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"On Massless Particle Decay"*

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by

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Abstract

A hamiltonian model of lorentz invariant particle interactions is used as a framework in an attempt at describing massless particle decay. By restricting to the two particle sector we find an exact solution for the time development of the survival probability. Several modes are obtained and discussed. In particular we show that the presence of a persistent mode, i.e., the case in which the survival probability has a non zero limiting value, has to be interpreted as indicating that the massless particle is a stable bound state.

1. Introduction

The possibility that zero mass particles, in particular the photon and neutrino, can display unstable features, has been considered in the literature.[1,2] The problem is challenging as its implications might be profound. Essentially all our knowledge of the Universe is the result of the interpretation of the flux of electromagnetic radiation impinging on the Earth, mostly in the optical but also in other portions of the spectrum. This interpretation is partly based on the assumption that the photon is a stable (massless) particle. By stable here is meant that spectroscopic analyses of a given beam of radiation, propagating in free space, carried out by observers at rest with respect to each other, and placed a distance apart along the beam, should not show any change as the observers separation is changed. Changes in intensity, on the other hand, should be fully explained by geometrical arguments.

Questions of stability of zero mass particles are intimately connected to relativistic invariance and to quantum mechanics. If we disregard quantum effects, and consider only a beam of classical massless particles, (propagating at the speed of light), and ask for a covariant prescription for the decay, (i.e., decrease in intensity), we end up with the results of reference 1. From relativistic invariance the only restriction is that the function describing the decay can depend only on the ratio t/E, where E is the energy of the particles and t is the time elapsed. The ratio t/E is the generalization of proper time appropriate to massless particles.

The situation is considerably more complicated when quantum mechanics is introduced. Our intuition on unstable particles is intimately connected to the exponential decay law, in which the half life is simply the inverse of the transition rate per unit time. Experimentally only massive unstable particles have been observed. It is also an experimental observation that for unstable particles the mass does not have a well defined value, but rather a certain intrinsic spread is seen, which can be characterized by a "width". For those particles that live long enough for the exponential decay law to have an experimental meaning, it is observed that the mass, which can be defined for instance as the mean value in the mass distribution, is always much larger than the width. A further important observation which seems to be verified in general is the proportionality of the width and the transition rate per unit time, i.e., particles decaying faster show a larger width. All these observations can be explained within a quantum mechanical framework, provided certain approximations are applicable. In particular the narrowness of the mass distribution, i.e., mass much larger than width, seems to play an important role. A closer look at the problem shows, however, that the exponential law cannot hold exactly in quantum mechanics for all times.[3] It has been indicated that the non-exponential behaviour can be traced to the non-vanishing probability of recombination of the decay products into the initial state.[4] We notice that recent explanations of the exponential decay law,[5] avoid this problem by simply not allowing the system to follow a purely quantum mechanical evolution, but rather amending the dynamical laws by the addition of random interactions ("measurements"). Whether this is the correct prescription or not the fact remains that an isolated (i.e., free) system will not follow the exponential law at all times.

In the case of massless particles, we must first of all recognize that no evidence is available on which our intuition could be built upon, although a few general statements can be made. Consider, for example, photons decaying according to an exponential law. From conservation of energy, if photons are also included in the decay products, they will appear at longer wavelengths than the initial one. Since in this case, from relativistic invariance, the transition rate per unit time is inversely proportional to the energy of the decaying photon, we expect the following effect to be observable; the ratio of the intensities of two different spectral lines, emitted under similar conditions, should show a correlation with the distance from the source to the observer. Astronomical observations,[6] however, indicate no anomalies in, e.g., the Balmer series, up to distances, computed on the basis of the red shift, of up to 10^9 - 10^{10} light years. A more stringent test is given by the absence of a conspicuously small intensity in the 21 cm wavelength radiation from neutral hydrogen, from astronomical objects as far away as 10^7 to 10^8 light years.

This places a lower limit for the half life for photons on the visible range, on the assumption of an exponential decay law, of about 10^{13} years. This is orders of magnitude larger than current estimates of the age of the Universe, and would rule out the possibility of observing exponential law decays of optical photons. For photons of much lower energies, the situation is not so hopeless but so far no experimental evidence is available.

Another important question is whether we should expect a width associated with a finite transition rate for a photon. The analysis of Goldhaber and Nieto[7] can be interpreted as indicating that the mass spectrum of the photon has no important contributions beyond

$$m \gtrsim 10^{10}\text{cm}^{-1} \simeq 10^{-15}\ \text{eV} \quad .$$

If this is interpreted as a width, and in turn given the usual correlation to half life, we find for a photon "at rest" a half life of the order of one second. Unfortunately there is no simple way of interpreting this result. If for instance we chose the same value of m as a scale for photon energies, we find for optical photons a half life of the order of 10^8 years. Arbitrarily large values can be obtained, however, by assigning smaller mass values to the photon.

In conclusion, our knowledge of the properties of the decays of massive particles seems to be of little use in discussing massless particle decays. In fact it seems that the only indication that we are dealing with the decay of a massless system would be that only massless particles appear as decay products, since otherwise, the presence of a massive particle, of say mass μ, in the final state would have to be interpreted as indicating that the decaying state mass was at least equal to μ. It is then of interest to try and establish what properties appear in a model in which quantum mechanics and relativistic invariance are considered exactly and not within a perturbative scheme. In what follows we shall take the point of view that whatever a massless unstable particle is, it can only decay into massless particles. We shall interpret the requirement of Poincaré invariance in the sense that there exists a set of operators satisfying the appropriate commutation relations and under which the physical (stable) states transform irreducibly. One of this operators plays the role of the Hamiltonian H. We shall then assume that any state can be written as a linear superposition of eigenstates of H and then study its evolution in time.

2. The model

We shall follow a method based on work described in references (8) and (9). As a starting point we define a set of vectors

$$|m_1, \vec{p}_1 ; m_2, \vec{p}_2\rangle \quad , \tag{2.1}$$

describing states of two (distinguishable) spinless particles of masses m_1 and m_2, with well defined momenta $\vec{p}_1$ and $\vec{p}_2$, and satisfying the normalization conditions,

$$\begin{aligned} &\langle m_1, \vec{p}_1{}'; m_2, \vec{p}_2{}' | m_1, \vec{p}_1; m_2, \vec{p}_2\rangle \\ &= 2\, \omega_1\, \omega_2\, \delta(\vec{p}_1 - \vec{p}_1{}')\, \delta(\vec{p}_2 - \vec{p}_2{}') \quad , \end{aligned} \tag{2.2}$$

where $\omega_i = \sqrt{m_i^2 + p_i^2}$.

This set is assumed complete. Every possible state can then be represented by a vector

$$|\phi\rangle = \int \frac{d^3p_1}{2\omega_1} \frac{d^3p_2}{2\omega_2}\, \phi(\vec{p}_1, \vec{p}_2)\, |m_1, \vec{p}_1; m_2, \vec{p}_2\rangle \tag{2.3}$$

By the methods indicated in reference (9) one can construct a representation of the Poincaré group. The details are not important as only the formal properties will be used. Instead of (2.3) one can consider states described by the variables

$$M = [(\omega_1 + \omega_2)^2 - p^2]^{\frac{1}{2}},$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2, \quad \hat{e} = \vec{q} - (M + \omega_1 + \omega_2)^{-1} q_o \vec{P},$$

$$\lambda(M) = [M^2 - (m_1 + m_2)^2]\,[M^2 - (m_1 - m_2)^2] \tag{2.4}$$

$$\vec{q} = M\lambda^{-\frac{1}{2}}(M)\ \{\vec{p}_1 - \vec{p}_2 - [(m_1^2 - m_2^2)/M^2]\ \vec{P}\}$$

$$q_o = M\lambda^{-\frac{1}{2}}(M)\ \{\omega_1 - \omega_2 - [(m_1^2 - m_2^2)/M^2](\omega_1 + \omega_2)\}\ .$$

M is then the center of mass energy and $\vec{P}$ is the total momentum. The unit vector $\hat{e}$ can be used to perform a partial wave analysis of the state. However, to keep our discussion as simple as possible, we shall assume from the outset that only S-waves are involved in the decay. We are then led to consider the states

$$|M, \vec{P}> \quad , \tag{2.5}$$

which we interpret as states of particles 1 and 2 with center of mass energy M, total momentum $\vec{P}$ and zero total angular momentum in their center of mass. Their normalization is

$$<M, \vec{P}|M', \vec{P}'> = 2\ W\ \delta(M - M')\ \delta^3(\vec{P} - \vec{P}') \quad , \tag{2.6}$$

where

$$W = \sqrt{M^2 + P^2} \quad .$$

We now consider another set of states

$$|\mu, \vec{\kappa}, o\rangle = \int_{M_o}^{\infty} dM \left(\frac{\mu^2 + \kappa^2}{M^2 + K^2}\right)^{\frac{1}{2}} \frac{F(M)}{\alpha_+(M)} |M, \vec{K}\rangle \quad , \tag{2.7}$$

where

$$\alpha_\pm(M) = M - \mu - \int_{M_o}^{\infty} \frac{F^2(M')}{M - M' \pm i\varepsilon} dM' \quad , \tag{2.8}$$

and F(x) is a real function of x for x in $(M_o \leqslant x < \infty)$. Our definition is clearly motivated by the relation between the "bare" V and the "physical" N-O particles in the Lee model. It guarantees that for any choice of F(x) we have[10]

$$\langle\mu, \vec{\kappa}, o|\mu, \vec{\kappa}', o\rangle = 2\sqrt{\mu^2 + \kappa^2}\ \delta^3(\vec{\kappa} - \vec{\kappa}'). \tag{2.9}$$

This normalization suggests that the vectors $|\mu, \vec{\kappa}, o\rangle$ could be treated as representing states of momentum $\vec{\kappa}$ of a single spinless particle of mass μ. We notice however, that our definition (2.7) is not invariant under boosts, and therefore the interpretation of (2.7) is frame dependent. One might then question the point of considering frame dependent objects in a relativistically invariant description. Our argument is as follows. We are interested in the description of an unstable system. Such a system has to be created a finite time in the past before observation. The creation of the system implies setting up an apparatus that will serve as a "source". For instance in the case of a photon it could be an excited atom or molecule. The apparatus then defines a special frame attached to the unstable particle. It is appealing to assume that if such particle is created, at say $t = 0$, in that particular frame, its corresponding wave function at $t = 0$, again in this special frame, will have the simple form (2.7). Because of the non-invariance of simultaneity, the description in other frames may be considerably more complicated. We remark that because of our definition the decaying state has always well defined spin (equal to zero), and, as we shall see below, for sufficiently narrow width, its mass spectrum is essentially frame independent.

Having defined our unstable state at $t = 0$, we now proceed to study its time development. Following the usual reasoning, we look at the quantity $A(t,\vec{K})$ defined by

$$2(\mu^2 + \kappa^2)^{\frac{1}{2}} \delta^3(\vec{\kappa} - \kappa') \, A(t, \vec{\kappa}) = \langle \mu, \vec{\kappa} | e^{-iHt} | \mu, \vec{\kappa}' \rangle \quad , \qquad (2.10)$$

where H is the tctal hamiltonian. $|A(t, \kappa)|^2$ is the quantum mechanical survival probability. From (2.7)

$$A(t, \vec{\kappa}) = \int_{M_0}^{\infty} \frac{F^2(M)\, e^{-i\sqrt{M^2 + \kappa^2}\, t}}{\alpha_+(M)\, \alpha_-(M)} \, dM \quad . \qquad (2.11)$$

The exponential in the integrand has branch points at $M = \pm i\kappa$. Using the relation

$$2i\pi F^2(M) = \alpha_+(M) - \alpha_-(M) \quad , \qquad (2.12)$$

we can write

$$A(t, \vec{\kappa}) = \frac{1}{2\pi i}\int_C \frac{e^{-i\sqrt{M^2 + \kappa^2}\ t}}{\alpha(M)}\ dM \quad , \tag{2.13}$$

where the contour C is indicated in figure 1. If we now assume that $\alpha(z)^{-1}$ has a pole on the second sheet (reached by continuation from above the cut from M_o to $+\infty$), located $z = \bar{M} - i\Gamma$, the contour can be deformed as indicated in figure 1, where AB runs on the first sheet and BD runs on the second sheet. The residue of the pole is picked up in the process and we obtain,

$$A(t, \vec{\kappa}) = \frac{1}{\alpha'(\bar{M} - i\Gamma)} \exp\{-it\sqrt{(\bar{M} - i\Gamma)^2 + \kappa^2}\} + R(\vec{\kappa}, t) \quad , \tag{2.14}$$

where $\alpha' = d\alpha/dz$. $R(\kappa,t)$ gives the well known departure from the exponential decay law. For suitable choices of F^2, we have $\alpha' \simeq 1$, $R << 1$, and if $\bar{M} \simeq \mu >> M_o$, $0 < \Gamma << \bar{M}$, we find

$$A(t, \kappa) \simeq \exp\left\{-it\sqrt{\mu^2 + \kappa^2} - \frac{\mu}{\sqrt{\mu^2 + \kappa^2}}\Gamma t + O\left(\frac{\mu^2}{\mu^2 + \kappa^2}\right) t\sqrt{\mu^2 + \kappa^2}\right\} \quad . \tag{2.15}$$

The factor $\mu/\sqrt{\mu^2 + \kappa^2}$ is precisely the appropriate relativistic correction factor for a particle of mass μ and momentum $\vec{\kappa}$ decaying in its rest frame with a half life $2\Gamma^{-1}$.

If instead of (2.7) we study the evolution of a state obtained by "boosting" (2.7) so that its momentum is $\vec{\kappa}_b$, we find

$$A(t,\ \kappa_b) = \left[\left(\frac{\mu^2+\kappa_b^{\ 2}}{\mu^2+\kappa^2}\right)^{\frac{1}{2}} + O\left(\frac{\mu}{\sqrt{\mu^2+\kappa^2}}\right) + O\left(\frac{\Gamma}{\sqrt{\mu^2+\kappa_b^{\ 2}}}\right)\right]$$

$$\times \exp\left[-it\sqrt{\mu^2+\kappa_b^{\ 2}} - \frac{\mu^2}{\sqrt{\mu^2+\kappa_b^{\ 2}}}\,\Gamma t \right. \tag{2.16}$$

$$\left. + O\left(\frac{\Gamma^2}{\mu^2+\kappa_b^{\ 2}}\right) t\sqrt{\mu^2+\kappa_b^{\ 2}}\right] + R_b(\kappa,\ t) \quad ,$$

where $R_b(\kappa,t)$ will make a small contribution in the same sense as $R(\kappa,t)$ does. The description then gives an exponential decay law, "invariant" to lowest order in Γ/μ.

After this review of the massive case we now study the massless case. In accordance with our previous discussion we let $m_1 = m_2 = M_o = 0$. Since in Nature massless particles are associated with the weaker interactions, we may assume if necessary or convenient that μ (the "bare" mass) and $F(x)$ (the transition matrix element) are small. On the other hand we cannot have $\Gamma << \mu$, for we would then recover the massive case. Considering again (2.14), we may write, for $|\kappa| >> \bar{M}$ and $\bar{M} \sim \Gamma$,

$$[(\bar{M}-i\Gamma)^2 + \kappa^2]^{\frac{1}{2}} \simeq |\kappa| + \frac{1}{2}\,\frac{(\bar{M}-i\Gamma)^2}{|\kappa|} + |\kappa|\ O\left[\left(\frac{M-i\Gamma}{|\kappa|}\right)^4\right] \quad , \tag{2.17}$$

This suggests that we set $M = \Gamma$, for then, replacing in (2.14), we find,

$$A(t, \kappa) = \frac{1}{\alpha'(\bar{M}-i\Gamma)} \exp\left[-it|\kappa| - \frac{\bar{M}}{|\kappa|}\bar{M}t - it|\kappa|\, O\left(\frac{\bar{M}^4}{\kappa^4}\right)\right] + R(\kappa, t) \quad , \tag{2.18}$$

and if we further assume that $|R|$ is small, the leading terms in (2.18) will have precisely the appropriate form for a massless unstable particle. The terms of order $\bar{M}^4/\kappa^4$ reflect the non invariance of the wave function, while $R(\kappa,t)$ gives the departure from the exponential law expected from the positivity of H.

If we take (2.18) seriously, from previous experience with separable potential models we expect $\bar{M} \approx \mu$. The exponential decay in intensity is then roughly given by $\exp[-(\mu/\kappa)\mu t]$. Since astronomical observations at a wavelength of 21 cm have shown no anomalies up to distances of 10^7 light years (i.e., time of the order of 10^7 years), we should have $\mu \lesssim 10^{-49}$ g. Here we may note as an amusing result that if we identify this "bare" mass with a purely electromagnetic photon mass, for instance in the sense of reference (7), the upper limit obtained agrees closely with the present experimental upper limits for the photon mass.

We finally want to discuss some alternatives within the same model. So far we have only considered the case when the pole remains in the second sheet. We may ask what happens if we allow the pole to move to the point $M = 0$. We find that there are two possibilities; from (2.14) we would have

$$A(t, \kappa) = \frac{1}{\alpha'(0)} \exp\{-it|\kappa|\} + R(\kappa, t) \quad , \qquad (2.19$$

and therefore, since $|R(\kappa,t)| \to 0$ as $t \to \infty$, if $\alpha'(0)$ is finite, $|A(t,\kappa)|^2$ approaches a constant value as t goes to infinity (see figure 2). However, in order to have $\alpha'(0)$ finite, $F^2(x)$ has to vanish rather strongly as $x \to 0$. Clearly, this case corresponds to the presence of a stable bound state of zero mass, plus a continuum which does not couple to zero mass. If on the other hand $\alpha'(0)$ is infinite, the exponential term in (2.14) disappears, and $|A|^2$ goes to zero as an inverse power of t (dashed curve in figure 2). The time evolution of $|A|^2$, which becomes dependent on the details of $F^2(M)$, does not display simple transformation properties under boosts. For these reasons it is difficult to think of this case as describing an unstable particle at all.

We notice that these features are not peculiarities of the massless case, but rather they will show up in this model whenever we consider a state whose rest energy coincides with the threshold for its decay channels. Clearly the arguments about the behaviour of $\alpha'(0)$ apply equally well to $\alpha'(M_o)$, ($M_o \neq 0$), with similar consequences for $|A(t, \vec{\kappa})|^2$.

In the first case, i.e., decoupling, one might very well question the interpretation of the initial decrease in the survival probability as indicative of instability. A simpler interpretation is obtained by saying that in this case all particles involved are stable, and the decrease is due to the admixture of multiparticle states in the definition of the initial state. In fact, a closer look at equations (2.6) and (2.9), shows that in this case they cannot simultaneously hold, unless a new set of states, satisfying

$$\langle M_o, \vec{P} | M_o, \vec{P}' \rangle = 2\,\omega_o\,\delta^3(\vec{p} - \vec{p}') \quad , \tag{2.20}$$

where $\omega_o = \sqrt{(m_1+m_2)^2 + p^2}$, is added to (2.6). In other words, a bound state of mass equal to the threshold mass must be added to the physical states for consistency.

3. Conclusions

Because of the lack of experimental evidence, the question of instability of massless particles is at this moment mainly of theoretical interest. A statement of the problem in the framework of a solvable model which incorporates quantum mechanics and Poincaré invariance is then of relevance. It is quite possible that because of the presence of zero mass particles, multiparticle effects, (i.e., infrared divergences), play a major role. Since no scheme is presently known in which all these questions can be considered outside a perturbative approach, we have studied a model combining Hamiltonian dynamics and relativistic invariance, but restricting the treatment to the two particle sector. Within the context of this model a particle whose mass is strictly zero is stable. States which display an approximate invariance and to the same order of approximation show zero mass and exponential decay law, can also be constructed. A third class of states, corresponding to a transition matrix element which is non-vanishing at threshold, fail to show recognizable particle features.

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References and Footnotes

1. P. Havas, Amer. J. Phys. 34, 753 (1966).

2. M. T. Parkinson and D. P. Vasholz, University of Florida preprint, March, 1973.

3. See, for example, L. A. Khalfin, J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 1371 (1957), (Sov. Phys. JETP 6, 1053 (1958)).

4. L. Fonda and G. C. Ghirardi, Nuovo Cimento 7A, 180 (1972).

5. H. Ekstein and A.J.F. Siegert, Ann. Phys. (N. Y.) 68, 509 (1971).

6. I am grateful to Prof. G. H. De Vaucouleurs for an enlightening discussion on these points.

7. A. S. Goldhaber and M. M. Nieto, Rev. of Mod. Phys. 43, 277 (1971).

8. A. J. Macfarlane, Jour. of Math. Phys. 4, 490 (1963).

9. T. F. Jordan, A. J. Macfarlane and E.C.G. Sudarshan, Phys. Rev. 133B, B487 (1964).

10. We shall assume that $F(x) \to 0$ as $x \to M_o$, and that $\mu > M_o$, in such a way that $\alpha(M) \neq 0$ for all M for which (2.8) is defined. See also the discussion at the end of this section.

Figure Captions

Fig. 1. The contours of integration for eqs. (2.13) and (2.14)

Fig. 2. Time dependence of $|A|^2$

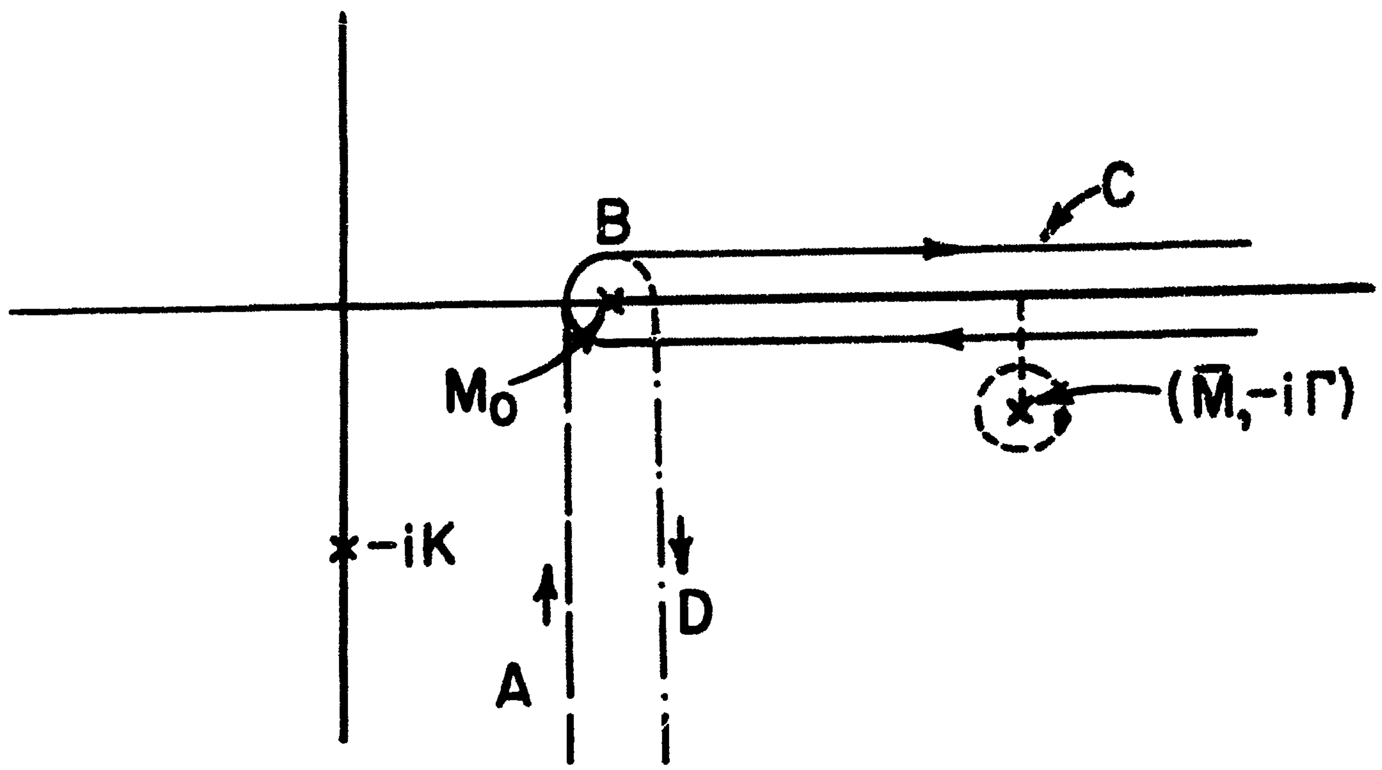

FIG. I

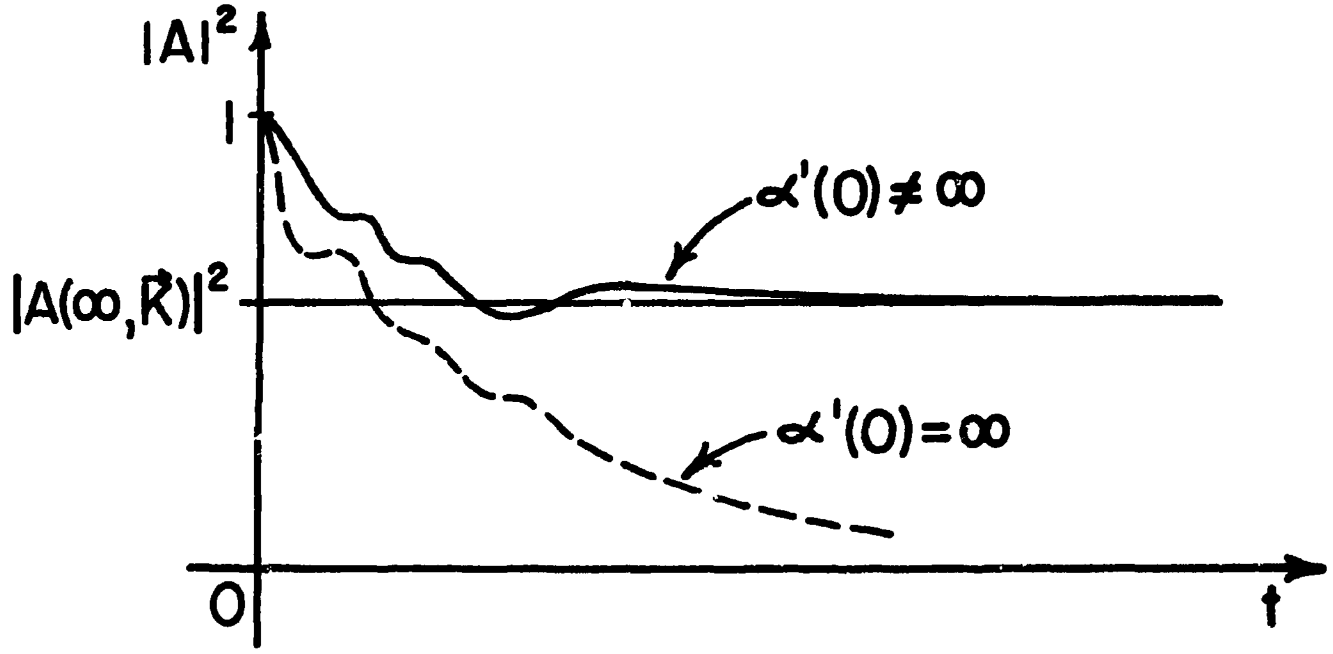

FIG. 2