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**EXISTENCE PROOFS
FOR WIGHTMAN TYPE FUNCTIONALS**

1973

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ТЕОРЕТИЧЕСКОЙ ФИЗИКИ**

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**EXISTENCE PROOFS
FOR WIGHTMAN TYPE FUNCTIONALS**

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1. Introduction

In the algebraic description of a quantum field, first given in [1,8], the Wightmann functions $W_n(x_1, \dots, x_n)$ are collected to one functional $W(f)$ on the algebra $\hat{U} = \hat{U}_0 + i\hat{U}_0$ of test-functions. Hence the Wightmann functional is uniquely determined by the hyperplane $E_w = \ker W$, the kernel of W . The positivity of the Wightmann functional W leads to the property that the cone K of positive elements of \hat{U}_0 lies on one side of E_w . The other properties of W , the locality, the spectrality and the invariance lead to the fact that E_w contains a certain linear space L , the so-called Wightmann kernel.

In this way the existence problem for quantum fields is transformed to the problem of finding all hyperplanes E_w in \hat{U}_0 with the properties mentioned before. Therefore to prove the existence of Wightman (type) functionals with certain properties one can apply the well-known separation theorems for convex sets in locally convex spaces. This method was outlined already in [6,7], where it was proved the existence of certain functionals with the property of positivity only. Further results in this direction were obtained in [2,3], where the existence of certain Lorentz invariant positive functionals could be proved. Recently in [9] a general extension theorem for linear functionals on a subspace M of \hat{U}_0 to a Wightman (type) functional on \hat{U}_0 was proved. In this paper we describe a method to prove the existence of fields with certain properties by applying this general extension theorem. We give a simple proof on the existence of a local invariant quantum

field (without spectrality) different from a generalized free field. We are convinced of the possibility to prove in analogous way also the existence of quantum fields with more difficult properties (non-trivial fields).

2. Geometric Interpretation of the Wightman Functionals

A quantum field $\phi(x)$ (neutral, scalar) is uniquely determined by the Wightman functional

$$W(f) = \sum_{n \geq 0} \int W_n(x_1, \dots, x_n) f_n(x_1, \dots, x_n) dx_1 \dots dx_n \quad (1)$$

$$W_n(x_1, \dots, x_n) = \langle 0 | \phi(x_1) \dots \phi(x_n) | 0 \rangle,$$

$$f_n(x_1, \dots, x_n) \in \mathcal{S}_n = \mathcal{S}(\mathbb{R}^{4n}),$$

where

$$f = \{ f_0, f_1(x_1), \dots, f_n(x_1, \dots, x_n), 0, 0 \dots \}$$

is an element of the field algebra

$$\mathcal{A} = \mathcal{S}_0 \oplus \mathcal{S}_1 \oplus \mathcal{S}_2 \oplus \dots, \quad \mathcal{S}_0 = \mathbb{C}. \quad (2)$$

\mathcal{A} is the tensor algebra over the Schwartz space $\mathcal{S}_1 = \mathcal{S}(\mathbb{R}^4)$. \mathcal{A} is a locally convex topological *-algebra. The involution is defined by

$$(f_n^*)(x_1, \dots, x_n) = \overline{f_n(x_n, \dots, x_1)}, \quad (3)$$

the multiplication by

$$(f \cdot g)_n(x_1, \dots, x_n) = \sum_{\ell+k=n} f_\ell(x_1, \dots, x_\ell) g_k(x_{\ell+1}, \dots, x_n) \quad (4)$$

and the topology τ by the system of seminorms

$$\tau: \|f\|_{(\gamma_n)(\nu_n)} = \sum_{n \geq 0} \gamma_n \|f_n\|_{\nu_n}, \quad (5)$$

where (γ_n) is an arbitrary sequence of positive numbers, (ν_n) an arbitrary sequence of nonnegative integers and $\|f_n\|_\nu$, $\nu=0,1,2,\dots$, the denumerable system of seminorms defining the topology of \mathcal{S}_n . Some properties of the topology τ and of another important topology τ_∞ in \mathcal{G} are investigated in [4,5].

The hermitean part \mathcal{G}_0 of the field algebra \mathcal{G} is given by

$$\mathcal{G}_0 = \{f \in \mathcal{G} ; f^* = f\}. \quad (6)$$

By K_0 we denote the cone of positive elements generated by $f^* \cdot f$, $f \in \mathcal{G}$, and by $K = \bar{K}_0$ its topological closure. K is also a cone [6].

The Wightman functional $W(f)$ is characterised by the following properties [1,8].

1. Positivity

$$W(f) \geq 0 \quad \text{for } f \in K \quad (7)$$

and $W(1)=1$, where $1 = \{1,0,0,\dots\}$ is the unity element of \mathcal{G} . In [9] it is proved that the continuity of W is a consequence of the positivity.

2. Locality

$$W(f) = 0 \quad \text{for } f \in I_{loc}, \quad (8)$$

where I_{loc} is the two-sided ideal generated by the elements $f_n(x_1, \dots, x_n) = g_n(\dots, x_j, x_{j+1}, \dots) - g_n(\dots, x_{j+1}, x_j, \dots)$

with $g_n(x_1, \dots, x_n) = 0$ for $(x_{j+1} - x_j)^2 \geq 0$.

3. Spectrality

$$W(f) = 0 \quad \text{for } f \in I_{sp}, \quad (9)$$

where I_{sp} is the linear space generated by $f_n(x_1, \dots, x_n)$

with $\tilde{f}_n(p_1, \dots, p_n) = \frac{1}{(2\pi)^{2n}} \int e^{i \sum p_i x_i} f_n(x_1, \dots, x_n) dx_1 \dots dx_n = 0$

for $\sum_{\ell=1}^k p_\ell \in \overline{V^+}$ (the closed forward cone), $\sum_{\ell=1}^n p_\ell = 0$,

$k=1, 2, \dots, n-1$,

4. Poincare invariance

$$W(f) = 0 \quad \text{for } f \in I_{inv}, \quad (10)$$

where I_{inv} is the linear space generated by the elements

$$\begin{aligned} f_n(x_1, \dots, x_n) &= g_n(x_1, \dots, x_n) - g_n(\Lambda^{-1}(x_1 - a), \dots, \Lambda^{-1}(x_n - a)) \\ &= g_n - (\Lambda, a)g_n. \end{aligned}$$

Since $W(f)$ is a positive functional, it is hermitean and therefore a real positive functional on the semiordered space \hat{G}_0 with the closed cone K .

Let $I = \overline{\mathcal{L}(I_{loc}, I_{sp}, I_{inv})}$ be the closed linear subspace generated by these three linear spaces. Since $f^* \in I$ if $f \in I$, the linear space I has the decomposition

$$I = L + iL, \quad (11)$$

where L is a real subspace of \hat{G}_0 . We call I respectively L the Wightman kernel. Collecting all mentioned properties we can say that a Wightman functional $W(f)$ is a (real) linear functional on the real linear space \hat{G}_0 , satisfying the following properties.

Wightman functional

- a) $W(f) \geq 0$ for $f \in K$
- b) $W(f) = 0$ for $f \in L$
- c) $W(1) = 1$

(12)

Let be $E_w = \ker W = \{f \in \mathcal{G}_0; W(f) = 0\}$.

Then E_w is a hyperplane in \mathcal{G}_0 . Since we assume the normality $W(1)=1$ the correspondence between E_w and W is unique. So we have proved

Lemma 1

There is a one-to-one correspondence between a quantum field $\phi(x)$ with the Wightman functional W and a hyperplane E_w of \mathcal{G}_0 with the properties (Fig. 1)

- a) the cone K lies on one side of E_w ,
 b) $L \subset E_w$.

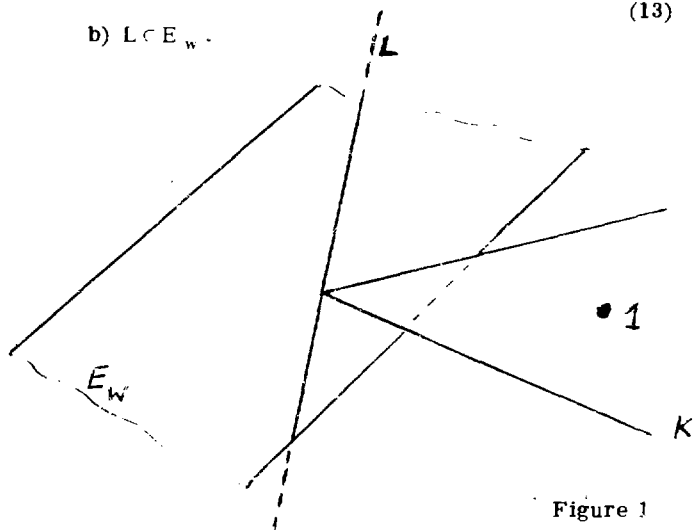


Figure 1

Thus the problem to determine all quantum fields $\phi(x)$ (neutral, scalar) is equivalent to the problem to determine all hyperplanes E of \mathcal{G}_0 with the properties (13). This conception of determining of the Wightman fields we had already formulated some years ago in [7]. Some results in this direction have been obtained in [2,3], where properties of positive functionals of the Wightman - type

(satisfying not all conditions) are proved. Recently Wyss¹⁹ proved some theorems about the extension of linear functionals in \mathcal{G}_0 to positive functionals. Let M be a subspace of \mathcal{G}_0 containing the identity 1 and $T(f)$ be a linear real functional on M normed by $T(1)=1$. T is then uniquely determined by its kernel

$$\ker T = \{ f \in M ; T(f) = 0 \}.$$

Question: Under which conditions does there exist a Wightman functional $W(f)$ being an extension of T , i.e. $W(f) = T(f)$ for $f \in M$?

That is equivalent to the question under which conditions there is a hyperplane E_w satisfying (13) and containing L and $\ker T$. The answer to this question is given by the following

Theorem 1¹⁹

There exists an extension of T to a Wightman functional W if and only if one of the following equivalent conditions is satisfied:

$$\begin{aligned} \text{i) } \mathcal{G}_0 &\neq \overline{K + L + \ker T}, \\ \text{ii) } -1 &\notin \overline{K + L + \ker T}, \end{aligned} \tag{14}$$

where the closure in $\overline{K + L + \ker T}$ is taken with respect to the topology τ (5).

In what follows we shall demonstrate how one can prove the existence of quantum fields with certain properties, but we cannot yet exclude the trivial fields.

3. Existence of Wightman Type Functionals

The next lemmas are important for the method proving the existence of certain Wightman functionals which we shall demonstrate in what follows.

Lemma 2

i) If $f_n \in I$ then

$$\int f_n(x_1, \dots, x_n) dx_1 \dots dx_n = 0 \quad \text{or equivalent}$$

$$\tilde{f}_n(0, \dots, 0) = 0. \quad (15)$$

ii) If $f_n \in I' = \tilde{\mathcal{L}}(I_{loc}, I_{inv})$ then

$$\int f_n(x_1, \dots, x_n) dx = 0 \quad (16)$$

Proof:

i) For $f \in I_{loc}$ or I_{sp} or I_{inv} it follows $\tilde{f}_n(0, \dots, 0) = 0$.
For instance, if $f \in I_{loc}$ has the form

$$f = f_n(x_1, \dots, x_n) = g_n(\dots x_j, x_{j+1} \dots) - g_n(\dots x_{j+1}, x_j \dots)$$

then it is $\tilde{f}_n(0, \dots, 0) = \frac{1}{(2\pi)^{2n}} \int f_n(x_1, \dots, x_n) dx_1 \dots dx_n = 0$.

Analogous for $f \in I_{inv}$. If $f \in I_{sp}$, $\tilde{f}_n(0, \dots, 0) = 0$ holds by definition. Since $f \rightarrow \tilde{f}(0, \dots, 0)$ is a linear continuous functional on $\tilde{\mathcal{L}}$, $\tilde{f}_n(0, \dots, 0) = 0$ holds for any $f \in I = \tilde{\mathcal{L}}(I_{loc}, I_{sp}, I_{inv})$.

ii) If

$$f = f_n(x_1, \dots, x_n) = g_n(\dots x_j, x_{j+1} \dots) - g_n(\dots x_{j+1}, x_j \dots) \in I_{loc}$$

then $\int f_n(x_1, \dots, x_n) dx = 0$

and analogous for $f = f_n \in I_{inv}$.

In consequence of the continuity and linearity of the functional $f \rightarrow \int f_n(x_1, \dots, x_n) dx$

we have proved (16) for any $f \in I' = \tilde{\mathcal{L}}(I_{loc}, I_{inv})$.

Let \mathcal{P} be the algebra of all real polynomials $p = \sum_{\nu=0}^{\infty} \alpha_{\nu} t^{\nu}$, $\alpha_{\nu} \in \mathbb{R}$ in one real variable, equipped with the strongest locally convex topology $\tau_{\mathcal{P}}$. The set $K_{\mathcal{P}}$ of all positive polynomials, $p(t) \geq 0$ for all real t , is a cone in \mathcal{P} . With this cone \mathcal{P} becomes a semiordered space.

Lemma 3:

Both the linear mappings of \mathcal{G}_0 into \mathcal{P} defined by

$$f = \{ f_0, f_1(x_1), f_2(x_1, x_2), \dots \} \in \hat{\mathcal{G}}_0$$

$$\begin{aligned} f \rightarrow \hat{f} = \hat{f}(t) &= \sum_{n \geq 0} \frac{t^n}{(2\pi)^{2n}} \int f_n(x_1, \dots, x_n) dx_1 \dots dx_n \\ &= \sum_{n \geq 0} \bar{f}_n(0, \dots, 0) t^n \end{aligned} \quad (17)$$

and

$$f \rightarrow \check{f} = \check{f}(t) = \sum_{n \geq 0} t^n \int f_n(x_1, \dots, x_n) dx \quad (18)$$

are continuous and positive, i.e. if $f \in K$ then $\hat{f}, \check{f} \in K_{\mathcal{P}}$.

Proof:

The mappings $f_n \rightarrow \hat{f}_n$ and $f_n \rightarrow \check{f}_n$ are continuous on the subspace $\mathcal{S}_{n0} = \hat{\mathcal{G}}_0 \cap \mathcal{S}_n$ of the homogeneous elements of degree n . Since $\hat{\mathcal{G}}_0[\tau]$ is the topological direct sum of the \mathcal{S}_{n0} , the continuity of these mappings on $\hat{\mathcal{G}}_0$ is proved. Further, since the cones K and $K_{\mathcal{P}}$ are closed, the positivity of the mappings (17) and (18) is proved, if we show $(f^+ \cdot f) \in K_{\mathcal{P}}$ and $(f^+ \cdot f) \in K_{\mathcal{P}}$ for every $f \in \hat{\mathcal{G}}$. Now it is $(f^+ \cdot f)(0, \dots, 0) = \bar{f}_n(0, \dots, 0) \check{f}_n(0, \dots, 0)$

and therefore

$$f^+ \cdot f = \sum_{n \geq 0} t^n \sum_{k+l=n} \bar{f}_k(0, \dots, 0) \check{f}_l(0, \dots, 0) = \left| \sum_m t^m \bar{f}_m(0, \dots, 0) \right|^2 \geq 0$$

for all t , i.e. $f^+ \cdot f \in K$. (19)

For $(f^+ \cdot f)$ we can conclude on the following way

$$\begin{aligned} (f^+ \cdot f) &= \sum_{n \geq 0} t^n \sum_{k+l=n} \int \bar{f}_k(x, \dots, x) \check{f}_l(x, \dots, x) dx \\ &= \int \sum_{n \geq 0} \sum_{k+l=n} t^k \bar{f}_k(x, \dots, x) t^l \check{f}_l(x, \dots, x) dx \end{aligned}$$

$$= \int \left| \sum_{m \geq 0} t^m \bar{f}_m(x, \dots, x) \right|^2 dx \geq 0 \quad (20)$$

for all t .

Now we can state and prove

Theorem 2

Let

$$M = \{ g = \lambda 1 \oplus \mu \tilde{f}_2; \lambda, \mu \in \mathbb{R} \}, \quad f = (0, 0, f_2, 0, \dots) \in K$$

with $\tilde{f}_2(0, 0) > 0$ and T the linear functional on M defined by $T(f) = \beta > 0$, $T(1) = -1$.

Then there exists a Wightman functional $W(g)$ (local, convex, invariant) which is an extension of $T(g)$.

Proof:

In consequence of Theorem 1 we have only to prove

$$-1 \notin K + L + \ker T.$$

Now $\ker T = \{ \lambda(-\beta 1 \oplus \tilde{f}_2), \lambda \in \mathbb{R} \}$.

If $-1 \in K + L + \ker T$, then it would exist a sequence

$$k^\alpha + \ell^\alpha + v^\alpha \in K + L + \ker T, \quad k^\alpha \in K, \quad \ell^\alpha \in L,$$

$$v^\alpha = \lambda^\alpha (-\beta 1 \oplus \tilde{f}_2) \in \ker T \quad \text{with} \quad k^\alpha + \ell^\alpha + v^\alpha \longrightarrow -1.$$

$\{\alpha\}$ is a directed set of indices. By applying the mapping (17) we obtain

$$\hat{k}^\alpha + \hat{\ell}^\alpha + \hat{v}^\alpha \longrightarrow -1 \quad (21)$$

in \mathcal{P} . From Lemma 2 (15) we get $\hat{v}^\alpha = 0$ and therefore it follows

$$\hat{k}^\alpha(t) + \lambda^\alpha \tilde{f}_2(0) t^2 - \lambda^\alpha \beta \longrightarrow -1 \quad (22)$$

with respect to the topology $\tau_{\mathcal{P}}$ of \mathcal{P} .

In consequence of Lemma 3 the zero-component of

\hat{k}^a is positive and therefore $\lambda^a \geq 0$ for $a \geq a_0$.
 Further we take a_0 such that $f_2(0,0) - \beta \geq 0$.
 Then $\hat{k}^a(t) = \lambda^a (f_2(0,0) - \beta) \geq 0$ for $a \geq a_0$.
 This is a contradiction to (22), since from the convergence with respect to $\tau \varphi$ it follows the convergence for any t .
 Thus the Theorem is completely proved.

Theorem 2 states the existence of a Wightman functional and consequently also of a field. Of course this Theorem says nothing about the existence of nontrivial fields, since we can find a free field $\phi_0(x)$ the Wightman functional \mathbb{W}_0 of which is an extension of \mathbb{T} , i.e. $\mathbb{W}_0(f) = \mathbb{T}(f)$. The proof of Theorem 2 is only the simplest example to demonstrate, how one can prove the existence of fields with the help of Theorem 1.

A more general result we will obtain with the next Theorem. We start with a test function $g_4(x_1, x_2, x_3, x_4) \in \mathcal{D}_0$ of the form

$$g_4(x_1, x_2, x_3, x_4) = h(x_1)u(x_2)u(x_3)h(x_4) \quad (23)$$

with the properties

$$\tilde{h}(p) = \tilde{h}(-p) \geq 0, \quad \tilde{u}(p) = \tilde{u}(-p) \geq 0 \quad (24)$$

Further we take h and u in such a way that $\text{supp } \tilde{h}$ and $\text{supp } \tilde{u}$ are compact and

$$\text{supp } \tilde{h} \subset \{p; p^2 > 0\}, \quad \text{supp } \tilde{u} \subset \{p; p^2 < 0\} \quad (25)$$

We can choose g_4 in such a way that

$$\int g_4(x, x, x, x) dx = (2\pi)^2 \int \delta(p_1 + p_2 + p_3 + p_4) \tilde{h}(p_1) \tilde{u}(p_2) \times \\ \times \tilde{u}(p_3) \tilde{h}(p_4) dp_1 \dots dp_4 > 0, \quad (26)$$

since for the point (p_1, p_2, p_3, p_4) in Fig. 2 the integrand on the right-hand side is positive.

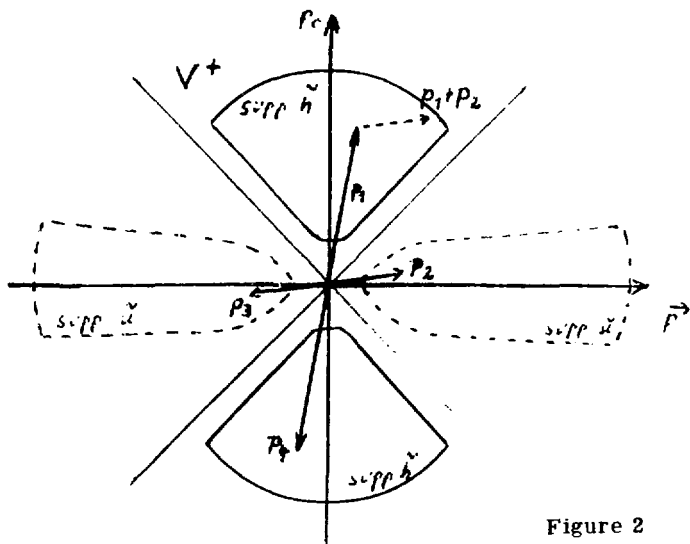


Figure 2

Theorem 3

Let $M = \{f = \lambda 1 \oplus \mu g_4; \lambda, \mu \in \mathbb{R}\}$ be the linear subspace of \hat{U}_0 generated by 1 and g_4 and $T(f) = \lambda + \mu\beta$, $T(g_4) = \beta > 0$ a linear functional on M . Then there exists positive functional $W(f)$ on \hat{U}_0 , which is a continuation of T and satisfies $W(f) = 0$ for

$$f \in L; \quad L' = L' + iL' = \overline{\mathcal{L}}(I_{loc}, I_{inv}).$$

$W(f)$ is then a Wightman-like functional. It is local and invariant. By W we can construct a field $\phi(x)$ which satisfied all assumptions of the Wightman axioms except the spectrality. The field $\phi(x)$ obtained on this way is different from the generalized free field, since

$$W(g_4) = \langle 0 | \phi(h)\phi(u)\phi(u)\phi(h) | 0 \rangle > 0,$$

but for a generalized free field ϕ_{gen} it holds $\phi_{gen}(u) = 0$.

Proof:

Analogous to the proof of Theorem 2 we have only to prove

$$-1 \notin \overline{K + L' + \ker T}, \quad (27)$$

where $\ker T = \{\lambda(-\beta) \oplus g_4\}; \lambda \in \mathbb{R}\}$.

Let $k^a + \ell^a + v^a \in K + L' + \ker T$, $k^a \in K$, $\ell^a \in L'$ and $v^a = \lambda^a(-\beta) \oplus g_4 \in \ker T$ be a sequence with

$$k^a + \ell^a + v^a \rightarrow -1 \quad (28)$$

in (i_0) with respect to the topology τ . Now we apply the mapping $f \rightarrow \check{f}$ (18) and get

$$\check{k}^a + \check{\ell}^a + \check{v}^a \rightarrow -1 \quad (29)$$

By Lemma 2 ii) it is $\check{v}^a = 0$. Therefore

$$\check{k}^a(t) + \lambda^a(ct^4 - \beta) \rightarrow -1. \quad (30)$$

with $c = \int g_4(x, x, x, x) dx > 0$. By the same conclusions as in the end of the proof of Theorem 2 (30) leads to a contradiction. Thus (28) is impossible and the Theorem is completely proved.

Since for the points p_1, p_2, p_3, p_4 of Fig. 2 $\bar{g}_4(p_1, p_2, p_3, p_4) > 0$ and $p_1, p_1 + p_2, p_1 + p_2 + p_3 \in V^+$, $p_1 + p_2 + p_3 + p_4 = 0$, g_4 is not an element of $I_{\sim p}$. Therefore one can hope with methods analogous to those demonstrated before also to prove the existence of an extension $W(f)$ of $T(f)$, which satisfies $W(f) = 0$ for $f \in I_{sp}$. Then $W(f)$ would be a full Wightman functional of a field different from the generalized free fields.

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