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S Matrix Elements in the Presence of Standing Wave States, the Bjorken Limit, and Shadow-quarks*

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ABSTRACT

When a scattering reaction occurs, it is usually assumed that the complete set of collision states is entirely composed of traveling waves, hence any constituent, in principle, is observable. Should, however, either quarks regulator photons or lepton satellites be standing wave states, i.e. only shadow states, we would not expect to have observed them. In this paper we discuss the new terms which modify the usual dispersion relations and Low equation and argue that present two-body experiments do not provide strong tests for their presence. In the Bjorken limit these piecewise analytic terms vanish. Yet, in a shadow indefinite-metric theory, the logarithmic terms breaking this limit are still absent. The triangle annomalies, therefore, should only be taken seriously in low energy reactions. A stronger constraint on the presence of shadow states can be provided by verification of the Callen-Gross sum rule. By means of a simple version of the Kuti-Weisskopf model, containing valence quarks and quark-antiquark pairs, we demonstrate explicitly how quarks can be shadow states. In an appendix, in formal scattering theory with shadow states, the correct formal solutions for the exact asymptotic states, both physical and and shadow, are obtained.

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I. Introduction

When a scattering reaction occurs in high energy physics, the assumption is usually made that all the asymptotic collision states are entirely composed of traveling waves. These asymptotic collision states, together with the vacuum, are supposed to be capable of giving a complete and correct framework[1,2] upon which to base measurements, analyze the data, and to phenomenologically describe the actual scattering process as it occurs in nature. Hence, any particle no matter how heavy, say regulator photons or lepton satellites, intermediate bosons, or fundamental constituents, say quarks, should be produced if a target is only struck by a sufficiently energetic projectile consistent with the selection rules. Systems in atomic and nuclear physics, and results from exploration at ever increasing energies of the high energy resonance spectrum have given support to this assumption.

However, searches for quarks, which appear light and free objects in hadrons, have remained fruitless.[3] Also if the regulator particles, introduced[4] to make the theory finite and meaningful, were produced, they would contradict unitarity. For these reasons then, we wish to consider from the viewpoint of properties of the scattering matrix the consequences of asymptotic states, containing more than one particle, which are of the standing wave type.

Such states have been introduced elsewhere[5] with the motivation of solving the problem of insuring the conservation of probability in the framework of indefinite metric theories.[4-10] Because these states do not contribute to the physical unitarity relation, even though like quarks they are essential to the dynamics, they are called "shadow states." To demonstrate the manner in which shadow states influence the dynamics, to study analytic properties, and to settle questions of interpretation, a a number of quantum field theory models have been studied without recourse to perturbation theory.[11] It has been shown there,[11] in low-order perturbation graphs,[12,13] and by a simple general argument[14] that such states always lead to a scattering theory which is unitary and Lorentz invariant.

As stressed in the first paragraph, when the possibility of shadow states is allowed in the dynamics of the scattering process, one has a more inclusive framework than is usually assumed. Thus, the natural topic for an initial investigation is to discover the necessary modifications of the standard S matrix formalism - preferably in the form of correction terms. This is useful qualitatively in showing where the corrections would be expected to be large and in showing what constraints present experiments in the explored energy regions put on the presence of shadow states. In Sections III and IV respectively, we derive the new terms which modify the usual dispersion relations and the Low equation. These terms are a consequence of the piecewise analyticity of the shadow theory's scattering amplitude. Each time a shadow pseudo threshold is crossed, the amplitude changes continuously from one analytic function to a new analytic function.

We find that present two-body experiments do <u>not</u> provide strong tests for the presence of shadow states. Consequently, the conclusion of Gundzik and Sudarshan[15] is not surprising: From an analysis of the data from pion-nucleon scattering, they find that a piecewise analytic structure cannot yet be distinguished experimentally from that predicted by Mandelstam analyticity.

By means of an ansatz, the analogue of Low's relation for replacing a (physical) particle in an asymptotic state by the (physical) current it is coupled to, we are able to rewrite the modified Low equation as the usual reduction formula (with contributions only from physical asymptotic states) plus the piecewise analytic terms. These piecewise analytic terms, however, vanish in the scaling region[16] of deep-inelastic electron-nucleon scattering, so this kinematic region is of special interest as to effects originating from possible standing wave structures in hadrons.

Secondly, shadow states are of special importance in the Bjorken limit since in positive metric field theory, logarithmic terms[17] violate scaling sum rules.[18] In a shadow indefinite metric theory, the shadow states can be regarded as providing a unitary cutoff to the phenomenologically successful parton models. With shadow states present, the triangle annomalies will not cause trouble in this "high energy" limit (i.e. break down the scaling behavior). Arguments in the literature, based on perturbation theory, have studied the behavior of only leading order terms in the regulator mass when the Bjorken limit is taken. These terms, however, are not expected to dominate when the regulator mass is finite because of the kinematic dependence in the Bjorken limit of the non-leading terms in the finite regulator mass. At the same time, the annomalies[19] will contribute in the low energy region, say for $\pi^0 \to 2\gamma$, since they can be derived as the contribution which survives to leading order in the regulator mass.

In Section V, therefore, we study the derivation of the Callen-Gross sum rule[20] in the presence of shadow states. Despite the vanishing of the piecewise analytic terms, because of presence of the shadow states in the asymptotic completeness relation, we find that experimental verification of this sum rule provides a stronger constraint on the presence of shadow states than tests of dispersion relations by experiment. Of course, if the thresholds of quarks or negative metric regulators is at higher energies than are accessible to present accelerators, whether producing time-like or space-like projectiles, the difference between the constraints in these two regions is at present only of academic interest.

In Section VI, we construct a simple version of the Kuti-Weisskopf model, containing valence quarks and quark-antiquark pairs, and explicitly demonstrate how quarks can be standing wave structures in hadrons.

As theoretical background, so as to keep this paper self-contained, we briefly review in Section I the in and out formalism of the scattering matrix when shadow states are present. To clarify some technical aspects, in an appendix we obtain the correct formal solutions in formal scattering theory for the exact asymptotic states, both physical and shadow. These states, orthonormal and complete, are necessary for a complete framework in which to describe the time evolution of the scattering process. In matrix elements between physical states involving some operators different from transition operators, the shadow states, in general, will contribute as intermediate states.

II. Review

The reader already familar[5,11-15] with the basic concepts and ideas of shadow states should go directly to Section III.

It is probably helpful to review briefly the relation between the "physical" scattering operator, S, in a theory with standing wave states and the closely associated, "conventional" scattering operator, $\mathcal{S}$, which would result if the $+i\epsilon$ prescription were assumed for all the asymptotic states. This review will serve to introduce our notations, to define the "physical" scattering amplitude, and to set the stage for our investigation of the effects of the shadow states.

In a theory with standing wave states, the essential difference is that in specifying the boundary condition for the shadow channels, there are no traveling waves, but only standing waves. This is achieved by the choice of a time-symmetric, half-retarded and half-advanced propagation function for the shadow states. A consequence is that from such a state there is no radiated flux of shadow particles, for example in a quark model no shadow-quarks will be observed by experiment as mass-shell particles. It is important to note that the standing wave prescription applies to the entire shadow <u>state</u>, not only to the shadow particle in that state.

Since regulator photons or lepton satellites may be standing wave states, we will work in the framework of a Lorentz invariant, indefinite-metric quantum field theory.[4-10] We begin by considering the Hamiltonian operator, H, for convenience in the conventional in-state representation.[21] It is only pseudo-Hermitian (η is the metric operator)

$$H^{*} = \eta H \eta \qquad (\eta^{2} = 1 \ , \ \eta^{*} = \eta) \tag{2.1}$$

but not Hermitian, with respect to the inner product $\langle n;\mathcal{J}|\ \eta\ |m;\mathcal{J}\rangle$ for two in-states $|n;\mathcal{J}\rangle$ and $|m;\mathcal{J}\rangle$ which are eigenstates of a (mass renormalized) free

Hamiltonian H_{in}. The "conventional" scattering operator, assuming the $+ i\epsilon$ prescription for all the asymptotic states can be calculated, for instance by covariant perturbation theory in terms of the in-fields

$$\mathcal{S} = 1 + i\,\mathcal{T}$$
$$= \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dt_1 \cdots dt_n \; T\left[H_I^{in}(t_1) \cdots H_I^{in}(t_n) \right] \tag{2.2}$$

where the script letters, $\mathcal{S}$ and $\mathcal{T}$, remind us of the dependence of these objects on the purely in and out boundary conditions. They are not in a theory with shadow states the "physical" operators. In fact, $\mathcal{S}$ is not unitary, but only pseudo-unitary

$$\mathcal{S}^* \eta\, \mathcal{S} = \eta \tag{2.3}$$

and therefore, is an unacceptable candidate to associate with the scattering process as observed by experiment. If instead of Eq. (2.2), we employ time-dependent perturbation theory to calculate $\mathcal{S}$, this pseudo-unitarity can be made visible at every step ($H = H_{in} + H_I^{in}$)

$$\langle f;\vartheta|\eta\,\mathcal{S}|i;\vartheta\rangle = \langle f;\vartheta|\eta|i;\vartheta\rangle - 2\pi i\,\delta(E_f - E_i) \times$$
$$\left[\langle f;\vartheta|\eta\, H_I^{in}|i;\vartheta\rangle + \sum_m \frac{\langle f;\vartheta|\eta\, H_I^{in}|m;\vartheta\rangle\langle m;\vartheta|\eta\, H_I^{in}|i;\vartheta\rangle}{E_i - E_m + i\epsilon} + \cdots \right] \tag{2.4}$$

This expansion more closely parallels the actual, time-dependent scattering reaction as it evolves in experiment. Notice that now the choice of boundary conditions for the asymptotic states is <u>manifest</u>. Hence, to arrive at the "physical", unitary scattering operator, S, in a theory with standing wave states, we merely change the boundary condition for the shadow states to that of the standing wave type: Change

$$\frac{1}{E_i - H_{in} + i\epsilon} \longrightarrow \frac{\Pi^P}{E_i - H_{in} - i\epsilon} + \mathcal{P}\frac{\Pi^S}{E_i - H_{in}}$$

$$= \frac{1}{E_i - H_{in} + i\epsilon} + i\pi\delta(E_i - H_{in})\Pi^S \tag{2.5}$$

where the projection operators Π^P and Π^S define, respectively, the orthogonal physical and shadow Hilbert spaces, $\mathcal{H} = \mathcal{H}(\text{physical}) \oplus \mathcal{H}(\text{shadow})$, and

$$\Pi^P + \Pi^S = 1$$

Of course, all the negative metric asymptotic states are assumed to be a subset of the standing wave states.

Notice, first, that after this change, Eq. (2.4) or equivalently Eq. (2.3) give immediately the perturbation solution for the scattering amplitude in the presence of the standing wave states. Second, these expansions can be resummed[14] in terms of $\mathcal{T}$, the associated indefinite metric theory's transition operator, to obtain

$$T = \mathcal{T} + T'$$

$$T' = -\frac{i}{2}\mathcal{T}\left[1 + \frac{i}{2}\Pi^S\eta\mathcal{T}\right]^{-1}\Pi^S\eta\mathcal{T} \tag{2.6}$$

where <u>only</u> the shadow intermediate states contribute to T'. Observe that T' represents the "modification" to the pseudo-unitary $\mathcal{T}$. The physical transition operator T is manifestly Lorentz invariant when $\mathcal{T}$ is written as in Eq. (2.2). This equation can be used as a framework to study the dynamical effects of standing wave states. The inverse operator in Eq. (2.7) can be written as an iterative series in the shadow states

$$T' = -\tfrac{i}{2}\, \mathcal{T} K \pi^s \eta \mathcal{T}$$

$$K = 1 - \tfrac{i}{2} \pi^s \eta \mathcal{T} K \tag{2.7}$$

The summation proceedure which led to Eq. (2.6) can also be exploited for many other physical objects in the shadow theory in order to relate them to quantities, often more easily calculated, in the associated theory based on purely in and out boundary conditions.

It is easy to show[14] from Eq. (2.5), for example via Eq. (2.6), that T will be unitary

$$i\,(T^* - T) = T^* \pi^P T \tag{2.8}$$

and therefore an acceptable scattering amplitude, if $\mathcal{T}$ is only pseudo-unitary. If Eq. (2.8) is sandwiched between two "physical" in-states, say $|m;\mathcal{J}\rangle$ and $|n;\mathcal{J}\rangle$ of the $\mathcal{T}$ theory, and if a complete set of intermediate states both physical and shadow, is inserted on the right-hand side, then the projection operator π^P in Eq. (2.8) is superfulous as T does not connect the physical in-states to the shadow states! This occurs because the shadow states are standing wave states which cannot radiate or absorb. In the formalism transitions from $|p,;\mathcal{J}\rangle \rightarrow |s;\mathcal{J}\rangle$ or $|s;\mathcal{J}\rangle \rightarrow |s';\mathcal{J}\rangle$ vanish, when explicitly calculated, since the boundary condition results in a purely real wave function for the shadow states; therefore by orthonormality there are no transitions.[22,23] Hence, Eq. (2.7) gives the optical theorem

$$\mathrm{Im} \langle p;\mathcal{J}|\eta T|p;\mathcal{J}\rangle = \sum_{p'=\text{physical}} |\langle p';\mathcal{J}|\eta T|p;\mathcal{J}\rangle \tag{2.9}$$

III. Piecewise Analyticity and the Bjorken Limit

We want to study the consequences of the shadow states' standing wave boundary condition for deep inelastic electron-nucleon scattering, in particular for the Callen-Gross sum rule. To do this, we shall need to use a few technical results which are derived in this and the following sections. These results consist essentially of the modifications of well-known expressions for scattering matrix elements in order to accomodate for the presence of the shadow states.

We begin by giving our notation for the kinematics.

<u>Kinematics</u>

The inclusive reaction is $e^-(E) + N(P) \to e^-(E',\theta) + p(P')$ where p denotes any possible final physical state of four-momentum P' produced from the target nucleon , N, of four-momentum P. Experimentally the initial electron energy, E, is known and only the final electron is observed so as to measure the final electron energy, E', and scattering angle, θ. We will assume the reaction proceeds by a single vector exchange and ignore any effects from a heavy photon exchange, so the matrix element for this process is proportional to the "vertex function" $V_\lambda(\gamma N \to p)$. Equivalently then, see Fig. 1, the process consists of the fragmentation of the target nucleon of rest mass M by a virtual photon of energy $\nu = E - E' = q\cdot P/M$ and $(\text{mass})^2 = q^2 = -4EE' \sin^2(\frac{\theta}{2})$. It will be useful to introduce their ratio $\omega = -\frac{q^2}{M\nu}$.

To describe this process as measured by experiment, we take the matrix element $V_\lambda(\gamma N \to p)$, square it, average over nucleon spins, and sum over the physical final states. Experiment, therefore, determines

$$A_{\mu\nu}(\nu, q^2) = \sum_p \overline{V_\mu(\gamma N \to p)}\, V_\nu(\gamma N \to p)\, (2\pi)^4\, \delta^4(P' - P - q) \tag{3.1}$$

where $\bar{\Sigma}$ implies the average over nucleon spins. By Lorentz and gauge invariance, this has the simple representation

$$A_{\mu\nu}(\nu, q^2) = W_1(\nu, q^2)\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)$$
$$+ W_2(\nu, q^2)\frac{1}{M^2}\left(P_\mu - \frac{\nu M}{q^2} q_\mu\right)\left(P_\nu - \frac{\nu M}{q^2} q_\nu\right) \quad (3.2)$$

which defines the two structure functions $W_{1,2}(\nu,q^2)$. In terms of these, the differential cross section for inelastic electron-nucleon scattering is given by

$$\frac{d\sigma^2}{d\Omega\, dE'} = \frac{\alpha^2}{4E^2 \sin^4(\frac{\theta}{2})}\left[2W_1(\nu, q^2)\sin^2(\tfrac{\theta}{2}) + W_2(\nu, q^2)\cos^2(\tfrac{\theta}{2})\right] \quad (3.3)$$

Since the optical theorem is unmodified by the presence of shadow states, $A_{\mu\nu}(\nu,q^2)$, e.g. as given by Eq. (3.1), is the imaginary part of the amplitude for forward Compton scattering of virtual photons on the target nucleon. Hence we consider the Lorentz invariant, gauge invariant amplitude for virtual elastic Compton scattering on nucleons.

$$T_{\mu\nu}{}^*(\nu, q^2) = T_1(\nu, q^2)\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)$$
$$+ T_2(\nu, q^2)\frac{1}{M^2}\left(P_\mu - \frac{\nu M}{q^2} q_\mu\right)\left(P_\nu - \frac{\nu M}{q^2} q_\nu\right) \quad (3.4)$$

which defines the invariant amplitudes $T_{1,2}(\nu,q^2)$.

Consequences of the Standing Wave Boundary Condition for Dispersion Relations

Because of the standing wave boundary condition for the shadow state, the scattering amplitude for the physical states is no longer a single analytic function. Instead, it is piecewise analytic. Below the first shadow threshold, say, it is one analytic function and just above it is another analytic function. For a dispersion relation analysis it is convenient, therefore, to write the unitary physical amplitude as the sum of the analytic function below the first shadow threshold plus the non-analytic pieces which occur each time a shadow pseudo threshold is crossed. So we write[15,24] for an invariant an amplitude $T(\nu,q^2)$, which could be either of the above T_1 or T_2 for virtual Compton scattering,

$$T(\nu, q^2) = \mathcal{T}(\nu, q^2) + T'(\nu, q^2) \tag{3.5}$$

where

$$T'(\nu, q^2) = \sum_i \left[\theta(s - s_i) + \theta(-s - s_i) \right] t_i(\nu, q^2) \tag{3.6}$$

with i running over the shadow pseudo thresholds at $s_i = w_i^2 = 2M\nu_i + M^2 + q^2$. For fixed q^2, Eq. (3.6) can be rewritten in terms of the ν variable as

$$T'(\nu, q^2) = \sum_i \left[\theta(\nu - \nu_i) + \theta(-\nu - \nu_i) \right] t_i(\nu, q^2) \tag{3.7}$$

where $\nu_i(q^2) = (s_i - M^2 - q^2)/2m$.

Notice that $T(\nu,q^2)$ is unitary but only piecewise analytic, whereas $\mathcal{T}(\nu,q^2)$ and the $t_i(\nu,q^2)$ are analytic as a consequence of the principal value boundary conditions. Note also if some of the shadow states have negative metric, the $\mathcal{T}(\nu,q^2)$ (the corresponding indefinite metric theory's invariant amplitude) is not unitary but only pseudo unitary and thus unacceptable unless modified by contributions from $T'(\nu,q^2)$.

Our procedure now will be to employ the analyticity of t_i and $\mathcal{T}$ to constrain the piecewise analytic T. To write dispersion relations for t_i and the $\mathcal{T}$ we need to know their asymptotic behavior. We will make the natural assumption that the asymptotic behavior of $\mathcal{T}(\nu,q^2)$ is the same as in the usual theory. Then according to the usual Regge phenomenology, see Harari,[24A] concerning the asymptotic behavior for this process, $\mathcal{T}_2(\nu,q^2)$ requires no subtractions and $\mathcal{T}_1(\nu,q^2)$ requires one subtraction.

Therefore, for $\mathcal{T}_2(\nu,q^2)$ we have

$$\mathcal{T}_2(\nu,q^2) = \frac{1}{\pi}\int_0^\infty d\nu'^2 \, \frac{\mathrm{Im}\,\mathcal{T}_2(\nu',q^2)}{\nu'^2-\nu^2-i\epsilon} \tag{3.8}$$

and using first Eq. (3.5) and then Eq. (3.7), we obtain

$$T_2(\nu,q^2) = T_2'(\nu,q^2) + \frac{1}{\pi}\int_0^\infty d\nu'^2 \, \frac{\mathrm{Im}\,T_2(\nu',q^2)}{\nu'^2-\nu^2-i\epsilon} - \frac{1}{\pi}\int_0^\infty d\nu'^2 \, \frac{\mathrm{Im}\,T_2'(\nu',q^2)}{\nu'^2-\nu^2-i\epsilon}$$

$$= \frac{1}{\pi}\int_0^\infty d\nu'^2 \, \frac{\mathrm{Im}\,T_2(\nu',q^2)}{\nu'^2-\nu^2-i\epsilon}$$

$$+ \left[T_2'(\nu,q^2) - \frac{1}{\pi}\sum_\lambda \int_{\nu_\lambda^2(q^2)}^\infty d\nu'^2 \, \frac{\mathrm{Im}\,t_{2,\lambda}(\nu',q^2)}{\nu'^2-\nu^2-i\epsilon}\right] \tag{3.9}$$

If the term in the bracket were to vanish, this would be of the form of a dispersion relation for the physical amplitude $T_2(\nu,q^2)$! We choose to re-express Eq. (3.9) in terms of the variables q^2 and ω. Note that (because of the optical theorem)

$$\frac{1}{\pi}\,\mathrm{Im}\,T_2(\nu,q^2) = W_2(\nu,q^2) \tag{3.10}$$

and (q^2 fixed)

$$\omega_\lambda(q^2) = 2 \Big/ \left[1 - \left(\frac{s_\lambda - M^2}{q^2}\right)\right] \tag{3.11}$$

Therefore,

$$\frac{1}{\pi}\int_0^\infty d\nu'^2 \, \frac{\mathrm{Im}\,T_2(\nu,q^2)}{\nu'^2-\nu^2-i\epsilon} = -\omega^2\int_0^4 d\omega'^2 \, \frac{W_2(\omega',q^2)}{\omega'^2(\omega'^2-\omega^2+i\epsilon)}$$

and

$$\int_{\nu_\lambda^2}^{\infty} d\nu'^2 \, \frac{\mathrm{Im}\, t_{2,\lambda}(\nu'^2, q^2)}{\nu'^2 - \nu^2 - i\epsilon} = -\,\omega^2 \int_0^{\omega_\lambda^2} d\omega'^2 \, \frac{\mathrm{Im}\, t_{2,\lambda}(\omega', q^2)}{\omega'^2(\omega'^2 - \omega^2 + i\epsilon)} \tag{3.12}$$

For the $t_{2,i}(\nu,q^2)$ we assume the same asymptotic behavior as for $\mathcal{T}(\nu,q^2)$, so[25]

$$\begin{aligned} t_{2,\lambda}(\nu, q^2) &= \frac{1}{\pi} \int_{\bar{\nu}_\lambda(q^2)}^{\infty} d\nu'^2 \, \frac{\mathrm{Im}\, t_{2,\lambda}(\nu', q^2)}{\nu'^2 - \nu^2 - i\epsilon} \\ &= -\,\frac{\omega^2}{\pi} \int_0^{\bar{\omega}_\lambda^2} d\omega'^2 \, \frac{\mathrm{Im}\, t_{2,\lambda}(\omega', q^2)}{\omega'^2(\omega'^2 - \omega^2 + i\epsilon)} \end{aligned} \tag{3.13}$$

where $\bar{\nu}_\lambda(q^2)$ is the threshold for the physical particle which is the partner of the "heavy" shadow particle, $\bar{\nu}_\lambda(q^2) = (\bar{s}_i - M^2 - q^2)/2M$ and $\bar{\omega}_i(q^2) = 2/\left[1 - \left(\frac{\bar{s}_i - M^2}{q^2}\right)\right]$. Therefore, we finally have from Eq. (2.9)

$$\begin{aligned} T_2(\omega, q^2) &= -\,\omega^2 \int_0^1 d\omega'^2 \, \frac{W_2(\omega', q^2)}{\omega'^2(\omega'^2 - \omega^2 + i\epsilon)} \\ &+ \sum_j \theta(-\omega + \omega_j)\left[-\frac{\omega^2}{\pi} \int_0^{\bar{\omega}_j^2} d\omega'^2 \, \frac{\mathrm{Im}\, t_{2,j}(\omega', q^2)}{\omega'^2(\omega'^2 - \omega^2 + i\epsilon)}\right] \\ &- \sum_\lambda \left[-\frac{\omega^2}{\pi} \int_0^{\omega_\lambda^2} d\omega'^2 \, \frac{\mathrm{Im}\, t_{2,\lambda}(\omega', q^2)}{\omega'^2(\omega'^2 - \omega^2 + i\epsilon)}\right] \end{aligned} \tag{3.14}$$

From this expression we see that for q^2 finite, because of the presence of the shadow states, the usual dispersion relation for $T_2(\omega,q^2)$ is modified by terms of two types: for the k's such that $\omega_k > \omega$ (i.e. $\nu_k < \nu$ so this is the contribution from a shadow pseudo threshold which lies below the projectile energy ν)

$$\int_{\omega_k^2}^{\bar{\omega}_k^2} d\omega'^2 \, \frac{\mathrm{Im}\, t_{2,k}(\omega', q^2)}{\omega'^2(\omega'^2 - \omega^2)} \tag{3.15}$$

and for the k's such that $\omega_k < \omega$ (i.e. $\nu_k > \nu$ so this is from a shadow pseudo threshold lying above the projectile energy)

$$\int_0^{\omega_k^2} d\omega'^2 \quad \frac{\mathrm{Im}\ t_{2,k}(\omega', q^2)}{\omega'^2(\omega'^2 - \omega^2)} \tag{3.16}$$

In both Eq. (3.15) and Eq. (3.16) no $+i\epsilon$ is needed in the denominator of the integrand as ω lies outside the ranges of integration so the denominators cannot vanish. From these expressions, then, it would not be surprising if in general the phenomenological effects of shadow states in the sense of modifying dispersion relations were relatively mild.[15] The strongest effects, Eq.s (3.15) and (3.16) indicate, should occur when ω (or ν) is near a shadow pseudo threshold, i.e. when ω pinches the end point of one of the above integrals.

We now consider what happens in the deep inelastic region where ν and $-q^2$ tend to infinity with ω fixed. Since $q^2 \to -\infty$, for any k both ω_k and $\bar{\omega}_k$ approach 2 so the two summations in Eq.(2.14) cancel to yield

$$T_2(\omega, q^2) = -\omega^2 \int_0^4 d\omega'^2 \ \frac{W(\omega', q^2)}{\omega'^2(\omega'^2 - \omega^2 + i\epsilon)} \tag{3.17}$$

which has the same form as a dispersion relation for $T_2(\omega, q^2)$!

Similarly from the once subracted $\mathcal{T}_1(\nu, q^2)$ we find

$$T_1(\nu, q^2) = \mathcal{T}_1(0, q^2) + \frac{\nu^2}{\pi} \int_0^\infty d\nu'^2 \ \frac{\mathrm{Im}\ T_1(\nu', q^2)}{\nu'^2(\nu'^2 - \nu^2 - i\epsilon)}$$

$$+ \left[T_1'(\nu, q^2) - \frac{\nu^2}{\pi} \sum_\lambda \int_{\nu_\lambda(q^2)}^\infty d\nu'^2 \ \frac{\mathrm{Im}\ t_{1,\lambda}(\nu', q^2)}{\nu'^2(\nu'^2 - \nu^2 - i\epsilon)} \right] \tag{3.18}$$

and as before

$$\frac{\nu^2}{\pi} \int_0^\infty d\nu'^2 \ \frac{\mathrm{Im}\ T_1(\nu', q^2)}{\nu'^2(\nu'^2 - \nu^2 - i\epsilon)} = -\int_0^4 d\omega'^2 \ \frac{W_1(\omega', q^2)}{(\omega'^2 - \omega^2 + i\epsilon)}$$

$$\nu^2 \int_{\nu_\lambda(q^2)}^\infty d\nu'^2 \ \frac{\mathrm{Im}\ t_{1,\lambda}(\nu', q^2)}{\nu'^2(\nu'^2 - \nu^2 - i\epsilon)} = -\int_0^{\omega_\lambda^2} d\omega'^2 \ \frac{\mathrm{Im}\ t_{1,\lambda}(\omega', q^2)}{(\omega'^2 - \omega^2 + i\epsilon)} \tag{3.19}$$

also

$$t_{1,\lambda}(\nu, q^2) = t_{1,\lambda}(0, q^2) + \frac{\nu^2}{\pi} \int_{\bar{\nu}_\lambda(q^2)}^{\infty} d\nu'^2 \frac{\mathrm{Im}\, t_{1,\lambda}(\nu', q^2)}{\nu'^2(\nu'^2 - \nu^2 - i\epsilon)}$$

$$= t_{1,\lambda}(\infty, q^2) - \int_0^{\bar{\omega}_\lambda^2} d\omega'^2 \frac{\mathrm{Im}\, t_{1,\lambda}(\omega', q^2)}{(\omega'^2 - \omega^2 + i\epsilon)} \tag{3.20}$$

Therefore, we have

$$T_1(\omega, q^2) = T_1(\infty, q^2) - \int_0^4 d\omega'^2 \frac{W_1(\omega', q^2)}{(\omega'^2 - \omega^2 + i\epsilon)}$$

$$- \sum_j \left[\theta(\omega - \omega_j)\, t_{1,j}(\infty, q^2) + \theta(-\omega + \omega_j) \frac{1}{\pi} \int_0^{\bar{\omega}_j^2} d\omega'^2 \frac{\mathrm{Im}\, t_{1,j}(\omega', q^2)}{(\omega'^2 - \omega^2 + i\epsilon)} \right]$$

$$+ \sum_\lambda \left[\frac{1}{\pi} \int_0^{\omega_\lambda^2} d\omega'^2 \frac{\mathrm{Im}\, t_{1,\lambda}(\omega', q^2)}{(\omega'^2 - \omega^2 + i\epsilon)} \right] \tag{3.21}$$

where

$$\mathcal{T}_1(\infty, q^2) + \sum_j \theta(-\omega + \omega_j)\, t_{1,j}(\infty, q^2)$$

$$= T_1(\infty, q^2) - \sum_j \theta(\omega - \omega_j)\, t_{1,j}(\infty, q^2) \tag{3.22}$$

has been used.

As in the subtracted case, the integrals in the two summations can be combined and the preceeding discussion applies as to the probable magnitude of their modification of the dispersion relation. The subtraction constant, $T_1(\infty, q^2)$, is also modified by a contribution from $t_{1,k}(\infty, q^2)$ from each k such that $\omega_k < \omega$ (i.e. $\nu_k > \nu$ so this is the contribution from a shadow pseudo threshold which lies above the projectile energy ν). The net result is that for $\omega \simeq 2$ ($\nu \simeq 0$) the subtraction constant is one value, $\mathcal{T}_1(\infty, q^2)$, and for $\omega \simeq 0$ ($\nu \to \infty$) the subtraction constant is another, $T_1(\infty, q^2)$. From the point of view of the present analysis, the size of this difference is a dynamical matter.[26,27]

Again in the deep inelastic region with $q^2 \to -\infty$, the integrals in the two summations cancel to give

$$T_1(\omega, q^2) = T_1(\infty, q^2) - \int_c^q d\omega' \; \frac{W_1(\omega', q^2)}{(\omega'^2 - \omega^2 + i\epsilon)} \tag{3.23}$$

which has the same form as that of a subtracted dispersion relation for $T_1(\omega, q^2)$.

The onset of a shadow channel is a point of join of two different analytic functions. Along the real energy axis the scattering amplitude is continuous, but only piecewise analytic, and at the shadow pseudo threshold there will be a cusp behavior in physical quantities, such as the physical total cross sections. Evidence[28,29] indicates that such cusps would also be difficult to observe.

IV. Modified Low Equation

The derivation of Callen and Gross of their sum rule employs the Bjorken-Johnson-Low theorem which follows formally from, for example, the Low equation. Therefore, we now attempt to generalize the arguments of the preceeding section, concerning the modification of dispersion relations by the presence of the shadow states, to the level of the Low equation for the Compton scattering amplitude. To reduce out the currents in the matrix elements of the shadow theory, we will find it necessary to make an ansatz, analogous to Low's relation in the normal quantum field theory framework. This ansatz is consistent with the piecewise analytic properties of a shadow theory (and hence with Section III) and with the physically expected contributions from the shadow intermediate states to the spectral function of the Low equation. The shadow intermediate states will contribute to the real part, but not to the imaginary part. The ansatz will be proven to be correct in the static limit.

As in the preceeding section (Eq. (3.5)) we decompose the Lorentz invariant amplitude of the shadow theory by

$$T_{\mu\nu}{}^{*} = \mathcal{T}_{\mu\nu}{}^{*} + T'_{\mu\nu}{}^{*} \tag{4.1}$$

where $\mathcal{T}_{\mu\nu}{}^{*}$ is the amplitude in the associated indefinite metric theory in which all of the boundary conditions are of the traveling wave type. Then for the well-studied Lorentz invariant $\mathcal{T}_{\mu\nu}{}^{*}$ we write

$$\mathcal{T}_{\mu\nu}{}^{*} = \mathcal{T}_{\mu\nu} + \text{polynomials in } q \tag{4.2}$$

where $\mathcal{T}_{\mu\nu}$ is the time-ordered product

$$\mathcal{T}_{\mu\nu}(q, P) = i\int d^4x\, e^{iqx} \langle \mathcal{P}|\eta\, T(J_\mu(x)\, J_\nu(0))|\mathcal{P}\rangle$$

where $|\mathcal{P}\rangle$ is the physical proton state[30] of the $\mathcal{T}_{\mu\nu}$ theory and single particle states are normalized $\langle \mathcal{P}'|\eta|\mathcal{P}\rangle = \frac{E}{M}(2\pi)^3\delta^3(\underset{\sim}{\mathcal{P}}'-\underset{\sim}{\mathcal{P}})$. The polynomials in q are possible Schwinger terms. The usual argument of positive metric quantum field theory that there must be Schwinger terms breaks down because of the indefinite metric. For the sake of generality, though, we will include them. Defining the absorbtive parts

$$\eta_{\mu\nu}(q, P) = \int d^4x\, e^{iqx} \langle \mathcal{P}|\eta\, J_\mu(x)\, J_\nu(0)|\mathcal{P}\rangle$$
$$= \sum_m (2\pi)^4\, \delta^4(q + P - p_m)\langle \mathcal{P}|\eta\, J_\mu(0)|m;\mathcal{O}\rangle\langle m;\mathcal{O}|\eta\, J_\nu(0)|\mathcal{P}\rangle$$

$$\bar{\eta}_{\mu\nu}(q, P) = \int d^4x\, e^{iqx} \langle \mathcal{P}|\eta\, J_\mu(0)\, J_\nu(x)|\mathcal{P}\rangle$$
$$= \sum_m (2\pi)^4\, \delta^4(q + p_m - P)\langle \mathcal{P}|\eta\, J_\mu(0)|m;\mathcal{O}\rangle\langle m;\mathcal{O}|\eta\, J_\nu(0)|\mathcal{P}\rangle \qquad (4.4)$$

where the $|m\mathcal{P}\rangle$ are a complete set of traveling wave (physical and shadow) "out-states" of the $\mathcal{T}_{\mu\nu}$ theory, the Low equation follows from Eq. (3.3)

$$\mathcal{T}_{\mu\nu}(q, P) = -\int_{-\infty}^{\infty} \frac{dq_0'}{(2\pi)}\left[\frac{\eta_{\mu\nu}(q_0', \underset{\sim}{q}, P)}{q_0 - q_0' + i\epsilon} - \frac{\bar{\eta}_{\nu\mu}(q_0', \underset{\sim}{q}, P)}{q_0 - q_0' - i\epsilon}\right] \qquad (4.5)$$

Notice that by the kinematics for the physical regions, for the first term the integral is only for $q_0'>0$ and for the second term, only for $q_0'<0$. By the decomposition $T_{\mu\nu} = \mathcal{T}_{\mu\nu} + T'_{\mu\nu}$ the absorptive parts are related according to

$$\eta_{\mu\nu} = 2\,\mathrm{Im}\, T_{\mu\nu} - 2\,\mathrm{Im}\, T'_{\mu\nu}, \quad q_0' > 0 \qquad (4.6)$$

$$\bar{\eta}_{\mu\nu} = 2\,\mathrm{Im}\, \bar{T}_{\mu\nu} - 2\,\mathrm{Im}\, \bar{T}'_{\mu\nu}, \quad q_0' < 0 \qquad (4.7)$$

with

$$2\,\mathrm{Im}\,T_{\mu\nu} = A_{\mu\nu}(\nu, q^2) = \bar{\sum_p}\, \overline{V_\mu(\gamma N \to p)}\; V_\nu(\gamma N \to p)(2\pi)^4\delta^4(P' - P - q)$$

and similarly for 2 Im $\bar{T}_{\mu\nu}$, so

$$T_{\mu\nu} = -\int_{-\infty}^{\infty}\frac{dq_0'}{(2\pi)}\left[\frac{2\,\mathrm{Im}\,T_{\mu\nu}(q_0', \underset{\sim}{q}, P)}{q_0 - q_0' + i\epsilon} - \frac{2\,\mathrm{Im}\,\bar{T}_{\nu\mu}(q_0', \underset{\sim}{q}, P)}{q_0 - q_0' - i\epsilon}\right] + R_{\mu\nu} \qquad (4.8)$$

with

$$R_{\mu\nu} = T'_{\mu\nu} + \frac{1}{\pi}\int_{-\infty}^{\infty} dq_0'\left[\frac{\mathrm{Im}\,T'_{\mu\nu}(q_0', \underset{\sim}{q}, P)}{q_0 - q_0' + i\epsilon} - \frac{\mathrm{Im}\,\bar{T}'_{\mu\nu}(-q_0', \underset{\sim}{q}, P)}{q_0 + q_0' - i\epsilon}\right] \qquad (4.9)$$

where in the second integral for $R_{\mu\nu}$ the $-i\epsilon$ is in the unphysical region. Therefore we change it to $+i\epsilon$ so as to have the dispersion relation choice. Writing a dispersion relation for $T'_{\mu\nu}$ we find, in the same manner as in Section II, that $R_{\mu\nu} \to 0$ in the Bjorken limit because of the special kinematics. To obtain a "modified Low equation" from Eq. (4.7) we must re-introduce the currents in the "vertex function" $V_\mu(\gamma N \to p)$.

Consistent with the above piecewise analyticity, we now make the ansatz ($J_\nu(0)$ is in momentum space).

$$\langle a; i|\eta\, T\, |b\nu; i\rangle = \langle a; o|\eta\, J_\nu(o)\,|b; i\rangle \qquad (4.10)$$

where $|a;i\rangle,\ldots$ are physical in-states of the shadow theory and J_ν, the physical current coupled to the renormalized vector potential A^ν. Although, as discussed in Section II, $\langle b_s;o|\eta|b_\nu;i\rangle = 0$, notice that with this ansatz we do not assume $\langle b_s;o|\eta J_\nu|b;i\rangle = 0$. In the static limit, $\langle b_s;out|\eta J_\nu|b;in\rangle = 0$ is equivalent to $\langle b_s;out|\eta H_I|b_2;free\rangle = 0$ which implies Im T' = 0 above the shadow pseudo threshold which is unsatisfactory. Since in the static limit,[31] $\langle a;free|\eta T|b\nu;free\rangle = \langle a;out|\eta H_I|b\nu;free\rangle$ for $\mathcal{H}_I(x)$ containing a term $\mathcal{H}_I(x) = J_\nu(x)A^\nu(x)$ it follows that $\langle a;free|\eta T|b\nu;free\rangle = \langle a;out|\eta J_\nu|b;free\rangle$

and the above ansatz is valid. In the ordinary positive metric quantum field theory, the analogous reduction relation of Low can be proven from, for instance, the LSZ scattering formalism. Unfortunately, here we have to appeal to the static limit since the proper set of asymptotic assumptions when shadow states are present is not clear. We conjecture that is possible to prove the ansatz from perturbation theory but have only verified it for a few low order diagrams. Because Eq. (4.10) makes no explicit statement regarding the action of J_ν on $|b;i>$ with regard to the shadow states, the presence of the shadow states in the asymptotic completeness statement retains its significance. This is particularly relevant in the Bjorken limit (see Section V).

Using the ansatz, we find from Eq. (4.8) that

$$T_{\mu\nu}(q, P) = -\int_{-\infty}^{\infty} \frac{dq_0'}{(2\pi)} \left[\frac{\rho_{\mu\nu}(q_0', \underset{\sim}{q}, P)}{q_0 - q_0' + i\epsilon} - \frac{\bar{\rho}_{\nu\mu}(q_0', \underset{\sim}{q}, P)}{q_0 - q_0' - i\epsilon} \right] + R_{\mu\nu} \tag{4.11}$$

with

$$\begin{aligned} \rho_{\mu\nu}(q, P) &= \bar{\sum_p} (2\pi)^4 \delta^4(q + P - p_p) < P|\eta\, J_\mu(0)|p;o><p;o|\eta\, J_\nu(0)|P> \\ \bar{\rho}_{\mu\nu}(q, P) &= \bar{\sum_p} (2\pi)^4 \delta^4(q + p_p - P) < P|\eta\, J_\mu(0)|p;o><p;o|\eta\, J_\nu(0)|P> \end{aligned} \tag{4.12}$$

where the $|p;o>$ are the set of physical "out-states" which are complete in the physical subspace[23] of the shadow $T_{\mu\nu}$. These are all traveling wave states and therefore do not constitute a complete set in the total Hilbert space $\mathcal{H} = \mathcal{H}(\text{physical}) \oplus \mathcal{H}(\text{shadow})$. If we introduce a projection operator π_o^P onto this physical out subspace, then the absorptive parts can be written as

$$\rho_{\mu\nu}(q, P) = \int d^4x \; e^{iqx} < P | \eta \; J_\mu(x) \, \Pi_0^P \; J_\nu(0) | P >$$

$$\bar{\rho}_{\mu\nu}(q, P) = \int d^4x \; e^{iqx} < P | \eta \; J_\mu(0) \, \Pi_0^P \; J_\nu(x) | P > \qquad (4.13)$$

and the modified time-ordered product as

$$T_{\mu\nu}(q, P) = i \int d^4x \; e^{iqx} < P | \eta \; T(J_\mu(x) \, \Pi_0^P \, J_\nu(0)) | P > + R_{\mu\nu}(q, P) \qquad (4.14)$$

where

$$T(A(x) \, \Pi_0^P \, B(0)) = \theta(x_0) \, A(x) \, \Pi_0^P \, B(0) + \theta(-x_0) \, B(0) \, \Pi_0^P \, A(x) \qquad (4.15)$$

with the usual sign prescription for Fermion fields.

V. Consequences for the Scaling Region

With the results of the preceeding two sections on the modifications to well-known formal expressions for scattering matrix elements by the presence of shadow states, we now explore the consequences for the scaling region. We begin by following the argument of Callen and Gross for their sum rule.

We set $q = (q_0, \underset{\sim}{0})$ and use Eq.s (3.14) and (2.21), so Eq. (3.4) becomes

$$T_{ij}^* = \delta_{ij}\Big\{T_1(\infty, q^2) - \int_0^4 d\omega'^2 \frac{W_1(\omega', q^2)}{\omega'^2 - \omega^2 + i\epsilon} - \frac{1}{\pi}\sum_{\omega_j > \omega} \int_{\omega_j^2}^{\bar{\omega}_j^2} d\omega'^2 \frac{\mathrm{Im}\, t_{1,j}(\omega', q^2)}{\omega'^2 - \omega^2}$$

$$- \sum_{\omega_i < \omega} \Big[t_{1,i}(\infty, q^2) - \frac{1}{\pi}\int_0^{\omega_i^2} d\omega'^2 \frac{\mathrm{Im}\, t_{1,i}(\omega', q^2)}{\omega'^2 - \omega^2}\Big]\Big\}$$

$$- \frac{P_i P_j}{M^2}\Big\{\omega^2 \int_0^4 d\omega'^2 \frac{W_2(\omega', q^2)}{\omega'^2(\omega'^2 - \omega^2 + i\epsilon)} + \frac{\omega^2}{\pi}\sum_{\omega_j > \omega} \int_{\omega_j^2}^{\bar{\omega}_j^2} d\omega'^2 \frac{\mathrm{Im}\, t_{2,j}(\omega', q^2)}{\omega'^2(\omega'^2 - \omega^2)}$$

$$- \frac{\omega^2}{\pi}\sum_{\omega_i < \omega} \int_0^{\omega_i^2} d\omega'^2 \frac{\mathrm{Im}\, t_{2,i}(\omega', q^2)}{\omega'^2(\omega'^2 - \omega^2)}\Big\} \qquad (5.1)$$

Taking $q_0 \to i\infty$, P^0 and $\hat{P}$ (unit vector in $\underset{\sim}{P}$ direction) temporarily fixed, so $\omega = -\frac{q^2}{q_0 P_0} \to -\frac{i|q_0|}{P_0} \to -i\infty$, we find

$$T_{ij}^*(P) = \delta_{ij}\Big[\lim_{q^2 \to -\infty} T_1(\infty, q^2) - \frac{P_0^2}{|q_0^2| M}\int_0^4 d\omega'^2 \Big\{\lim_{q^2 \to -\infty} M\, W_1(\omega', q^2)\Big\}\Big]$$

$$+ \frac{P_i P_j}{|q_0^2| M}\int_0^4 \frac{d\omega'^2}{\omega'} \lim_{q^2 \to -\infty}\big[\nu' W_2(\omega', q^2)\big] \qquad (5.2)$$

since ω_i and $\bar{\omega}_i \to 2$. Therefore, this expression is unmodified by the presence of shadow states. Hence, for $C_{ij}(P) = \lim_{q^2 \to \infty} q^2\, T^*_{ij}$, as shown by Callen and Gross for $\hat{P}$ fixed

$$\lim_{P_0 \to \infty} \frac{M}{P_0^2}\, C_{ij}(P) = \int_0^4 \frac{d\omega'^2}{\omega'}\Big\{\omega' F_1(\omega')\big(\delta_{ij} - \hat{P}_i \hat{P}_j\big)$$

$$- \big[F_2(\omega') - \omega' F_1(\omega')\big]\hat{P}_i \hat{P}_j\Big\} \qquad (5.3)$$

where $F_1(\omega) = \lim_{q^2\to-\infty} M\, W_1(\omega,q^2)$, $MF_2(\omega) = \lim_{q^2\to-\infty} \nu W_2(\omega,q^2)$.

From Eq. (4.11), by the Bjorken- Johnson-Low Theorem we also have for $|q_0|$ large

$$T_{\mu\nu} - R_{\mu\nu} = - \Big[\frac{1}{q_0} \int d^3x \; e^{-i\mathbf{q}\cdot\mathbf{x}} < P| \eta \, [J_\mu(0,\mathbf{x}), J_\nu(0)]_{\pi_0^P} |P>$$

$$+ i \left(\frac{1}{q_0}\right)^2 \int d^3x \; e^{-i\mathbf{q}\cdot\mathbf{x}} < P| \eta \, [\tfrac{\partial}{\partial t} J_\mu(0,\mathbf{x}), J_\nu(0)]_{\pi_0^P} |P>$$

$$+ \; \ldots \Big] \tag{5.4}$$

where the π_0^P subscripts imply this projection operator acts on the intermediate states. Thus, for $\mathbf{q} = 0$ and $q^2 \to -\infty$, since $R_{ij} \to 0$, the object $C_{ij}(P) = \lim_{q^2\to-\infty} (q^2 T^*_{ij})$ will diverge unless the shadow states give a vanishing contribution to the coefficient of the $\frac{1}{q_0}$ term in Eq. (5.4). In order for the limit to be finite, we must have[32]

$$\int_\mu^\infty dq_0' \; \mathrm{Im}\; T_{ij}'(q_0') = 0 \tag{5.5}$$

Which provides a constraint on the shadow states. The $(\frac{1}{q^0})^2$ term gives

$$C_{ij}(P) = -i \int d^4x \; \delta(t) < P| \eta \, [\tfrac{\partial}{\partial t} J_i(t,\mathbf{x}), J_j(0)]_{\pi_0^P} |P>$$

$$+ \; \text{polynomial} \tag{5.6}$$

If, as for the $(\frac{1}{q^0})$ term, the shadow states do not contribute when inserted and integrated over, the Callen-Gross result is obtained:

$$C_{ij}(P) = -i \int d^4x \; \delta(t) < P| \eta \, [\tfrac{\partial}{\partial t} J_i(t,\mathbf{x}), J_j(0)] |P>$$

$$+ \; \text{polynomial}$$

$$= B\,(P_i P_j - \delta_{ij} P^2) + B' \tag{5.7}$$

where B, B' are Lorentz scalars and the last line follows[20] in the "gluon model", i.e. the quark model bound by neutral vector meson exchange. Then using Eq. (5.3) and the positivity of

$$F_2(\omega) - \omega F_1(\omega) = \frac{1}{4\pi^2\alpha} \lim_{q^2 \to -\infty} \left(q^2 \sigma_L(\omega, q^2) \right) \tag{5.8}$$

where σ_L is the total photoabsorption cross section for longitudinal virtual photons, Eq. (5.7) yields

$$\lim_{q^2 \to -\infty} q^2 \sigma_L(\omega) = 0 \tag{5.9}$$

Experimentally[33] σ_L is 20% or less of σ_T. If Eq. (5.9) is exactly true, then

$$\int_{\mu'}^{\infty} dq_0' \, q_0' \, \mathrm{Im}\, T'_{\lambda j}(q_0') = 0 \tag{5.10}$$

forcing the vanishing of a higher moment of the absorbtive part, $\mathrm{Im}\, T'_{\lambda j}(q'_0)$. Notice that the positivity of $\mathrm{Im}\, T'_{\lambda j} = \mathrm{Im}(T_{\lambda j} - \mathcal{T}_{\lambda j})$ provides some net measure of the amount of the negative-metric's violation of unitarity which is corrected by the shadow boundary condition. Notice also that this says $\mathrm{Im}\, T'_{\lambda j} \to \left(\frac{1}{q_0}\right)^{2+\varepsilon}$ as $q_0 \to \infty$ which is the limit relevant for making the divergences finite. This should be compared with the weaker conclusions which can be drawn from the leading order piecewise analytic modifications to the dispersion relations which are

$$\int_{\mu'}^{\infty} dq_0' \left(\frac{1}{q_0'}\right)^{+1} \mathrm{Im}\, T'_{\lambda j}(q_0') \tag{5.11}$$

which follows as $q_0' \to 0$ from the modified Low equation associated with

Eq. (3.16) and

$$\int_{\mu}^{\mu'} dq_0' \; \mathrm{Im}\, t_{\lambda\gamma}(q_0') \tag{5.12}$$

where the range of integration is finite, which follows for $q_0 \gg \mu'$ from the modified Low equation associated with Eq. (3.15).

VI. A Shadow Quark Model

The symmetric quark model has led to many successes in particle physics: in classification of mesons and baryons into supermultiplets, in mass formulas for these multiplets, in magnetic moment ratios for baryons, in description of decay processes, and in relations for high energy scattering.[34] Despite these results, the mysteries associated with the bound state picture in the quark model have remained. In particular, phenomenological applications of the model to magnetic moments,[35] the mass spectrum,[36] and high energy scattering[37] indicate that quarks are "light" objects in hadrons (the effective mass of the bound quark is the order of a few hundred MeV) which move about independently of one another, as if "free" particles (this is indicated, for instance, by the success of the "additivity hypothesis"). Hence, the suggestion, experimentally motivated, that physical unbound quarks are very massive (greater than 5 GeV) is inconsistent: For if quarks were strongly bound, how could they move freely and non-relativistically, without large corrections to quark-antiquark pairs? How could they be relevant for low lying hadronic structure when both dispersion theory and bootstrap theory suggest that low mass states should dominate? However, should the quarks be shadow particles,[14] i.e. standing wave structures in hadrons, then they could <u>never</u> be produced and, yet, they could move as free, light objects in hadrons.

In this section, we demonstrate this possibility by means of a simple soluble model and study the qualitative consequences of the boundary conditions which would be expected to generalize to more realistic descriptions. To make our model moderately realistic we, first, recall that the symmetric quark model, which has been so remarkably successful phenomenologically, assumes that baryons are made out of three quarks and mesons out of a quark-antiquark pair. However, in the deep inelastic region of e-p scattering, when partons are directly identified with quarks, this leads to a violation of the mean squared charge sum rule. In general the evidence[38] from this process is that the partons here which carry most of the nucleon's momentum have $Q = Y = I = 0$. To resolve this, some time ago Paschos suggested that quark-antiquark pairs might also be present inside the proton. Recently, Kuti and Weisskopf[39] have studied the quantitative aspects of this idea that baryons, N, are approximately structures with three valence quarks, Q, having a symmetric SU(3) quark-antiquark pair distribution as a core. Since such a model resembles systems in atomic and nuclear physics, one would expect the valence quarks would be ionized by a sufficiently excited incoming hadron and therefore be observable.

The model that we consider is a familiar one but re-interpreted so as to simulate baryons as composite structures of three valence quarks, represented by the creation operator ψ^+_{3Q}, and quark-antiquark pairs, each represented by $\pi^+(\underset{\sim}{q})$. The lightest baryon with mass M has the simple, composite, single particle state

$$|N\rangle\rangle = \int d\underset{\sim}{q} \; \frac{f_r(\omega)}{(8\pi\omega)^{1/2}\,(M-\omega)} \; \pi^+(\underset{\sim}{q})\,|3Q\rangle \tag{6.1}$$

where $|3Q\rangle = \psi^+_{3Q}\,|0\rangle$, $|0\rangle$ is the vacuum, and $f_r(\omega)$ describes the relativistic energy distribution of the quark-antiquark pair of mass μ and energy $\omega(\underset{\sim}{q}) = (\mu^2 + \underset{\sim}{q}^2)^{1/2}$. To construct the model, we first introduce a field ψ^+_N for this lightest baryon but later, at the end of our calculation, remove it by taking the wave-function renormalization constant, Z_2, to zero. When this is done, $|N\rangle\rangle$ is entirely composite and the properties of ψ^+_N are completely determined by the "fundamental" ψ^+_{3Q} and $\pi^+(\underset{\sim}{q})$.

The Hamiltonian, defining the system, is $H = H_o + H_I$

$$H_o = m_N^o \psi_N^+ \psi_N + 3 m_Q \psi_{3Q}^+ \psi_{3Q} + \int d\underset{\sim}{q}\, \omega(\underset{\sim}{q})\, \pi^+(\underset{\sim}{q})\, \pi(\underset{\sim}{q})$$

$$H_I = \left(\frac{1}{4\pi Z_2^{1/2}}\right) \int d\underset{\sim}{q}\, \frac{f_r(\omega)}{(2\omega)^{1/2}} \left[\psi_N^+ \psi_{3Q}\, \pi(\underset{\sim}{q}) + \psi_{3Q}^+ \psi_N\, \pi^+(\underset{\sim}{q}) \right] \tag{6.2}$$

with

$$Z_2 = 1 - \int d\underset{\sim}{q}\, \frac{f_r(\omega)^2}{(8\pi\omega)(M-\omega)^2} .$$

This is a static system which may not be a bad approximation since the quark-antiquark pairs, like pion pairs, may be essentailly massless. The single particle state, Eq. (6.1) then follows as a stable discrete eigenstate when $f_r(\omega)$ leads to $M < 3m_Q + \mu$. The continuum shadow state of three free valence quarks plus a free quark-antiquark pair, described in Fig. 2(a), leads to no scattering because the denominator function

$$D_{2q}(\omega) = (\omega - M)\left[1 - (\omega - M) \int d\underset{\sim}{q}'\, \frac{f_r(\omega')^2}{(8\pi\omega')(M-\omega')^2}\, \mathcal{P}\left(\frac{1}{\omega-\omega'}\right)\right] \tag{6.3}$$

has no right-hand cut as a consequence of the principal value prescription for the shadow state.

Let us now consider what happens when a quark-antiquark pair, like a pion, is incident upon the baryon $|N\rangle\rangle$. In this case the second diagram, Fig. 2(b), contributes. If the shadow boundary condition were <u>not</u> imposed, then $\pi N \to 3Q\pi\pi$ and the scattering matrix for $\pi N \to \pi N$ would be

$$\langle\langle N\, \pi(\underset{\sim}{q}')\ \text{out} \,|\, N\, \pi(\underset{\sim}{q})\ \text{in}\rangle\rangle = \delta(\underset{\sim}{q}' - \underset{\sim}{q}) \left[1 + i\, \mathcal{T}(\omega)\right]$$

$$\mathcal{T}(\omega) = - \frac{\pi q\, f_r(\omega)^2\, \rho(\omega + M + i\epsilon)}{\mathcal{D}_2(\omega + M + i\epsilon)} \tag{6.4}$$

with the denominator function

$$\mathcal{D}_2(\omega + M) = \mathcal{D}_1(\omega)\, \rho(\omega + M) + 2 \tag{6.5}$$

having both a right-hand $N\pi$ cut, $\mu < \omega < \infty$, from

$$\mathcal{D}_1(\omega \pm i\epsilon) = (\omega - M)\left[1 - (\omega - M)\int d\underset{\sim}{q}' \frac{f_r(\omega')^2}{(8\pi\omega')(M-\omega')^2(\omega - \omega' \pm i\epsilon)}\right] \tag{6.6}$$

and a $3Q\pi\pi$ cut, $2\mu + 3m_Q < \omega + M < \infty$, because

$$\rho(\omega + M \pm i\epsilon) = -\frac{1}{\omega - M + 3m_Q} + \iint d\underset{\sim}{p}_1\, d\underset{\sim}{p}_2 \frac{|g_r(\underset{\sim}{p}_1)|^2\, |g_r(\underset{\sim}{p}_2)|^2}{[\omega + M - 3m_Q - \omega(p_1) - \omega(p_2) \pm i\epsilon]} \tag{6.7}$$

where $g_r(\underset{\sim}{q}) = f_r(\omega)/(8\pi\omega)^{\frac{1}{2}}\mathcal{D}_1(\omega - i\epsilon)$. But when the quarks are only shadow-quarks, we have,[40] instead of $\mathcal{T}(\omega)$, the transition amplitude

$$T_q(\omega) = -\frac{\pi q\, f_r(\omega)^2\, \rho_q(\omega + M)}{D_{2q}(\omega + M + i\epsilon)}$$

$$D_{2q}(\omega + M + i\epsilon) = \mathcal{D}_1(\omega + i\epsilon)\,\rho_q(\omega + M) + 2 \tag{6.8}$$

where now

$$\rho_q(\omega + M) = -\frac{1}{\omega - M + 3m_Q} + \mathcal{P}\iint d\underset{\sim}{p}_1\, d\underset{\sim}{p}_2 \frac{|g_r(\underset{\sim}{p}_1)|^2\, |g_r(\underset{\sim}{p}_2)|^2}{[\omega + M - 3m_Q - \omega(\underset{\sim}{p}_1) - \omega(\underset{\sim}{p}_2)]} \tag{6.9}$$

as a consequence of the standing wave boundary condition. Notice that the summation equation of Section II, Eq. (2.7), is now an expansion of the scattering process in terms of possible intermediate shadow-quark states. For the piecewise analytic $D_{2q}(\omega + M)$ there is only the physical $N\pi$ right-hand cut for $\mu < \omega < \infty$ and $T_q(\omega)$ can easily be shown to satisfy

$$i[T_q(\omega)^* - T_q(\omega)] = T_q(\omega)^*\, T_q(\omega)$$

by using Eq. (6.6).

Some comments are appropriate: In Eq. (6.8) for D_{2q}, we observe that if the 2 is neglected then the $\rho_q(\omega + M)$ factors would cancel in $T_q(\omega)$ yielding the well-known result for the scattering of two non-composite particles in the static limit. Thus, the presence of the composite shadow-quarks may not be easily gleaned, and the consequences of piecewise analyticity may be very slight. Also if quarks have fractional quantum numbers, as in the simplest model, the production of a single quark would even be forbidden by selection rules, hence the tell-tale shadow pole, which[11] normally would appear "as a dip" without corresponding physical thresholds at high energies, would be absent.

Notice that in Fig. 2(b) there are actually two kinds of standing wave states - the discrete, ordinary $|N\rangle\rangle$ "bound" state and the continuum $|3Q, \pi(q)\rangle\rangle$ shadow state. The shadow state under interacton always remains a standing wave structure as a direct consequence of the choice of boundary condition. On the other hand, the standing wave property for $|N\rangle\rangle$ was a consequence of the form factor $f_r(\omega)$ being such that $M < 3m_Q + \mu$ where $\mathcal{D}_1(M) = 0$. In general $|N\rangle\rangle$ under interaction with another $\pi(\underset{\sim}{q}')$, as in Fig. 2(b), will dissociate into a free $3Q\pi$ system for $\omega(\underset{\sim}{q}')$ sufficiently large. But when the quarks were shadow objects, this did not happen.

If we push this comparison further, though, we will see that these two types of standing waves lead to essentially the same system. For example, the shadow state of Fig. 2(a), consisting of 3Q and a π, can be achieved by first "confining" the valence quarks and the quark-antiquark pair to a spherical box of radius R in coordinate space and then taking R to infinity. Notice that if we did not take the limit, we would have an infinite set of discrete shadow state energy levels, say, as in the harmonic oscillator well. Similarly, for $\hat{f}_r(\underset{\sim}{r}) = \int d\underset{\sim}{q}\, e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}} f_r(\omega)$ sufficiently singular as $|\underset{\sim}{r}| \to \infty$, the ordinary $|N\rangle\rangle$ "bound" state would be so tightly bound as not to be dissociated no matter how energetic a π was incident. Such an $f_r(\omega)$ would lead to an infinite number of standing wave levels.[41]

With the inclusion of the proper dynamics this infinite set of discrete standing wave levels, whether the consequence of a "box" of radius R or a singular $\tilde{f}_r(\underset{\sim}{r})$, might be a good framework to approximate the experimental hadronic resonance spectrum. That it is phenomenologically appropriate is clear from it's strong similarity with the nuclear physics framework used in phenomenological applications of the symmetric quark model. The results of the other sections of this paper would apply in this case, but also would be well-known: The basic change is that an infinite, discrete continuum from the shadow states replaces the previously considered continuous one. These composite bound states and resonances would, as usual, contribute to the completeness statement in momentum space and replace the piecewise analytic modifications to the dispersion relations.

It should be emphasized, though, that neither the "box" of radius R nor the singular $\tilde{f}_r(\underset{\sim}{r})$ <u>explains</u> how hadronic systems saturate as 3Q, 3$\bar{Q}$, or Q$\bar{Q}$ valence quark structures.[42] Especially in the case of the $\tilde{f}_r(\underset{\sim}{r})$ mechanism, which acts as an infinite well over an infinite range, it is difficult to understand why quarks in one hadron would not be strongly effected by those in another hadron, even when hadrons are far apart. Also quarks "bound" by $\tilde{f}_r(\underset{\sim}{r})$ may tend <u>not</u> to move as free, independent objects inside the hadron, though quarks should if kept in a "box" of radius R. From a fundamental framework such as relativistic quantum field theory, it is not at all clear how such a singular $\tilde{f}_r(\underset{\sim}{r})$ or a "box" of radius R can be obtained.

VII. Conclusions

When the possibility of standing wave objects, i.e. shadow states, is allowed in the dynamics of a scattering process, there is a more inclusive framework. In this paper we have derived the new terms which modify the usual dispersion relations and the Low equation. From the structure of these terms, we have concluded that the verification[15] of analytic results by present two-body scattering experiments does not put strong constraints on the presence of shadow states, even if they are quite light. The piecewise analytic terms also vanish in the Bjorken limit of deep--inelastic electron-nucleon scattering while the shadow states can still provide the unitary cutoff of the parton models. Thus, the triangle graph annomalies should only be taken seriously in the low energy region. A stronger constraint on the presence of shadow states, in principle, is provided by verification of the Callen-Gross sum rule.

Finally, by means of a simple model, we have demonstrated how this standing wave mechanism can "bind" quarks in hadrons as standing wave, shadow-quarks. In this case, if the boundary condition only applies over a separation distance R, the spectrum of the shadow-quarks would not be continuous, but only a series of discrete, countably infinite levels such as in the harmonic oscillator potential.

Appendix A: Formal Scattering Theory

The simplest framework for the purpose of studying the effects of mixed boundary conditions in high energy physics, i.e. both standing wave and in-going (or out-going) traveling wave states in continuum channels, is in formal scattering theory. In this appendix we will exploit this simplicity to derive some results on this level, to show how to properly othonormalize both the shadow and physical eigenstates of the full Hamiltonian, and to point out which approaches, because of consequences of the mixed boundary conditions, should be the most promising in a quantum field-theoretic framework.

Because shadow states are assumed present, it should be stressed that a correct generalization of the standard formulation of formal scattering theory is essential to showing that the concepts and definitions of objects, such as the scattering matrix, in the presence of mixed boundary conditions are in agreement with scattering phenomena as presently observed in particle physics. That the scattering matrix is indeed properly defined has been shown elsewhere[11] in the context of exactly soluble quantum field theory models.

We begin by investigating the matter of the correct orthonormalization of the shadow and physical eigenstates of the full Hamiltonian. As is well-known,[43] when all the boundary conditions are standing wave, even when there are no degeneracies in the system, the eigenstates

$$|\psi_a^P\rangle = |\phi_a\rangle + \mathcal{P}\left(\frac{1}{E_a - H_0}\right) H_I \, |\psi_a^P\rangle \qquad \text{(A:1)}$$

satisfying $H\,|\psi_a^P\rangle = E_a\,|\psi_a^P\rangle$ with $H = H_0 + H_I$

are not orthonormal. The unperturbed eigenstate $|\phi_a\rangle$ satisfies $H_0|\phi_a\rangle = E_a|\phi_a\rangle$ and is normalized $\langle\phi_b(E_b')|\phi_a(E_a)\rangle = \delta_{ab}$. For the sake of clairity we will suppress the continuous delta function $\delta(E_b' - E_a)$. The free state $|\phi_a\rangle$ is also the eigenstate of some operator A which commutes with H_0 with eigenvalue "a". While the full state $|\psi_a^P\rangle$ is labeled by the same quantum numbers, "a", as $|\phi_a\rangle$ these are not, in general, the eigenvalues of operators which commute with H. Their significance for $|\psi_a^P\rangle$ resides in the integral equation for $|\psi_a^P\rangle$, i.e. Eq. (A.1), including the boundary conditions!

As stated, $|\psi_a^P\rangle$ is not orthonormal, in fact, in terms of it the completeness statement is (since bound states enter in the usual way, we will omit their consideration)

$$1 = \sum_{a,b} |\psi_a^P\rangle \, [1 + K^2]_{ab}^{-1} \langle\psi_b^P| \tag{A.2}$$

where $K_{ba} \equiv -\pi\langle\phi_b|H_I|\psi_a^P\rangle$ so the usual choice for the orthonormalized standing wave state vector is

$$|\psi_\alpha^{P'}\rangle \equiv \sum_b \left[(1 + K^2)^{-1/2}\right]_{ba} |\psi_b^P\rangle \tag{A.3}$$

An alternative choice, however, is more convenient when there are mixed boundary conditions: It has the advantage of being easily generalized to multichannel scattering[11] to yield directly the physical states associated with the scattering matrix as defined in Section II. We only treat this choice in this paper. It consists of introducing for each standing wave component, $|\psi_a^P\rangle$, a non-interacting (free) standing wave component, $|\chi_a\rangle$, of opposite metric so as to effect the orthonormalization of the interacting standing wave states by subtraction, instead of by division! Specifically, with $|\psi_a^P\rangle$ given as above by Eq. (A.1), we take

$$|\tilde{\psi}_a^P\rangle \equiv |\psi_a^P\rangle + \pi^{s'}\sum_e K_{ea}|\chi_e\rangle \tag{A.4}$$

so $\langle\tilde{\psi}_b^P|\tilde{\psi}_a^P\rangle = \delta_{ab}$ where $H_0|\chi_e\rangle = E_e|\chi_e\rangle$

and $\Pi^{S'}$ projects onto the non-interacting standing wave subspace normalized by $\langle\chi_b|\chi_a\rangle = -\delta_{ba}$. The completeness statement now reads

$$1 = \sum_c |\tilde{\psi}_c^P\rangle\langle\tilde{\psi}_c^P| - \sum_{c,d}|\tilde{\psi}_c^{P'}\rangle[1+K^2]^{-1}_{cd}\langle\tilde{\psi}_d^{P'}| \tag{A.5}$$

with the "shadow-ghost" state

$$|\tilde{\psi}_c^{P'}\rangle = \sum_e K_{ec}|\psi_e^P\rangle + \pi^{s'}\sum_e[1+K^2]_{ec}|\chi_e\rangle \tag{A.6}$$

By construction $|\tilde{\psi}_c^{P'}\rangle$ is orthogonal to $|\tilde{\psi}_a^P\rangle$ and it obviously satisfies $H|\tilde{\psi}_c^{P'}\rangle = E_c|\tilde{\psi}_c^{P'}\rangle$. Notice that in the no-interaction limit $|\tilde{\psi}_c^P\rangle \to |\phi_c\rangle$ and $|\tilde{\psi}_c^{P'}\rangle \to \Pi^{S'}|\chi_c\rangle$.

It is straight forward to generalize this proceedure to scattering with both physical and shadow states: Using the projection operators of Section II, we have the orthonormal physical states $(H|\hat{\psi}_{1a}\rangle = E_a|\hat{\psi}_{1a}\rangle)$[42,43]

$$|\hat{\psi}_{1a}\rangle = |\tilde{\psi}_{1a}\rangle + \pi^{s'}\sum_e K_{ea}|\chi_e\rangle \tag{A.7}$$

with $K_{ba} = -\pi\langle\phi_b|H_I|\tilde{\psi}_{1a}\rangle$ where

$$|\tilde{\psi}_{1a}\rangle = \pi^P\left[|\phi_a\rangle + \frac{1}{E_a - H_0 + i\epsilon}H_I|\tilde{\psi}_{1a}\rangle\right] + \pi^S\mathcal{P}\left(\frac{1}{E_a - H_0}\right)H_I|\tilde{\psi}_{1a}\rangle \tag{A.8}$$

Eq. (A.7) should be compared with Eq. (A.4). The second term in Eq. (A.7) is clearly required since

$$\langle \hat{\Psi}_{1b} | \hat{\Psi}_{1a} \rangle = \delta_{ba} + \pi^2 \sum_c \langle \tilde{\Psi}_{1b} | H_I | \phi_c \rangle \langle \phi_c | H_I | \tilde{\Psi}_{1a} \rangle$$

$$= \delta_{ba} + \sum_c K^{\dagger}_{bc} K_{ca}$$

$$= (N_{11})_{ba}$$

which in general is not even a diagonal matrix in the "a,b" indices. From the argument in Section IV, the transition matrix element for the physical states is given by

$$T_{ba} = - \langle \phi_b | H_I | \hat{\Psi}_{1a} \rangle \tag{A.9}$$

which is the same as the singular part of Eq. (A.7) since

$$\langle \hat{\Psi}_{1c} | \phi_a \rangle = \delta_{ac} + \langle \tilde{\Psi}_{1c} | H_I | \phi_a \rangle \frac{1}{E_c - E_a - i\epsilon}$$

The state vector, Eq. (A.7), also gives the same transition matrix when it is Fourier transformed into coordinate space and the limits $t \to \pm \infty$ are taken. Because of both the shadow and shadow ghost terms in Eq. (A.7) there is no shadow contribution to the state in either of the limits $t \to \pm \infty$. Obviously there will be no net shadow flux. If the second term in Eq. (A.7) were omitted there would still be zero net shadow flux but the physical states would not be orthonormal and the shadow contributions would not vanish in the limits $t \to \pm \infty$. Because of the lack of orthonormality, this is not acceptable.

The shadow states and "shadow-ghost" states are also easy to construct:

(In this appendix, to simplify the formalism we take all shadow states to have positive metric.) While the solution

$$|\psi_{2a}\rangle = \pi^{P} \frac{1}{E_a - H_0 + i\epsilon} H_I |\psi_{2a}\rangle + \pi' \left[|\phi_a\rangle + \mathcal{P}\left(\frac{1}{E_a - H_0}\right) H_I |\psi_{2a}\rangle \right] \tag{A.11}$$

is not real nor orthogonal to $|\tilde{\psi}_{1b}\rangle$, by Schmid orthogonalization we construct

$$|\tilde{\psi}_{2a}\rangle = |\psi_{2a}\rangle - \sum_{f,e} (N_{12})_{fa} \left(\frac{1}{N_{11}}\right)_{ef} |\hat{\psi}_{1e}\rangle \tag{A.12}$$

which is purely real where

$$(N_{12})_{ba} = \langle \hat{\psi}_{1b} | \psi_{2a} \rangle$$

$$= \delta_{ba}\, i\, L_{aa} + 3\, K^{+}_{bc} L_{ca}$$

with

$$L_{ba} \equiv -\pi \langle \phi_b | H_I | \psi_{2a} \rangle$$

Since $|\tilde{\psi}_{2a}\rangle$, unlike $|\hat{\psi}_{1a}\rangle$, does not depend explicitly on the shadow-ghost components $|\chi_c\rangle$ and since $\langle \tilde{\psi}_{1b}|\psi_{2a}\rangle = \langle \hat{\psi}_{1b}|\psi_{2a}\rangle = (N_{12})_{ba}$, the shadow-ghost states will be orthogonal to $|\tilde{\psi}_{2a}\rangle$ if constructed out of the $|\tilde{\psi}_{1a}\rangle$ and $|\chi_a\rangle$. Constructing them orthogonal to $|\tilde{\psi}_{1a}\rangle$, we have the purely real

$$|\hat{\psi}_{2'a}\rangle = \sum_{e} (K^{+})_{ea} |\tilde{\psi}_{1e}\rangle + \pi^{2'} \sum_{e} [1 + K^{+}K]_{ea} |\chi_e\rangle \tag{A.13}$$

to be compared with Eq. (A.6).

The completeness statement is finally

$$1 = \sum_{c} |\hat{\psi}_{1c}\rangle\langle\hat{\psi}_{1c}| + \sum_{c,d} |\tilde{\psi}_{2c}\rangle \left(\frac{1}{\tilde{N}_{22}}\right)_{cd} \langle\tilde{\psi}_{2d}|$$

$$- \sum_{c,d} |\hat{\psi}_{2'c}\rangle \left(\frac{1}{\hat{N}_{2'2'}}\right)_{cd} \langle\hat{\psi}_{2'd}| \qquad \text{(A.14)}$$

where the normalization factors are given by

$$\langle\hat{\psi}_{2'b}|\hat{\psi}_{2'a}\rangle = -(\hat{N}_{2'2'})_{ba} = -\delta_{ba} - \sum_{c} K_{bc} K^{+}_{ca}$$

$$\langle\tilde{\psi}_{2b}|\tilde{\psi}_{2a}\rangle = (\tilde{N}_{22})_{ba}$$

$$= (N_{22})_{ba} - \sum_{e,f} (N_{12}^{+})_{be} \left(\frac{1}{N_{11}}\right)_{ef} (N_{12})_{fa}$$

$$\langle\psi_{2b}|\psi_{2a}\rangle = (N_{22})_{ba}$$

$$= \delta_{ba} + 3 L^{+}_{bc} L_{ca}$$

Again Eq. (A.14) should be compared with Eq. (A.5).

Knowing the state vectors, $|\hat{\psi}_{1a}\rangle$, $|\tilde{\psi}_{2a}\rangle$ and $|\hat{\psi}_{2'a}\rangle$, and also the completeness statement, we can derive explicit expressions for modifications to scattering matrix elements when there are shadow states. For example, consider the matrix element

$$H_{I,ba} \equiv \langle\phi_b|H_I|\phi_a\rangle$$

$$= \sum_{c} \langle\phi_b|H_I|\hat{\psi}_{1c}\rangle\langle\hat{\psi}_{1c}|\phi_a\rangle + \sum_{c,d} \langle\phi_b|H_I|\tilde{\psi}_{2c}\rangle \left(\frac{1}{\tilde{N}_{22}}\right)_{cd} \langle\tilde{\psi}_{2d}|\phi_a\rangle$$

$$- \sum_{c,d} \langle\phi_b|H_I|\hat{\psi}_{2'c}\rangle \left(\frac{1}{\hat{N}_{2'2'}}\right)_{cd} \langle\hat{\psi}_{2'd}|\phi_a\rangle \qquad \text{(A.15)}$$

Notice that the similar transition matrix element $T_{ba} = -\langle\phi_b|H_I|\hat{\psi}_{1a}\rangle$

occurs in Eq. (A.15) as the singular part of $\langle\hat{\psi}_{1c}|\phi_a\rangle$, see Eq. (A.10). From the other state vectors, we have

$$\langle\tilde{\psi}_{2d}|\phi_a\rangle = \langle\tilde{\psi}_{2d}|H_I|\phi_a\rangle\,\mathcal{P}\left(\frac{1}{E_d-E_a}\right)$$

$$+\,\delta_{ad}\left[i\pi\langle\tilde{\psi}_{2d}|H_I|\phi_a\rangle - \sum_g (N_{12}{}^{+})_{dg}\left(\frac{1}{N_{11}}{}^{+}\right)_{ga}\right]$$

$$\langle\hat{\psi}_{2'd}|\phi_a\rangle = \sum_f K_{df}\left\{\langle\hat{\psi}_{1f}|H_I|\phi_a\rangle\,\mathcal{P}\left(\frac{1}{E_d-E_a}\right)\right.$$

$$\left.+\,\delta_{fa}\left[1 + i\pi\langle\hat{\psi}_{1f}|H_I|\phi_a\rangle\right]\right\}$$

so we easily obtain

$$T_{ba} = -\langle\phi_b|H_I|\phi_a\rangle$$

$$+\sum_c \frac{\langle\phi_b|H_I|\hat{\psi}_{1c}\rangle\langle\hat{\psi}_{1c}|H_I|\phi_a\rangle}{E_c-E_a-i\epsilon}$$

$$+\sum_{c,d}\mathcal{P}\left[\frac{\langle\phi_b|H_I|\tilde{\psi}_{2c}\rangle\left(\frac{1}{\tilde{N}_{22}}\right)_{cd}\langle\tilde{\psi}_{2d}|H_I|\phi_a\rangle}{E_d-E_a}\right]$$

$$-\sum_{c,d}\mathcal{P}\left[\frac{\langle\phi_b|H_I|\hat{\psi}_{2'c}\rangle\left(\frac{1}{\hat{N}_{2'2'}}\right)_{cd}\langle\hat{\psi}_{2'd}|H_I|\phi_a\rangle}{E_d-E_a}\right]$$

$$+\sum_{c,d}\langle\phi_b|H_I|\tilde{\psi}_{2c}\rangle\left(\frac{1}{\tilde{N}_{22}}\right)_{cd}\left[i\pi\langle\tilde{\psi}_{2d}|H_I|\phi_a\rangle - \sum_g (N_{12}^{+})_{dg}\left(\frac{1}{N_{11}}^{+}\right)_{ga}\right]$$

$$-\sum_{c,d,f}\langle\phi_b|H_I|\hat{\psi}_{2'c}\rangle\left(\frac{1}{\hat{N}_{22}}\right)_{cd}K_{df}\left[\delta_{fa} + i\pi\langle\hat{\psi}_{1f}|H_I|\phi_a\rangle\right] \tag{A.16}$$

as the modified Low equation. The second term comes only from the physical states and is all that survives (except for the potential matrix element) when there are no shadow states. Using the analytic results of section III, Eq. (A.14) can be rewritten in the simpler form

$$T_{ba} = -\langle \phi_b | H_I | \phi_a \rangle + \sum_c \frac{\langle \phi_b | H_I | \hat{\psi}_{1c} \rangle \langle \hat{\psi}_{1c} | H_I | \phi_a \rangle}{E_c - E_a - i\epsilon} + R_{ba}$$

where (A.17)

$$R_{ba} = \sum_c S_c(E_a, E_c)\, \mathcal{P}\left(\frac{1}{E_c - E_a}\right)\Big[\langle \phi_b | H_I | \tilde{\psi}'_{2c} \rangle \langle \tilde{\psi}'_{2c} | H_I | \phi_a \rangle$$

$$- \langle \phi_b | H_I | \hat{\psi}'_{2'c} \rangle \langle \hat{\psi}'_{2'c} | H_I | \phi_a \rangle \Big]$$

(A.18)

with $|\tilde{\psi}'_{2c}\rangle \equiv \sum_d \left(\frac{1}{N_{22}}\right)^{1/2}_{da} |\tilde{\psi}_{2d}\rangle$ and

$|\hat{\psi}'_{2'b}\rangle \equiv \sum_a \left(\frac{1}{\hat{N}_{2'2'}}\right)^{1/2}_{ab} |\hat{\psi}_{2'a}\rangle$ the properly orthonormalized statevectors. The weight function, $S_c(E,E')$, determined by the previous analytic arguments, has the values $S_c(E,E') = 1$, for $E < E^s_{th,c} < E'$; -1, for $E^p_{th,c} < E' < E^s_{th,c} < E$; and is zero otherwise, where $E^p_{th,c}$ is the "c" physical channel threshold and $E^s_{th,c}$ the "c" shadow channel threshold. The important point to notice in Eqs. (A.17) and (A.18) is that H_I acting on the unperturbed eigenstate $|\phi_b\rangle$ connects with the shadow state $|\tilde{\psi}'_{2c}\rangle$ and the shadow-ghost state $|\hat{\psi}'_{2'c}\rangle$. Hence, when generalized to a static field-theoretic framework having interaction terms of the common source type

$$\mathcal{H}_I(x) = J_{(b)}(x)\,[\,\phi_{(b)}(x) + \phi'_{(b)}(x)]$$

where the $\phi_{(b)}(x)$ ($\phi'_{(b)}(x)$) field creates and annihilates physical (shadow) quanta, the currents $J_{(b)}(x)$ <u>also</u> couple to shadow states $|\tilde{\psi}'_{2c}\rangle$ and $|\hat{\psi}'_{2'c}\rangle$.

Therefore, the presence of the mixed boundary conditions is easily incorporated into the formal perturbative expansions involving the propagation functions for eigenstates of the unperturbed Hamiltonian. This is the case in Section II, in Eq. (2.4), by the change Eq. (2.5) giving the perturbative solution for T, and in this appendix, in Eqs. (A.7) and (A.8), giving the perturbative solution for the physical states $|\tilde{\psi}_{1a}\rangle$. However, in the presence of mixed boundary conditions the usual relation

$$G^{+} = G_0 + G_0 H_I G^{+} \tag{A.19}$$

between the full propagation function and the free one, given by Eq. (2.5), no longer holds. If all the boundary conditions were standing wave, the analogous equation is found simply by taking $G^P = \frac{1}{2}(G^+ + G^-)$ and has the simple form

$$G^P = G_0^P + G_0^P H_I G^P + G_0^S H_I G^S \tag{A.20}$$

where $G^S = \frac{1}{2}(G^+ - G^-)$,.... When the boundary conditions are mixed, though, the above trick does not work so we have used the approach of this appendix and the piecewise analytic procedure of Section III to obtain the modified Low equation.

Because of the mixed boundary conditions, other equations involving full propagation functions in formal scattering theory, and their analogues in quantum field theory no longer hold. Two examples are (i) $T \neq H_I + H_I G^+ H_I$ with

$$G^{+} = \frac{\Pi^P}{E - H + i\epsilon} + \mathcal{P}\frac{\Pi^S}{E - H} \tag{A.21}$$

and (ii) $|\tilde{\psi}_{1a}\rangle \neq \Pi^P|\ _a\rangle + G^+|\tilde{\psi}_{1a}\rangle$ which should be compared with Eq. (A.4).

Notice, however, that (i) will hold if the $S_c(E,E')$ weight function of Eq. (A.18) is "included" in the π^s of second term of Eq. (A.21). The effect of $S_c(E,E')$ is to <u>restrict</u> the spectral support of the shadow term in the <u>full</u> propagation function. We conjecture that this restriction on the shadow support generalizes to, for example, the renormalized in-fields of quantum field theories with shadow states.

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22. For explicit wave functions for the shadow states see Ref. 11 and the appendix of this paper.

23. The normalization factors, see Ref. 11 and the appendix, are crucial to construction in a theory with shadow states of the generalized Moller matrix which is related to the U(t',t) operator, $\Omega^{(\pm)} = U(0,\mp)$, that describe the time evolution of the system and connects the interacting Heisenberg fields to the in or out fields of the shadow theory. (See e.g.; ref. 21).

24. Compare Eq. (2.6)

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25. For shadow particles, different from "heavy" regulator particles, one should use the dispersion relation for the corresponding t_i. The same proceedure, used in Sections III and IV, should then be followed

26. Notice that the subtraction constant drops out in Eq. (5.3) when the Bjorken limit is taken.

27. The difference between the subtraction constants does not lead to a jump discontinuity at the shadow pseudo-threshold. While amplitude can have cusps, see reference 11, it is continuous and, as a consequence of threshold theorems, for other than s wave scattering these cusps are smoothed out (as they are if the shadow particle is unstable).

28. The similar Wigner cusps from the opening up of physical channels are difficult to observe. For a few examples of possible cusp behavior from physical thresholds see R. Dalitz, Proc. Roy. Soc. London A318, 279(1970).

29. In the simple static theory of low energy pion-nucleon scattering (See ref 11), the induced cusps from the shadow pseudo threshold were compatible with present limits on the s wave pion-nucleon scattering total cross sections.

30. Since the shadow boundary condition only effects the continuum states, the single particles states, such as $|\mathcal{P}\rangle$ and $|P\rangle$, are the same for $\mathcal{T}$ and T.

31. If we consider only the singular part of the physical state, the $|a;out\rangle = |a;free\rangle + (G_o^P + G_o^S)H_I|a;\ out\rangle$ where $G_o^P + G_o^S$ is given by Eq. (2.5). Then $|a;out\rangle = |a;free\rangle + (G_o^P + G_o^S)\ T|a;free\rangle$; hence, $\langle b\nu;free|\eta H_I\ |a;out\rangle = \langle b\nu,free\ |\eta T|a;free\rangle$.

32. The usual current algebra commutation relations will hold when the coupling is of the "common source" type, $H_I(\chi) = J(\chi)[\phi(\chi) + \phi'(\chi)]$ where the $\phi(\chi)$ ($\phi'(\chi)$) field creates and annihilates physical (shadow) quanta.

33. For $R = \frac{\sigma_L}{\sigma_T}$ held constant, fitting gives $R = 0.18 \pm 0.10$; also reported compatible are $R = 0.031$ (q^2/M^2), q^2/ν^2. See H. W. Kendall in "Proceedings 1971 International conference on Electron and Photon Interactions at High Energies", Cornell University, 1971.

34. R. H. Dalitz, in "proceedings of the Thirteenth International Conference in High Energy Physics", Berkeley, 1966 (U. of Calif. Press, Berkeley, 1967); H. Harari, in "Proceedings of the Fourteenth International Conference on High Energy Physics", Vienna, 1968. O. W. Greenberg, in "Proceedings of Fifth International Conference on Elementary Particles", Lund, Sweeden, 1969; J. J. Kokkeedee, "The Quark Model", (W. A. Benjamin, Inc., New York 1969), and references therein.

35. W. E. Thirring, Phys. Letters 16, 335(1965).

36. O. W. Greenberg and M. Resnikoff, Phys. Rev. 163, 1844(1967).

37. L. Van Hove, in "Particle Physics at High Energy", edited by T. W. Priest and L. L. J. Vick (Oliver & Boyd, Edinburgh, 1967). C. A. Nelson, Phys. Rev. D2, 2669(1970) and references therein.

38. C. H. Llewellyn Smith, report to Fourth International Conference on High Energy Collisions, Oxford, England (1972); SLAC - PUB - 1039 (1972).

39. Kuti and Weisskopf, Phys. Rev. D4, 3418(1971).

40. This could be shown, for example, by using the summation equation (Eq. 2.7) , rearranging it, and making use of the transition amplitude of the usual quark model, i.e. Eq. (6.4) and the amplitudes involving free quarks.

41. See also K. Johnson, to be published, Phys. Rev. (1972).

42. Since only a finite number of Fermi particles of a fixed energy can be restricted to a finite "box" of size R, there is, in this case a natural origin of saturation in this framework. This could explain the saturation at 3Q but not why 2Q structures are not seen.

43. See, for example, R. G. Newton, "Scattering Theory of Waves and Particles", (McGraw-Hill Book Co., New York, 1966) and references therein.

44. This is an "in state" because the physical component of $|\widetilde{\psi}_{1a}\rangle$ is a plane plus outgoing traveling wave.

Figure Captions

1. Kinematics for single photon exchange for deep inelastic electron-neutron scattering, $e^-(E) + N(P) \to e^-(E',\theta)$ + anything (P'), where with M the nucleon rest mass, $\nu = q\cdot P/M$ and $\omega = -q^2/M\nu$.

2. S matrix diagrams in a shadow-quark model for the forbidden transitions (a) $N \not\to 3Q\pi$ and (b) $\pi N \not\to 3Q\,\pi\pi$. The shadow-quarks remain always bound because they are standing wave structures in the baryon, N.

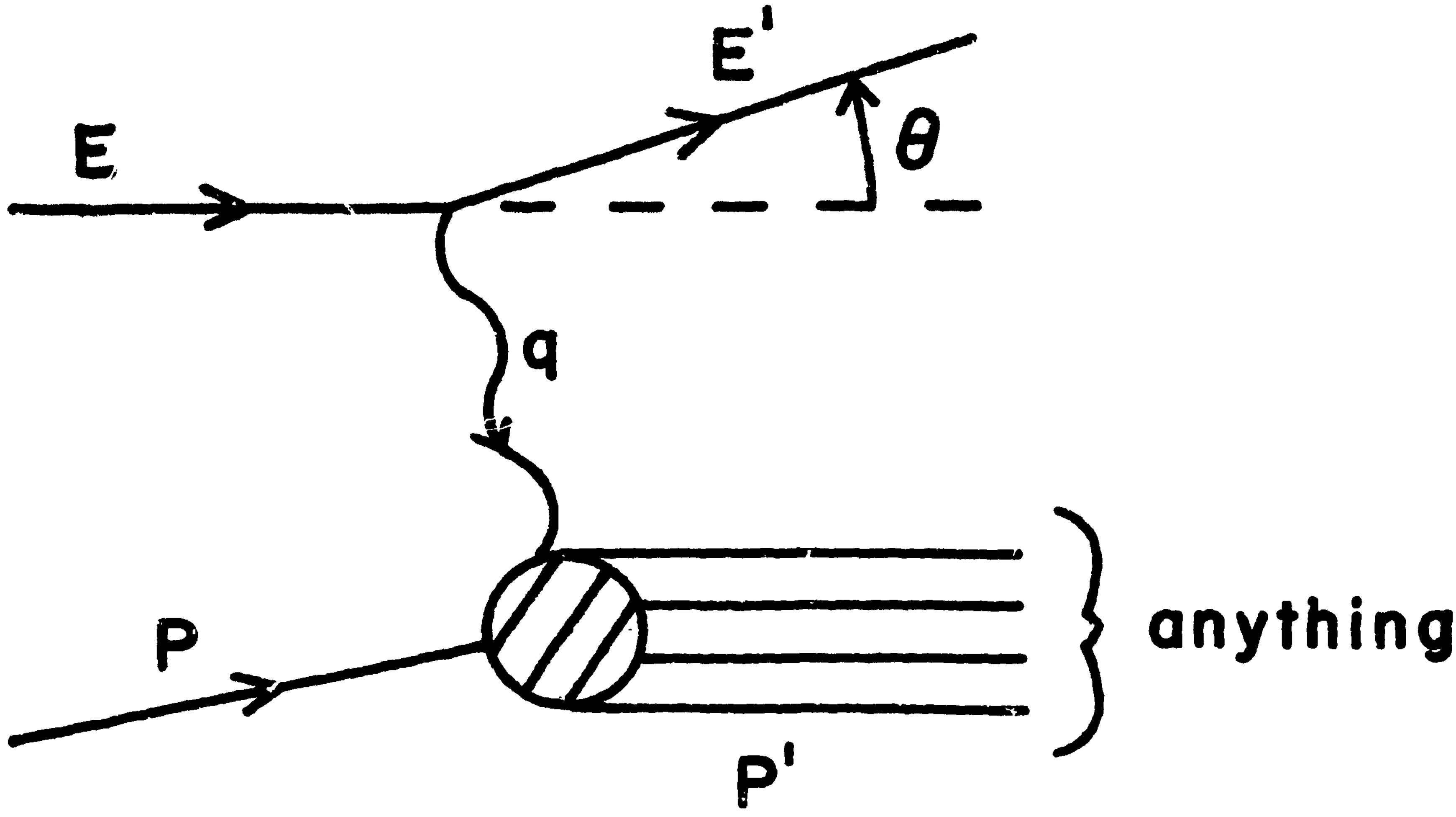

Fig. 1

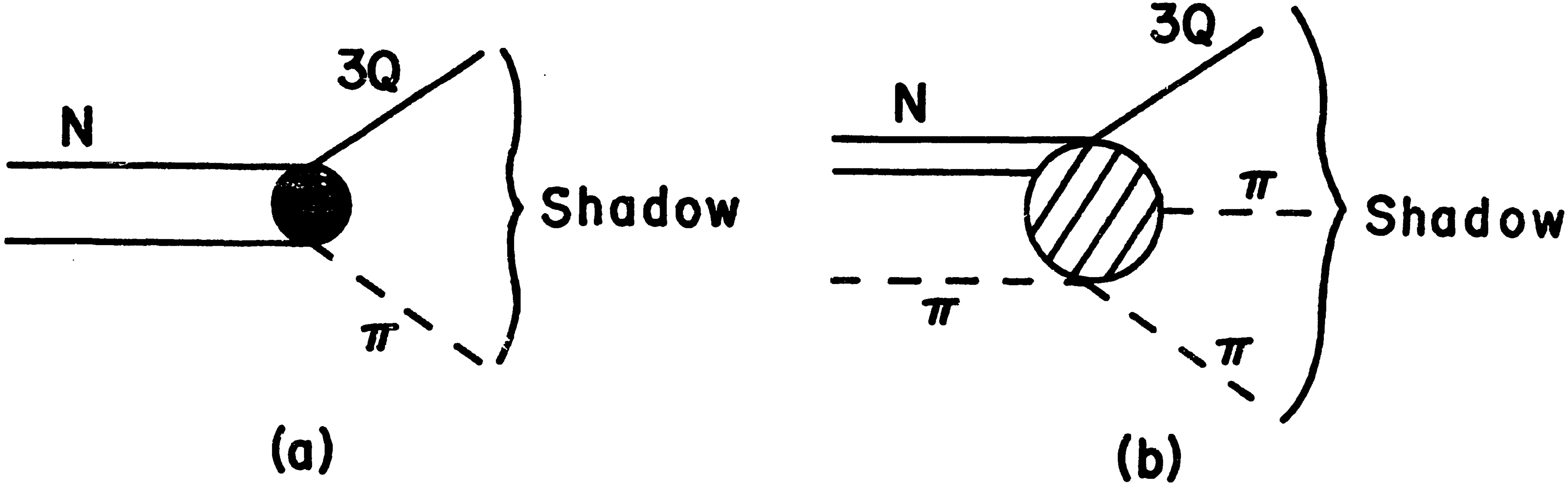

Fig. 2