

SMALL ANGLE SCATTERING OF HIGH ENERGY PHOTONS  
ON HEAVY NUCLEI (DELBRÜCK SCATTERING, PHOTON  
SPLITTING)

G. Jarlskog, L. Jönsson  
University of Lund, Lund

S. Prunster, H.D. Schulz, H.J. Willutzki, G.G. Winter  
DESY, Hamburg

(Preliminary version, presented at a CERN seminar,  
May 1972)

INTRODUCTION

During 1970 - 1972 an experiment has been carried out at the DESY electron synchrotron with the aim to study Delbrück scattering which is the elastic scattering of a photon in the Coulomb field of nuclei via virtual electron positron pairs. One of the corresponding Feynman diagrams is shown in fig. 1b. The incoming photon converts into a virtual electron positron pair, which is scattered twice in the static Coulomb field of the nucleus and subsequently annihilates into an outgoing photon.

There are at least two photons exchanged with the nucleus, since the so-called Furry theorem says that diagrams containing a fermion loop with an odd number of corners do not contribute to the cross section. Therefore one photon exchange with the nucleus gives no contribution to Delbrück scattering. The diagram contains 6 vertices which means that Delbrück scattering is of at least 6th order in the electro-magnetic coupling. This is fairly high compared to the more common processes like bremsstrahlung and pair production, that are only of 3rd order. The coupling of the two photons to the Coulomb field gives a factor  $Z^2$  in the Delbrück amplitude. Thus the cross section becomes proportioned to  $Z^4 \alpha^6$ .

As the diagram shows, Delbrück scattering is the direct consequence of vacuum polarization (fig. 1a), that is the part time existence of a photon as a virtual electron positron pair. This virtual electron positron pair may be scattered by external fields. If this field is the nuclear Coulomb field the process is called Delbrück scattering. Detection of Delbrück scattering is thus the most direct proof of the existence of vacuum polarization.

There are two more processes related to vacuum polarization, that is the scattering of light by light (fig. 1c) and the splitting of a photon into two by interaction with an external field (fig. 1d). Especially the first one, scattering of light by light has attracted the interest of physicists for a long time as a basic process predicted by QED, but not possible in linear electrodynamics. Up to now the experimental detection of scattering of light by light has not been possible because of the small cross section and the difficulty of getting dense targets. Delbrück scattering is a first approximation

to the scattering of light by light in the sense that the second photon is replaced by a virtual photon from a static Coulomb field. It is much easier to measure, since one can use solid target with high  $Z$  material.

Another aspect of the Delbrück process is its being the shadow scattering of photons resulting from photon absorption by electromagnetic pair production. The imaginary part of its forward amplitude is related to the total pair production cross section by the optical theorem

$$\text{Im } A_D(k, \vartheta = 0) = \frac{k}{4\pi} \sigma_{\text{tot}}(k)$$

Early approximate calculations of the Delbrück cross section used this relation. Starting from the known pair production cross section Bethe and Rohrlich (ref.1) calculated in 1952 the imaginary part of the Delbrück amplitude from the optical theorem and the real part via dispersion relations. These calculations gave results for small momentum transfer ( $\Delta \ll m_0$ ,  $m_0$  = electron mass) and high energies.

In spite of the fact that Delbrück predicted the process already 40 years ago (ref.2) it was only recently that a calculation of the cross section via conventional perturbation theory was published by Cheng and Wu (ref.3). Even this calculation is valid only for asymptotically high energies and not too small momentum transfer. This is a consequence of the high order of the process, which makes calculations extremely lengthy and complicated. On the other hand it is interesting to study very high order processes, since there is no compelling reason why perturbation theory need be correct to such high orders. Stimulated by Cheng and Wu's paper we proposed a new measurement of the Delbrück cross section at high energies (ref.4).

Earlier measurements had clearly shown the existence of the Delbrück process, but suffered from various

systematic errors. At energies of a few MeV, where most of the experiments were done, it was the interference with not well known low energy scattering processes that made quantitative interpretation difficult. There is one measurement of Moffatt and Stringfellow (ref. 5) at an energy of 100 MeV, which is clean in this respect, but there the contribution of degraded photons from showers developing in the target was not clear. Moffatt and Stringfellow gave their results including shower background. They made estimates on this shower background using two different methods and getting two different values as the answer, leaving it to the reader to choose between the two alternative values for the background. Depending on what value for the background you choose, you get two distinct data sets for the experimental cross section. Cheng and Wu (ref.3) picked out the set most favorable to their curve, whereas if you take the other, which is just as good, you find the experimental data are now in agreement with the Bethe Rohrlich prediction which is quite different from Cheng and Wu (fig. 2). In this situation it was clear that the data of Moffatt and Stringfellow could not be used to check Cheng and Wu's calculation and it seemed worthwhile to perform another measurement in the GeV region, where we expected the competing processes to be well known (however, see below):

In fig. 3 the differential cross section is plotted as a function of the scattering angle and photon energy for Delbrück scattering and for Compton scattering on atomic electrons as the competing processes. The target material is Uranium, which is most favorable for the Delbrück process, because this is proportional to  $Z^4$ , whereas Compton scattering is proportional to  $Z$  only. Other competing elastic processes one might think of, for example Compton scattering on nuclei, are negligible compared to these ones. Delbrück scattering shows a strong forward maximum and dominates at small scattering angles. For the

primary photon energies considered, that is between 1 and 7 GeV, it is above the Compton cross section only within a few milliradians from the beam. That means one has to measure scattered photons at very small angles, close to the primary beam. The differential cross section is large, but since the relevant solid angle is very small counting rates become nevertheless critical.

#### EXPERIMENTAL METHOD

The experimental set up used is shown in fig. 4. The sketch is not to scale, only to show the principle. A well collimated bremsstrahlung beam from the synchrotron hits the scatterer and passes without interaction through a magnetic pair spectrometer and is finally buried in a totally absorbing ionization chamber, that serves as the monitor. There are several sweeping magnets and collimators between target and spectrometer (not shown in fig. 4), that clean the beam from charged particles. The scattered photons are converted into electron positron pairs on a ring shaped aluminum converter and analyzed with a magnetic spectrometer.

To be able to measure scattered photons at 1 mr one has to make sure that the photon density of the primary beam drops by 8 to 9 orders of magnitude at a distance of 2 cm from the beam, which has a diameter of .8 cm. This reduction of the beam halo was achieved by a special system of 3 collimators and sweeping magnets. The condition on the collimation channel and the position of the converter ring was the following: The first collimator is the defining one, all the following ones are just to strip off the beam halo and must not touch the primary beam any more. The second collimator must not see the machine target. The third must not touch the secondary beam from the edges of the first collimator. The scattering target is just behind the third collimator. Scattered photons are detected by the conversion in a ring which in turn

must not see the target nor any edges of collimators except the last one. We found, that each collimator carefully positioned in this way reduces the beam halo by about three orders of magnitude. With this arrangement empty target background in our measurements ranged from 5 to 30% and has been subtracted in the following.

Another dangerous source of background are secondary processes within the scatterer. A photon produces a pair, the pair electrons change direction via multiple scattering and finally emit bremsstrahlung that mixes with the Delbrück scattered photons. This is an especially serious background since it is  $Z^4$ -dependent as is the Delbrück scattering. There are two ways to keep this background small:

1. The production of secondary photons is a two step process. Their contribution rises quadratically with the target thickness and can be kept small by using thin targets. Since this would mean small Delbrück rates too, a trick was used to overcome this limitation. The target was split into a number of sufficiently thin foils (0.02 r.l.) which were placed one behind the other inside a sweeping magnet with equal distances in between. The magnetic field between the foils has to sweep out the electrons and positrons produced in a previous foil so that the emitted secondary photons from the next foil are well outside the solid angle of the converter ring. In this way we could increase the effective target thickness by a factor of ten without getting a quadratic rise in the number of secondary photons.
2. The other way to keep the contribution of secondary photons small is based on the fact that secondary photons are degraded in energy, because they originate from an inelastic process and any inelastic process would move them downwards in energy, contrary to the elastically scattered Delbrück photons. If one uses photons only from the edge of the bremsstrahlung spectrum for the measurement, one can keep

the contribution of secondary photons small. We used only photons within 3% from the edge of the bremsstrahlung spectrum in the Delbrück measurement. The relative contribution of secondary photons in this bin was typically 10% and could be corrected for with good precision as will be shown below.

The scattered photons are detected with the pair spectrometer. They hit a ring made of 2 mm aluminum, through the central hole of which the primary beam passes. This ring converts a definite fraction of the scattered photons into electron positron pairs, which are bent in the magnetic field and detected by scintillation counters and Charpak chambers. The wire separation in the Charpak chambers was 2 mm and the gas used was a mixture of argon with carbon dioxide; two crossed planes were used in each position. The setup contained a total of 1000 wires. The efficiency of the chambers was better than 99.5 % at a time resolution of 120 ns. The system worked reliably and caused us no trouble.

The information from the Charpak chambers made possible a complete reconstruction of the event as to the energy and the conversion point of the photon. The photon energy was measured to 1%, the coordinates of the conversion point to  $\pm 3$  mm, which gave a geometrical angular resolution or  $\pm .15$  mrad.

If one plots the reconstructed starting points of pairs in the plane of the converter one gets a true picture of the ring as shown in fig. 5. This is the image of a small ring with 2 cm radius and .4 cm width. One clearly sees the ring, the holder and a few pairs produced by the primary beam in the rest gas of the spectrometer. We had to have rather good vacuum of  $10^{-5}$  torr within the spectrometer not to be flooded by those beam units. The reconstruction of the conversion points enabled us to cover in one run the total angular acceptance from 1 to 3.5 mrad using one large ring at the expense of giving up some angular resolution.

The reconstruction was also useful for the rejection of spurious tracks. Good events had to satisfy the constraint, that electron and positron must start from the same point within reconstruction errors.

To check the overall performance of the apparatus, especially the efficiency and acceptance of the spectrometer, we measured the primary bremsstrahlung spectrum using a conversion foil instead of the ring and compared the result with the theoretical prediction. This is shown in fig. 6. The line is the theoretical spectrum normalized with the monitor reading. The experimental points are event rates multiplied by the acceptance of the spectrometer. The agreement is satisfactory and gives a measure of the absolute precision of data obtained with this apparatus.

### DISCUSSION

Fig. 7 shows a typical spectrum of scattered photons for gold as scatterer and a ring shaped converter. All open points are event rates after subtraction of empty target background. The line is drawn by hand to guide the eye. The form of the spectrum is very much different from a normal bremsstrahlung spectrum which looks similar to the Compton contribution, shown here as the broken line. There are many more soft photons than in a bremsstrahlung spectrum.

The Delbrück contribution is obtained from the highest 3%-bin. The energy resolution is one third of the width of this bin. We proceed in the following way: The Compton contribution is subtracted, calculated from the Klein-Nishina formula, which in this case amounts to 16% of the counting rate in the Delbrück-bin. Further we subtract the contribution from secondary photons calculated by Monte Carlo methods from the known cross sections of pair production and bremsstrahlung. This amounts to 7% of the counting rate in this bin. After doing these



subtractions the full points are obtained. Since our acceptance covers down to roughly 75% of maximum energy the calculation of secondary photons is extended down to that point and it can be seen how fast the number of secondary photons is rising towards lower energies.

To check experimentally the correct description of secondary photons by the Monte Carlo method the thickness of the scatterer was varied. Fig. 8 shows the ratio between counting rates for double-thickness and single-thickness gold foils versus photon energy. For elementary processes this ratio would be 2. If secondary processes are present it comes out larger than 2. The open points are the ratio of counting rates without subtraction, the full points after subtraction of calculated secondary photons. As expected at the edge of the bremsstrahlung spectrum at the Delbrück-bin, there are practically only elementary processes. At lower energies there is an appreciable contribution from secondary processes, but after subtraction of the Monte Carlo simulated secondary photons there are only elementary processes left within the full energy range considered. That means the Monte Carlo simulation of secondary photons is correct over the full energy range.

Fig. 9 shows the spectrum as we have seen before (fig.7) after subtraction of Compton and secondary photons has been done. We know that only elementary processes contribute to these counting rates and we would expect this to be the Delbrück signal. Let us compare it to the original prediction of Cheng and Wu given here as the broken line. There are two things, one can say at once: The counting rate in the Delbrück-bin is at least a factor of 3 smaller than predicted by Cheng and Wu. In addition the form of the spectrum is quite different from what one would expect for Delbrück scattering.

At the end of the spectrum there are only elastic processes contributing. We are able to measure the elastic contribution in the other parts of the spectrum by varying the maximum energy of the bremsstrahlung spectrum experimentally. We find then the heavy line as the elastic contribution, that is the Delbrück contribution in this spectrum. We see that with growing distance from the edge of the bremsstrahlung spectrum the counting rate is increasing much faster than expected from the elastic contribution alone. Thus we know that the extra photons giving the rise in the soft part of the spectrum 1) originate from elementary processes in the target, 2) are inelastically scattered photons. To find out what process might produce these photons we plotted the number of extra photons, that is number of photons above the heavy line, versus the atomic number  $Z$  of the target as shown in fig. 10. The ordinate shows the number of photons per target atom divided by  $Z^2$ , the abscissa is  $Z$ . The experimental points are within the errors independent of  $Z$  in this plot, which means that the cross section for production of extra photons is proportional to  $Z^2$ .

When visiting DESY some time ago professor von Dardel mentioned that the photon splitting process might give a contribution to our counting rate. Based on an estimate found in the literature we then believed this contribution to be small, until our experimental results raised the question anew. The photon splitting process exactly has the properties we are looking for to explain our extra photons, that is the scattering is inelastic and proportional to  $Z^2$ .

If one looks into the literature about the photon splitting process, one finds the following (refs 6,7):

- 1) There exists no experimental data up to this time;
- 2) there are some complete calculations, which come out with some 100 terms, but have neither been evaluated numerically nor integrated;
- 3) there are a few rough

estimates on the total cross section which differs by a factor of 10. There was one very relevant statement among the rough estimates saying that the cross section  $d\sigma/dk'$  is rising linearly with  $k'$  for small  $k'$ . This looks somewhat strange since one is used from radiative corrections that the probability of soft photon emission is proportional to  $1/k'$ . The different behavior here is the interesting consequence of the fact, that photon splitting is not a radiative correction to some elastic process, since this elastic process is not allowed (the Furry theorem).

The statement of the linear rise of the cross section with  $k'$  of the splitted photon spectrum allows a simple prediction as to the form in our experiment, i.e. the number of extra photons should rise quadratically with the distance from the edge of the bremsstrahlung spectrum. This is because the contribution of splitted photons originating from primary photons in the highest energy bin of the Bremsstrahlung spectrum is rising linearly with the distance from this bin. Further the number of photons in the beam is rising linearly too, as a first approximation, with the distance from the edge.

Let us compare this behavior of splitted photons with our counting rates of extra photons as shown in fig. 11, where the spectrum of extra photons is fitted with a parabola. There is indeed the quadratic rise present. Together with the  $Z^2$  dependence we take this behavior as evidence for the extra photons to be produced by the photon splitting process. This is the first experimental evidence for this process.

From the fitted parabola we get numbers for the photon splitting cross section integrated over the angles of one of the outgoing photons:

$$\frac{d\sigma}{dk' d\Omega_{k'}} = C(k, k') * (1 - \frac{k'}{k})$$

The results are shown in fig. 12. Here the slope  $C$  divided by  $Z^2$  is plotted as a function of the angle of the detected photon with the primary photon energy as parameter. There is a steep rise of the cross section with decreasing angle of the detected photon and with decreasing energy. At the moment there are no theoretical data available for these results to be compared with.

To compare our data on photon splitting with estimates of the total cross section existing in the literature, we integrated our results over the accepted solid angle. The expected energy distribution of splitted photons is shown in fig. 13. The photon splitting distribution looks different from the energy distribution in pair production, which is almost rectangular at high energies. Now there is a linear rise from the edges, the slope of which we have measured. Nobody knows what it looks like in between, whether there is a dip or a maximum. The region measured in our experiment, hatched in the diagram, integrates up to 2.3 mb for gold, which has to be compared to the total cross sections of .87 mb given by Bolsterli (ref. 8) and 4.5 mb as given by Bukhwostov (ref. 9). Considering the fact that the angular integration was not complete in our experiment even the second value for the total cross section seems to be low.

The contribution of photon splitting to the counting rate in the Delbrück-bin, the 3% energy bite at the edge of the bremsstrahlung spectrum, is typically less than 5% and will be neglected in the following, since the statistical errors are larger than this contribution. Thus there is almost no interference in our experiment between the

elastic Delbrück scattering and the inelastic photon splitting processes, as long as one stays close to the edge of the bremsstrahlung spectrum for the Delbrück analysis as we do.

Spectra of scattered photons like the one of fig. 7 have been measured for a variety of angles, energies and scatterer materials. The first check on the consistency of the data from the so-called Delbrück-bin is shown in figs 14 and 15 for two momentum transfer intervals. The experimentally observed cross section is plotted versus atomic number  $Z$  of the scatterer. Both scales are logarithmic. The straight line is the  $Z^4$ -dependence of the Cheng and Wu cross section (ref. 3). The experimental points of copper, silver, gold and uranium are consistently lower than the Cheng and Wu prediction by a factor of about 3 and the  $Z$ -dependence is somewhat less steep than  $Z^4$  especially at high  $Z$ .

We were able to discuss this experimental evidence with Cheng and Wu who then spent some time at DESY. There was the suspicion that higher order effects not taken into account in Cheng and Wu's original paper might produce these effects. The effective coupling constant of the virtual electron or positron to the target nucleus is  $Z\alpha$ , which for large  $Z$  is no longer small compared to one. The Born approximation then breaks down, as is well known from pair production and bremsstrahlung where the actual cross section for uranium is about 20% less than the Born approximation gives. This can be accounted for by the so called Coulomb correction, which in the picture of Feynman diagrams means that one has to take into account the exchange of an infinite number of virtual photons between electron and nucleus. Learning about our results Cheng and Wu introduced the Coulomb correction into their Delbrück cross section and indeed got an impressive reduction (ref.10) of their cross section up to a factor of 5 for several MeV/c momentum transfers and high  $Z$ . The broken lines of figs 14 and 15 shows the new

theoretical prediction for the  $Z$  dependence of this cross section. It comes much closer to the experimental values.

Fig. 16 shows our complete experimental results on the Delbrück cross section for gold as the target, compared to the theory with and without Coulomb correction. The measured values are a factor 3 to 5 below the Born approximation, called CWI, but are essentially in agreement with the theory including the Coulomb correction CWII. To our knowledge this is the largest effect ever observed for this special kind of higher order corrections. Indeed the Coulomb correction in this case is no minor correction but a drastic modification of the cross section.

A closer inspection of our data reveals that there might be some systematic deviations from the Cheng and Wu prediction, but our statistical errors do not allow a definite statement on this point. Besides, the Cheng and Wu calculation is for infinite primary energy. They neglect the real part of the amplitude altogether. What they calculate is the limiting value

$$\lim_{k \rightarrow \infty} \frac{1}{k} M_0^D$$

From the well-known forward scattering amplitude given by Rohrlich and Gluckstern (ref.13) we see that even at 1 GeV photon energy the real part still contributes 10% to the forward cross section. Thus it might be that our energy is not yet high enough to compare directly with Cheng and Wu, without having an estimate of the real part of the amplitude.

Fig. 17 shows results for uranium, together with some data from earlier experiments at low energy. The crosses are measurements of Bösch et al. (ref. 11) from Zürich in 1962 using 9 MeV photons, the rectangle is of Jackson et al. (ref. 12) at Argonne 1969 with 10 MeV photons. The open points are our data. There is about a factor of two between Cheng and Wu predictions and the low energy data. Our data are in rough overall agreement with Cheng and Wu, as it was with gold.

SUMMARY

From the results given above we conclude:

- 1) The Delbrück cross section at high energy, high  $Z$  and a few MeV/c momentum transfer is a factor of 3 to 5 smaller than calculated in the lowest nonvanishing order of perturbation theory. This drastic reduction is due to a special kind of higher order corrections, called Coulomb correction, which fully accounts for the observed difference. Thus the result is that perturbation expansion gives correct results provided the calculation is extended to high enough orders.
- 2) There is strong evidence for the photon splitting process to be present in our data. The cross section deduced from these data is of the expected order of magnitude, but at present there are no precise theoretical numbers to compare with. Experimentally the next step should be to do a coincidence experiment on the photon splitting process and from our experience such an experiment is feasible.

Finally one has to admit, that wherever one tries to check QED once more it comes out as an astonishing good theory.

## Acknowledgements.

We acknowledge the support from several members of the group F39 in DESY, especially Prof. Walochek. Further we acknowledge stimulating and helpful discussions with Prof Guy von Dardel. During this experiment there has been an extraordinary feedback between experiment and theory, here represented by the Professors H Cheng and T T Wu.

Finally two of us (G J and L J) wishes to thank the Swedish Atomic Research Council, the Knut and Alice Wallenberg foundation and the fund of Eilert and Emma Ekberg for grants which made our participation possible, and DESY for their kind hospitality.



## References.

1. H A Bethe and F Rohrlich, Phys Rev 86, 10 (1952)
2. M Delbrück, Z Phys, 84, 144 (1933)
3. H Cheng and T T Wu, Phys Rev, 182, 1873 (1969)
4. G Jarlskog et al, Proposal F39, DESY (1970)
5. J Moffatt and M W Stringfellow, Proc Roy Soc A254, 242 (1960)
6. Y Shima, Phys Rev 142, 944 (1966)
7. V Constantini, B De Tollis and G Pistoni, N Cim 2A, 733 (1971)
8. M Bolsterli, Phys Rev 94, 367 (1954)
9. A P Bukhvostov, V Ya Frenkel and V M Shekhter, Zhur Eksptl' i Teoret Fiz 43, 655 (1962)
10. H Cheng and T T Wu, DESY 71-69
11. R Bösch et al, Phys Lett 2, 16 (1962), Helv Phys Acta 36, 625 (1963)
12. H E Jackson et al, Phys Rev Lett 22, 1008 (1969)
13. F Rohrlich and R L Gluckstern, Phys Rev 86, 1 (1952)

## Figure captions.

1. Feynman diagrams showing a) vacuum polarization b) Delbrück scattering c) scattering of light by light d) photon splitting.
2. Early results on the differential cross section of Delbrück scattering compared with predictions of a) Cheng and Wu (ref 3) b) Bethe and Rohrlich (ref 1).
3. Differential cross section for Delbrück scattering and Compton scattering on uranium.
4. Experimental arrangement (not to scale).
5. The converter ring as mapped by reconstructed events.
6. Measured Bremsstrahlung spectrum compared with the calculated spectrum normalized by the Quantameter reading.
7. Spectrum of photons scattered in gold and converted in a ring-shaped converter.
8. Ratio in counting rate of scattered photons in gold foils of thicknesses  $2t$  and  $t$  versus photon energy  $\bigcirc$  unsubtracted result  $\bullet$  after the subtraction of secondary photons.
9. Spectrum of scattered photons from gold after the secondary and Compton photons have been subtracted compared to predictions of Cheng and Wu (refs 3, 10)
10. Excess photons (see fig 9) per target atom and divided by  $Z^2$  as a function of  $Z$ .
11. Energy spectrum of excess photons fitted with a parabola.
12. Differential cross section for photon splitting integrated over one of the outgoing photons a) versus scattering angle  $\theta$  b) versus primary photon energy  $k$ .
13. Energy distribution of splitted photons.
14. Differential cross section of Delbrück scattering as function of  $Z$  for the scatterer for a given momentum transfer  $\Delta = 3.24$  MeV/c compared with predictions from Cheng and Wu I) without Coulomb corrections (ref 3) II) with Coulomb corrections (ref 10).
15. Same as fig 14 for  $\Delta = 4.28$  MeV/c.
16. Differential cross section of Delbrück scattering in gold compared with Cheng and Wu predictions I) without Coulomb corrections II) with Coulomb corrections.
17. Same as fig 16 for uranium.



Fig 1a

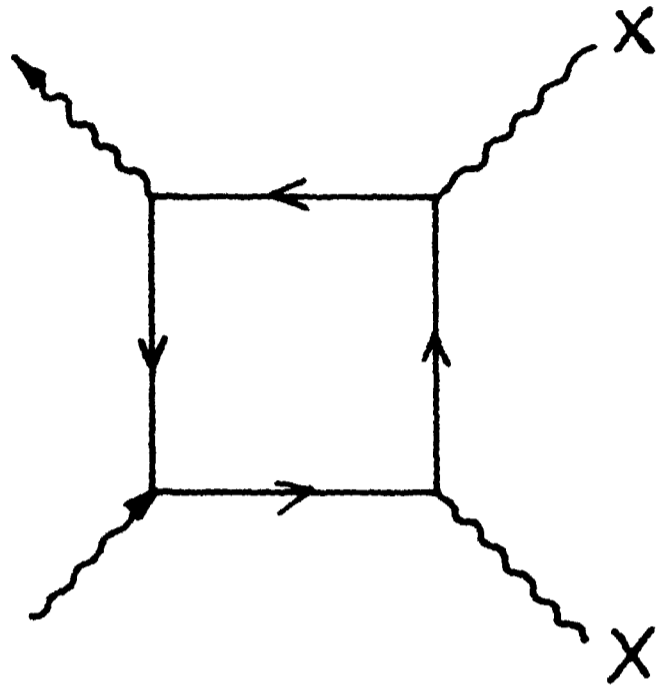


Fig 1b

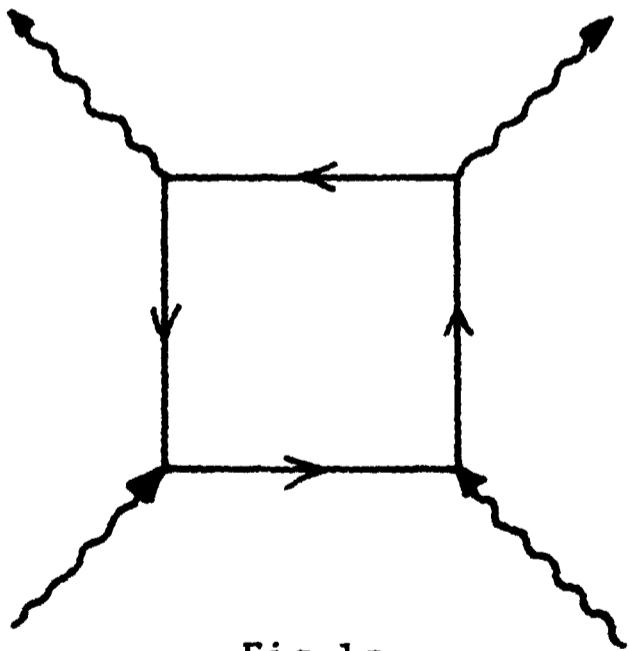


Fig 1c

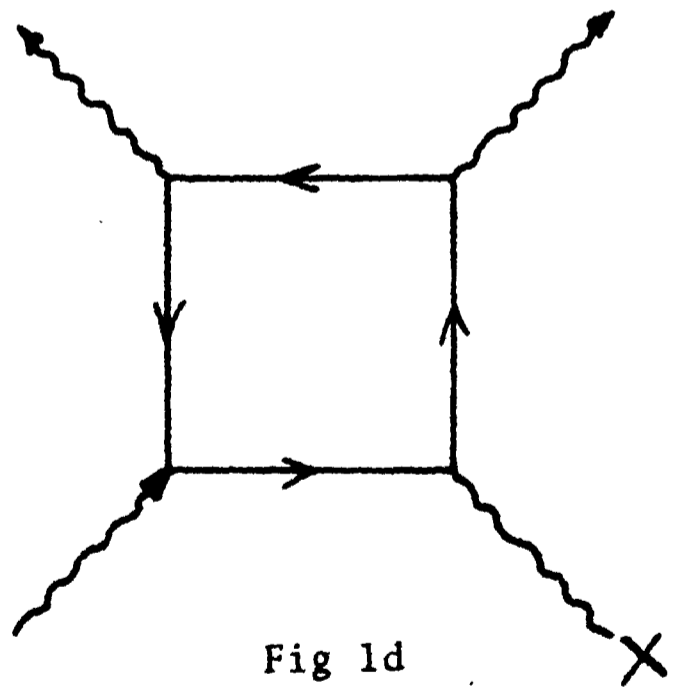


Fig 1d

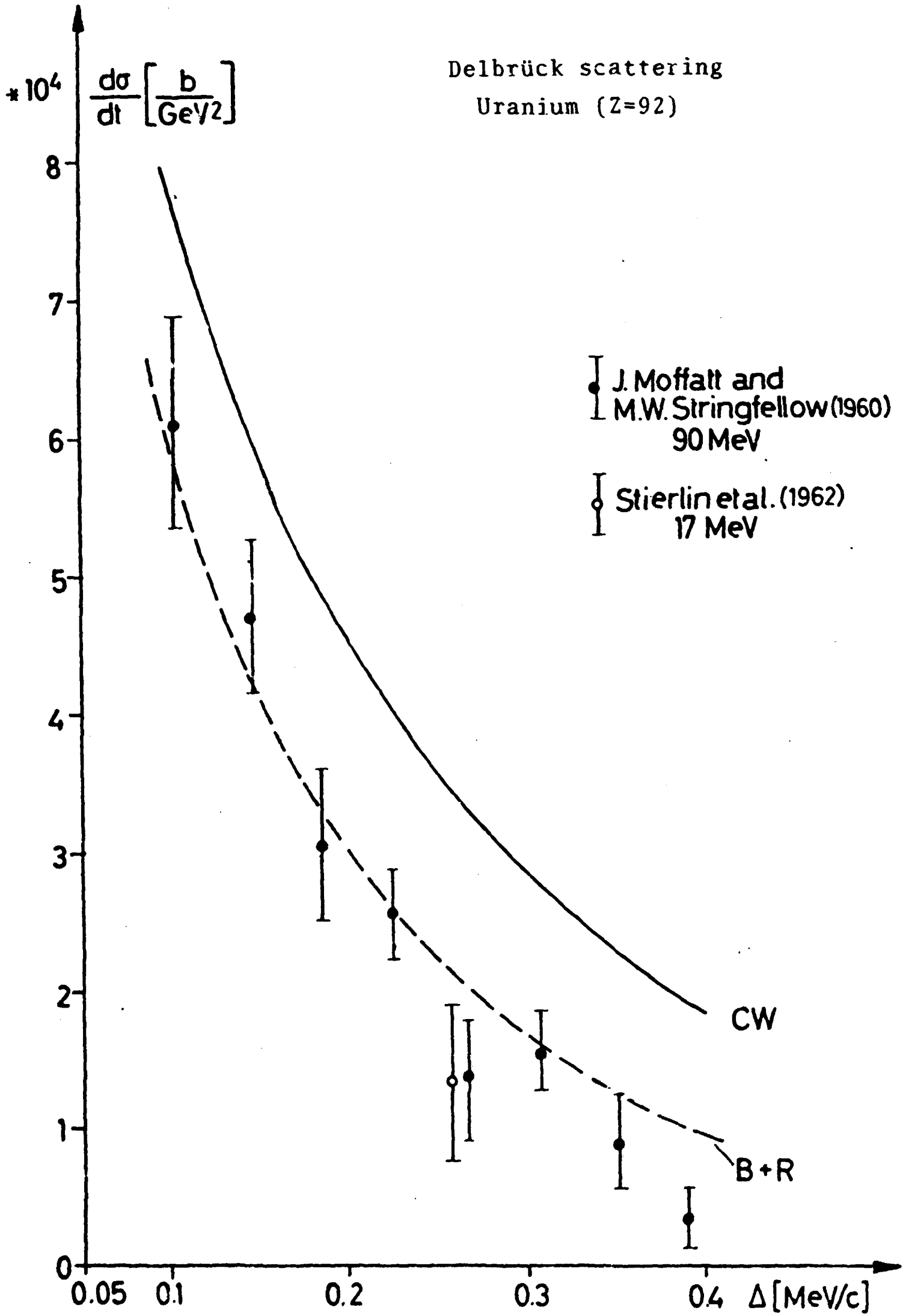


Fig 2

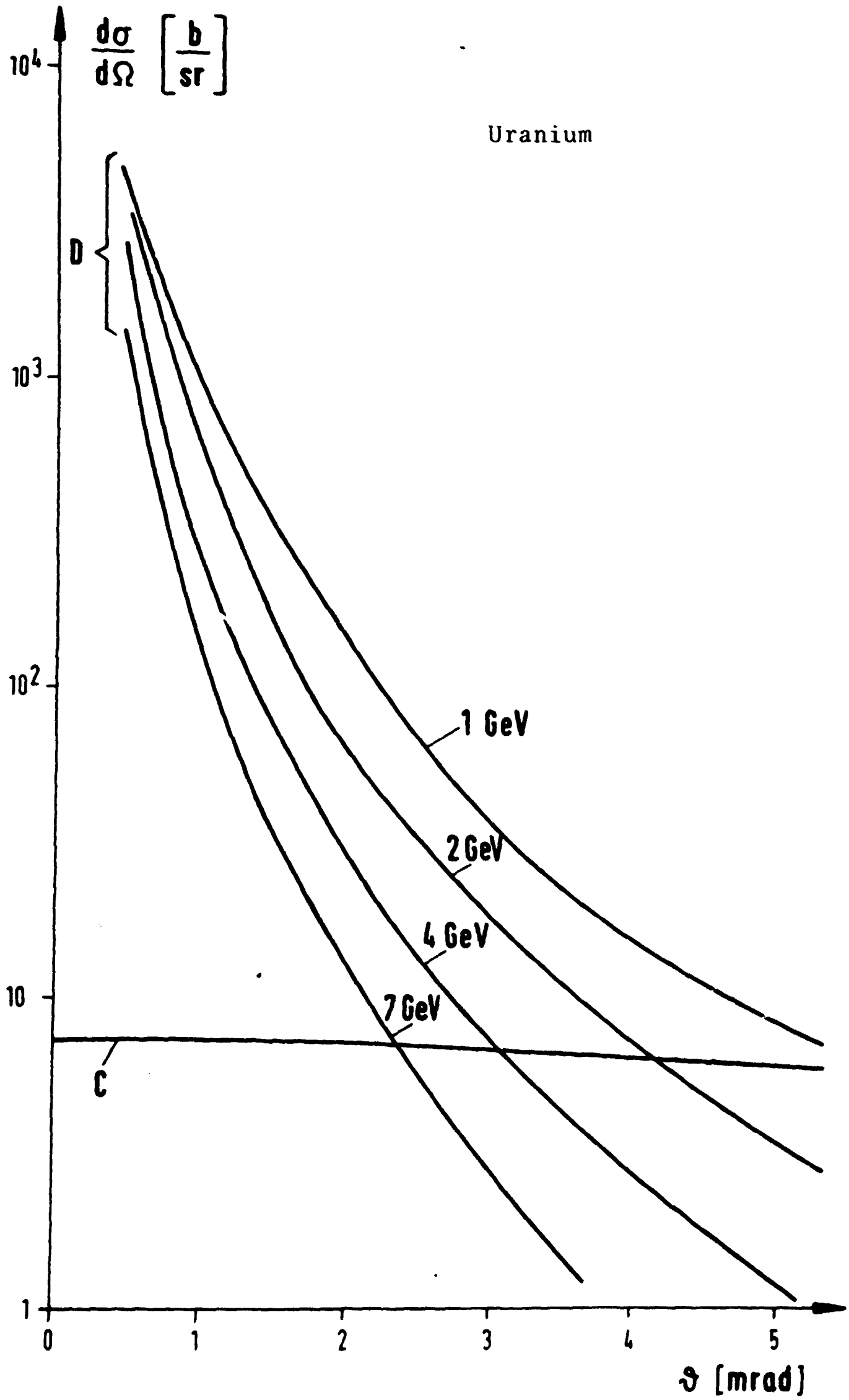
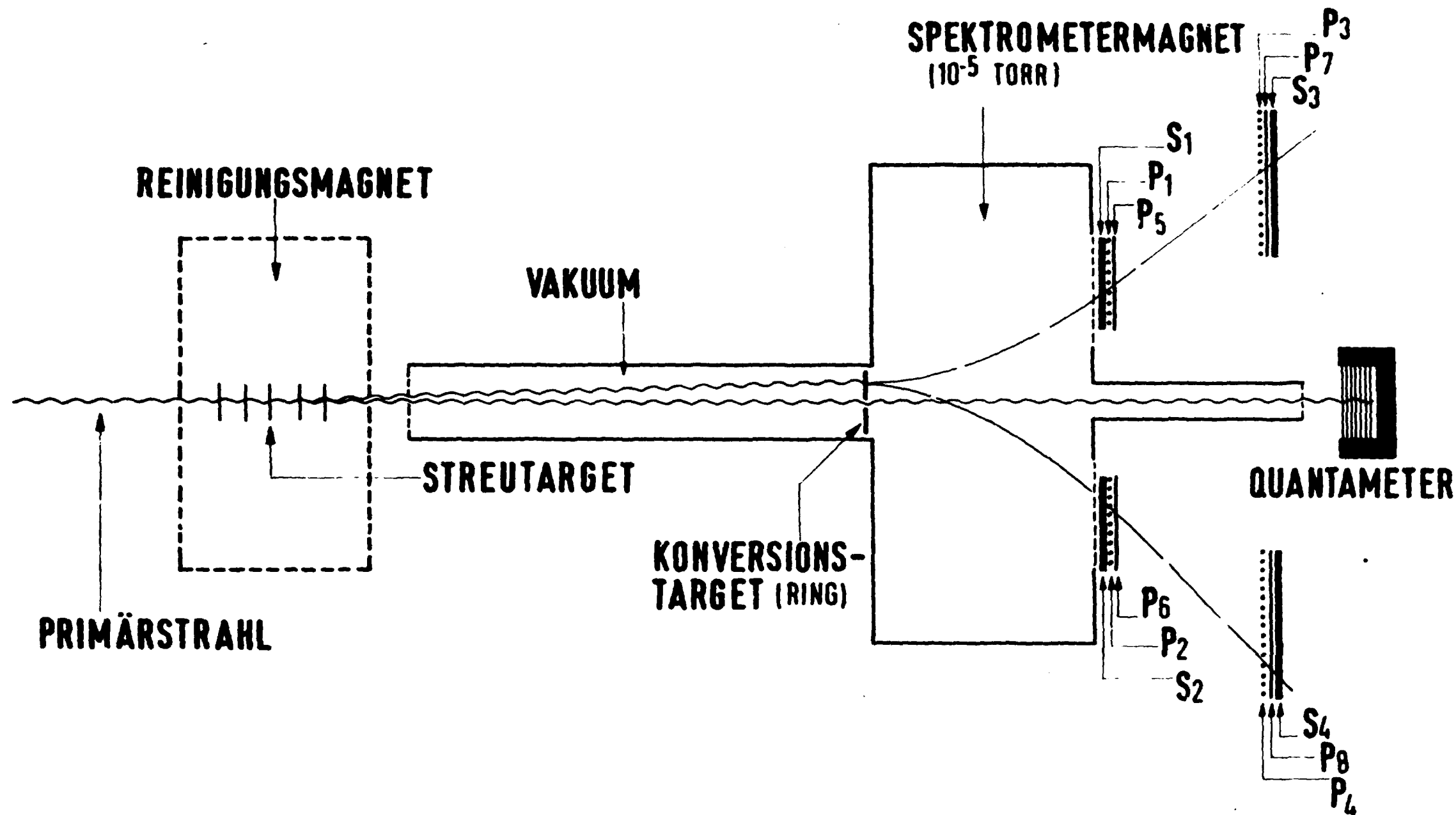


Fig 3



**PRINZIP DER APPARATUR**  
 (NICHT MASSTABGETREU)

S<sub>1</sub> - S<sub>4</sub> SZINTILLATIONSZÄHLER  
 P<sub>1</sub> - P<sub>8</sub> PROPORTIONALKAMMERN

Fig 4

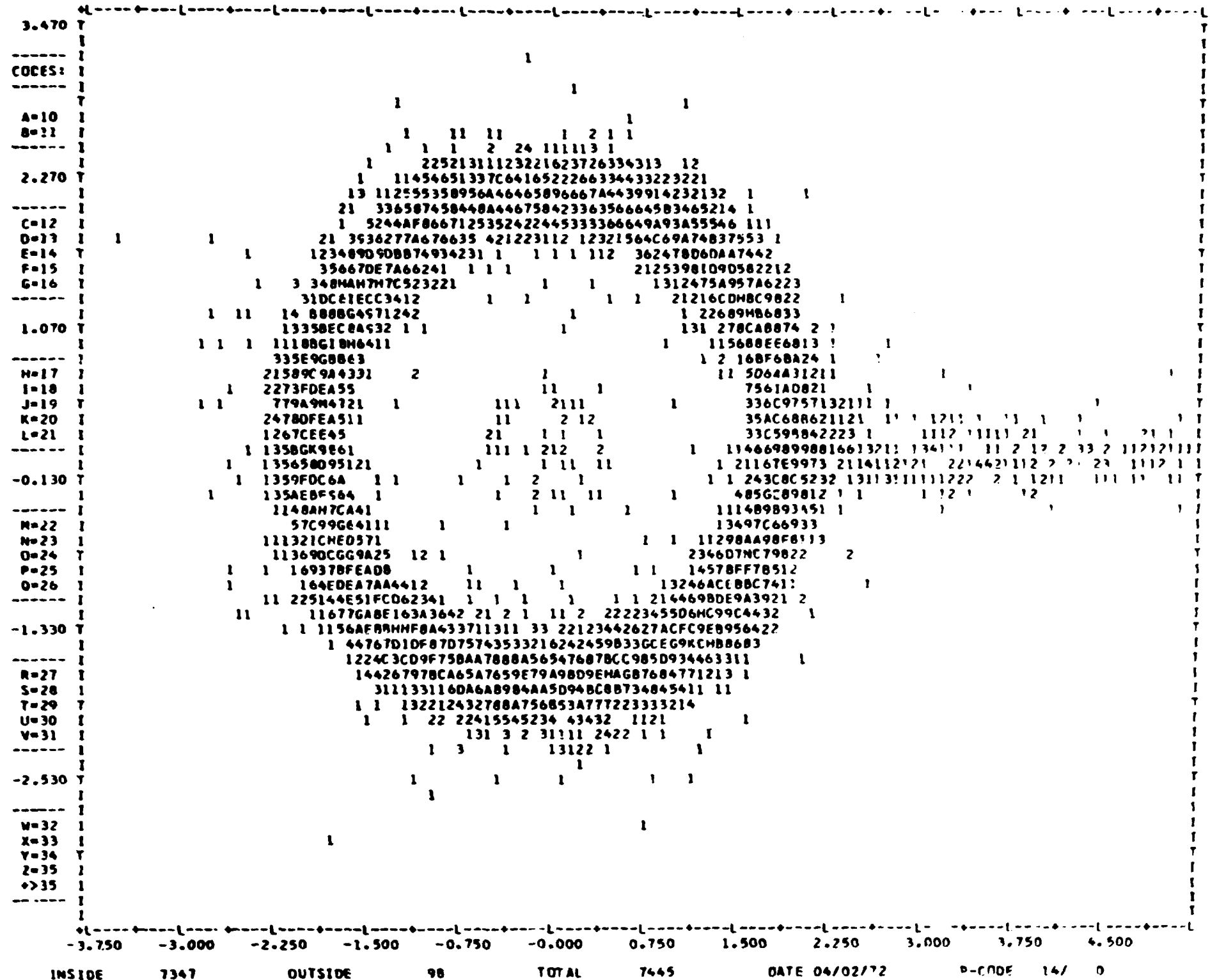


Fig 5

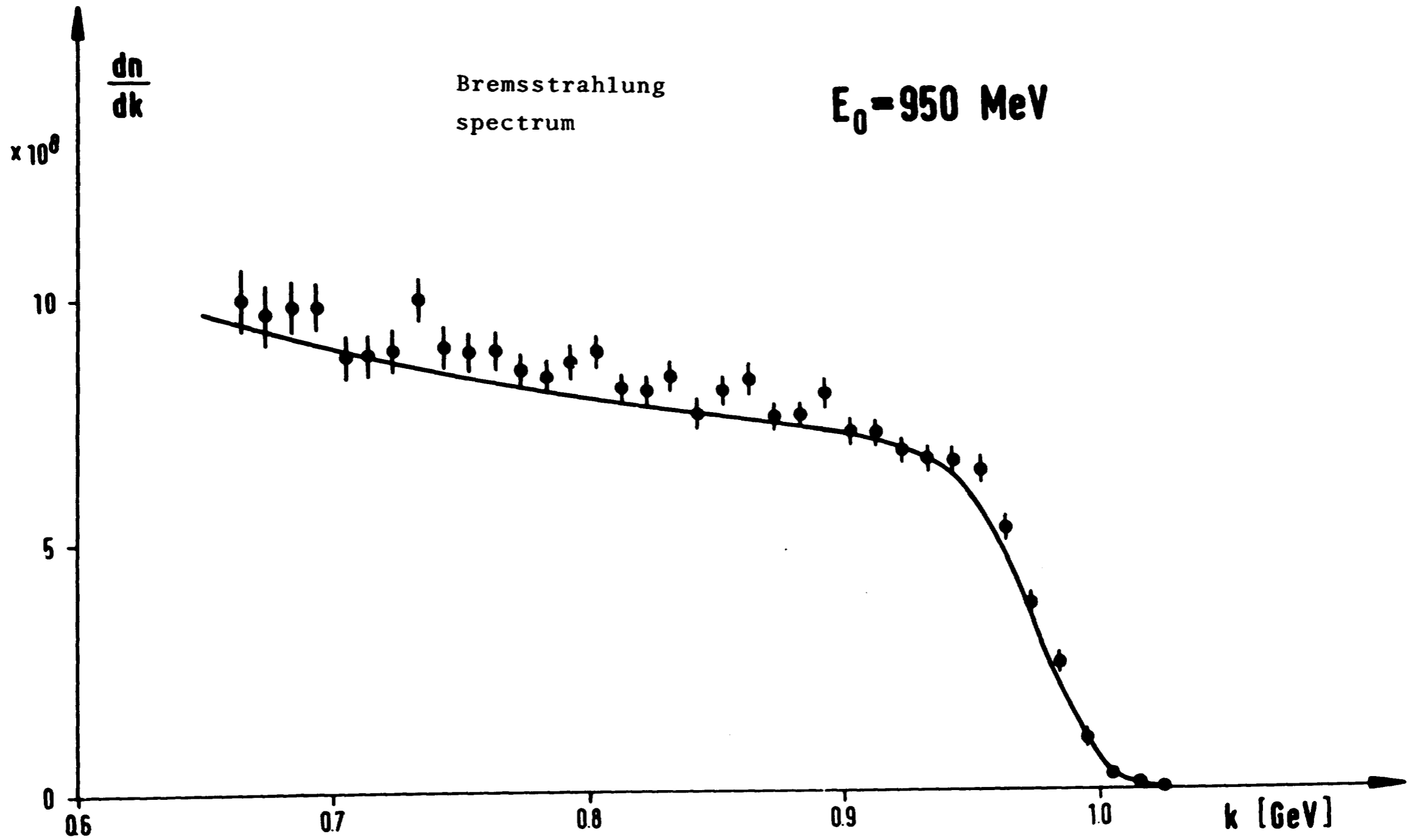


Fig 6



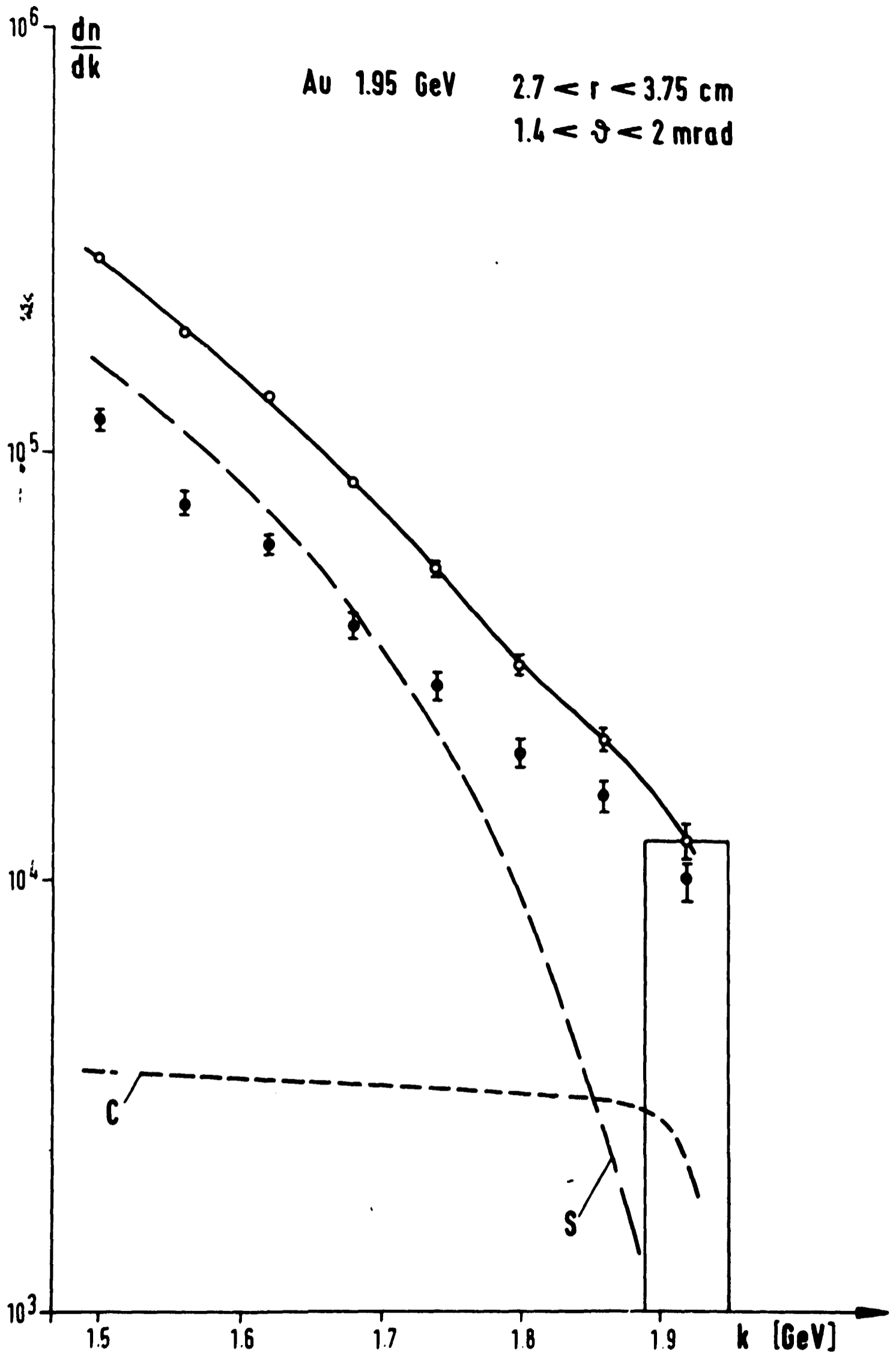


Fig 7

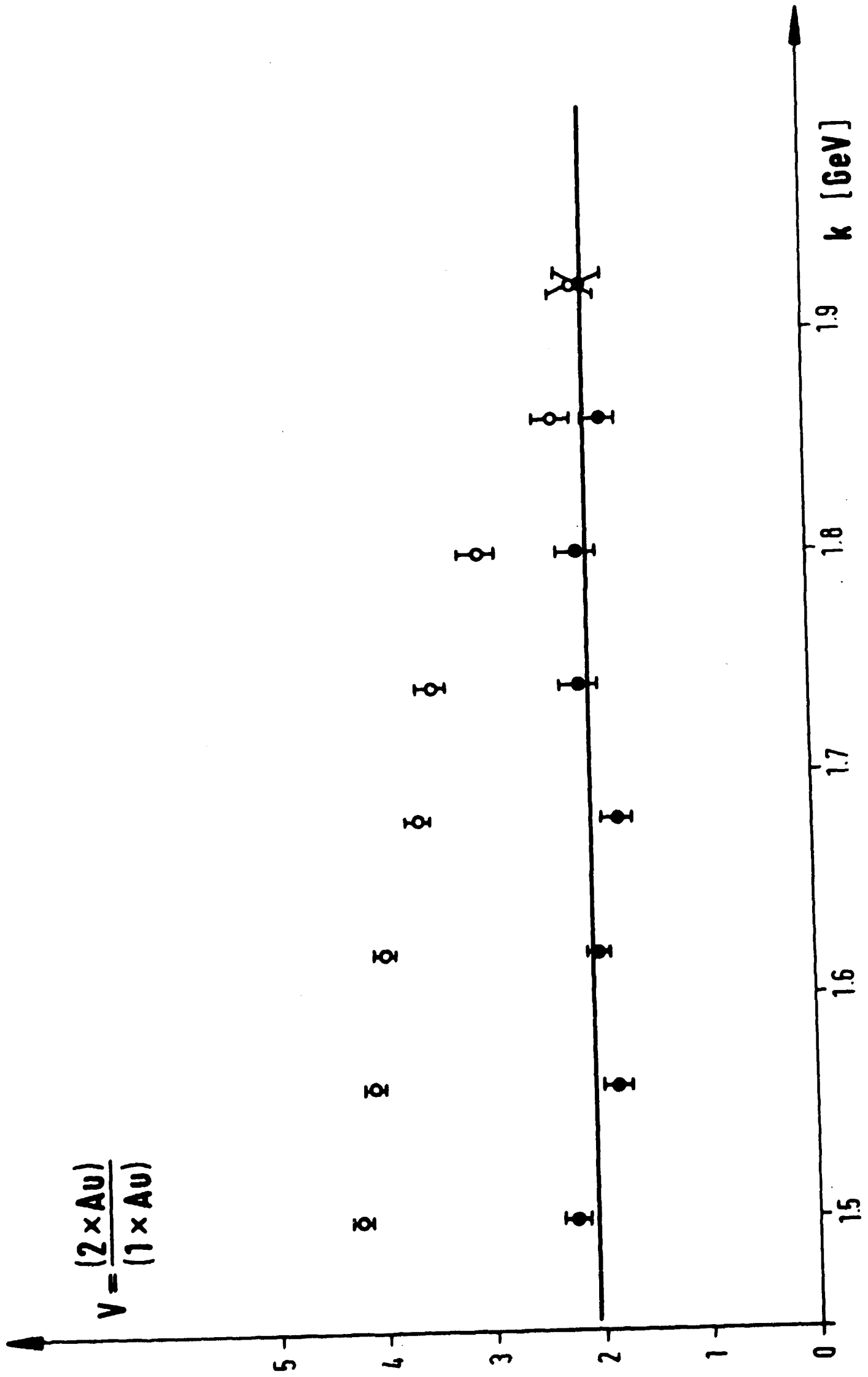


Fig 8

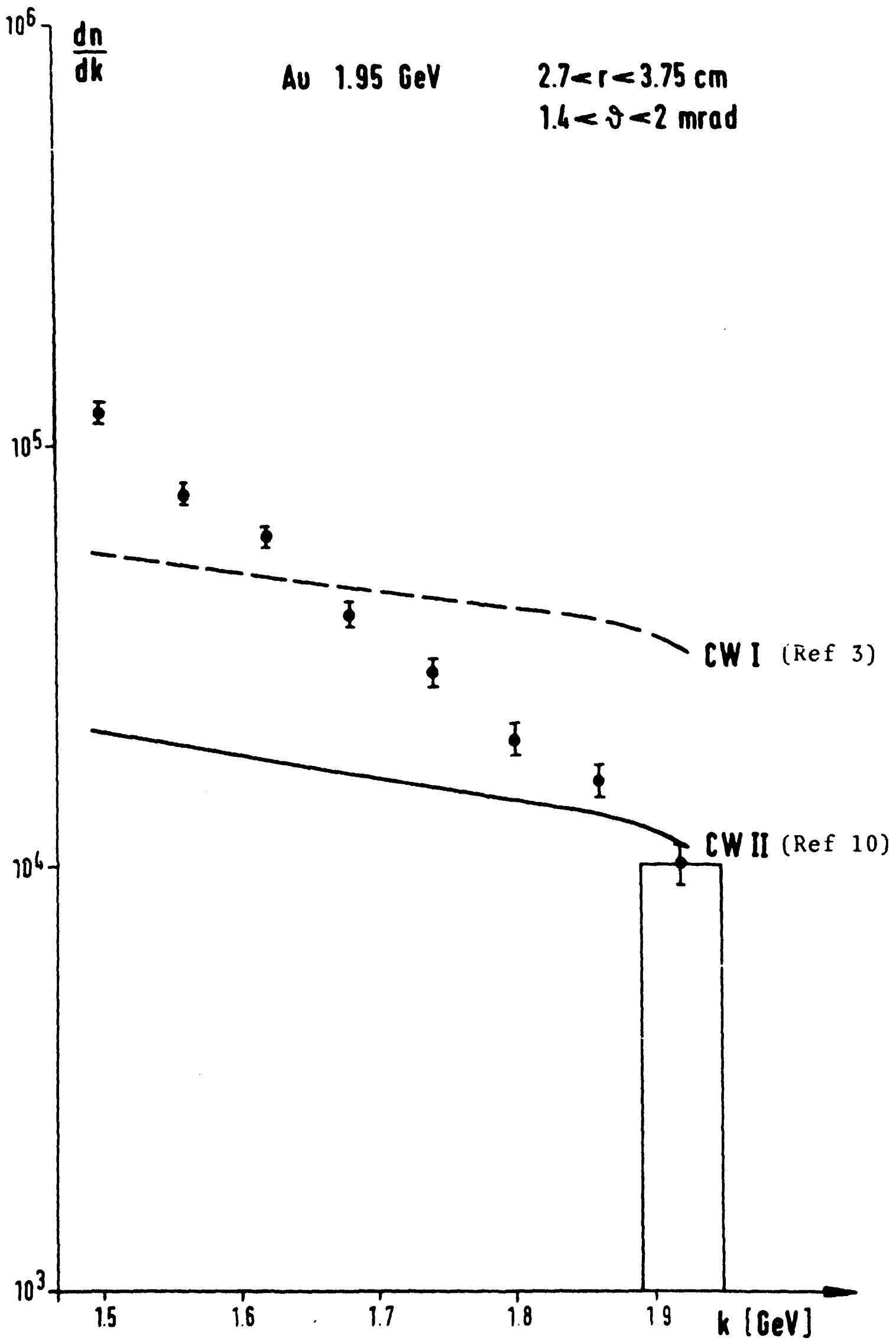


Fig 9

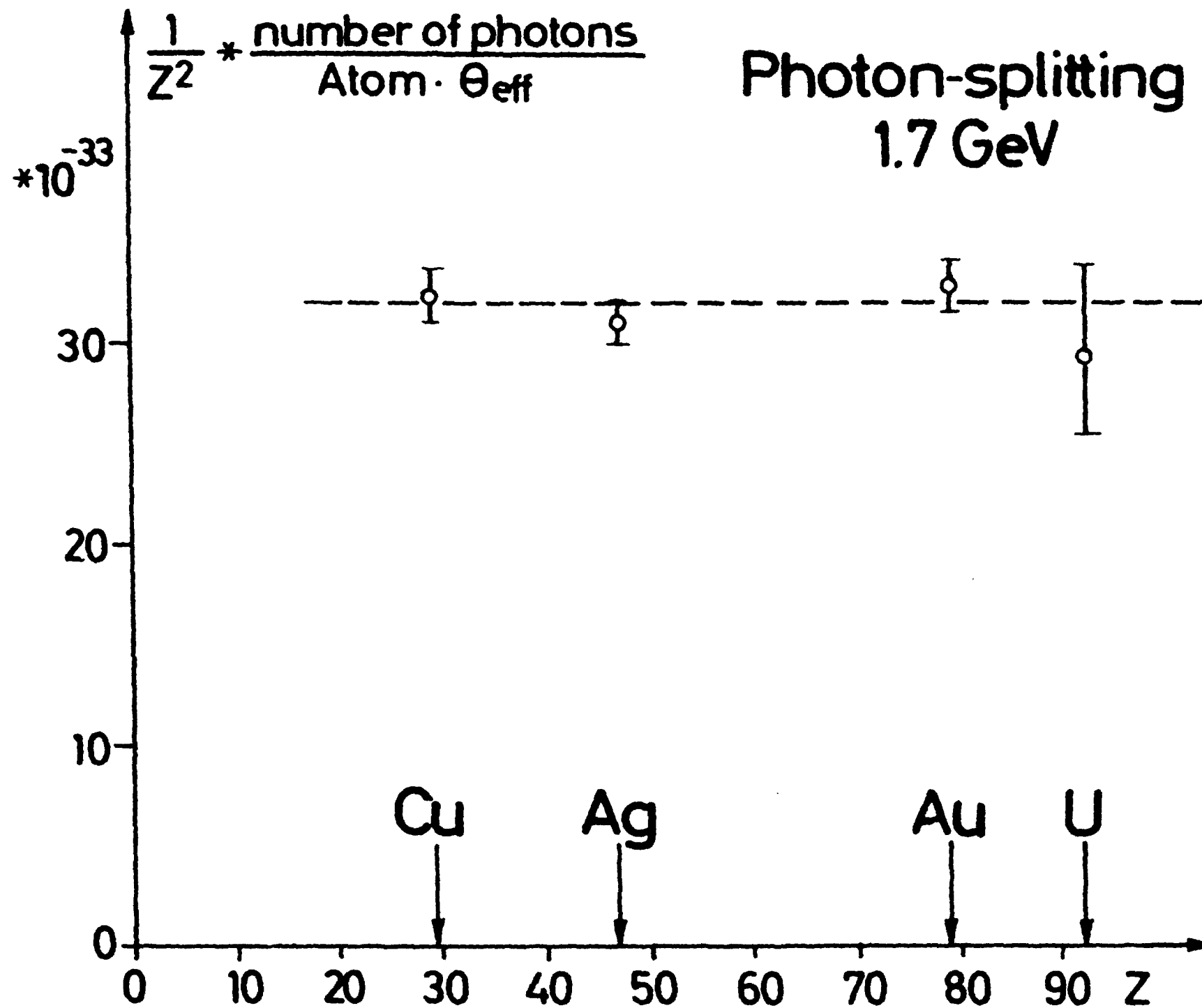


Fig 10

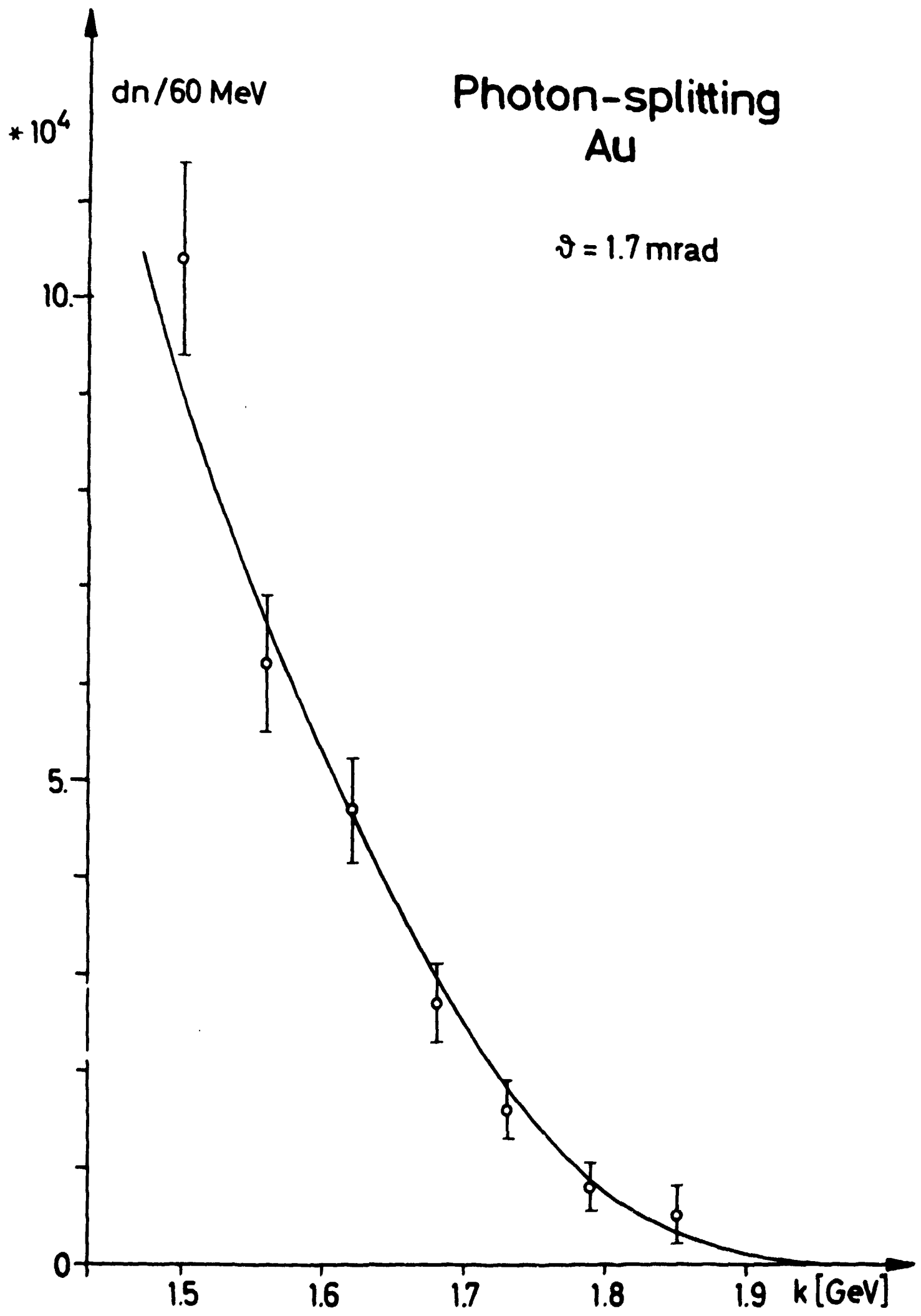


Fig 11

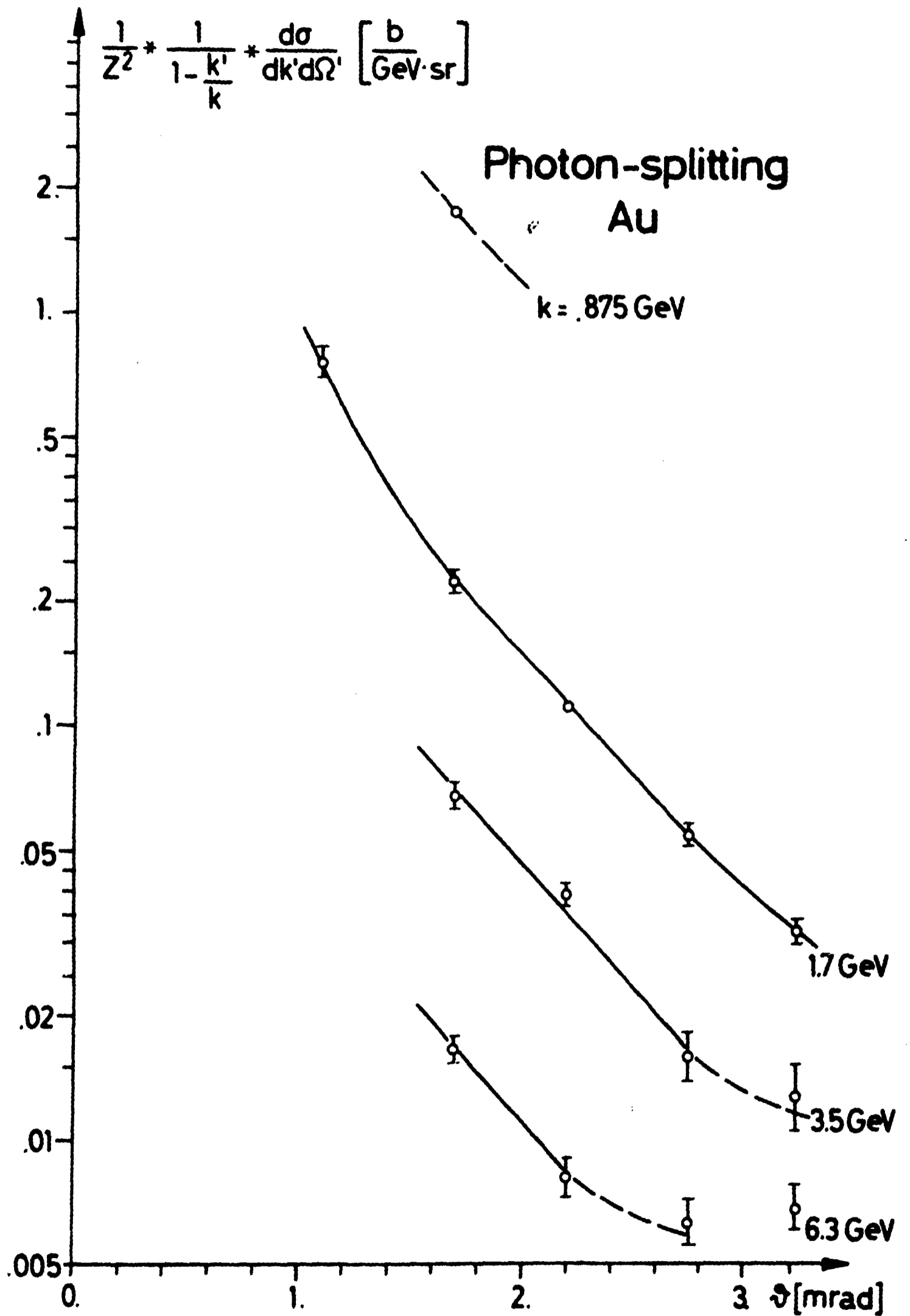


Fig 12

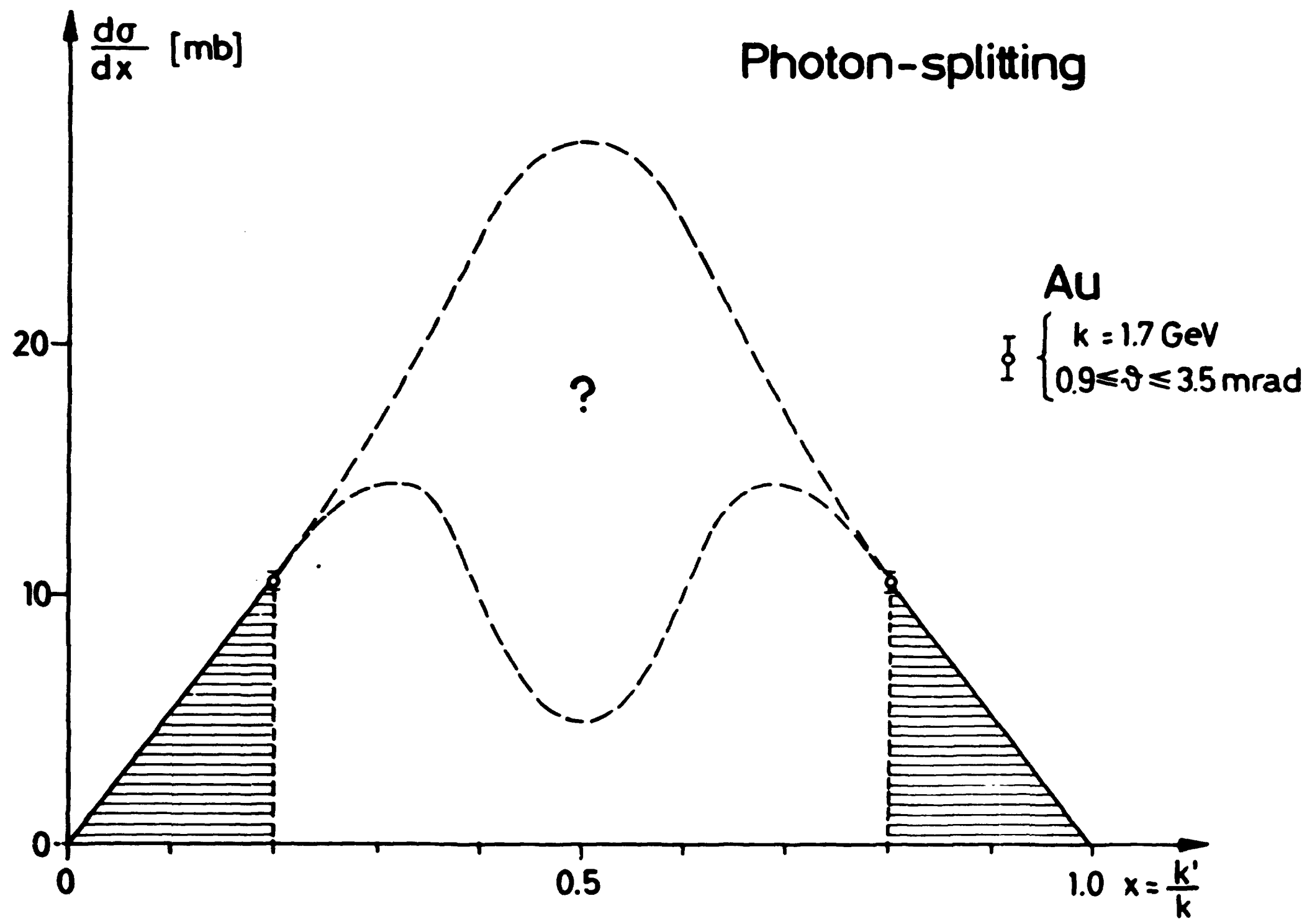


Fig 13

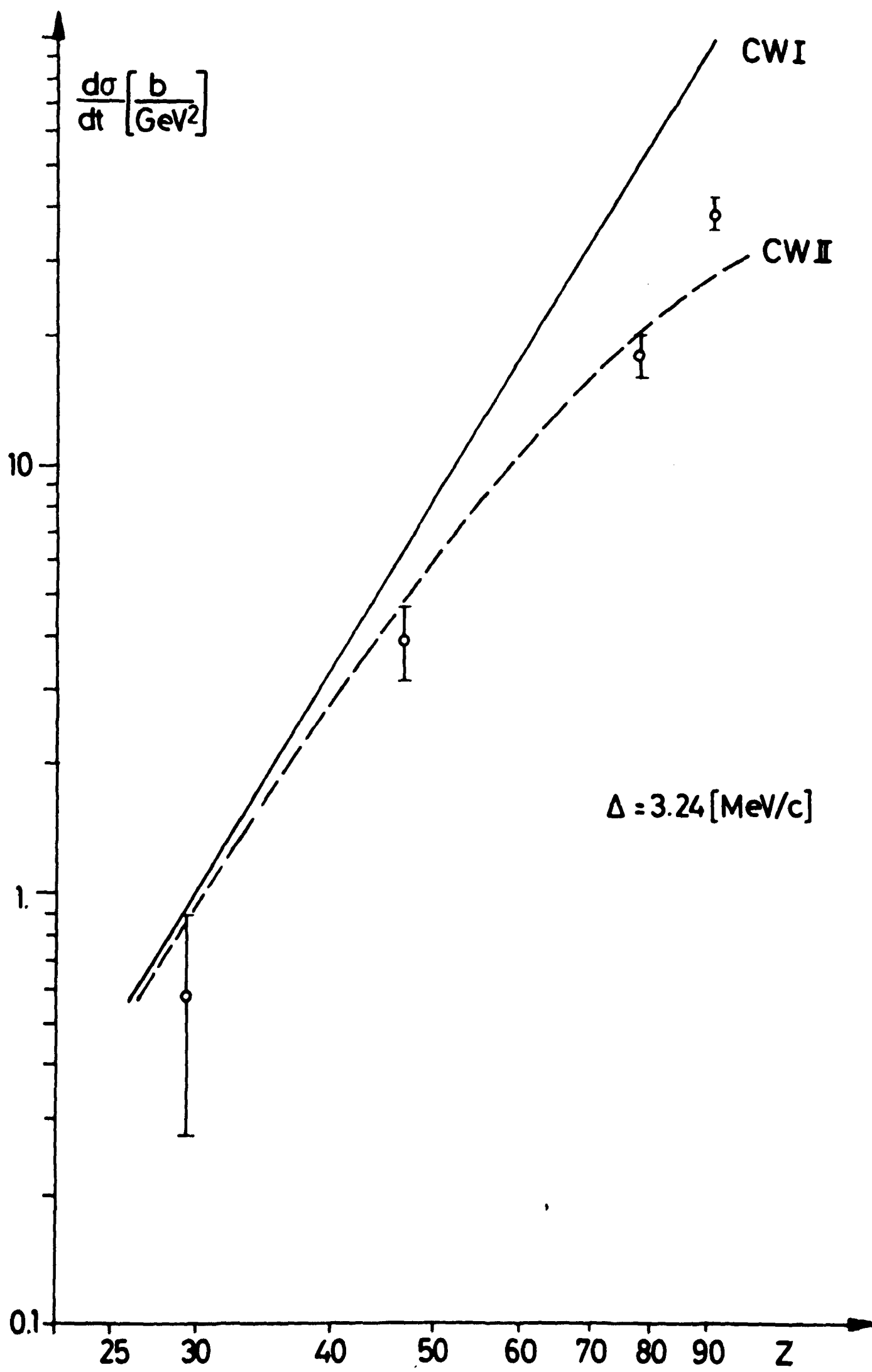


Fig 14



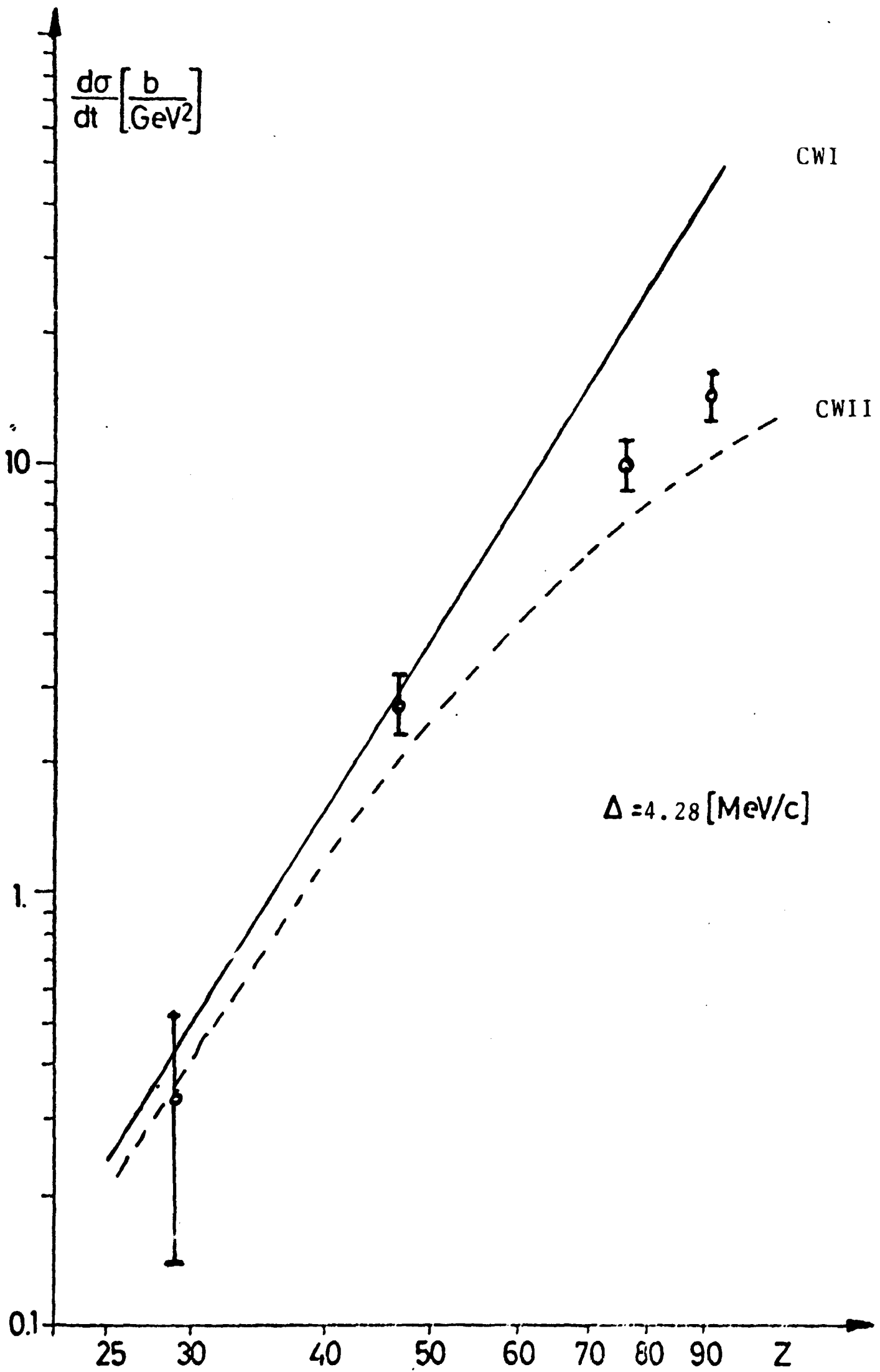


Fig 15

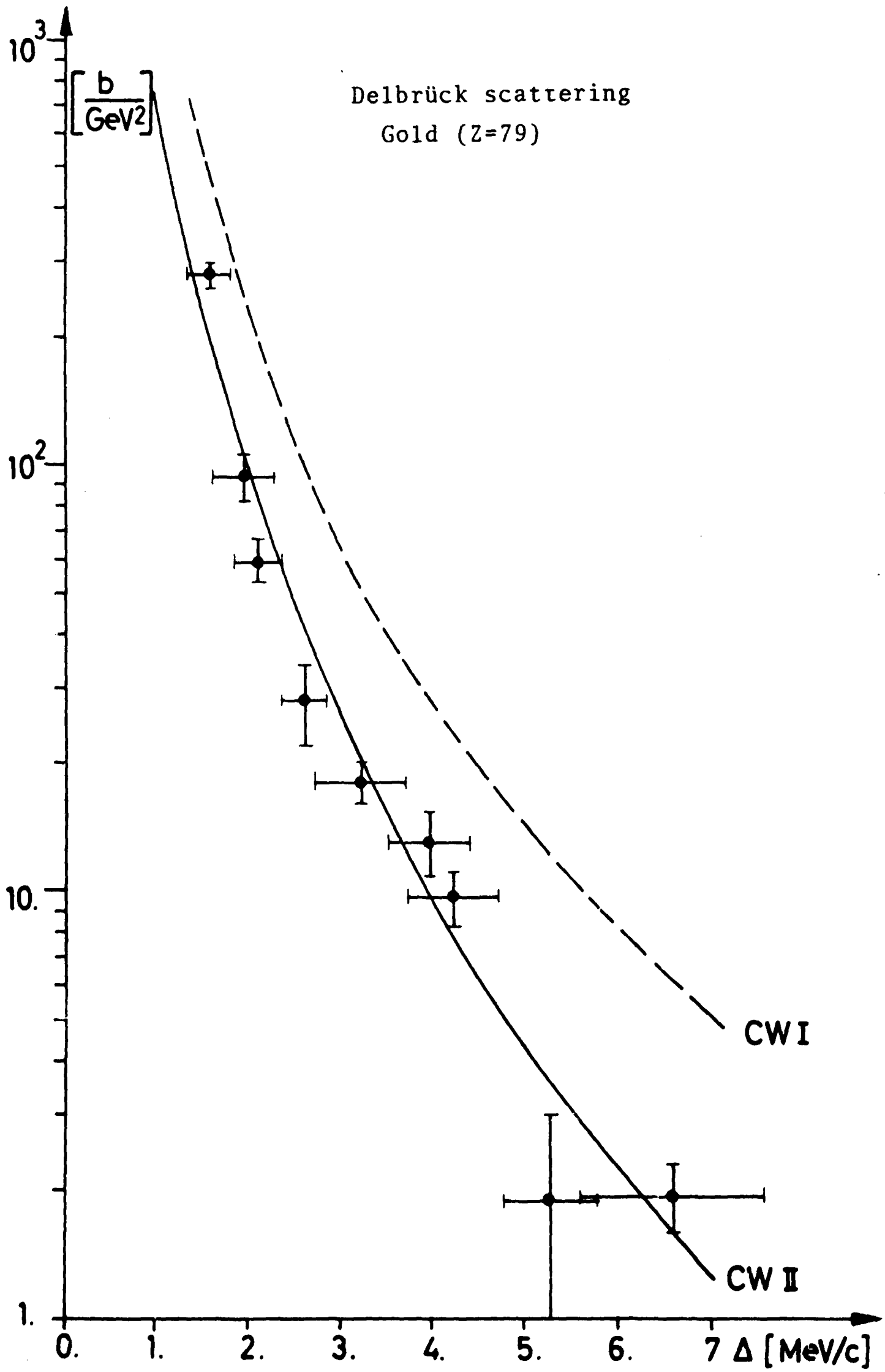


Fig 16

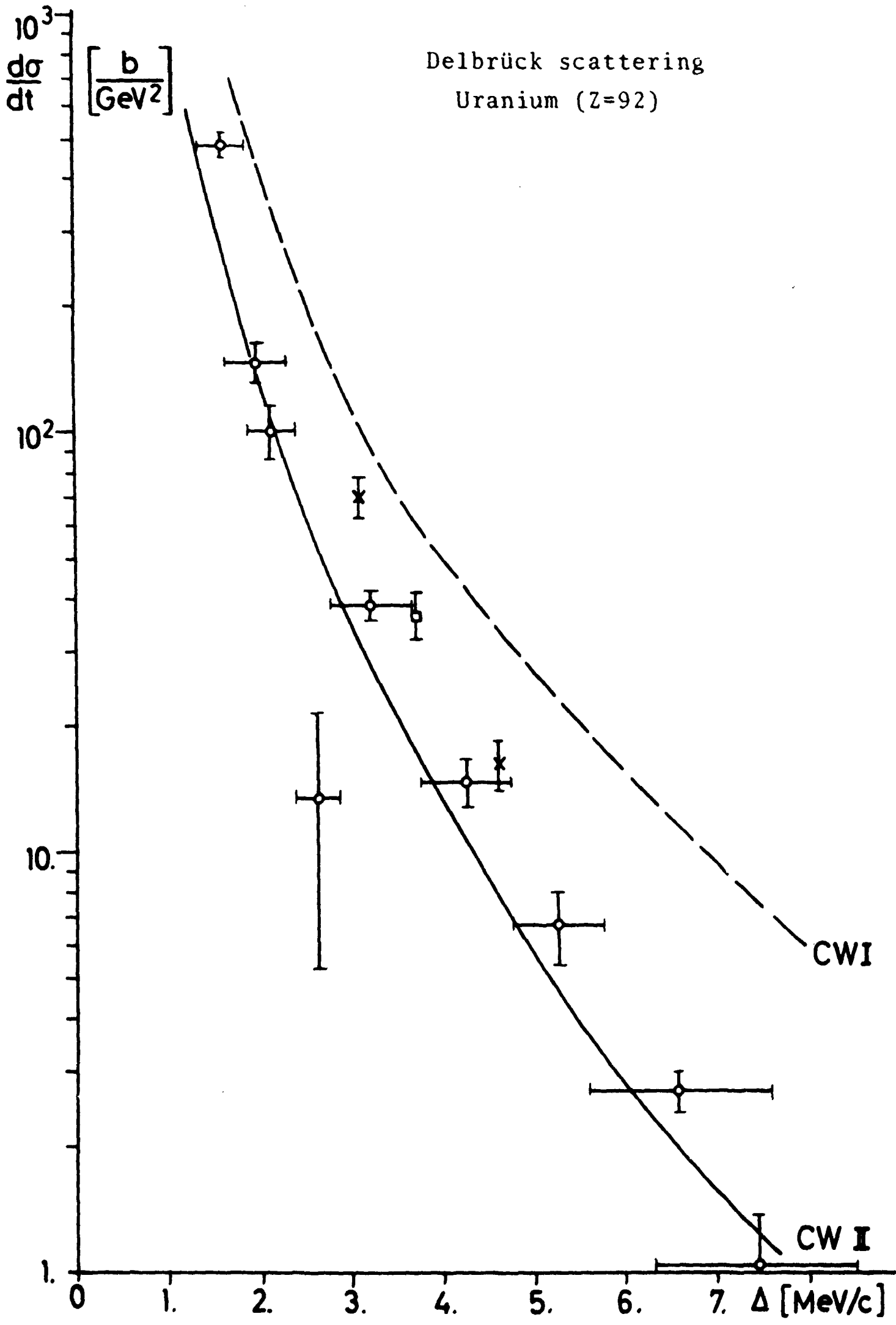


Fig 17

