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# PERTURBED ORBIT THEORY AND NUMBERICAL SIMULATIONS OF PARAMETRIC HEATING

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**Perturbed Orbit Theory and Numerical Simulations of Parametric Heating***

by

J. DeGroot, J. Katz, J. Weinstock, R. Faehl, and W. Kruer

Numerical simulations of the anomalous plasma heating caused by an electric field oscillating at the plasma frequency were discussed in the last paper. We are mainly interested in the case $\eta \lesssim 1$ ($\eta = E_0/\sqrt{4\pi n k T_{e0}}$) for laser fusion. It was shown that for this case the electron heating results in the production of extensive supra-thermal tails. In this paper, I shall show that the dominant part of this heating is due to diffusion in velocity space.

(slide 1) In these weak cases we notice very little coherent acceleration (trapping). This is shown in Slide 1, which is phase space for a weak case ($\eta$ = .36, $v_0 = v_{pe}$) somewhat after saturation. By examining phase space as a function of time it is clear that the motion of the particles is diffusive. There is some trapping orbits at high phase velocity, but the main heating process is random. This suggests using the Dupree-Weinstock perturbed orbit theory to calculate the heating.

(slide 2) To verify the diffusive heating process we have followed the motion of test particles in the electric fields that exist in the plasma after the instability has saturated. We store the electric fields, then follow 100 test particles through these fields and calculate ensemble averages of the mean and mean square velocity. These 100 particles have the same initial velocity and their initial positions are chosen randomly. A typical result for the time evolution of $[(v(t) - v_0)^2]$ is shown in slide 2, during a time shortly after the wave energy has saturated. The time interval must be short enough so that the plasma temperature is essentially stationary, but long enough so diffusion is the correct picture of the motion. We see that $\langle \Delta v^2 \rangle$ follows a straight line, ie the motion is diffusive. We can define the diffusion coefficient: $D = \frac{\langle \Delta v^2 \rangle}{t}$ .

(slide 3) The diffusion coefficient derived in this way is shown in slide 3. We see that D(v) is large over a large range in velocity. Also, D(v) is small at low velocities since the linearly unstable waves have very high-phase velocities. One result from this slide is that above about $2v_{th_0}$ , D is linear with v until about $5v_{th_0}$ , ie D = a(v).

(slide 4) Since we calculate $E_{k,\omega}$ in the simulation code, we can calculate the diffusion coefficient directly from the Dupree-Weinstock perturbed orbit theory. The equation for D (v) is shown in slide 4. This is a non linear equation for D(v).

(slide 5) To use this theory we need the $|E_{k,\omega}|$ for each mode. A typical short wavelength mode ( $k\lambda_{De_0}$=.3 ) is shown. We see that the width of the mode is very wide so that we have $\omega_B \tilde{\tau}_E \ll 1$ , where $w_b$ is the bounce frequency. $\tilde{\tau}_E$ is the correlation time of the electric field. Thus, perturbed orbit theory should be applicable.

(slide 6) The ( $E_{k,\omega}$) for a very long wave length mode ( $k\lambda_{De_0}$ = .06) is shown in slide 6. Here the width is only $\tilde{\tau}_E$=.1 and thus $\omega_B \tilde{\tau}_E \gtrsim 1$. Thus these long wavelength modes can trap some particles. However most of the heating occurs at lower velocities where diffusion theory should be applicable.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

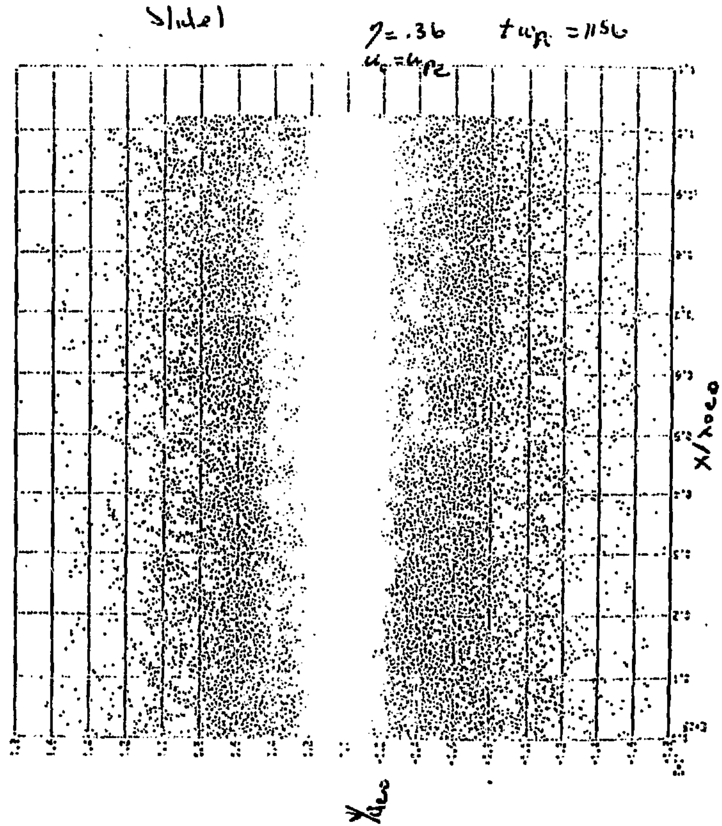
(slide 7) The electric field averaged over one plasma period is shown in slide 7. The linearly unstable modes occur at long wavelengths. We see that mode coupling has produced a broad spectrum of modes.

(slide 8) Using these electric fields, we have solved the nonlinear equation for D(v) and this is shown in slide 8. We see quite a lot of structure, part of which is dependent on the time interval over which the ($E_{k,\omega}$) was taken from. Thus we should average over a longer period to find better D(v). However, this longer time scale is close to the heating time so the plasma conditions are changing.

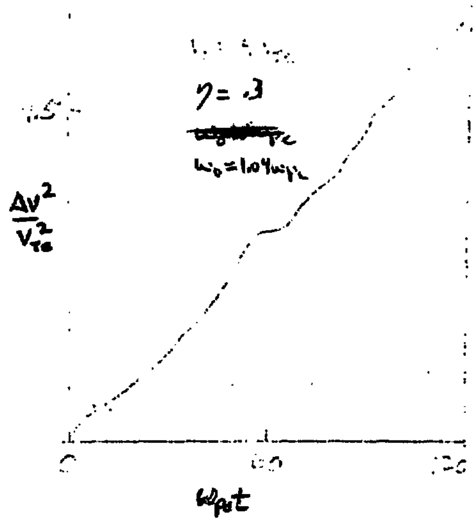
(slide 9) The diffusion equation was solved with the D(v) from slide 8. The resulting electron distribution function is shown in slide 9 at a time of $t\omega_{pe} = 1065$ compared with the spacially averaged distribution from the simulation. We see that over the range $2V_{the} \lesssim V \lesssim 10 V_{the}$ the curves are in very good agreement. The heating rate from the diffusion solution within 10% of the simulations result. There are two clear discrepencies in the comparison:

(1) Diffusion theory predicts too much diffusion at low velocities, and (2) Diffusion theory predicts too little diffusion at high velocities.

Recently, Jeff Thompson has shown that the D(v) we gave above is not correct because in the derivation we neglected the driving field. He finds that when the driving field is included there should be less diffusion at low velocities and more at high velocities and little change in between. We are now performing calculations with this D(v).



Slide 2

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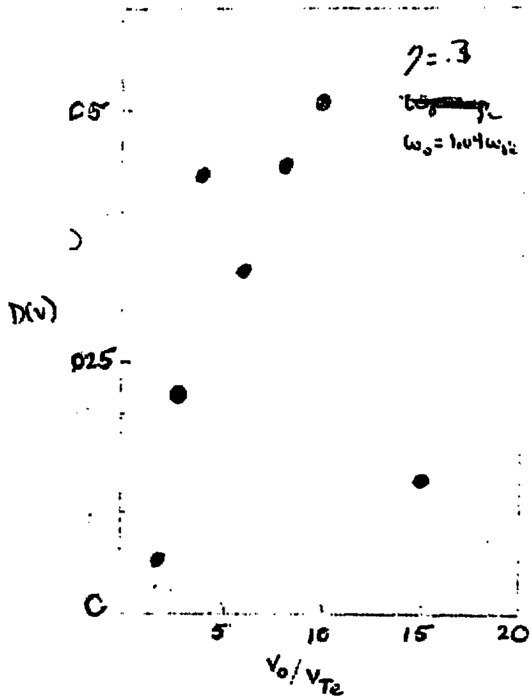
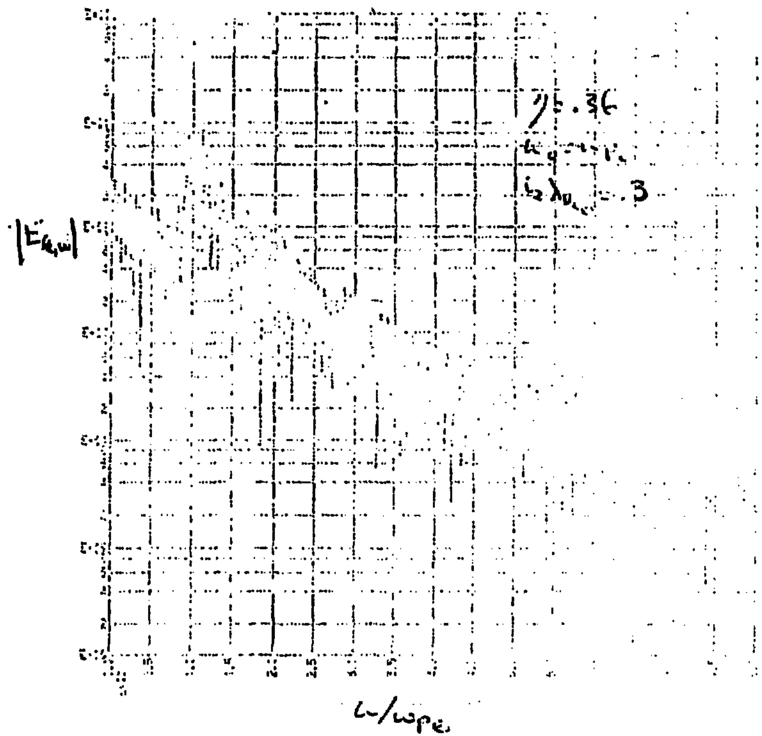


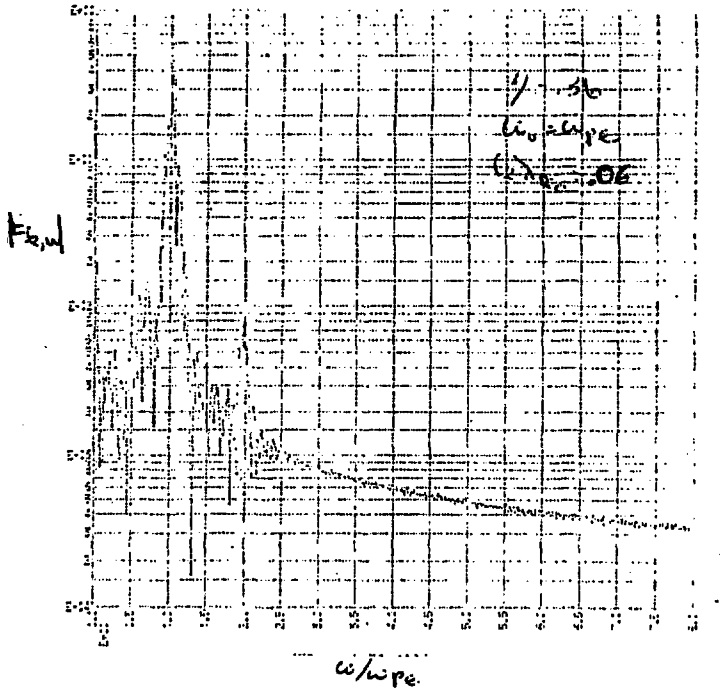
Slide 4

$$D(v) = \frac{e^2}{m^2} \sum_k \sum_\omega |E_{k,\omega}|^2 \frac{\sqrt{\pi}}{\sqrt{2}\,(k^2 D)^{1/2}} \, e^{-\frac{(\omega - k v)^2}{(k^2 D)^{1/2}}}$$

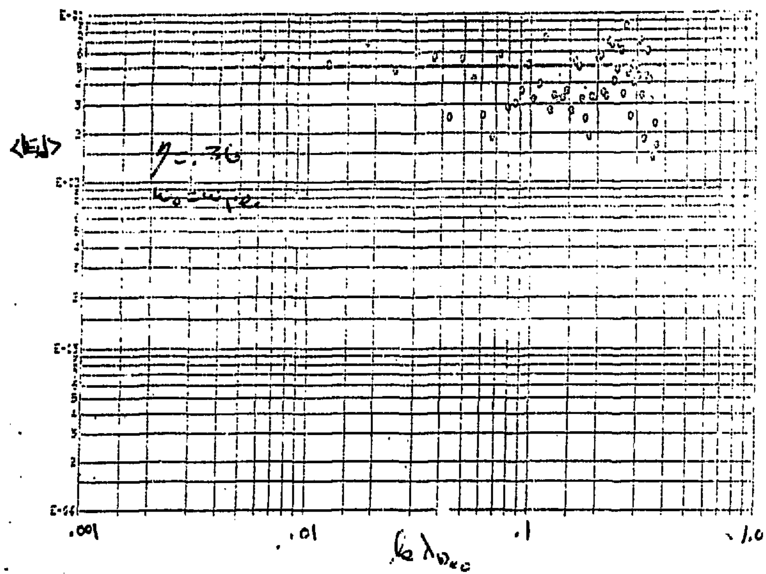
Slide 5



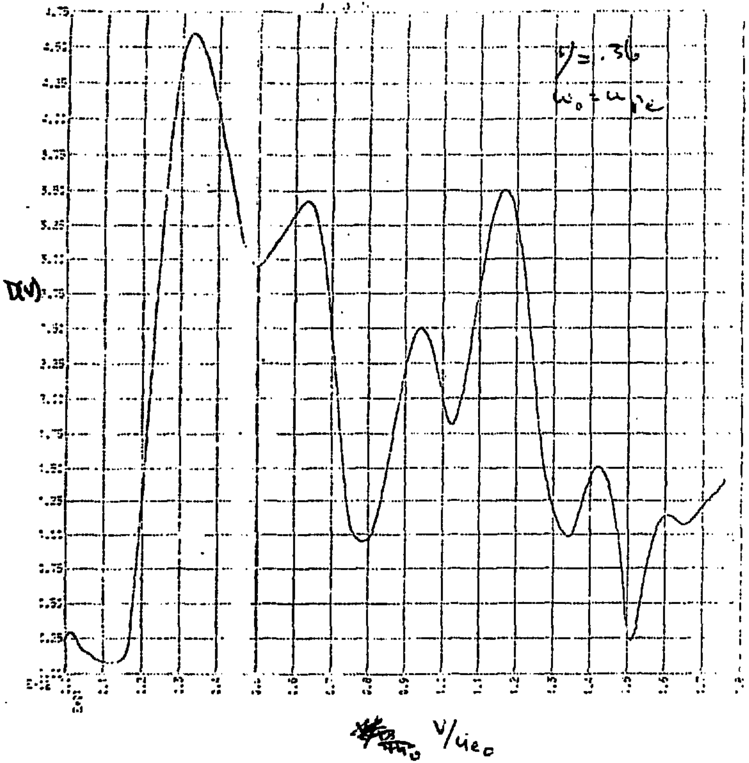
Slide 6




Slide 7



Slide 8




Slide 9