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Luminosity Measurements for Colliding Beams

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LUMINOSITY MEASUREMENTS FOR COLLIDING BEAMS

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Many experiments with colliding beam facilities require rather precise measurements of the luminosity. These experiments are of two general types, those which measure absolute reaction rates (e.g. total cross section) and those which require relative luminosity measurements in order to normalize a series of runs (e.g. studies of s dependence in a particular channel which requires runs at a variety of machine energies). The problem of luminosity measurement is a serious one in that such a measurement may be the limiting factor for a given experiment.

The following is a representative set of experiments which could, in principle, be limited in their physics output by inaccuracies in the luminosity determination.

a) Total cross section

b) S dependence of total cross section

c) Scaling in N particle inclusive reactions (hadrons, $\ell^+\ell^-$, $\ell\nu$)

d) S dependence of elastic peak and structure in $d\sigma/dt$

In the case of σ_T, for example, there are many models which predict s dependance at asymptotic energies.

a) Simple Regge Pole Models with the Pomeron dominating predict a constant σ_T = 38.8 mb.

b) Regge Pole + Cut models predict a slowly rising σ_T which reaches 42 mb a 80 Tev lab equivalent.

c) Complex Regge Pole models predict

$$\sigma_T = 38.8 - 4.3 \cos(0.7 \ln s + 2) = 36 \text{ mb at } 80 \text{ Tev}$$

d) Field Theoretical Models which saturate the Froissard bound predict $\sigma_T = 38.8 + c\ (\ln s/s_o)^2$. A recent analysis of cosmic ray data by Yodh et al fits a $(\ln s)^2$ rise with C=0.4. This gives σ_T = 52 mb at 80 Tev.

Thus we see that in the case of the S dependance of σ_T we need a normalization which is accurate to at least 2 or 3% in order to differentiate between these models to three standard deviations (neglecting errors in the determination of the total interaction rate). A measurement of the luminosity to 1% represents a useful goal.

There are two approaches to this problem which we will consider: measurements of the beam characteristics and measurements of an interaction channel whose absolute cross section can be calculated in a believable way. In the first case one may require a perturbation in the beam orbits and so a continuous luminosity measurement is difficult, though not impossible. In the second case one need not acquire data in the normalization channel concurrently with data from the reaction being studied though in some cases, at the cost of extra apparatus or nonideal beam operation, this may be desirable.

We first consider methods of luminosity measurement which rely on a normalization channel with a calculable cross section.

1) Measurement of the Coulomb Region in P-P Collisions:

As is well known, at sufficiently low values of t $(t < .002\ \mathrm{GeV/c})^2)$ the differential cross section for P-P is dominated by coulomb scattering

$$\left.\frac{d\sigma}{dt}\right|_{\mathrm{coulomb}} = \frac{2.6 \times 10^{-4}}{t^2} \quad \mathrm{mb/Gev}^2$$

where $t = E_1\ E_2\ (1-\cos\ \theta)$

If one could study the small t region with sufficient resolution, this calculable cross section can be used to calibrate the luminosity.

Unfortunately, as we will show, this idea imposes prohibitive constraints on the angular divergence of the colliding beams. Let us assume that one wishes a 1% measure of the luminosity.

$$\Delta\, d\sigma/dt/d\sigma/dt = 2\,\Delta\, t/t = 1\%$$

thus one must determine $\Delta\, t/t$ to 0.5%. From the relation between t, E_1, E_2, and θ

$$\Delta\, t/t = 2\left[\left(\frac{\Delta E}{E}\right)^2 + \left(\frac{\Delta \theta}{\theta}\right)^2\right]^{1/2}$$

where we have assumed $E_1=E_2$. The design machine energy spread ($\Delta E/E = \pm .7 \times 10^{-3}$) and our limits on $\Delta\, t/t$ determine the upper limit for $\Delta\theta/\theta$, the beam divergence

$$\frac{\Delta\theta}{\theta} = \left[\frac{1}{4}\left(\frac{\Delta t}{t}\right)^2 + \left(\frac{\Delta E}{E}\right)^2\right]^{1/2} = 2.5 \times 10^{-3}$$

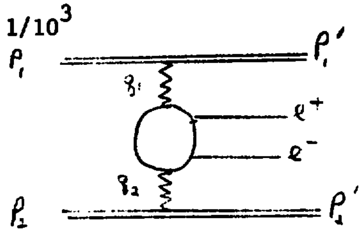
$E_1=E_2=200$ Gev and $t = .001\ (\mathrm{gev}/c)^2$, for example, correspond to $\theta = 0.15$ mr. Thus the largest tolerable beam divergence is $\Delta\theta < .4\ \mu$r. This requirement is prohibitive, in fact, a great deal of beam gyrations are required to limit the divergence to the 25 μr required for the study of the low t nuclear elastic scattering at ISABELLE.

2) Coulomb Dissociation of $N^*(1236)$

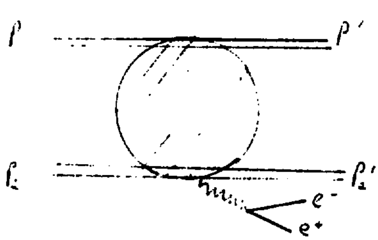
This scheme, proposed by Gobbi and Rosen, is discussed in detail in another Crisp note. Here we only summarize the pertinent features of their idea. In the reaction $p + p \to p + \Delta \to N\,\pi^+$ where the Δ is produced in the coulomb field of one of the protons, the momentum impulse required is $q_L = \frac{M_\Delta^2 - M_p^2}{2\,p} = 3$ Kev/c. In terms of an impact parameter $b = \hbar/q_L \sim 0.7$A° so we see that the interaction occurs so far from the proton that we may regard it as a point particle and neglect both the strong interactions and the contribution of the proton magnetic moment to the electromagnetic interaction. Since the coulomb photons are quite near the mass shell, the cross section for the coulomb production of deltas can be determined by measuring low energy delta photoproduction. Gobbi and Rosen estimate a cross section for coulomb production of the Δ(1236) of about 0.1 μb which gives 100 events/sec for a luminosity of 10^{31} $cm^{-2}sec^{-1}$. This reaction produces a forward going neutron of average energy 150 Gev and a π^+ at about 4 mr with 40 Gev of energy. They propose to measure the π^+ with a classic spectrometer and the neutron direction and energy in a calorimeter with a matrix of detection elements. This method is limited by the accuracy of the low energy photoproduction experiment and by background. It is proposed to suppress the 40 mb of strong interaction backgrounds with veto counters and kinematic constraints. In this sample there is 100 μb of $N\pi^+$ diffractive production by Pomeron exchange. This background is eliminated by kinematics. Exchanges of trajectories other than the Pomeron are expected to be greatly suppressed at ISABELLE energies.

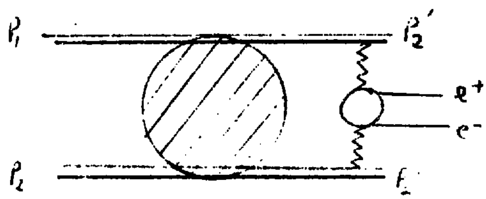
3) <u>Electron Pair Production in P-P Collisions</u>

Budnev et al have suggested the use of the reaction $P+P \rightarrow P+P+e^+ + e^-$ in the appropriate kinematic region in order to measure luminosity. In the region of small lepton four-momentum the protons in the final state do not leave the beam. Here, the cross section is dominated by diagrams of the sort studied by Brodsky and which can be calculated from pure QED to accuracies of $1/10^3$



Diagrams involving strong vertices contribute only at the level of $1/10^4$





Radiative corrections, which depend on the details of the method of detection, can be ignored at the 1% level. At the level of 0.1% they must only be calculated to 10%.

The kinematic region of interest is the region of proton scattering angles $< \frac{Me}{E} \sim 2\ \mu r$. For this region the cross section for $p+p \rightarrow p+p\ e^{+}+e^{-}$ from QED is calculated to be

$$\sigma\ (\delta) = \frac{28}{27\pi}\ \frac{\alpha^4}{M_e^{\ 2}}\ \left[\left(\ln \frac{4E\delta}{M_p^{\ 2}} - 1\right)^3 + \frac{3}{2}\ \ln \frac{4E^2\delta}{M_p^{\ 2}} - 2\right]$$

where δ is a parameter which defines the region of phase space to be considered:

$$\left| q^2_{\ell} \right| < \omega^2\delta \text{ where } \omega^2 = 4\ \omega_1\omega_2$$

$\omega_1\omega_2$ = energy loss by protons

For $\delta = .1$, for example, the cross section is $\sigma > 1$ mb. The kinematics of the electron pairs is as follows: if Σ is the sum of the electron energies

$$\frac{dN}{d\Sigma} \sim \frac{1}{\Sigma} \text{ if } \Sigma \le \frac{m_e}{M_p} E \qquad E - 200 \text{ Gev}$$

$$\frac{dN}{d\Sigma} \sim \frac{m_e}{M_p}\ \frac{E}{\Sigma^2} \text{ if } \Sigma \gg \frac{m_e}{M_p} E$$

The transverse momenta are both of order m_e and their sum is near zero, i.e. e^+ and e^- are emitted symmetrically around the beam. Thus we must detect electron pairs of typical momenta of 50 Mev and emission angles of 10 mr with respect to the beams. Since the cross section is >1 mb we can avoid being swamped by strong interaction background. In fact, such a luminosity measurement could be made with two small shower counters and vetos to eliminate events with large angle hadrons. One could work with luminosities of 10^{30} $cm^{-2}sec^{-1}$ and see 1000 events/sec and a background counting rate from hadrons of only 10^4 - 10^5/sec.

Budnev et al have also considered various contributions to the backgrounds. Lepton pair production on residual gas with charge Ze can be calculated by the substitution of 2E/M for $4E^2/M^2$ and a factor Z^2 in the formula for the beam-beam pair production cross section. For a vacuum of 10^{-10} torr, beam currents of 10 amps, a luminosity of 10^{33} $cm^{-2}sec^{-1}$ and Z=7 this background is 0.02%. Pair production in inelastic pp scattering can contribute a cross section 10% of the signal, however, the final state hadrons have a wide angular distribution ($< \theta > -$ 1mr $< p_\perp > \sim 300$ Mev/c) as compared to the final state protons in the signal reaction. One can kill this background by detecting lepton pairs in anticoincidence with hadrons. The contribution of $p + p \rightarrow \pi^o + \ldots\ldots$, $\pi^o \rightarrow \gamma\, e^+e^-$ is small in this kinematic region ($\sigma \sim \sigma_{pp}^{inel}\ K_\perp^2/m_\pi^2$). If however, it should be that the electron pair production in the appropriate kinematic region suffers from some residual background problem one can reduce the kinematic region. This does not substantially decrease the elastic pair production cross section and the accuracy. For example, decreasing δ by a factor of 3 decreases the cross section by only 25% while the background should scale down as δ.

We next consider methods which rely on a determination of beam characteristics:

4) <u>The Van der Meer Method</u>

For two beams of current I_1 and I_2 crossing at an average angle of θ, the luminosity can be written

$$L = \frac{I_1\ I_2}{e^2 c \tan(\theta/2)} \cdot \frac{1}{H}$$

If one measures the beam currents by either induction or by observing gas scattering at a point away from the interaction region, then a determination of H constitutes a measure of the luminosity. H is the vertical overlap integral of the two beams

$$\frac{1}{H} = \int \rho_A(X)\ \rho_B(X)\ dx \qquad \text{where} \int \rho_{Ab}(X)\ dx = 1$$

Van der Meer considers a relative measurement of the interaction rate as a function of δ, the vertical separation of the two beams

$$R(\delta) = c \cdot \int \rho_A(X)\, \rho_B(X + \delta)\, dx \qquad R(0) = C/H$$

To determine C we measure the area of the curve F (δ)

$$\int R(\delta)\, d\delta = c \int d\delta \int \rho_A(X)\, \rho_B(X + \delta)\, dx = C \cdot \int dX\, \rho_A(X) \int dX\, \rho_B(X + \delta)$$

since $\rho_A(X)$, $\rho_B(X)$ are normalized to unit area $\int R(\delta)\, d\delta = C$. Thus

$$1/H = R(0) / \int R(\delta)\, d\delta$$

This calculation assumes the independence of beam shape and detection efficiency on the displacement δ. The shape changes only if the betatron function is locally modified by a beam displacement. The Van der Meer method has been used at the ISR by Rubbia et al who quote a 5% error in their measurements of elastic pp scattering due to uncertainty in the luminosity. This method may be pushed to the level of 1% accuracy but one must be careful to worry about changes in the beam characteristics (shape, emittance) during the displacement. In addition it is inconvenient to measure the luminosity continually in this way because of the duty cycle factor introduced.

5) Diffuse Beam Method

Y.Y. Lee has suggested a method for determining luminosities which is dependent on the ability of the machine to generate one beam which has a large cross section compared to the other beam and a uniform particle density. We discuss this method for the case of 0^o crossing even though a varient of the method can be used for a finite angle crossing region. The specific luminosity for a 0^o crossing section is

$$\frac{dL}{dZ} = \frac{2I_1\, I_2}{e^2\, c} \quad \frac{1}{A_{eff}}$$

where I_1 and I_2 are the beam currents

$1/A_{eff}$ is the normalized two dimensional density overlap integral.

$$\frac{1}{A_{eff}} = \int \rho_1 (X, Y) \rho_2 (X, Y) dx\, dy \int \rho_{12} (X,Y) dx\, dy = 1$$

If beam #1 is blown up so that its cross section is much larger than beam #2 and if its density is uniform over the region of overlap between the two beams, then

$$\frac{1}{A_{eff}} = \rho_1 \int \rho_2 (x,y) dx\, dy = \rho_1$$

and so the luminosity becomes

$$L = \frac{2 I_1 I_2 \ell_o}{e^2 c \rho_1}$$

where ℓ_o is the length of the interaction region.

One must measure only the particle density in the diffuse beam. This density can be measured by plotting the points of origin for secondaries for beam-gas collisions with beam #2 turned off, or alternatively, the density of beam #1 can be probed with beam #2 in a way analogous to the Van der Meer method. This technique for luminosity measurement then depends very strongly on the diffuse beam having a highly uniform density throughout the interaction region. Unfortunately one doesn't know precisely where beam #2 strikes beam #1. The density must be uniform over the region of uncertainty. In addition, the β function for the pencil beam will vary over the length of the interaction region

$$\beta (Z) = \beta (0) + \frac{Z^2}{\beta(0)}$$

so that the size of the pencil beam will vary as

$$r (Z) \alpha \left[1 + \left(\frac{Z}{\beta(0)} \right)^2 \right]^{1/2}$$

For the high luminosity region $|Z| \leq 3m$, $\beta (0) \sim 1m$, so that the size of the pencil beam varies by a factor of 3. This puts a tighter constraint on the region of nonuniform particle density in the diffuse beam. For the high luminosity region, for example, the beam half height is expected to be 0.7 mm so that one needs the diffuse beam to be uniform, say to 1%, over a radius of 5mm. The success of this technique for luminosity measurement depends on yet unproven capabilities for the control of beam characteristics, especially over the whole length of the interaction region. There is, in addition, the serious problem of end effects in that the two beams do not always cross at 0^o - they must separate at some point. In this region of separation the pencil beam will probe the region of the diffuse beam where the density of particles is rapidly changing. Such end effects could contribute errors or at least several per cent if the beam density is gaussian (as observed at the ISR). Only if the density of the diffuse beam is relatively flat will the end effects be negligible.

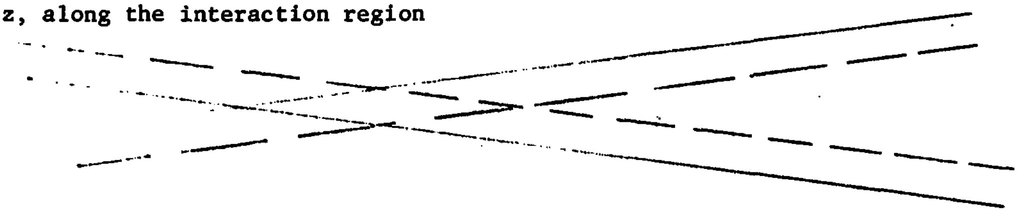
3) <u>Attenuation Method for Relative Luminosity Measurements</u>

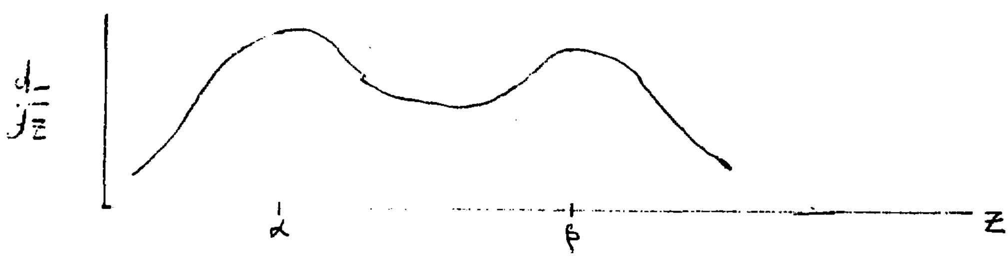
Sandweiss has suggested a scheme for measuring the total interaction rate by observing the attenuation in one of the proton beams. This is usually difficult because the attenuation is $1/10^6$ per second and because it is difficult to separate beam-beam from beam-ga and beam-wall loss. If, however, one were to sinusoidally sweep one beam across the other, the attenuation due to beam-beam collisions will also be sinusoidal with a known frequency. This signal can then be easily extracted with phase sampling techniques. Although this approach is useful for monitoring the luminosity or for normalizing a series of runs, it doesn't measure absolute luminosity. In fact, this method together with an independent measure of the luminosity may be the best technique for measuring total cross sections at colliding beam facilities. A detailed discussion of the method will be presented in another Crisp note in which just such an experiment to measure σ_T will be proposed.

4) <u>Luminosity Measurement for Collinear Regions</u>

Palmer and Marx have devised a varient of the method of Y.Y. Lee discussed above which does away with its most serious problems, i.e. end affects and varying beam size. This technique as described below requires the measurement of two distributions in order to determine the luminosity. Each of these distributions can, in principle, be determined in two independent ways thus providing a useful check on the accuracy and reliability of the technique.

Consider two beams crossing at 0°, each focused at a different point, z, along the interaction region





The luminosity per unit length has the two humped distributions shown above. The peaks in luminosity occur at points $z = \alpha$ and $z = \beta$ where one beam is very small and the other is relatively large. If one knows the shape of the curve dL/dz and the absolute specific luminosity at any point z_o, then one simply obtains the total luminosity by intergrating dL/dz and using the specific luminosity at z_o as a normalization. At the points $z = \alpha$ and $z = \beta$ we use this method of Y.Y. Lee to calculate the specific luminosity

$$\frac{dL}{dz} = \frac{2\, I_1\, I_2}{e^2 c\, A_{eff}} = \frac{2\, I_1\, I_2}{e^2 c} \int \rho_1\,(x,y,z)\;\rho_2\,(x,y,z)\;d\,x\;dy$$

where ρ_j (x,y,z) is the two dimensional density of the beam j as a function of z, $\int \rho_1 \rho_2 \, d\,x\,d\,y$ is the normalized area density overlap integral (the inverse of the effective overlap area of the beams). The normalization is such that $\int \rho_j \, (x,y,z,) \, d\,x\,dy = 1$

At the points z = α, β where one beam is much larger than the other the density of the diffuse beam is made to be uniform over the area intercepted by the pencil beam. Then, if this assumption holds

$$dL/dz_{\;z = \alpha,\beta} = \frac{2\,I_1\,I_2}{e^2\,c}\;\rho_1\;(z = \alpha \text{ or } \beta)$$

Formally this is equivalent to Lee's method but here we only require the uniformity conditions at one value of z. There are no serious inaccuracies due to changing beam size with z and to end effects as discussed above. We thus see the first required measurement to be made - we must know the density of the diffuse beam in the region of intersection with the pencil beam and we must demonstrate that the density is uniform over this region. The diffuse beam density can be measured in two independent ways (and at two points z = α and z = β). First, one can observe secondary interaction products from beam-gas scattering (with the pencil beam off). If one then projects these observed secondaries back to their origin one generates a profile of the diffuse beam. Given a set of chambers surrounding the region at z = α or z = β with sufficient lever arm one can measure the density with adequate spacial resolution. One can also use the pencil beam to probe the diffuse beam and so to study the relative interaction rate as a function of the position of overlap of the beams. This method had a resolution of the order of the size of the pencil beam cross section which is sufficient. I emphasize that 4 independent measurements of the diffuse beam density (2 ways for each beam) can be made and so produce consistency checks on this input to the luminosity measurement.

Once the diffuse beam density has been determined, the distribution of specific luminosity in z must be measured or calculated. In principle from the beam optics one knows the form of the β functions as a function of z. The β function determines the beam size and so one can calculate the shape of the two dimensional overlap of the beams and thus the shape of the specific luminosity curve in z. This calculation is uncertain due to many assumptions - among them the assumption of a constant beam density distribution along the interaction region and the assumption of a good knowledge of beam positions so that end effects can be included. This calculation should thus be viewed only as a check on the measurement of the shape of dL/dz which is done as follows: If several counter telescopes at 90^{o} to the interaction region were constructed with relatively small solid angle acceptance, each at the same z but different azmuthal angles, one could measure the relative luminosity by observing coincidence rates between these telescopes. The telescopes would be moved along the interaction region length in order to map the relative luminosity as a function of z (i.e. dL/dz).

The errors in this technique for luminosity measurement come from two sources. The first is the uncertainty in the density distribution of the diffuse beam in the region intercepted by the pencil beam. Since this region is of the order of 1 mm^2 in cross section, it should be possible to set up the diffuse beam with sufficiently uniform characteristics and even if this cannot be done, one can measure this density distribution at two points in z in two different ways. The second source of error is in the shape of dL/dz. Since we only rely on relative measurements many systematic problems are not important. By using several telescopes in coincidence we average over uncertainties in solid angle and we eliminate accidental problems (which can also be eliminated by running at lower beam currents). If we aim for a 1% luminosity measurement the difficulty is not in the shape of dL/dz, where we need less accuracy than 1% at each z since we integrate over the full range of z. End effects are minimal since both beams are diffuse and diverging with the result that the luminosity decreases rapidly at the ends of the interaction region. These ends can be made to contribute less than a 1% uncertainty to the total luminosity. The real limit is in the measure of the normalization (specific luminosity at $z = \alpha$ or $z = \beta$). This measurement can probably be made to better than the required 1% with sufficient care in beam preparation and in density measurement.

Conclusion:

In this report we have discussed several suggestions for the measurement of the luminosity of colliding beams. Certainly, in time, better methods will be suggested. The point that must be emphasized, however, is that some of these methods could give accuracies to the 1% level or better under carefully controlled circumstances. This is in no way a drawback. One can measure absolute luminosities under such controlled circumstances in order to calibrate one or more 90° telescopes. These telescopes would then provide the luminosity monitor in that interaction region when the physics program proceeds. During this time the method suggested by Sandweiss for measuring beam attenuation with a detector far from the interaction region could be used to periodically monitor the relative luminosity and thus the stability of the calibrated 90° telescopes. The advantage of this approach is that an extensive effort with optimal equipment could be mounted to measure the absolute luminosity at convenient beam currents and for all beam energies of interest. In fact, one could employ several of the methods suggested above to serve as a cross check. One could then have 90° telescopes calibrated for all beam currents. These telescopes could then be used for all experiments (some experiments may calibrate a still more convenient monitor with respect to the 90° telescope).