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TWO-NUCLEON FORM FACTORS FOR HEAVY-ION DWBA CALCULATIONS*

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At first sight the calculation of two nucleon form factors in DWBA for heavy ion reactions seems to lend itself to two alternate approaches. One approach would be to use an oscillator separation into relative and center of mass coordinates, and then describe the center of mass motion in a Woods-Saxon well. This method has two drawbacks. First, the DWBA interaction must be represented as acting on the center of mass of the cluster, when it is clear that the two particles are not always constrained to be together in a projectile as large as, for example, ¹⁸O. Secondly, the use of an oscillator separation into relative and c of m coordinates certainly fails to reproduce the proper structure in the tails of the bound state wave functions, which, as we will see, is crucial in heavy ion reactions.

The alternate approach, which we follow here, is to integrate directly over each nucleon coordinate. Although we will only discuss our own specific method¹ of evaluating the integrals, based on the Sawaguri-Tobocman expansion,² it should be pointed out that this problem has been the subject of parallel work by groups at several different institutions.^{3,4}

The reactions considered are of the type $(A + 2N) + B \rightarrow A + (B + 2N)$ where 2N stands for two like nucleons that are transferred and A, B are inert cores. The exact DWBA amplitude for such a process may be schematically written

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$$\begin{aligned}
 M_{if} = & \sum a_{1'2'} b_{12} C(\ell_1 \ell_2 \ell_1' \ell_2' m_1 m_2 m_1' m_2') \\
 & \times \langle \psi^{(-)}(k_f, \tilde{\mathbf{r}}_f) \varphi_{m_1}^{\ell_1}(\tilde{\mathbf{r}}_{1B}) \varphi_{m_2}^{\ell_2}(\tilde{\mathbf{r}}_{2B}) | V(\tilde{\mathbf{r}}_{1A}) + V(\tilde{\mathbf{r}}_{2A}) | \\
 & \times \varphi_{m_1}^{\ell_1'}(\tilde{\mathbf{r}}_{1A}) \varphi_{m_2}^{\ell_2'}(\tilde{\mathbf{r}}_{2A}) \psi^{(+)}(k_i, \tilde{\mathbf{r}}_i) \rangle
 \end{aligned} \quad (1)$$

where C incorporates standard normalization and angular-momentum coupling factors; $\psi^{(+)}, \psi^{(-)}$ are respectively initial and final optical wave functions, while the $\varphi_{m_i}^{\ell_i}$ are the transferred nucleon bound state wave functions. $a_{1'2'}$ and b_{12} are the structure factors which define the parentage of two particle configurations in the incoming projectile ($A + 2N$) and the final residual nucleus ($B + 2N$):

$$\psi_{(A+2N)}^J = \sum_{1,2} a_{1'2'} [j_1' j_2' J]^J \psi_{(A)}^0 \quad (2)$$

$$\psi_{(B+2N)}^{J'} = \sum_{1',2'} b_{12} [j_1 j_2 J']^{J'} \psi_{(B)}^0 \quad (3)$$

To evaluate the nine dimensional DWBA integral, we choose the three independent vectors $\tilde{\mathbf{r}}_{1B}, \tilde{\mathbf{r}}_{2B}, \tilde{\mathbf{r}}_i$ as variables of integration (Figure 1).

Then the following exact relations may be stated

$$\tilde{\mathbf{r}}_f = \frac{M_B}{(M_B + 2M_N)} \tilde{\mathbf{r}}_i + \frac{M_A + M_B + 2M_N}{(M_A + 2M_N)(M_B + 2M_N)} (\tilde{\mathbf{r}}_{1A} + \tilde{\mathbf{r}}_{2A}) \quad (4)$$

$$\tilde{\mathbf{r}}_{1A} = \frac{(M_A + 2M_N)}{M_A} \tilde{\mathbf{r}} - \frac{(M_A + M_N)}{M_A} \tilde{\mathbf{r}}_{1B} - \frac{M_N}{M_A} \tilde{\mathbf{r}}_{2B} \quad (5)$$

$$\tilde{\mathbf{r}}_{2A} = \frac{(M_A + 2M_N)}{M_A} \tilde{\mathbf{r}}_i - \frac{(M_A + M_N)}{M_A} \tilde{\mathbf{r}}_{2B} - \frac{M_N}{M_A} \tilde{\mathbf{r}}_{1B} \quad (6)$$

We make the standard "no-recoil" approximation

$$\tilde{\epsilon}_f = \frac{M_B}{(M_B + 2M_N)} \tilde{\epsilon}_i, \quad (7)$$

and the additional approximations

$$\tilde{\epsilon}_{1A} = \frac{(M_A + 2M_N)}{M_A} (\tilde{\epsilon}_i - \tilde{\epsilon}_{1B}) \quad (8)$$

$$\tilde{\epsilon}_{2A} = \frac{(M_A + 2M_N)}{M_A} (\tilde{\epsilon}_i - \tilde{\epsilon}_{2B}). \quad (9)$$

If the transferred mass $2M_N$ is small relative to M_A, M_B these relations become exact. With these substitutions the DWBA amplitude becomes

$$\begin{aligned} M_{if} = & \sum a_1' b_{12} C(\ell_1 \ell_2 \ell_1' \ell_2' m_1 m_2 m_1' m_2') \\ & \times \langle Y^{(-)}(k_f, \frac{M_B}{(M_B + 2M_N)} \tilde{\epsilon}_i) \rangle_{m_1}^{\ell_1} (\tilde{\epsilon}_{1B}) \langle \rangle_{m_2}^{\ell_2} (\tilde{\epsilon}_{2B}) \\ & \times \left| V\left(\frac{M_A + 2M_N}{M_A} [\tilde{\epsilon}_i - \tilde{\epsilon}_{1B}]\right) + V\left(\frac{M_A + 2M_N}{M_A} [\tilde{\epsilon}_i - \tilde{\epsilon}_{2B}]\right) \right| \quad (10) \\ & \times \langle \rangle_{m_1}^{\ell_1'} \left(\frac{M_A + 2M_N}{M_A} [\tilde{\epsilon}_i - \tilde{\epsilon}_{1B}]\right) \langle \rangle_{m_2}^{\ell_2'} \left(\frac{M_A + 2M_N}{M_A} [\tilde{\epsilon}_i - \tilde{\epsilon}_{2B}]\right) \\ & Y^{(+)}(k_i, \tilde{\epsilon}_i) \rangle. \end{aligned}$$

The integral is now tractable by the Sawaguri-Tobocman method,² i.e. an expansion of functions of the difference of two variables in terms of modified harmonic oscillator functions of each variable. For the two-nucleon case a double expansion is required. We have suitably modified and

expanded the Tobocman finite range code RDR⁶ to perform these calculations.

It is useful to observe that in two particle transfer reactions with light projectiles, the structure of the projectile is generally suppressed. In the case of heavy ions, the structure factors for both target and projectile enter explicitly. The total form factor is, in fact, a coherent sum over many two particle form factors arising from various initial and final state two nucleon configurations

$$M_{if} = \langle \psi^{(-)}(k_f, \tilde{r}_i) F(\tilde{r}_i) \psi^{(+)}(k_i, \tilde{r}_i) \rangle \quad (11)$$

$$F(\tilde{r}_i) = \sum_{1,2,1',2'} a_{1'2'} b_{12} C(l_1 l_2 l_1' l_2' m_1 m_2 m_1' m_2') \\ \times \left\{ \begin{aligned} &\langle \varphi_{m_1}^{l_1} | V | \varphi_{m_1}^{l_1'} \rangle \langle \varphi_{m_2}^{l_2} | \varphi_{m_2}^{l_2'} \rangle \\ &+ \langle \varphi_{m_1}^{l_1} | \varphi_{m_1}^{l_1'} \rangle \langle \varphi_{m_2}^{l_2} | V | \varphi_{m_2}^{l_2'} \rangle \end{aligned} \right\} \quad (12)$$

Note that each term of the form factor is the product of a usual DWBA form factor with an interaction for one particle and an overlap form factor for the other particle.

To compare this work with experiment we have analyzed recent Brookhaven data from the (¹⁸O, ¹⁶O) reaction on the molybdenum isotopes⁷ and on the nickel isotopes.⁸ For both cases we assumed ¹⁸O wave functions of the form⁹

$$\psi(0^{18}) = [a_{dd} (1d_{\frac{5}{2}})^{2 0+} + a_{ss} (2s_{\frac{1}{2}})^{2 0+}] \psi(0^{16}) \quad (13)$$

The transferred neutrons were bound in ^{18}O and the various final state nuclei with one-half the appropriate separation energy and in a well having $r = 1.25A^{1/3}$ fm, $a = .65$ fm.

To avoid a structure calculation for the molybdenum nuclei we allow the valence neutrons to occupy only the $3s_{1/2}$ and $2d_{5/2}$ shells. Then the heavier isotopes are generated by adding correlated pairs of neutrons to an inert ^{92}Mo core. Thus the wave function for the ^{94}Mo ground state is

$$\psi_{94} = (b_{ss}(3s^2)^{0+} + b_{dd}(2d^2)^{0+}) \psi_{92} = A^+ \psi_{92} \quad (14)$$

and subsequently $\psi_{94+2n} = N_n (A^+)^{n+1} \psi_{92}$. The magnitudes of b_{ss}, b_{dd} are selected to reproduce the single neutron pickup data for $^{94}\text{Mo}(d,t)^{93}\text{Mo}$.¹⁰

The phases were assigned to yield coherence in the two-neutron transfer as one might expect from an attractive neutron-neutron interaction. Table 1 contains the two-particle spectroscopic factors deduced from the above assumptions for ^{18}O and $^{92+2n}\text{Mo}$.

The optical potentials were chosen to be consistent with elastic ($^{18}\text{O}, ^{94}\text{Mo}$) scattering and are $V_0 = 100$ MeV, $W_{\text{vol}} = 10$ MeV, $R = 1.3$ ($A_1^{1/3} + A_2^{1/3}$) fm and $a_0 = 0.5$ fm.

We have presented our results in Table 2 in the form of peak differential cross-sections for varying energy and isotope. The angular distributions seen in Figure 2 are single-peaked in shape, typical of strongly absorbed grazing collisions. There is little dependence of the shape on the shell model states in which the transferred neutrons are bound.

The trends of the ground state-ground state peak cross sections with isotope and energy are reasonably well reproduced by our DWBA calculations.

The theoretical cross sections in Table 2 were obtained after a single overall normalization factor $N = 7$ had been applied. Such an underestimate of the cross-section magnitude in the present situation is consistent with that observed in light-projectile two neutron transfer.¹¹ Indeed the presence of two spectroscopic factors in the heavy-ion transfer suggests a greater sensitivity to omitted small components in the two-nucleon wave functions.

Perhaps the most interesting feature of the heavy-ion two particle transfer is a surprisingly strong dependence on the nucleon bound states. Even in light projectile reactions direct transfer is expected to take place at the nuclear surface or further out, resulting in an enhancement for transfer out of (or into) those states extending furthest. However, the strong absorption in heavy ion-reactions exaggerates and dramatizes this dependence on wave-function tails. To give a quantitative measure of this state dependence we have presented in Table 3 the cross sections obtained when the transferred neutrons are placed in pure configurations. Evidently the cross-sections are largest when the (2s) and (3s) wave functions are used, reflecting their extra-noded and therefore long-tailed aspect. The choice of ^{18}O configuration is, however, most crucial, with for example the $(2s)^2 \rightarrow (2d)^2$ cross sections being some 17 times the $(1d)^2 \rightarrow (2d)^2$ cross section. Thus the relatively small amount of $(2s)^2$ in the ^{18}O wave function dominates the two neutron transfer. It is difficult to see how such a striking state dependence would ensue from the cluster approach to the multi-nucleon transfer. For example, if an effective interaction acting only on the center of mass of the transferred pair is used in Eq. (1) and if the center of mass wave function corresponding only to maximum nodes is allowed, then the above ratio reduces from 17 to 2 or 3.

A second calculation was performed for the ($^{18}_{0}, ^{16}_{0}$) reaction on nickel isotopes. Here the nickel structure factors were taken from a previous pairing model calculation.¹² Since the elastic scattering data was quite good, the distorted wave parameters were obtained from fits to the elastic data.⁸ With $V = 70$, $a = .4$ fm the real radius was determined for each channel of each reaction $^{58,60,62,64}_{\text{Ni}}(^{18}_{0}, ^{16}_{0})$. There was some freedom then to adjust W and the radius of W while maintaining a good χ^2 value in fitting the elastic data. Figure 3 shows the calculated angular distributions with $W = 9$ MeV $r_I = r_R$, $a_I = a_R$ for $^{62,64}_{\text{Ni}}(^{18}_{0}, ^{16}_{0})$ and $W = 8$ MeV, $a_I = .5$, $r_I = 8.2$ for $^{58,60}_{\text{Ni}}(^{18}_{0}, ^{16}_{0})$. Aside from the apparent strong oscillations, a reasonable fit to the shape of the 65 MeV reaction data was obtained. Although complete elastic data was not available at incident energies of 50,57 MeV, we have presented theoretical angular distributions for the $^{64}_{\text{Ni}}(^{18}_{0}, ^{16}_{0})$ reaction at these energies to illustrate the excitation systematics (which are similar to the molybdenum). For $^{62,64}_{\text{Ni}}(^{18}_{0}, ^{16}_{0})$ the overall underestimate of the absolute cross section at 65 MeV is about a factor of 3, while it is about 1.5 for $^{58,60}_{\text{Ni}}(^{18}_{0}, ^{16}_{0})$.

The difference in normalization between molybdenum and nickel can be explained, to some extent at least, by the different projectile energies in the two cases. If one normalizes at approximately the same energy above the Coulomb barrier ($E = 70$ for $^{94}_{\text{Mo}}$, $E = 57$ for $^{64}_{\text{Ni}}$) then both normalizations are close to 5. Of course, we have not yet attempted to vary the imaginary optical potential with projectile energy. This might well both improve angular fits and give a consistent energy independent normalization.

Several general observations should be made at this point. The first is that a somewhat weaker absorption than is usually employed for

heavy ion transfer reactions seems to be appropriate for the analysis of these two-nucleon transfer reactions. We have used $W = 8-10$ MeV while $W = 40$ is often used for single nucleon transfer calculations. Of course for the extreme single peaked grazing collisions where "strong absorption" is produced by $W = 10$ MeV, then the use of $W = 40$ MeV will produce little change in shape.

Secondly, in comparing two nucleon form factors with one nucleon form factors, one observes a much more rapid radial fall off in the tail region for two nucleon transfer. That is, a normal $\langle \varphi | V | \varphi' \rangle$ one particle type form factor is multiplied by the $\langle \varphi | \varphi' \rangle$ bound state wave function overlap form factor. Physically, this indicates that unless there is a large build up due to two nucleon coherence, the contribution of the grazing part of the cross section will be substantially reduced from the one particle case.

The effect of the "no-recoil" approximation is another question of some concern. In a real sense the approximation is twice as bad as it would be for a similar single particle calculation. The sensitivity of cross section magnitudes to the "no recoil" approximation may be explored somewhat by altering the ratio between r_f and r_A . The approximation in Eq. (7) is exact when the transfer takes place at the center of A in Fig. 1. Increasing the multiplicative factor in Eq. (7) roughly corresponds to the transfer taking place closer to the surface of A. With the choice $A \equiv {}^{16}\text{O}$, i.e. in the "post" version of the reaction considered, the cross section showed variations of 10-15% in Mo and ~30% in Ni with this factor.

We have performed preliminary calculations, exactly evaluating the first order Taylor series approximation to recoil to investigate angular shape dependence. The method is similar to that of Nagarajan¹³ except that

we exactly evaluate the gradient and use the more exact Tobocman form factor. In qualitative agreement with exact recoil calculations of DeVries¹⁴ we find that angular oscillations tend to be damped. However overall single peak grazing shapes are little modified. Thus our expectation is that proper recoil calculations might somewhat damp the oscillations in the Ni angular distributions while preserving the grazing and forward peaked structure.

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Table I. Ground state parentage coefficients used in the two neutron transfer calculations.

Overlap	$(\frac{1^+}{2})^2$	$(\frac{5^+}{2})^2$
$(0^{18}, 0^{16})$	0.4500	0.8930
$(\text{Mo}^{94}, \text{Mo}^{92})$	0.2887	0.9574
$(\text{Mo}^{96}, \text{Mo}^{94})$	0.4432	1.1136
$(\text{Mo}^{98}, \text{Mo}^{96})$	0.7107	1.2142
$(\text{Mo}^{100}, \text{Mo}^{98})$	0.8864	0.8018

Table II. Peak cross sections for the ($0^{18}, 0^{16}$) reaction on even Molybdenum isotopes.

Target	Lab Energy (MeV)	$(d\sigma/d\Omega)_{EXP}$ ($\mu\text{b}/\text{sr}$)	$(d\sigma/d\Omega)_{DWBA}$ (x 7) ($\mu\text{b}/\text{sr}$)
Mo ⁹²	60	51 ⁺¹³ ₋₉	43
Mo ⁹⁴	55	117 ⁺³⁶ ₋₂₇	80
	60	170 \pm 34	135
	70	180 ⁺⁵⁵ ₋₃₉	256
Mo ⁹⁶	60	274 \pm 86	296
Mo ⁹⁸	60	412 \pm 120	393
	70	250 \pm 55	495

Table III. Comparison of peak DWBA two particle transfer cross sections for pure configurations. Quantities are in μb

0^{18} Mo ⁹⁶	$2d^2$	$3s^2$
$1d^2$	0.84	1.52
$2s^2$	14.66	30.31

Figure Captions

1. Coordinates used in evaluating the DWBA integral.
2. Angular distributions for $^{94}\text{Mo}(^{18}\text{O}, ^{16}\text{O})^{96}\text{Mo}(\text{g.s.})$. Solid curves are DWBA calculations.
3. Angular distributions for $(^{18}\text{O}, ^{16}\text{O})$ reactions on even Ni isotopes. Solid curves are DWBA calculations.

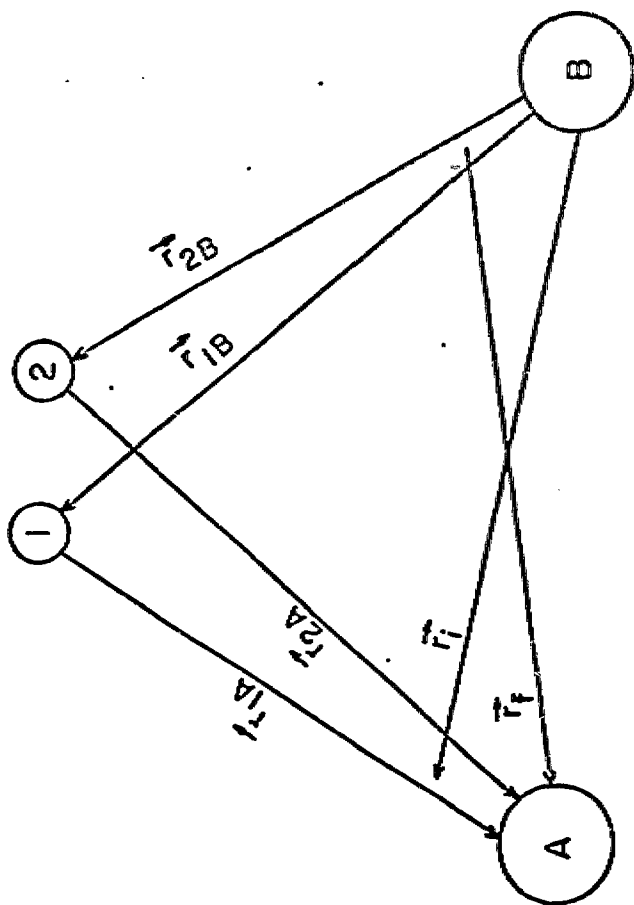


FIGURE 1

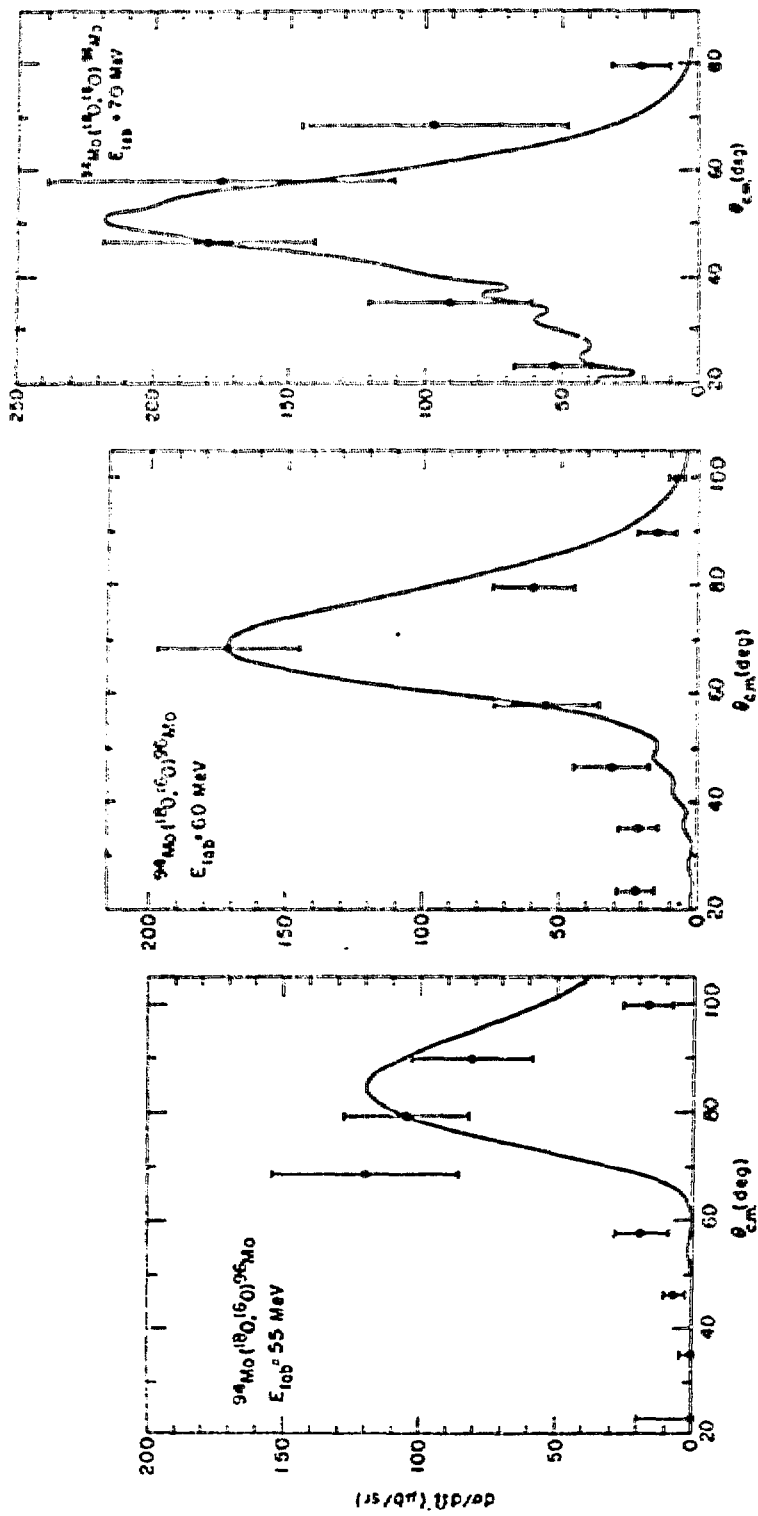


FIGURE 2

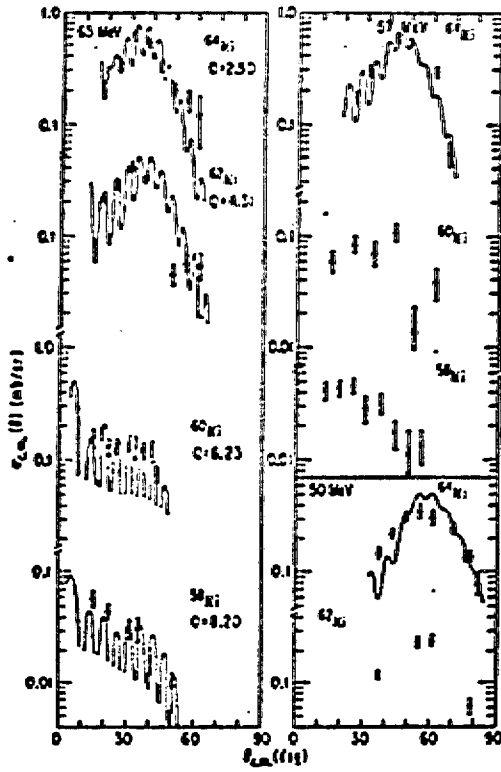


FIGURE 3