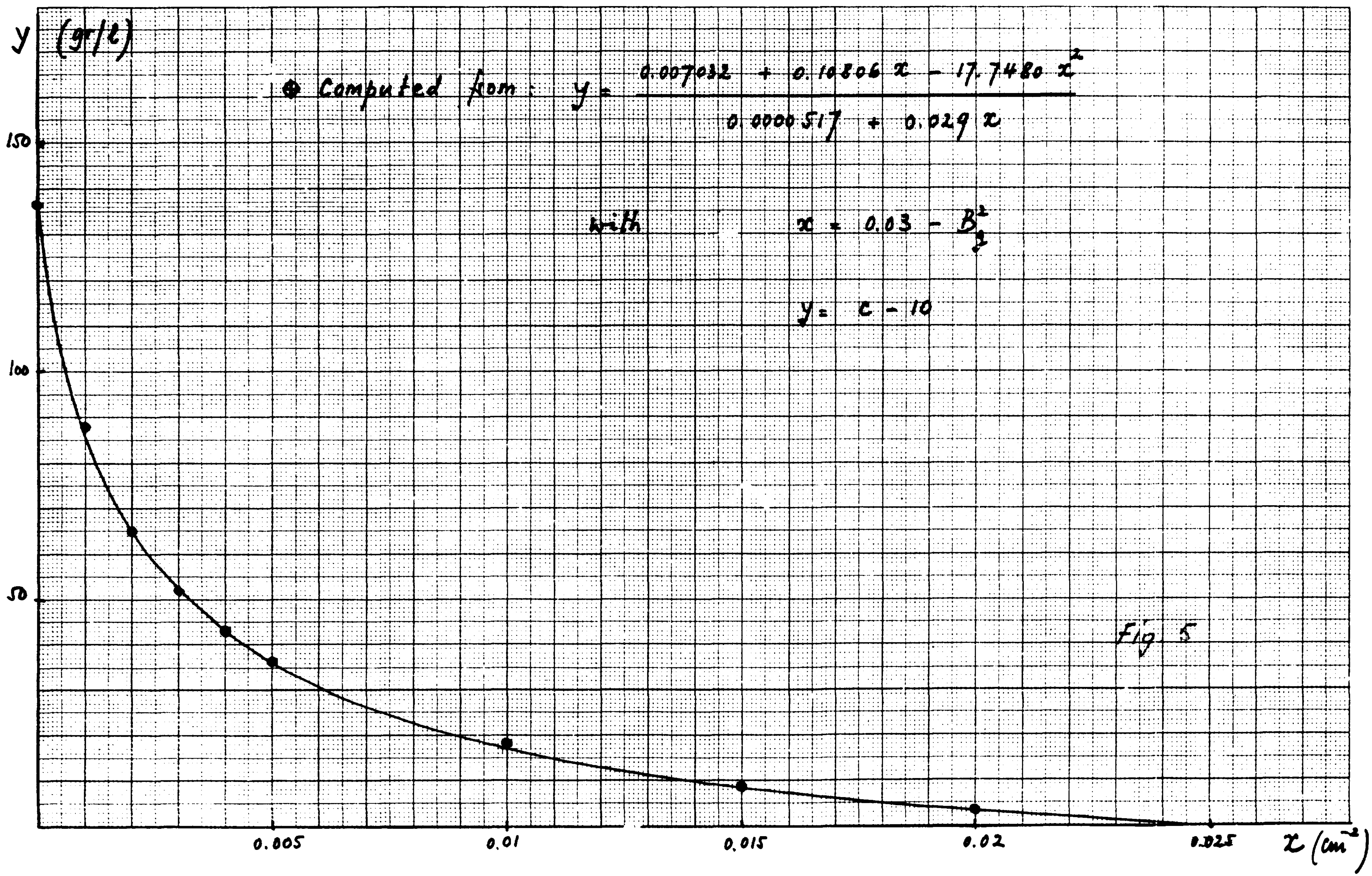
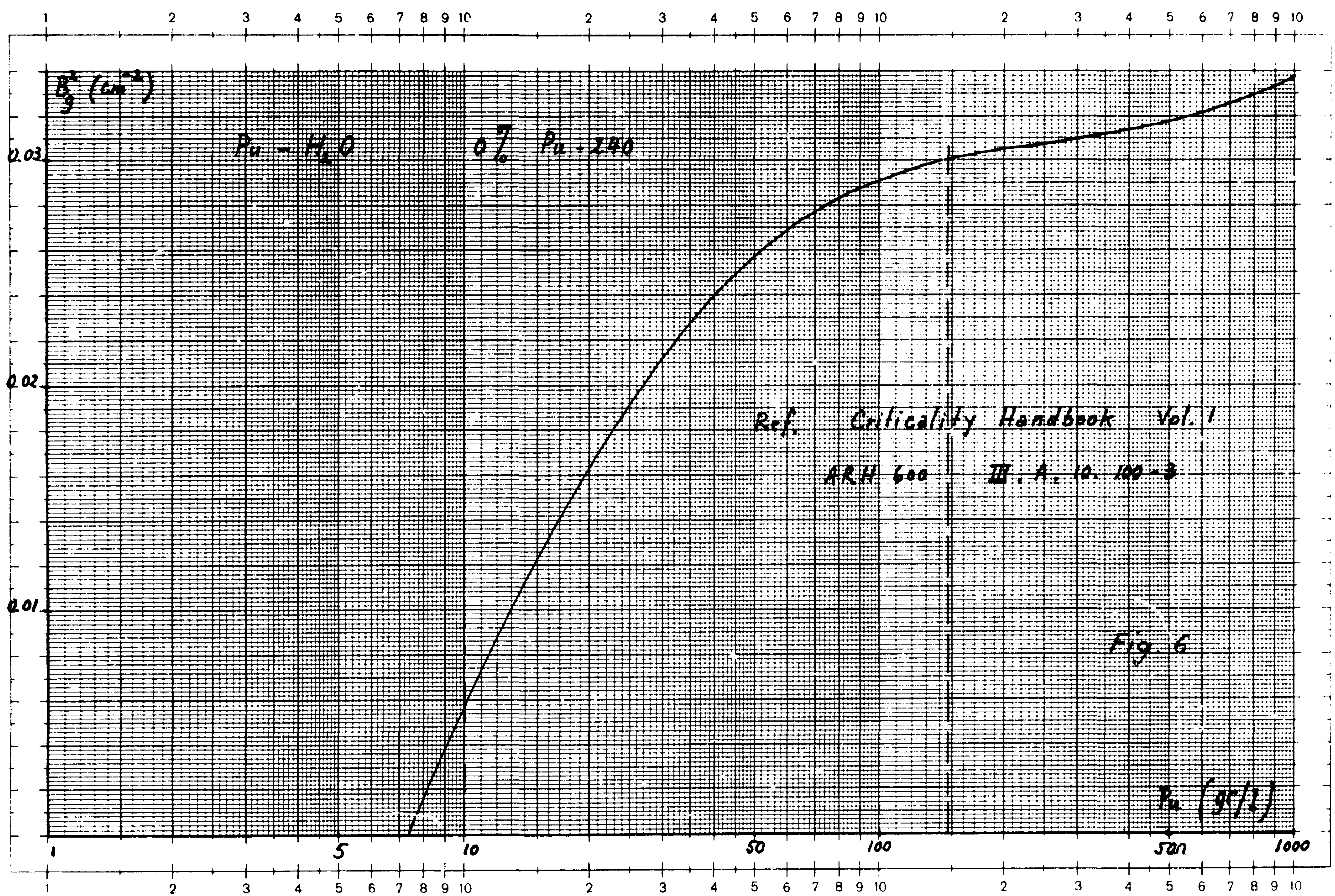



Fig. 4 bis

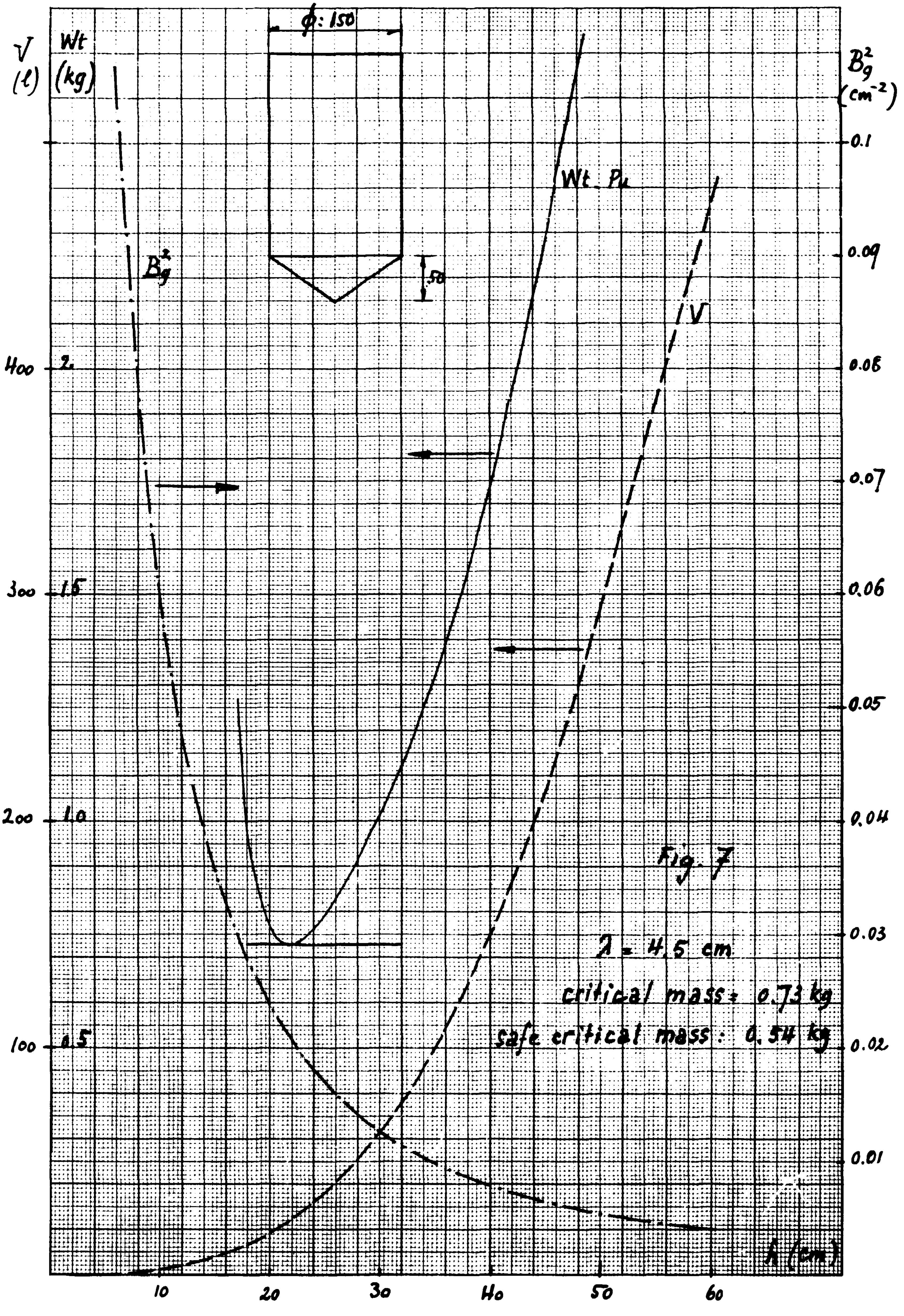


y (gr/l)
⊕ Computed from: y = (0.007032 + 0.10806 x − 17.7480 x²) / (0.0000517 + 0.029 x)
with x = 0.03 − B²g
y = c − 10
150
100
50
0.005
0.01
0.015
0.02
0.025
x (cm⁻²)


Fig. 5



$B_g^2$ ($cm^{-2}$)
Pu - $H_2O$
0 % Pu - 240
Ref. Criticality Handbook Vol. 1
ARH 600 III. A. 10. 100 - 3
Fig. 6
Pu (gr/l)
0.03
0.02
0.01
1
5
10
50
100
500
1000



φ: 150
50
V (l)
Wt (kg)
$B_g^2$ (cm$^{-2}$)
Wt. Pu
$B_g^2$
V
400 2.
300 1.5
200 1.0
100 0.5
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
10
20
30
40
50
60
h (cm)
Fig. 7
λ = 4.5 cm
critical mass: 0.73 kg
safe critical mass: 0.54 kg

# *Appendix 1*

```
      DIMENSION H(30),V(30),BUCK(30)
1     FORMAT (I1,I3,6F6.2)
2     FORMAT (/,7H LEVEL,8X,3HVOL,8X,9HBUCKLING,/)
3     FORMAT (3E12.6)
      READ (1,1) ID,N,HU,RC,RS,HC,X,A,B,HP,EXT
      ID (ID-2) 4,5,6
4     IF (RS) 7,8,7
7     HM=RS-SQRT(RS*RS-RC*RC)
      M=0
      DO 9 I=1,N
      M=M+1
      F=I
      H(I)=HU+(F-1.)*HU
      IF (H(I)-HM) 10,10,11
10    V(I)=1.0472*H(I)*H(I)*(3.*RS-H(I))
      GOTO 12
11    V(I)=1.0472*HM*HM*(3.*RS-HM)+3.1416*(H(I)-HM)*RC*RC
12    REQ=SQRT(V(I)/(3.1416*H(I)))
      C=(2.405/(REQ+EXT))**2
      D=(3.1416/(H(I)+2.*EXT))**2
      BUCK(I)=C+D
9     CONTINUE
      GOTO 13
8     IF (HC) 21,22,21
21    M=0
      DO 23 I=1,N
      M=M+1
      F=I
      H(I)=HU+(F-1.)*HU
      IF (H(I)-HC) 24,24,25
24    V(I)=1.0472*RC*RC*H(I)*H(I)*H(I)/(HC*HC)
      GOTO 26
25    V(I)=3.1416*RC*RC*(H(I)-0.6666*HC)
26    REQ=SQRT(V(I)/(3.1416*H(I)))
      C=(2.405/(EXT+REQ))**2
      D=(3.1416/(2.*EXT+H(I)))**2
      BUCK(I)=C+D
23    CONTINUE
      GOTO 13
22    M=0
      DO 14 I=1,N
      M=M+1
      F=I
      H(I)=HU+(F-1.)*HU
      V(I)=3.1416*H(I)*RC*RC
```

```
      C=(2.405/(RC+EXT))**2
      D=(3.1416/(H(I)+2.*EXT))**2
      BUCK(I)=C+D
14    CONTINUE
      GOTO 13
5     M=0
      DO 15 I=1,N
      M=M+1
      F=I
      H(I)=HU+(F-I.)*HU
      IF (H(I)-HP) 16,16,17
16    Q1=H(I)*A/HP
      Q2=H(I)*B/HP
      V(I)=H(I)*Q1*Q2/3.
      GOTO 18
17    V(I)=HP*A*B/3.+(H(I)-HP)*A*B
18    W1=SQRT(A*V(I)/(B*H(I)))
      W2=W1*B/A
      U1=3.1416/(H(I)+2.*EXT)
      U2=3.1416/(W1+2.*EXT)
      U3=3.1416/(W2+2.*EXT)
      BUCK(I)=U1*U1+U2*U2+U3*U3
15    CONTINUE
      GOTO 13
6     R=RS-SQRT(RS*RS-RC*RC)
      M=0
      DO 19 I=1,N
      M=M+1
      F=I
      H I)=HU+(F-1.)*HU
      DIF=RC-H(I)
      IF (DIF) 13,13,20
20    T=SQRT(RC*RC-DIF*DIF)
      S1=RC*RC*ATAN(T/DIF)
      S2=T*DIF
      S3=S1-S2
      Y=X+H(I)*R/RC
      V(I)=S3*Y
      W=V(I)/(Y*H(I))
      Z1=(3.1416/(Y+2.*EXT))**2
      Z2=(3.1416/(W+2.*EXT))**2
      Z3=(3.1416/(H(I)+2.*EXT))**2
      BUCK(I)=Z1+Z2+Z3
19    CONTINUE
13    WRITE (1,2)
      WRITE (1,3) (H(I),V(I),BUCK(I),I=1,M)
      CALL EXIT
      END
```

# Appendix 2

G
1 30 2 75 0 50 0 0 0 0 4.5

| LEVEL | VOL | BUCKLING |
|---|---|---|
| 2.00 | 0.188496E+02 | 0.230493E+00 |
| 4.00 | 0.150797E+03 | 0.149592E+00 |
| 6.00 | 0.508939E+03 | 0.105387E+00 |
| 8.00 | 0.120637E+04 | 0.784378E-01 |
| 10.00 | 0.235620E+04 | 0.607363E-01 |
| 12.00 | 0.407151E+04 | 0.484601E-01 |
| 14.00 | 0.646541E+04 | 0.395858E-01 |
| 16.00 | 0.965099E+04 | 0.329569E-01 |
| 18.00 | 0.137414E+05 | 0.278716E-01 |
| 20.00 | 0.188496E+05 | 0.238835E-01 |
| 22.00 | 0.250888E+05 | 0.206971E-01 |
| 24.00 | 0.325721E+05 | 0.181103E-01 |
| 26.00 | 0.414126E+05 | 0.159813E-01 |
| 28.00 | 0.517233E+05 | 0.142077E-01 |
| 30.00 | 0.636174E+05 | 0.127145E-01 |
| 32.00 | 0.772079E+05 | 0.114454E-01 |
| 34.00 | 0.926080E+05 | 0.103576E-01 |
| 36.00 | 0.109931E+06 | 0.941809E-02 |
| 38.00 | 0.129289E+06 | 0.860105E-02 |
| 40.00 | 0.150797E+06 | 0.788607E-02 |
| 42.00 | 0.174566E+06 | 0.725679E-02 |
| 44.00 | 0.200710E+06 | 0.670003E-02 |
| 46.00 | 0.229343E+06 | 0.620504E-02 |
| 48.00 | 0.260577E+06 | 0.576301E-02 |
| 50.00 | 0.294525E+06 | 0.536664E-02 |
| 52.00 | 0.329927E+06 | 0.501875E-02 |
| 54.00 | 0.365270E+06 | 0.471904E-02 |
| 56.00 | 0.4006.3E+06 | 0.445715E-02 |
| 58.00 | 0.435956E+06 | 0.422595E-02 |
| 60.00 | 0.471299E+06 | 0.402011E-02 |

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