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A GENERALIZATION OF THE MULTIGROUP APPROACH FOR CALCULATING THE NEUTRON SLOWING DOWN LENGTH

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A generalization of the multigroup approach for calculating the neutron slowing down length

Uogólnienie wielogrupowej metody obliczania długości spowalniania neutronów

Обобщение многогрупповой методы расчёта длины замедления нейтронов

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Summary

An original method for calculating the neutron slowing down length, using a generalized multigroup approach, is presented. Unlike other multigroup methods, this method takes into account all the basic phenomena involved in the neutron transport, i.e. the elastic scattering, the inelastic scattering and the neutron absorption.
The procedure for calculating L_s for polyenergetic neutron source is given together with its theoretical substantiation. The procedure applies for soils and rocks of any chemical composition and for all neutron sources which energies do not exceed 10,5 MeV. The FORTRAN programs for computations are enclosed.
Results of calculations the media being water-saturated sandstone and limestone of different porosity (from 0 % up to 100 %) are presented for monoenergetic neutrons and for radioisotope Ra-Be and Pu-Be neutron sources.
A good agreement between the calculated L_s values and available experimental values of other authors can be observed both for water and for weakly slowing down media, i.e. dry sandstone and dry limestone. For the media containing relatively small amounts of hydrogen a non-monotonic dependence of the slowing down length on the neutron initial energy has been observed.

Streszczenie

W pracy przedstawiono oryginalną metodę obliczania długości spowalniania neutronów, opartą na wielogrupowej teorii transportu neutronów. Pozwala ona, w odróżnieniu od dotychczas stosowanych metod wielogrupowych, uwzględnić rozproszenia sprężyste na wodorze, rozproszenia niesprężyste, oraz absorpcję neutronów prędkich.
Podano i uzasadniono teoretycznie sposób obliczania długości spowalniania neutronów dla źródeł polienergetycznych. Opisano procedurę obliczania L_s dla dowolnego składu chemicznego skał i gruntów, oraz dowolnego źródła neutronów w zakresie energii do 10,5 MeV. Załączone zostały programy obliczeń numerycznych napisane w języku FORTRAN. Niniejsza praca zawiera wyniki obliczeń dla piaskowca i wapienia o różnej porowatości w zakresie 0-100 %, nasyconego wodą, dla neutronów monoenergetycznych i źródeł radioizotopowych Ra-Be i Pu-Be.
Uzyskano dobrą zgodność obliczonych wartości L_s z dostępnymi danymi doświadczalnymi innych autorów, zarówno dla wody, jak i dla ośrodków słabo spowalniających (suchego piaskowca i wapienia).
Stwierdzono, że w przypadku ośrodków zawierających mało wodoru zależność długości spowalniania neutronów od ich energii początkowej nie jest monotoniczna.

Резюме .

В работе представлено оригинальный метод расчёта длины замедления нейтронов, базирующий на многогрупповой теории переноса . В отличие от применяемых до сих пор многогрупповых методов, разрешает он учитывать упругое рассеяние на ядрах водорода, неупругое рассеяние и захват быстрых нейтронов. Приведено способ расчёта длины замедления нейтронов полиенергетических источников. Изложено процедуру расчёта длины замедления для произвольного химического состава грунтов и произвольного источника нейтронов в диапазоне энергии до 10,5 Мэв. Приложено вычислительные программы на языке FORTRAN 4900. Приведено результаты вычислений для песчаника и известняка насыщенного водой, с пористостью изменяющей в диапазоне 0- 00 % для моноэнергетических нейтронов, а также Ra-Be и Pu-Be источников. Получено величины хорошо совпадающие с экспериментальными значенями других авторов.

Результаты расчётов указывают, что зависимость длины замедления нейтронов от их начальной энергии не всегда монотонная.

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1. Introduction

During recent years some works have been carried on in the Geophysics Departament of Institute of Nuclear Techniques (Academy of Mining and Metallurgy, Cracow) concerning calibration of neutron gauges for determining moisture content. The works aim at creating a semi-empirical calibration method, based on a theory of the neutron transport which could describe in a possibly most faithfull manner the neutron transport in the media one deals usually with in the geophysical practice, i.e. very heterogeneous from the point of view of their chemical composition. A great deal of importance is therefore attached to the question of a proper method to calculate the neutron parameters of the media. These parameters play the most important and conspicuous part in any theory which lies at the basis of calibration methods for calibrating the neutron moisture gauges.

The following paper is the result of investigations for a proper procedure to calculate the neutron slowing down length for rocks and soils. The widely used multigroup approach for calculating the neutron slowing down length has been here criticized and a generalization of this method has been proposed. The generalization makes possible to take into account a greater number of physical phenomena accompanying the neutron slowing down

process in the media. FORTRAN computer programs for direct computations of neutron slowing down length have been elaborated, allowing to obtain the neutron slowing down length value for any geological medium and for any type of neutron radioisotope source.

2. General

One of the most efficacious methods for solving the neutron transport problems in the stationary case is the multigroup method. At its basis lies the neutron transport equation in the diffusion approximation:

$$\frac{\partial q(r,u)}{\partial u} + \Sigma_a(u)\,\Phi(r,u) - D(u)\Delta\Phi(r,u) = S(r,u) \quad , \qquad (1)$$

where u is the neutron letargy, $q(r,u)$ is the neutron slowing down density, $\Phi(r,u)$ is the neutron flux, $\Sigma_a(u)$ is the neutron macroscopic absorption cross section of the given medium, $D(u)$ is the neutron diffusion coefficient and $S(r,u)$ is a function representing the neutron sources.

The multigroup approach consists in dividing the whole range of the neutron letargy into many intervals (energy groups). Integrating the diffusion equation over the separate intervals one obtains the following system of equations (cf Bakurts [1], p.164):

$$q(r,u_i)-q(r,u_{i-1}) + \int_{u_{i-1}}^{u_i} \Sigma_a(u)\,\Phi(r,u)du - \int_{u_{i-1}}^{u_i} D(u)\Delta\Phi(r,u)du =$$

$$= \int_{u_{i-1}}^{u_i} S(r,u)du \quad , \qquad (2)$$

for i = 1,2, ... N, where u_i is the lower boundary of the i-th energy group, N is the number of groups.

Assuming that for each group $\Phi_i(r,u) = \Phi_i(r)\,\Phi_i(u)$, the system (2) can be rewritten in the form :

$$q_i(r) - q_{i-1}(r) + \Sigma_{a,i}\Phi_i(r) - D_i\Delta\Phi_i(r) = S_i(r) \quad , \qquad (3)$$

where

$$\Sigma_{a,i} = \frac{\int_{u_{i-1}}^{u_i} \Sigma_a(u)\,\Phi_i(u)\,du}{\int_{u_{i-1}}^{u_i} \Phi_i(u)\,du} \quad ,$$

$$D_i = \frac{1}{3\,\Sigma_{tr,i}} = \frac{\int_{u_{i-1}}^{u_i} \frac{1}{3\,\Sigma_{tr}(u)}\,\Phi_i(u)\,du}{\int_{u_{i-1}}^{u_i} \Phi_i(u)\,du} \quad , \qquad (4)$$

$$q_i(r) = q(r,u_i) \quad ,$$

$$S_i(r) = \int_{u_{i-1}}^{u_i} S(r,u)\,du \quad ,$$

$\Sigma_{tr}(u)$ is the macroscopic transport cross section.

In the multigroup approach one assumes that the neutron belonging to one of the energy groups undergoes

within its limits the diffusion process without any change of its energy. The slowing down process is thus considered as the transfer of the neutron from the higher group into the lower one. In equation (3) the neutron inflow into the i-th group from higher energy groups is represented by the difference of the neutron slowing down densities : $q_i(u) - q_{i-1}(u)$. The energy losses due to the neutron collisions with hydrogen nuclei and due to the neutron inelastic scattering processes correspond to the neutron transfers down by a number of groups. Thus the slowing down density $q_i(r)$ can be split up into the sum of the contributions due to the neutrons falling out all higher groups :

$$q_i(r) = \sum_{k=1}^{i} a_{k,i}\, \Phi_k(r) \quad , \tag{5}$$

where $a_{k,i}$ play the role of the macroscopic cross sections for slowing down the neutron from the k-th group to a value of the letargy $u \geq u_i$.

Making use of (5) we obtain :

$$q_i(r) - q_{i-1}(r) = \sum_{k=1}^{i} a_{k,i}\, \Phi_k(r) - \sum_{k=1}^{i-1} a'_{k,i-1}\, \Phi_k(r) =$$

$$= \Sigma_{sl,i}\, \Phi_i(r) - \sum_{k=1}^{i-1} \Sigma_{sl(k,i)}\, \Phi_k(r) \quad , \tag{6}$$

where $\Sigma_{sl,i} = a_{i,i}$ is the macroscopic cross section for the neutron slowing down outside the i-th group, $\Sigma_{sl(k,i)} = a'_{k,i-1} - a_{k,i}$ is the macroscopic cross

section for the neutron slowing down from the k-th group into the i-th group.

Taking into account (6) and introducing the neutron diffusion length L_i in all the energy groups, the system (3) becomes

$$L_i^2 \Delta\Phi_i(r) - \Phi_i(r) + \frac{\sum_{k=1}^{i-1} \Sigma_{sl(k,i)}\Phi_k(r) + S_i(r)}{\Sigma_{r,i}} = 0 \quad , \quad (7)$$

where $\Sigma_{r,i} = \Sigma_{a,i} + \Sigma_{sl,i}$,

and $L_i^2 = \frac{D_i}{\Sigma_{r,i}} = \frac{1}{3\,\Sigma_{tr,i}\,\Sigma_{r,i}}$.

As the first step in solving the system of equations (7) one has to establish the averaged values of the neutron macroscopic cross sections, according to the formulas (4). Then the equations of the system (7) can be solved one by one, starting from the first equation, as each next equation makes use of the preceding solutions. The solution of each equation of (7) is $\Phi_i(r)$ and at the end the flux of the slowed down neutrons $\Phi_N(r)$ is constructed.

The system of equations (7) undergoes a serious simplification for the case of a point monoenergetic neutron source situated in an infinite, non-absorptive medium, containing neither hydrogen nor any nuclei which interact with the neutrons in inelastic collisions. In this

case $\Sigma_{s,i} = 0$, $S_1(r) = \delta(r)$, $S_i(r) = 0$ for $i \neq 1$ and $\Sigma_{sl(k,i)} = 0$ for $k \neq i-1$, the latter meaning that all the neutrons in the slowing down process pass subsequently through all the energy groups. Under the assumptions formulated above an interesting relationship can be observed (cf BEKURTS [1], p.166) :

$$L_s^2 = \sum_{i=1}^{N} L_i^2 \quad , \tag{8}$$

where L_s is the neutron slowing down length.

The above relation being the consequence of the particular case of the multigroup theory of the neutron transport is applied in the, so called, multigroup methods of calculation of the neutron slowing down length (cf references [2,3,4]). Those methods, all of them basing on the relation (8), cannot be right in the case of a medium containing great amounts of hydrogen, or when inelastic scattering and fast neutron absorption occur. The true value of L_s should have been calculated directly from its definition, after having solved the system (7). However, the solution of the system (7) being a result of digital computations, there is no possibility of introducing a general formula which could be used to calculate L_s.

Such a formula, consistent with the system of equations (7) has been found as a result of some elementary

argumentation, starting up from the definition of the neutron slowing down length.

3. Physical principles of the method

The neutron slowing down length is usually defined with the help of the second moment, $\langle r^2 \rangle$, of the neutron distribution around a point isotropic neutron source in an infinite, homogeneous medium :

$$L_s^2 = \frac{\langle r^2 \rangle}{6} = \frac{\frac{1}{6} \int_0^\infty r^2 \, \Phi_0(r) \, 4\pi r^2 \, dr}{\int_0^\infty \Phi_0(r) \, 4\pi r^2 \, dr} \quad , \qquad (9)$$

where $\Phi_0(r)$ is the flux of the slowed down neutrons.

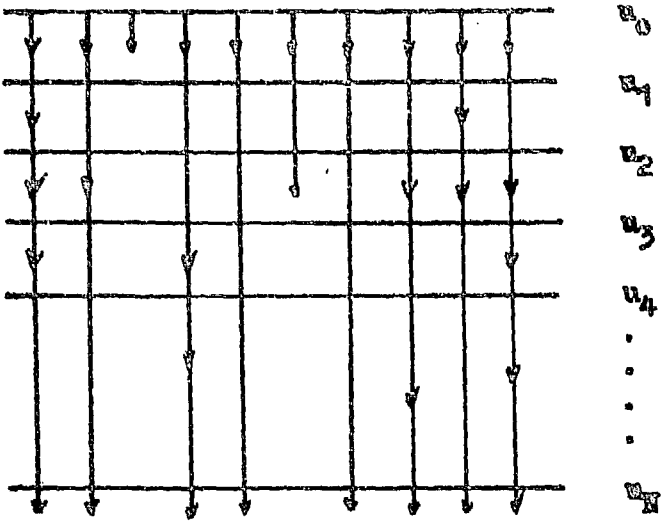

Fig. 1.

Let us have a closer look at the slowing down process in the multigroup pattern. A neutron can be slowed down from its initial energy (letargy u_0) into a letargy $u > u_M$ in many different manners, each of them corresponding to a different pattern of the energy groups travelled by the neutron (cf Fig.1.). Denoting by M the number of these combinations and by $\Phi_{0,k}(r)$ the flux of the neutrons which have been slowed down travelling by the k-th pattern of the diffusion groups we can write down an evident relation :

$$\Phi_0(r) = \sum_{k=1}^{M} \Phi_{0,k}(r) \quad . \tag{10}$$

Introducing (10) into the definition (9) we get

$$L_s^2 = \frac{\frac{1}{6} \sum_{k=1}^{M} \int_0^\infty r^2 \Phi_{0,k}(r)\, 4\pi r^2\, dr}{\int_0^\infty \Phi_0(r)\, 4\pi r^2\, dr} \quad . \tag{11}$$

The idea of the slowing down length can be introduced separately for each slowing down history. Denoting by $L_{s,k}$ the slowing down length corresponding to the k-th slowing down pattern we have

$$L_{s,k}^2 = \frac{\frac{1}{6} \int_0^\infty \Phi_{0,k}(r)\, 4\pi r^4\, dr}{\int_0^\infty \Phi_{0,k}(r)\, 4\pi r^2\, dr} \quad , \tag{12}$$

and the definition (11) becomes

$$L_s^2 = \frac{\sum_{k=1}^{M} L_{s,k}^2 \int_0^\infty \Phi_{0,k}(r)\, 4\pi r^2\, dr}{\int_0^\infty \Phi_0(r)\, 4\pi r^2\, dr} \quad . \qquad (13)$$

The integral $\int_0^\infty \Phi_0(r)\, 4\pi r^2\, dr$ represents the source output (cf BEKURTS [1], p.314), so

$$\frac{\int_0^\infty \Phi_{0,k}(r)\, 4\pi r^2\, dr}{\int_0^\infty \Phi_0(r)\, 4\pi r^2\, dr} = \frac{Q\, p_k}{Q\, p_0} = \frac{p_k}{p_0} \quad , \qquad (14)$$

where p_k is the probability that the neutron would be slowed down in the k-th history, p_0 is the probability that the neutron would be slowed down in any history (that it would be not captured during the slowing down process) and Q is the fast neutron source output.

Substituting (13) into (14) one obtains

$$L_s^2 = \sum_{k=1}^{M} L_{s,k}^2 \frac{p_k}{p_0} \quad . \qquad (15)$$

Let

$$L_{s,k}^2 = \sum_{i=1}^{N} A_i^k L_i^2 \quad , \qquad (16)$$

where $A_i^k = 1$ if the k-th slowing down history contains the i-th group,

$A_i^k = 0$ if the neutron undergoing the k-th slowing down history passes the i-th group by.

Transforming (15) according to the form of (16) one obtains finally a generalized formula which can be used to calculate L_s :

$$L_s^2 = \sum_{k=1}^{M} \left(\sum_{i=1}^{N} A_i^k L_i^2 \right) \frac{p_k}{p_o} = \sum_{i=1}^{N} L_i^2 \left(\sum_{k=1}^{M} A_i^k \frac{p_k}{p_o} \right) =$$

$$= \sum_{i=1}^{N} L_i^2 \frac{W_i}{p_o} \quad , \qquad (17)$$

where $W_i = \sum_{k=1}^{M} A_i^k p_k$ is the probability that the neutron during its slowing down history passes through the i-th group.

Let us turn our attention to the problem of determining W_i. The probability that the slowing down history includes the i-th energy group can be represented as the product of two probabilities : R_i- that the neutron emitted from the source enters the i-th group, and X_i- that the neutron belonging already to the i-th group will be slowed down. So,

$$W_i = R_i X_i \quad . \qquad (18)$$

Both the probabilities R_i and X_i can be expressed in terms of the appropriate macroscopic neutron cross sections :

$$R_i = \sum_{k=1}^{i-1} \frac{\Sigma_{sl(k,i)}}{\Sigma_{r,k}} R_k \quad , \quad X_i = \frac{1}{\Sigma_{r,i}} \sum_{k=i+1}^{N+1} \Sigma_{sl(i,k)} X_k \quad , \qquad (19)$$

where $R_1 = 1$ and $X_{k+1} = 1$.

From the argumentation presented above it follows immediately that $p_0 = W_1 = X_1$.

4. The procedure for calculation of L_s

4.1. Calculations of the values of neutron cross sections

The method for calculating the neutron slowing down length presented in the proceding chapter requires the knowledge of the appropriate set of averaged microscopic neutron cros sections $\sigma_{r,i}$, $\sigma_{tr,i}$, $\sigma_{al(i,k)}$ for all the elements which are to be met in the considered medium. Such a material has been gathered from the tables of neutron group constants of ARAGYAN et al [5]. These tables cover the neutron energy range fully adequate for the case of any neutron radioisotope source (from 0,215 eV to 10,5 MeV). The entire energy range is split up into 25 intervals. The schema of this energetic division is presented in Table I. The tables [5] contain the averaged values of the microscopic neutron cross sections relative to the fundamental neutron interactions, for different elements and for each energy group. The formulas for calculating some other microscopic cross sections, used in various multigroup calculation schemas, can be also found in the tables.

The microscopic neutron cross section, necessary for calculations of L_s, were calculated according to the following formulas :

$$\sigma_{r,i} = \sigma_{sl(e),i} + \sigma_{in,i} - \sigma_{in(i,i)} + \sigma_{c,i} \quad ,$$

$$\sigma_{tr,i} = \sigma_{e,i}(1-\mu_{e,i}) + \sigma_{in,i}(1-\mu_{in,i}) + \sigma_{c,i} \quad . \qquad (20)$$

$$\sigma_{sl(i,k)} = \sigma_{e(i,k)} + \sigma_{in(i,k)} \quad ,$$

where $\mu_{in,i} = \dfrac{\sum_{k=1}^{N} \sigma_{in(i,k)}\mu_{in(i,k)}}{\sigma_{in,i}}$ for the elements with $A < 20$ and $\mu_{in,i} = 0$ for the elements with $A \geq 20$; $\sigma_{sl(e),i}$, $\sigma_{in,i}$, $\sigma_{in(i,i)}$, $\sigma_{c,i}$; $\sigma_{e,i}$, $\sigma_{in(i,k)}$, $\sigma_{e(i,k)}$, $\mu_{e,i}$, $\mu_{in(i,k)}$ are the averaged values and they are to be found directly in the tables derived by ABAGYAN et al [5].

$\sigma_{e(i,k)}$ is the cross section for transfer from the i-th group to the k-th group caused by elastic scattering,

$\sigma_{sl(e),i}$ is the elastic slowing down cross section,

$\sigma_{in,i}$ is the inelastic scattering cross section,

$\sigma_{in(i,i)}$ is the inelastic scattering cross section at which the neutron remains in the group,

$\sigma_{c,i}$ is the capture cross section,

$\sigma_{e,i}$ is the elastic scattering cross section,

$\sigma_{in(i,k)}$ is the cross section of inelastic transfers from the i-th group to the k-th group,

$\mu_{e,i}$ is the average cosine of the elastic scattering angle,

$\mu_{in(i,k)}$ is the average value of cosine of inelastic scattering angle in a transfer from the i-th group to the k-th group.

On the basis of chemical analyses of different soils (cf references [2,6,7]) it has been assumed that in calculations of L_s values the following 13 elements should be taken into account :

H, B, C, N, O, Na, Mg, Al, Si, K, Ca, Ti, Fe .

The group values of the cross sections $\sigma_{r,i}$, $\sigma_{tr,i}$, $\sigma_{sl(i,k)}$ for the elements listed above has been stored with the help of the program FK01 (cf Appendix I), on a magnetic tape. These data have been used by the program FK03 (cf Appendix II) calculating L_s .
The macroscopic cross sections $\Sigma_{r,i}$, $\Sigma_{tr,i}$, $\Sigma_{sl(i,k)}$ for media of a given chemical composition and density have been calculated using the well known formula

$$\Sigma = \rho N_o \sum_{j=1}^{n} \frac{\nu_j}{A_j} \sigma_j \quad , \qquad (21)$$

where Σ is the macroscopic cross section, σ_j is the microscopic cross section for the j-th constituent element, A_j is the atomic mass of the j-th element, ν_j is the weight percentage of the j-th element, ρ is

the medium density, N_o is the Avogadro's number and n is the number of the constituent elements in the medium (in our case n = 13).

The values of σ_j were calculated directly from the formulas (20), without introducing any correction factors for self-shielding effects (this being equivalent to the assumption that the cross section relative to all the other constituent elements comprised in the medium is infinitely great in comparison with σ_j).

4.2. Calculation of L_s for monoenergetic neutron sources

The group values of the neutron cross sections prepared as described above, make possible to calculate the values of L_s for neutrons which initial energy does not overcome 10,5 MeV. Depending on neutron initial energy a different number of energy groups is used in the calculations of L_s, because the splitting into the energetic groups pattern never changes (cf Table I). For example , if the initial energy of neutrons is 4,0 MeV, the two highest groups drop out from the calculations and the L_s value is calculated according to the formula

$$L_s^2 = \sum_{i=3}^{25} \frac{W_i}{W_3} L_i^2 \quad . \qquad (22)$$

Table I

Division into the energetic groups used in the Abagyan's tables [5]

i	E_i	Δu_i
1	6,5 - 10,5 MeV	0,48
2	4,0 - 6,5 MeV	0,48
3	2,5 - 4,0 MeV	0,48
4	1,4 - 2,5 MeV	0,57
5	0,8 - 1,4 MeV	0,57
6	0,4 - 0,8 MeV	0,69
7	0,2 - 0,4 MeV	0,69
8	0,1 - 0,2 MeV	0,69
9	46,5 - 100 keV	0,77
10	21,5 - 46,5 keV	0,77
11	10 - 21,5 keV	0,77
12	4,65 - 10 keV	0,77
13	2,15 - 4,65 keV	0,77
14	1 - 2,15 keV	0,77
15	465 - 1000 eV	0,77
16	215 - 465 eV	0,77
17	100 - 215 eV	0,77
18	46,5 - 100 eV	0,77
19	21,5 - 46,5 eV	0,77
20	10 - 21,5 eV	0,77
21	4,65 - 10 eV	0,77
22	2,15 - 4,65 eV	0,77
23	1 - 2,15 eV	0,77
24	0,465 - 1 eV	0,77
25	0,215 - 0,465 eV	0,77

For the case when the initial energy of neutrons E_o does not correspond to any of the boundary values in the energy splitting pattern, the L_s has to be calculated twice for two boundary energies, one of them being smaller and the another being bigger than E_o ; the value $L_s\ E_o$ has to be found by means of the interpolation.

Let us assume that i_o is the number of the highest energy group which is being taken into account in calculation of L_s. The schema of this calculation is following: firstly, the macroscopic group cross sections $\Sigma_{r,i}$, $\Sigma_{tr,i}$ and $\Sigma_{sl(i,k)}$ have to be determined, according to the formula (21), for $i \geq i_o$, and than the group values of diffusion lengths L_i should be calculated according to the formula (7). The next step in the calculations consists in determination of the W_i, according to the formulae (18) and (19). Introducing the calculated values of values of L_i and W_i into the formula (17) one obtains the value of the slowing down length for monoenergetic neutrons

$$L_s^2 = \sum_{i=i_o}^{25} L_i^2 \frac{W_i}{W_{i_o}} \quad . \qquad (23)$$

4.3. Calculation of L_s for polienergetic neutron sources

Any radioisotope neutron source is characterized by continuous energy spectrum. This continuous spectrum $-\frac{\partial Q}{\partial E}$ can be approximated by a spectrum which contains a finite number of discrete energy values, so the condition

$$Q = \int_0^\infty -\frac{\partial Q}{\partial E}\, dE = \sum_{l=1}^{m} Q_l \qquad (24)$$

is fullfilled (Q_l being the partial neutron output, corresponding to the l-th energy value). Let us return once more to the definition (9) of $\langle r^2 \rangle$. Introducing the quantity $\Phi_l(r)$ which we shall call the partial neutron flux, originated by the neutron of the initial energy E_l, we have

$$\Phi(r) = \sum_{l=1}^{m} \Phi_l(r) \qquad . \qquad (25)$$

For each separate energy value in the discrete spectrum we candefine the value $\langle r_l^2 \rangle$:

$$\langle r_l^2 \rangle = \frac{\int_0^\infty r^2\, \Phi_l(r)\, 4\pi r^2\, dr}{\int_0^\infty \Phi_l(r)\, 4\pi r^2\, dr} \qquad . \qquad (26)$$

Substituting the relations (25) and (26) into (9) we obtain

$$\langle r^2 \rangle = \frac{\sum_{l=1}^{m} \langle r_l^2 \rangle \int_0^\infty \Phi_l(r)\, 4\pi r^2\, dr}{\int_0^\infty \Phi(r)\, 4\pi r^2\, dr} \quad . \qquad (27)$$

On the basis of an argumentation analogical to that used in the chapter 3 one can write

$$\int_0^\infty \Phi_l(r)\, 4\pi r^2\, dr \sim Q_l p_l \quad ,$$

where p_l is the probability that the neutron of the initial energy E_l has been slowed down (has not been captured).

From this one comes to the evident conclusion that

$$L_s^2 = \frac{\sum_{l=1}^{m} L_{s,l}^2\, Q_l p_l}{\sum_{l=1}^{m} Q_l p_l} \quad , \qquad (28)$$

where $L_{s,l}$ is the slowing down length of the monoenergetic neutrons of initial energy E_l.

The argumentation presented above establish the way of calculation of L_s for a neutron source of any energy spectrum. First of all the actual energy distribution has to be approximated by the sum of monoenergetic neutron sources each of them having the output Q_l and the initial energy being one of the boundary energies in Table I. In this paper it has been decided

to use the following set of initial energy values : 10,5 , 6,5 , 4,0 , 2,5 , 1,4 , 0,8 , 0,4 , 0,2 MeV. Then the values of $L_{s,l}$ have to be calculated, according to the procedure given in the section 4.2.. It can be easily seen that the probabilities p_l are equal to the quantities W_{l_o}, which one has to determine while caltulating $L_{s,l}$.

5. Results of calculations

The applicability of the presented procedure for calculating the neutron slowing down length has been tested in calculations of neutron slowing down length the medium being sandstone and limestone of different porosities, saturated with water. The choice of the media have been stimulated by the fact that they constitute most widely met compounds of soils and rocks and, on the other hand, only for these media some experimental data are available. The calculations were led both for the monoenergetic neutrons (of different energies) and for the radioisotope neutron sources Ra-Be and Pu-Be, for the final energy of slowed down neutrons equal to 0,215 eV. The continuous energy spectra of the isotope sources have been approximated with the discrete energy spectra listed in the Table II. The results of these calculations are presented in the Fig.2,3 and 4 and in the Table III and IV.

Table II

Discrete energy spectra of the radioisotope neutron sources Ra-Be and Pu-Be (the second and third column represent the neutron output in the relative units)

source / energy	Ra - Be *	Pu - Be **
10,5 MeV	0,04	0,07
6,5 MeV	0,27	0,32
4,0 MeV	0,32	0,28
2,5 MeV	0,22	0,15
1,4 MeV	0,12	0,08
0,8 MeV	0,02	0,06
0,4 MeV	0,01	0,03
0,2 MeV	0,00	0,01

* After the spectrum given in the Bak's paper [10].

** After the spectrum given in the Amaldi's paper [11].

Table III

Neutron slowing down length (assuming final energy of slowed down neutrons equal to 0,215 eV) for the water-saturated sandstone of different porosity, calculated for different initial energy of neutrons.

Porosity	L_s [cm]					
	10,5 MeV	6,5 MeV	4,0 MeV	2,5 MeV	Ra-Be	Pu-Be
0,00	30,16	30,35	29,54	28,68	29,19	29,19
0,01	26,31	26,48	25,56	24,58	25,18	25,18
0,02	23,95	24,08	23,08	22,02	22,68	22,69
0,03	22,30	22,39	21,32	20,20	20,92	20,93
0,05	20,11	20,11	18,94	17,72	18,53	18,55
0,10	17,20	16,98	15,65	14,31	15,27	15,31
0,15	15,65	15,23	13,80	12,40	13,46	13,53
0,20	14,64	14,04	12,55	11,11	12,24	12,33
0,25	13,92	13,16	11,63	10,17	11,35	11,46
0,30	13,37	12,47	10,90	9,43	10,66	10,78
0,35	12,93	11,91	10,31	8,84	10,10	10,23
0,40	12,57	11,44	9,82	8,34	9,64	9,77
0,45	12,28	11,04	9,40	7,93	9,24	9,39
0,50	12,02	10,69	9,03	7,57	8,91	9,06
0,70	11,32	9,66	7,96	6,51	7,92	8,10
1,00	10,77	8,73	6,97	5,56	7,04	7,26

Table IV

Neutron slowing down length (assuming final energy of slowed down neutron equal to 0,215 eV) for the water-saturated limestone of different porosity, calculated for different initial energy of neutrons.

Porosity	L_s [cm]					
	10,5 MeV	6,5 MeV	4,0 MeV	2,5 MeV	Ra-Be	Pu-Be
0,00	25,76	26,02	25,65	24,42	24,98	24,94
0,01	23,31	23,54	23,10	21,80	22,42	22,38
0,02	21,65	21,84	21,34	19,99	20,65	20,62
0,03	20,42	20,58	20,02	18,63	19,33	19,31
0,05	18,69	18,76	18,11	16,68	17,44	17,43
0,10	16,24	16,10	15,25	13,79	14,66	14,67
0,15	14,87	14,53	13,54	12,09	13,03	13,07
0,20	13,96	13,45	12,35	10,92	11,91	11,97
0,25	13,31	12,65	11,45	10,03	11,07	11,15
0,30	12,81	12,02	10,74	9,34	10,42	10,51
0,35	12,42	11,50	10,16	8,77	9,89	9,99
0,40	12,10	11,07	9,68	8,30	9,45	9,56
0,45	11,85	10,71	9,27	7,90	9,08	9,20
0,50	11,64	10,40	8,92	7,55	8,76	8,89
0,70	11,08	9,49	7,88	6,51	7,83	8,00
1,00	10,77	8,73	6,97	5,56	7,04	7,26

The results represent a material on the basis of which some dependences of the neutron slowing down length on the neutron energy, moisture content of the medium, and the chemical composition of the medium matrix can be investigated. As it follows from the obtained data, the slowing down length in media with a relatively low moisture content is not a monotonic function of the neutron energy, having a maximum in the region of the intermediate energies between 6 and 10 MeV. This is probably due to the increasing role of the inelastic scattering in the slowing down process at the higher neutron energies. It is worth to be noticed that the curves giving the dependence of L_s versus the moisture content for radioisotope sources intersect the respective curves for monoenergetic neutrons. Thus the postulation of KOZHEVNIKOV [12], concerning a proper way of taking into account the neutron energy spectrum in calculations of L_s for polyenergetic sources, is fully confirmed. The differences in the shape of the energy spectrum of isotope sources influence more strongly the values of L_s calculated for water or for fully water saturated media than the corresponding values of L_s for the case of dry media. This can be easily seen when one compares the values of L_s calculated for the same medium but for two different radioisotope sources (cf Tables III and IV). The chemical composition of the dry matrix of the medium

can influence the L_s value to a great extent, which is to seen in Fig.4.

For the case of water, dry sandstone and limestone a comparison between the theory and the experiment could have been done. For the media mentioned above the values of L_s have been calculated for the neutron energies chosen in such manner that they would match the neutron energies for which some experiments had been carried out. The comparison cannot however be a fully significant one as the energy spectra of the sources which had been used in the experiments are not exactly known, so the calculations could have been carried out actually for some different source spectra. The discrepancies between the theory and the experiment in the case of water are most probably due to that fact. On the other hand it seems that this effect does not manifest itself in a significant fashion in the case of dry SiO_2 and $CaCO_3$. All the experimental data together with the theoretical results are listed in Table V.

Finally, a comparison of the values of L_s, computed according to the method proposed in this paper with the values of L_s obtained with the help of other multigroup methods, developed by various authors, has been carried out. The corresponding data for the water-saturated sandstone of different porosity are presented in Table VI.

Table V

Comparison of the neutron slowing down length values calculated in this paper with the experimental data published by different authors. The L_s values (in cm) correspond to the slowing down to the final energy 1,46 eV (water) and 1,6 eV (SiO_2 and $CaCO_3$).

source	medium / Author	H_2O	SiO_2 $\rho=2,65 g/cm^3$	$CaCO_3$ $\rho=2,72 g/cm^3$
Ra-Be	J.Tittman	-	26,52±0,63	21,47±0,54
	B.Munn A.M.Pontecorvo *	6,80		
	D.Fieno *	6,96 7,09		
	H.L.Anderson *	6,93		
	J.H.Rush *	6,73		
	V.P.Duggal **	7,37		
	this paper ****	6,99	27,56	23,82
Pu-Be	F.Valente R.Sullivan ***	7,27±0,17		
	this paper ****	7,21		

* After AMALDI [11]

** After BEKURTS [1]

*** After KOZHEVNIKOV [12]

**** The values of L_s for the energies 1,46 eV and 1,6 eV have been obtained by means of the interpolation between the values of L_s calculated for 1,0 eV and 2,15 eV.

Table VI

Neutron slowing down length for water-saturated sandstone of different porosity, calculated with the help of multigroup methods developed by different authors (the column heads include author's name, type of the neutron source or the initial neutron energy and the final energy of slowed down neutrons). All the values of L_s are given in cm.

Porosity	ØLGAARD* 4,5 MeV $5kT_n$**	SEMMLER* 5,0 MeV 1,44 eV	GARDNER* 5,0 MeV 1,44 eV	this paper Ra-Be 0,215eV	this paper Pu-Be 0,215eV
0,00	33,20	38,90	42,10	29,19	29,19
0,01	29,09	34,4	37,2	25,18	25,18
0,03	24,75	28,8	31,1	20,92	20,93
0,05	22,24	25,4	27,4	18,53	18,55
0,10	17,64	20,2	21,8	15,27	15,31
0,15	15,39	17,3	18,7	13,46	13,53
0,20	13,81	15,3	16,5	12,24	12,33
0,30	11,75	12,8	13,8	10,66	10,78
0,50	8,807	10,0	10,8	8,91	9,06
1,00	7,16	6,98	7,63	7,04	7,26

* After Czubek [9].

** T_n is the thermal neutron temperature in the medium.

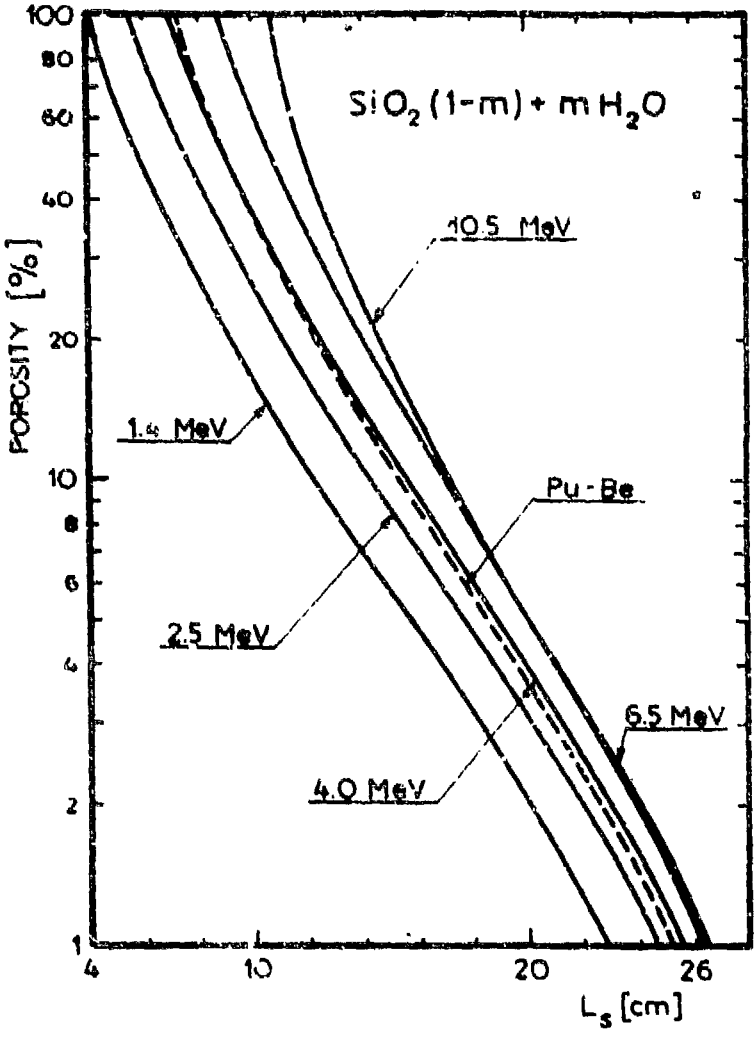

Fig.2. Slowing down lengths of neutrons in the water-saturated sandstone calculated for different initial energy of neutrons (assuming their final energy equal to 0,215 eV).

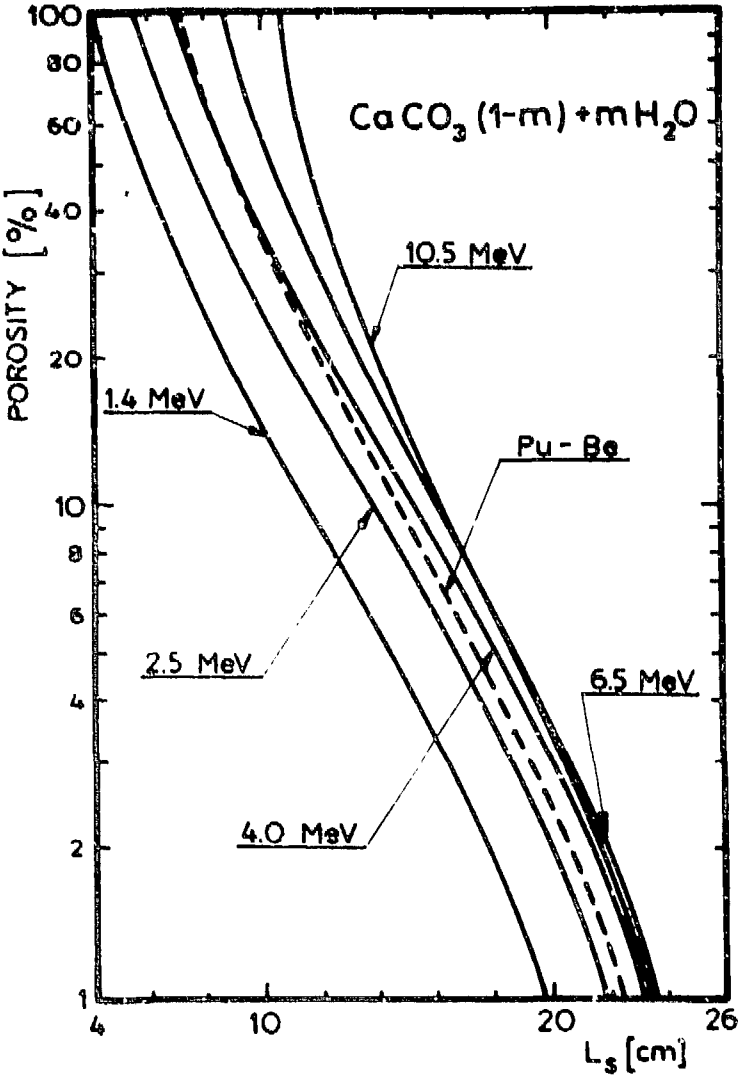

Fig.3. Slowing down lengths of neutrons in the water-saturated limestone calculated for different initial energy of neutrons (assuming their final energy equal to 0,215 eV).

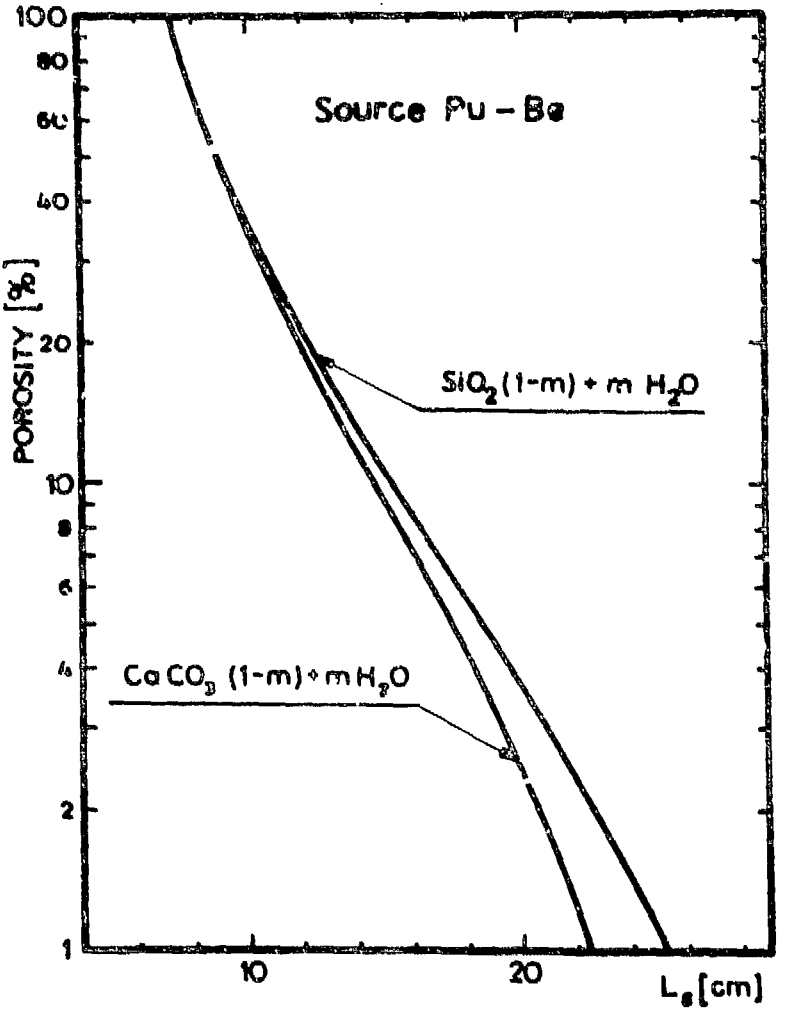

Fig.4. Slowing down lengths of neutrons in the water-saturated sandstone and limestone calculated for radio-isotope neutron source Pu-Be (assuming the final energy of neutrons equal to 0,215 eV).

6. Conclusions

The presented method of calculations of the neutron slowing down length allows to take into account all the fundamental phenomena which together form the slowing down process in rocks and soils, namely, the elastic scattering (including scattering on hydrogen nuclei), the inelastic scattering and the neutron absorption. The method can be succesfully used for any geological media for the whole range of neutron energies which one can meet in practice. The applicability of the method is affirmed by the excellent agreement of the computed L_s values with the experimental data both for water and some weakly slowing down media.

The procedure of computation of L_s in the case of a polyenergetic source allows for the energy spectrum of any radioisotope source. The eight-energies schema for approximating a continuous energy spectrum (arising from the energy pattern of the tables of neutron group constants) is perhaps not entirely adequate but it seems to approximate the energy spectra in a sufficient fashion (it should be noted that in practice an energy spectrum depends on the way the source has been manufactured and it is usually known only in an approximative way). One of the undeniable advantages of the method is thus its universality.

The results of the computations confirm a frequently met conclusion that when calculating the neutron slowing down length one cannot treat the polyenergetic source as monoenergetic one with an effective energy.
On the basis of the results obtained here one can form a conclusion that L_s is not always a monotonic function of the initial energy of neutrons. For the media containing small quantities of hydrogen and being at the same time the effective inelastic scatterers of neutrons the dependence of L_s versus the energy reaches a maximum in the 6-10 MeV energy region.

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Appendix I

Program FK01

The computer program FK01 is written in FORTRAN 1900 language. The program is used exclusively for creating the complete data system, which is used in the procedure for calculating L_s. The execution of the program requires no additional external data, all data being already stored in the program's internal structure. Running the program results in creating on a magnetic tape a file comprising properly ranged values of A, $\sigma_{tr,i}$, $\sigma_{r,i}$ $\sigma_{al(i,k)}$ for different chemical elements. Then the file can be used by the program FK03. The store space used by the program FK01 is 24946 words and the time needed for the execution of the program is very short. Program FK01 simplifies to a great extent handling the large set of the neutron group constants (about 10 000 numbers). Below the full listing of the program FK01 is given.

```
 LIST (LP)
 PROGRAM       (FK01)
 CREATE 3 = MT1(FK01)
 END
 MASTER DANE
 DIMENSION A(13),TR1(87),TR2(119),TR3(119),U1(87),U2(119),
1U3(119), S (13,25,25)
 DATA A/1.008, 10.82, 12.01, 14.01, 16.0 , 22.99, 24.32, 26.97,
1      28.086, 39.1 , 40.08, 47.9 , 55.85 /
 DATA TR1/  .4  ,  .907,  .852, 1.077,  .852, 1.3  , 1.204,
1  1.255,  1.215,  1.711,  1.773,  1.717,  1.744,   .55 ,   .95 ,
2  1.185,  1.268,  1.096,  1.475,  1.441,  1.388,  1.408,  1.896,
3  2.053,  1.922,  2.041,   .733,  1.176,  1.84 ,  1.345,  1.409,
4  1.82 ,  1.533,  1.739,  1.56 ,  2.224,  2.24 ,  2.218,  2.435,
5  1.   ,  1.735,  1.602,  1.703,  1.488,  2.177,  1.94 ,  2.1  ,
6  2.194,  2.189,  2.088,  2.322,  2.535,  1.365,  1.957,  2.218,
7  1.173,  3.956,  2.771,  2.084,  2.31 ,  2.304,  1.523,  2.624,
8  2.484,  2.297,  1.898,  2.539,  2.728,  2.183,  4.312,  4.151,
9  3.3  ,  3.201,  3.01 ,  1.557,  2.407,  2.302,  3.152,  2.697,
A  3.486,  3.68 ,  3.042,  3.686,  3.761,  6.88 ,  3.472,  4.929/
 DATA TR2/ 1.982,  2.639,  2.516,  2.757,  3.663,  3.627,  3.999,
1  3.896,  3.36 ,  3.649,  4.512,  4.891,  2.565,  2.337,  1.94 ,
2  3.014,  3.512,  4.662,  3.743,  4.23 ,  4.751,  3.408,  5.143,
3  8.245,  4.802,  1.94 ,  2.82 ,  1.47 ,  8.724,  5.138,  5.528,
4  3.931,  4.324,  5.797,  3.456,  4.174,  3.88 ,  7.174,  1.568,
5  2.692,  1.47 , 25.756, 14.227,  6.161,  4.288,  4.418,  6.842,
6  3.504,  4.851,  3.88 ,   .981,  1.764,  1.945,  1.47 , 60.41 ,
7  3.955,  6.427,  4.812,  4.418,  7.888,  3.552,  7.761,  3.395,
8  2.549,  2.45 ,  3.63 ,  1.862, 15.89 ,  8.32 ,  6.561,  5.534,
9  4.418,  8.356,  3.60 , 97.1  ,  3.395,  1.373,  2.45 ,  4.316,
A  2.45 , 23.786,  5.842,  6.661,  6.595,  4.418,  8.643,  3.6  ,
B  6.024,  3.395,  1.373,  2.45 ,  1.969,  2.842,  4.584,  7.422,
C  6.695,  8.076,  4.418,  8.932,  3.6  ,  3.206,  3.395,  1.373,
D  2.254,  1.973,  2.94 ,  4.292,  9.915,  6.73 , 10.076,  4.418,
E  9.023,  3.6  ,  3.013,  3.396,  1.374,  2.156,  1.978,  2.94 /
 DATA TR3/ 4.209, 10.918,  6.731, 13.451,  4.418,  9.144,  3.6  ,
1  3.014,  3.396,  1.375,  2.156,  1.987,  2.94 ,  4.234, 11.323,
2  6.766, 18.106,  4.418,  9.25 ,  3.6  ,  3.017,  3.396,  1.377,
```

```
3  2.156,  2.0  ,  2.94 ,  4.269, 11.339,  6.769, 24.756,  4.418,
4  9.362,  3.6  ,  3.022,  3.397,  1.379,  2.156,  2.018,  2.94 ,
5  4.32 , 11.358,  6.774, 35.016,  4.418,  9.387,  3.6  ,  3.029,
6  3.398,  1.382,  2.166,  2.045,  2.94 ,  4.397, 11.391,  6.78 ,
7 49.646,  4.418,  9.422,  3.6  ,  3.039,  3.399,  1.387,  2.166,
8  2.09 ,  2.94 ,  4.509, 11.44 ,  6.79 , 71.306,  4.418,  9.475,
9  3.6  ,  3.053,  3.401,  1.393,  2.176,  2.14 ,  2.94 ,  4.67 ,
A 11.506,  6.804,103.23 ,  4.418,  9.552,  3.6  ,  3.075,  3.404,
B  1.403,  2.176,  2.23 ,  2.94 ,  4.917, 11.616,  6.824,149.776,
C  4.419,  9.664,  3.6  ,  3.108,  3.408,  1.418,  2.186,  2.36 ,
D  2.94 ,  5.268, 11.776,  6.853,216.276,  4.419,  9.83 ,  3.6  ,
E  3.154,  3.414,  1.439,  2.206,  2.54 ,  2.94 ,  5.778, 12.006/
 DATA U1/  1.04 ,  .882,  .703,  .895,  .642,  .934,  1.01 ,
1  1.012,  .998,  1.276,  1.318,  1.349,  1.457,  1.36 ,  .615,
2  .644,  .762,  .46 ,  .795,  .902,  .871,  .953,  .936,
3  .982,  1.215,  1.317,  1.74 ,  .558,  .726,  .589,  .422,
4  .678,  .850,  .788,  .767,  .551,  .319,  .927,  1.066,
5  2.28 ,  .594,  .474,  .49 ,  .325,  .524,  .616,  .667,
6  .379,  .266,  .184,  .651,  .855,  3.12 ,  .654,  .653,
7  .454,  .868,  .720,  .314,  .412,  .288,  .160,  .231,
8  .323,  .484,  4.13 ,  .747,  .665,  .488,  .779,  .640,
9  .4  ,  .343,  .31 ,  .119,  .175,  .151,  .167,  5.87 ,
A  1.05 ,  .899,  .633,  .688,  .476,  .834,  .372,  .508/
 DATA U2/  .151,  .192,  .156,  .148,  7.97 ,  1.18 ,  .972,
1  .809,  .609,  .461,  .547,  .526,  .265,  .18 ,  .141,
2  .188,  .187,  9.77 ,  1.23 ,  .923,  .884,  .553,  .584,
3  .899,  .462,  .18 ,  .197,  .096,  .084,  .245, 11.6  ,
4  1.447,  .944,  1.082,  .561,  .474,  .421,  .693,  .146,
5  .193,  .096,  1.408,  .674, 12.9 ,  1.765,  .964,  1.272,
6  .569,  .550,  .421,  .095,  .163,  .135,  .096,  3.28 ,
7  .188, 13.5 ,  2.254,  .964,  1.473,  .577,  .879,  .369,
8  .298,  .227,  .273,  .121,  .904,  .389, 13.801,  2.933,
9  .964,  1.555,  .584, 11.07 ,  .369,  .132,  .227,  .475,
A  .16 ,  1.87 ,  .281, 14.001,  3.968,  .964,  1.618,  .584,
B  .69 ,  .369,  .132,  .227,  .14 ,  .185,  .276,  .445,
C 14.002,  5.435,  .964,  1.671,  .584,  .367,  .369,  .132,
D  .208,  .144,  .192,  .265,  .474, 14.103,  7.606,  .964,
E  1.697,  .584,  .346,  .37 ,  .133,  .199,  .149,  .192/
```

```
 DATA U3/   .275,   532, 14.104, 10.798,   .964,1.724,.584,.347,
1   .37 ,   .134,   .199,   .158,   .192,   .3  ,   .56 , 14.206,
2 15.452,   .964, 1.745,   .584,   .35 ,   .37 ,   .136,   .199,
3   .171,   .202,   .335,   .576, 14.209, 22.103,   .964, 1.782,
4   .584,   .355,   .371,   .138,   .199,   .189,   .202,   .386,
5   .595, 14.214, 32.363,   .964, 1.807,   .584,   .362,   .372,
6   .141,   .2  ,   .216,   .212,   .463,   .628, 14.22 , 46.992,
7   .964, 1.842,   .584,   .372,   .373,   .146,   .2  ,   .261,
8   .222,   .575,   .677, 14.23 , 68.652,   .964, 1.895,   .584,
9   .386,   .375,   .152,   .201,   .311,   .232,   .736,   .743,
A 14.244,100.721,   .964, 1.972,   .584,   .408,   .378,   .162,
B   .201,   .401,   .252,   .983,   .853, 14.264,147.12 ,   .965,
C  2.084,   .584,   .441,   .382,   .177,   .202,   .531,   .272,
D  1.334,  1.013, 14.293,213.62 ,   .965,  2.25 ,   .584,   .487,
E   .388,   .198,   .204,   .711,   .312,  1.844,  1.243 /
  DO 1 I=1,25
  DO 1 K=1,25
  DO 1 L=1,13
1 S(L,K,I)=0.
  S(1,1,1),S(2,1,1),S(7,6,5),S(13,1,3)=0.4
  S(1,1,2),S(8,12,12),S(10,3,3),S(10,12,12)=0.24
  S(1,1,3),S(10,8,8),S(10,10,10),S(11,1,1),S(11,6,6)=.176
  S(1,1,4),S(11,9,9),S(11,10,10),S(11,11,11)=.096
  S(1,1,5)=.064
  S(1,1,6),S(1,11,18),S(2,2,4)=.032
  S(1,1,7),S(1,7,14),S(1,12,20),S(1,13,21),S(1,14,22)=.016
  S(1,15,23),S(1,16,24),S(2,2,6)=.016
  S(1,1,8),S(1,3,10),S(1,4,11),S(1,8,16)=.009
  S(1,1,9),S(1,2,10),S(1,3,11),S(1,4,12),S(1,5,13)=.004
  S(1,5,14),S(1,6,15),S(1,7,16)=.004
  S(1,1,10),S(1,2,11),S(1,4,13)=.003
  S(1,2,2),S(6,4,4)=.512
  S(1,2,3),S(7,4,4)=.375
  S(1,2,4),S(10,1,1)=.205
  S(1,2,5)=.136
  S(1,2,6)=.068
  S(1,2,7),S(1,13,20)=.034
  S(1,2,8),S(1,3,9)=.018
```

S(1,2,9),S(1,8,17),S(2,3,6)=.008
S(1,3,3),S(2,10,10)=.767
S(1,3,4),S(5,3,3)=.418
S(1,3,5),S(12,2,3)=.279
S(1,3,6),S(7,1,2),S(13,3,4)=.139
S(1,3,7),S(3,1,2),S(4,1,4),S(8,2,5),S(2,1,2),S(12,2,6)=.07
S(1,3,8)=.037
S(1,3,12)=.002
S(1,4,4)=.975
S(1,4,5),S(2,6,6),S(3,5,5)=.65
S(1,4,6),S(5,4,4)=.325
S(1,4,7),S(1,16,21),S(1,17,22),S(9,11,11),S(12,1,1)=.163
S(1,4,8)=.087
S(1,4,9),S(6,3,6),S(8,1,6),S(8,3,6),S(9,3,5),S(2,2,3)=.04
S(10,1,6),S(10,2,6),S(12,5,7),S(13,1,7)=.04
S(1,4,10)=.019
S(1,5,5)=1.558
S(1,5,6),S(5,6,6),S(2,11,11)=.779
S(1,5,7),S(4,3,3)=.389
S(1,5,8),S(9,15,15)=.209
S(1,5,9)=.097
S(1,5,10)=.045
S(1,5,11)=.021
S(1,5,12),S(1,9,18),S(2,3,4),S(2,3,5),S(3,1,6),S(3,2,6)=.01
S(4,2,3),S(5,1,6),S(6,1,7),S(6,2,7),S(6,3,7),S(6,4,6),S(6,6,8)=.01
S(7,1,7),S(7,2,7),S(7,4,7),S(8,1,7),S(8,2,7),S(8,3,7),S(8,5,8)=.01
S(9,1,7),S(9,2,7),S(9,4,7),S(10,1,7),S(10,2,7),S(11,3,7)=.01
S(12,2,8),S(12,3,6),S(13,1,8),S(13,2,8),S(13,4,7),S(13,5,8)=.01
S(1,6,6)=2.065
S(1,6,7)=1.033
S(1,6,8),S(5,9,9),S(7,3,3)=.553
S(1,6,9)=.257
S(1,6,10),S(2,1,3),S(6,1,2)=.119
S(1,6,11)=.055
S(1,6,12)=.026
S(1,6,13)=.012
S(1,6,14)=.006
S(1,7,7)=2.936
S(1,7,8)=1.572

```
S(1,7,9)=.732
S(1,7,10),S(13,1,4),S(13,14,14)=.338
S(1,7,11)=.157
S(1,7,12)=.073
S(1,7,13),S(2,2,5)=.034
S(1,7,15)=.007
S(1,8,8)=4.27
S(1,8,9)=1.984
S(1,8,10)=.919
S(1,8,11),S(4,5,5)=.427
S(1,8,12),S(10,4,4)=.199
S(1,8,13),S(2,1,4)=.092
S(1,8,14)=.043
S(1,8,15),S(3,1,5),S(3,2,5),S(4,1,6),S(5,1,2),S(10,3,6)=.02
S(11,1,7),S(11,2,7),S(11,3,5),S(11,3,6),S(12,1,7),S(12,4,6)=.02
S(12,5,8)=.02
S(1,9,9)=5.738
S(1,9,10)=2.43
S(1,9,11)=1.128
S(1,9,12),S(8,8,8)=.524
S(1,9,13)=.243
S(1,9,14)=.113
S(1,9,15)=.052
S(1,9,16),S(2,1,6)=.024
S(1,9,17),S(1,10,19)=.011
S(1,10,10)=6.21
S(1,10,11)=2.882
S(1,10,12)=1.338
S(1,10,13)=.621
S(1,10,14),S(9,5,5),S(11,1,4),S(12,1,4)=.288
S(1,10,15),S(10,5,5)=.134
S(1,10,16),S(2,1,5)=.062
S(1,10,17)=.029
S(1,10,18),S(1,11,20),S(1,12,21)=.013
S(1,11,11)=6.921
S(1,11,12)=3.212
S(1,11,13)=1.491
S(1,11,14),S(6,5,5)=.692
S(1,11,15)=.321
```

S(1,11,16),S(6,1,4),S(8,1,2),S(10,1,2),S(10,2,4),S(13,5,6)=.149
S(12,7,7)=.149
S(1,11,19)=.015
S(1,12,12)=7.22
S(1,12,13)=3.35
S(1,12,14),S(4,13,13)=1.556
S(1,12,15)=.722
S(1,12,16)=.335
S(1,12,17)=.156
S(1,12,18)=.072
S(1,12,19)=.033
S(1,13,13)=7.369
S(1,13,14)=3.42
S(1,13,15)=1.588
S(1,13,16)=.737
S(1,13,17),S(8,6,6)=.342
S(1,13,18),S(7,1,4),S(12,4,5)=.159
S(1,13,19)=.074
S(1,13,22),S(1,14,23),S(1,15,24),S(1,16,25)=.014
S(1,14,14)=7.482
S(1,14,15)=3.472
S(1,14,16),S(4,14,14)=1.612
S(1,14,17)=.748
S(1,14,18)=.347
S(1,14,19),S(10,2,3),S(11,13,13)=.161
S(1,14,20),S(1,15,21)=.075
S(1,14,21),S(1,15,22),S(1,16,23),S(1,17,24)=.035
S(1,15,15)=7.52
S(1,15,16)=3.489
S(1,15,17)=1.62
S(1,15,18)=.752
S(1,15,19)=.349
S(1,15,20),S(13,6,6)=.162
S(1,16,16),S(1,17,17)=7.557
S(1,16,17),S(1,17,18)=3.506
S(1,16,18),S(1,17,19)=1.628
S(1,16,19),S(1,17,20)=.756
S(1,16,20),S(1,17,21),S(3,1,1)=.351
S(1,16,22),S(1,17,23),S(1,18,24)=.076

```
S(1,17,25),S(4,2,6),S(5,1,5),S(6,1,6),S(6,2,6),S(6,5,6)=.03
S(6,6,7),S(7,1,6),S(7,2,6),S(7,3,5),S(8,2,6),S(8,5,7),S(9,1,6)=.03
S(9,2,6),S(9,4,6),S(12,2,7),S(12,3,5),S(13,2,7),S(13,3,7)=.03
S(13,4,6)=.03
S(1,18,18),S(1,19,19),S(1,20,20),S(1,21,21),S(1,22,22)=7.594
S(1,23,23),S(1,24,24)=7.594
S(1,18,19),S(1,19,20),S(1,20,21),S(1,21,22),S(1,22,23)=3.524
S(1,23,24)=3.524
S(1,18,20),S(1,19,21),S(1,20,22),S(1,21,23),S(1,22,24)=1.636
S(1,18,21),S(1,19,22),S(1,20,23),S(1,21,24)=.759
S(1,18,22),S(1,19,23),S(1,20,24)=.352
S(1,18,23),S(1,19,24)=.164
S(1,18,25)=.066
S(1,19,25)=.142
S(1,20,25)=.306
S(1,21,25)=.658
S(1,22,25)=1.417
S(1,23,25)=3.053
S(1,24,25)=6.577
S(1,25,25)=14.171
S(3,1,4),S(3,2,4),S(4,1,5),S(4,2,4),S(5,1,3),S(5,1,4),S(5,2,5)=.05
S(13,3,6),S(13,5,7)=.05
S(7,2,5),S(7,4,6),S(8,5,6),S(10,3,5),S(11,1,5),S(12,1,6)=.06
S(4,2,5)=.06
S(4,1,2),S(6,1,5),S(9,1,5),S(9,2,4),S(9,5,7),S(10,3,4)=.08
S(11,2,6),S(12,5,6),S(13,1,6)=.08
S(6,2,5),S(6,3,4),S(6,3,5),S(7,1,5)=.09
S(7,2,4),S(8,1,5),S(8,3,5),S(10,1,5),S(10,2,5),S(13,2,6)=.1
S(4,1,3),S(8,2,4),S(9,1,2),S(12,3,4)=.1
S(7,2,3),S(9,1,4),S(8,3,4),S(10,11,11),S(13,3,5)=.11
S(6,2,3),S(7,4,5),S(11,2,3)=.12
S(8,1,4),S(9,1,3),S(11,1,5),S(12,2,5),S(12,5,7),S(12,8,8)=.18
S(13,8,8)=.18
S(3,1,3),S(6,1,3),S(7,1,3)=.2
S(9,2,3),S(11,1,2),S(11,2,5),S(12,1,2)=.21
S(8,2,3),S(13,1,5)=.22
S(8,1,3),S(10,1,3),S(12,14,14),S(13,4,5)=.25
S(7,3,4),S(12,2,4),S(13,5,5),S(13,13,13)=.27
S(11,1,3),S(12,1,3),S(6,15,15),S(6,2,2)=.36
```

```
      S(8,4,5),S(10,1,4),S(10,9,9),S(13,1,2)=.19
      S(2,2,2)=.436
      S(2,3,3)=.482
      S(2,4,4)=.537
      S(2,5,5)=.612
      S(2,7,7)=.879
      S(2,8,8)=.876
      S(2,9,9)=.774
      S(2,12,12)=.791
      S(2,13,13)=.805
      S(2,14,14)=.814
      S(2,15,15)=.818
      DO 2 I=16,25
    2 S(2,I,I)=.823
      S(3,2,2)=.544
      S(3,3,3)=.726
      S(3,4,4)=.474
      S(3,6,6)=.665
      S(3,7,7)=.899
      S(3,8,8)=.972
      S(3,9,9)=.923
      S(3,10,10)=.944
      DO 3 K=11,25
    3 S(3,K,K)=.964
      S(4,1,1)=.315
      S(4,2,2)=.392
      S(4,4,4)=.405
      S(4,6,6)=.443
      S(4,7,7)=.631
      S(4,8,8)=.808
      S(4,9,9)=.883
      S(4,10,10)=1.08
      S(4,11,11)=1.27
      S(4,12,12)=1.47
      S(4,15,15)=1.66
      S(4,16,16)=1.68
      S(4,17,17),S(4,18,18)=1.7
      DO 4 K=19,25
    4 S(4,K,K)=1.73
```

```
      S(5,1,1)=.252
      S(5,2,2)=.385
      S(5,5,5)=.868
      S(5,7,7)=.688
      S(5,8,8)=.609
      S(5,10,10)=.561
      S(5,11,11)=.569
      S(5,12,12)=.577
      DO 5 K=13,25
    5 S(5,K,K)=.584
      S(6,1,1)=.294
      S(6,3,3)=.448
      S(6,6,6)=.6
      S(6,7,7)=.475
      S(6,8,8)=.46
      S(6,9,9)=.582
      S(6,10,10)=.472
      S(6,11,11)=.549
      S(6,12,12)=.878
      S(6,13,13)=10.97
      S(6,14,14)=.68
      DO 6 K=16,25
    6 S(6,K,K)=.34
      S(7,1,1)=.32
      S(7,2,2)=.569
      S(7,5,5)=.314
      S(7,7,7)=.834
      S(7,8,8)=.547
      S(7,9,9)=.895
      S(7,10,10)=.421
      S(7,11,11)=.421
      DO 7 K=12,25
    7 S(7,K,K)=.369
      S(8,1,1)=.187
      S(8,2,2)=.408
      S(8,3,3)=.506
      S(8,4,4)=.477
      S(8,5,5)=.312
      S(8,7,7)=.371
```

```
      S(8,9,9)=.46
      S(8,10,10)=.687
      S(8,11,11)=.094
      DO 8 K=13,25
    8 S(8,K,K)=.131
      S(9,1,1)=.158
      S(9,2,2)=.543
      S(9,3,3)=.467
      S(9,3,4)=.26
      S(9,4,4)=.259
      S(9,6,6)=.31
      S(9,7,7)=.508
      S(9,8,8)=.265
      S(9,9,9)=.179
      S(9,10,10)=.145
      DO 9 K=12,14
    9 S(9,K,K)=.227
      DO 10 K=16,25
   10 S(9,K,K)=.199
      S(10,2,2)=.236
      S(10,6,6)=.116
      S(10,7,7)=.147
      S(10,13,13)=.275
      DO 11 K=14,25
   11 S(10,K,K)=.131
      S(11,2,2)=.232
      S(11,2,4)=.26
      S(11,3,3)=.269
      S(11,4,4)=.184
      S(11,5,5)=.231
      S(11,7,7)=.192
      S(11,8,8)=.141
      S(11,12,12)=.121
      S(11,14,14)=.185
      DO 12 K=15,25
   12 S(11,K,K)=.192
      S(12,2,2)=.365
      S(12,3,3)=.774
```

```
      S(12,4,4)=.469
      S(12,6,6)=.147
      S(12,9,9)=.47
      S(12,10,10)=1.39
      S(12,11,11)=3.26
      S(12,12,12)=.854
      S(12,13,13)=1.25
      S(12,15,15)=.23
      DO 13 K=16,25
   13 S(12,K,K)=.224
      S(13,1,1)=.141
      S(13,2,2)=.302
      S(13,2,3)=.31
      S(13,2,4)=.33
      S(13,2,5)=.23
      S(13,3,3)=.714
      S(13,4,4)=.562
      S(13,7,7)=.142
      S(13,9,9)=.238
      S(13,10,10)=.657
      S(13,11,11)=.183
      S(13,12,12)=.385
      S(13,15,15)=.459
      DO 14 K=16,25
   14 S(13,K,K)=.523
      WRITE(3) A,TR1,TR2,TR3,U1,U2,U3,S
      ENDFILE 3
      STOP
      END
      FINISH
```

Appendix II

Program FKO3

The computer program FKO3 is written in FORTRAN 1900 language. It is used for computing the neutron slowing down length, according to the procedure presented in chapter 3 of this report. The program makes use of the data file which has to be created with the help of the program FKO1. The other data which should be also given at the input are: the chemical composition and the density of the medium and the energy spectrum of the neutron source. The chemical composition is to be determined by giving the values of wight percentages of the following elements:

H, B, C, N, O, Na, Mg, Al, Si, K, Ca, Ti, Fe .

The neutron energy spectrum is to be given as it is shown in Table II. The program structure allows to calculate the values of L_s for given chemical composition of the dry matrix and for 16 different values of moisture content. The latter has to be expressed in units of g/cm^3. The mixture of the dry skeleton with water can be realized in two different ways. First of them consists in changing the density of the dry skeleton and assuming the complete saturation of the medium with water; in the second case the density of the dry skeleton is kept constant whereas the moisture content varies.

All those data have to be introduced on punched cards in the following sequence :

1. 14 numbers in FORMAT(7F8.4). The first quantity is the density of the dry skeleton (in the first case it is the mineralogical density), the following 13 quantities are the weight percentages of the constituent elements.

2. 24 numbers in FORMAT(8F5.2). The first 8 quantities determine the neutron source energy spectrum, the following 16 numbers are the moisture content values which can be choosen in an arbitrary way.

3. One integer number in FORMAT(I1). If it is equal to 1 the first variant of the program will be executed, if the number is equal to 3 the second variant will be executed.

As the output of the program FKO3 the values of the neutron slowing down length (in cm), are obtained together with the neutron slowing down probabilities for the given dry skeleton and for the different values of moisture content.

The store space used is 26052 words. The execution time for one complete set of intial data is about 4 minutes for ODRA 1304 computer.

Below the full listing of the program FKO3 is given.

```
   LIST (LP)
   PROGRAM (FK03)
   INPUT 1=MT1(FK01)
   INPUT 2=CR1
   OUTPUT 3=LP1
   END
   MASTER SPOWALNIANIE
   INTEGER SWITCH
   REAL MPTR,MPU,MPSP
   DIMENSION A(13),COMP(13),PTR(13,25),PU(13,25),PSP(13,25,25),W(16),
  1MPTR(25),MPU(25),MPSP(25,25),P1(25),P2(26),WIEK(8),Q(8),TR(25),
  2DLS(16),H2OTR(25),H2OU(25),H2OS(25,25),P(8),DL(8),U(25),SP(25,25),
  3ESC(16)
   REWIND 1
   READ(1)A,PTR,PU,PSP
55 READ(2,1)HO,COMP
   READ(2,2)Q,W
   READ(2,13) SWITCH
13 FORMAT(I1)
 1 FORMAT(7F8.4)
 2 FORMAT(8F5.2)
   DO 3 K=1,13
 3 COMP(K)=COMP(K)*0.6023*HO/A(K)
   CH=0.6023*0.1119/A(1)
   CO=0.6023*0.8881/A(5)
   DO 5 J=1,25
   H2OTR(J)=CH*PTR(1,J)+CO*PTR(5,J)
   H2OU(J)=CH*PU(1,J)+CO*PU(5,J)
   MPTR(J)=0.
   MPU(J)=0.
   DO 4 K=1,13
   MPTR(J)=MPTR(J)+COMP(K)*PTR(K,J)
 4 MPU(J)=MPU(J)+COMP(K)*PU(K,J)
   DO 5 K=1,J
   MPSP(K,J)=0.
   H2OS(K,J)=CH*PSP(1,K,J)+CO*PSP(5,K,J)
   DO 5 L=1,13
 5 MPSP(K,J)=MPSP(K,J)+COMP(L)*PSP(L,K,J)
```

```
      DO 12 M=1,16
      IF(SWITCH-2)15,15,16
   15 DO 20 J=1,25
      TR(J)=MPTR(J)*(1.-W(M))+H2OTR(J)*W(M)
      U(J)=MPU(J)*(1.-W(M))+H2OU(J)*W(M)
      DO 20 K=1,J
   20 SP(K,J)=MPSP(K,J)*(1.-W(M))+H2OS(K,J)*W(M
      GO TO 25
   16 DO 21 J=1,25
      TR(J)=MPTR(J)+H2OTR(J)*W(M)
      U(J)=MPU(J)+H2OU(J)*W(M)
      DO 21 K=1,J
   21 SP(K,J)=MPSP(K,J)+H2OS(K,J)*W(M)
   25 AGE=0.
      WAGA=0.
      DO 11 N=1,8
      P1(N)=1.
      DO 6 J=N+1,N+10
      P1(J)=0.
      DO 6 K=N,J-1
    6 P1(J)=P1(J)+P1(K)*SP(K,J-1)/U(K)
      DO 7 J=N+11,25
      P1(J)=0.
      DO 7 K=J-10,J-1
    7 P1(J)=P1(J)+P1(K)*SP(K,J-1)/U(K)
      P2(26)=1.0
      DO 8 J=2,10
      P2(27-J)=0.
      DO 8 K=1,J-1
    8 P2(27-J)=P2(27-J)+P2(27-K)*SP(27-J,26-K)/U(27-J)
      DO 9 J=11,27-N
      P2(27-J)=0.
      DO 9 K=J-10,J-1
    9 P2(27-J)=P2(27-J)+P2(27-K)*SP(27-J,26-K)/U(27-J)
```

```
      P(N)=P2(N)
      WIEK(N)=0.
      DO10J=N,25
   10 WIEK(N)=WIEK(N)+P1(J)*P2(J)/(3.*TH(J)*U(J)*P2(N))
      DL(N)=SQRT(WIEK(N))
      WAGA=WAGA+Q(N)*P2(N)
   11 AGE=AGE+WIEK(N)*Q(N)*P2(N)
      ESC(M)=WAGA
      AGE=AGE/WAGA
   12 DLS(M)=SQRT(AGE)
      WRITE(3,100)
  100 FORMAT(////1H ,1HW,15X,2HLS,15X,1HP//)
      WRITE(3,101)(W(K),DLS(K),ESC(K),K=1,16)
  101 FORMAT(/1H ,F5.2,8X,F10.4,8X,F10.4)
      GO TO 55
      STOP
      END
      FINISH
```