IC/73/172

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

SPECTROSCOPIC FACTORS OF NEGATIVE-PARITY MULTIPLET STATES IN ODD Sn ISOTOPES

S. de Barros

M. J. Bechara

T. Borello-Lewin

and

V. Paar

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1973 MIRAMARE-TRIESTE

IC/73/172

International Atomic Energy Agency

and

United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

SPECTROSCOPIC FACTORS OF NEGATIVE-PARITY MULTIPLET STATES IN ODD Sn ISOTOPES *

S. De Barros,** M.J.Bechara,** T. Borello-Lewin ** and V. Paar***

International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

The spectroscopic factors of negative-parity multiplet states in odd Sn isotopes are calculated by including leading-order scattering and the ground-state correlation process. The ground-state correlation gives an important contribution to the spectroscopic factors.

The theoretical results are in good agreement with the available experimental data.

MIRAMARE - TRIESTE

November 1973

* A revised version will be submitted for publication.

** Permanent address: Instituto de Física, Universidade de São Paulo, São Paulo, Brazil.

*** Permanent address: Institute "Ruđer Bošković", Zagreb, Yugoslavia.

The treatment of the nuclear system in terms of elementary excitations provides a useful method for describing the nuclear properties. For spherical nuclei, the dominant role in creating the properties of low-lying states is played by the shell-model particles (holes) or quasiparticles in the valence shell as fermion-type and by the low-frequency quadrupole vibrations as boson-type elementary excitations.[1),2)] The interaction between these two fields is given by the familiar expression:

$$V_{int} = \kappa \sum_{\mu=-2}^{2} Y_2^{\mu *} \alpha_2^{\mu} = \kappa \sqrt{\frac{\hbar\omega_2}{2C_2}} \sum_{\beta\gamma} \langle \beta | Y_2^{\mu *} | \gamma \rangle a_\beta^+ a_\gamma \left(b_2^{\mu +} + (-)^\mu b_2^{-\mu} \right),$$

proportional to a simple scalar built from fermion current and boson amplitude. Here a_β^+ and a_γ are the creation and annihilation operators of the single-particle states $|\beta\rangle$ and $|\gamma\rangle$, respectively. The boson amplitude $\alpha_2^\mu = \sqrt{\frac{\hbar\omega_2}{2C_2}} \left[b_2^{\mu +} + (-)^\mu b_2^{-\mu} \right]$ is a linear combination of a creation operator of a phonon $|2\mu\rangle$ and an annihilation operator of a phonon $|2-\mu\rangle$.

The quantity $\sqrt{\frac{\hbar\omega_2}{2c_2}} = \frac{4\pi}{3ZR_0^2}\sqrt{B(E2)(2_1^+ \rightarrow 0_1^+)}$ is given by the $B(E2)(2_1^+ \rightarrow 0_1^+)$, Z and A of the nucleus which plays the role of the basic vibrator.[1)] The strength $\langle k \rangle \approx \langle r\frac{dV}{dr} \rangle$ can be determined from inelastic scattering or calculated from the Woods-Saxon potential.[1)]

First-order processes in H_{int} are represented by emission or absorption of a phonon by single-particle (hole); creation or destruction of a particle-hole pair and a phonon, and destruction (creation) of a phonon by creating (destructing) a particle-hole pair. The BCS quasiparticle can be considered as a hybrid of particle and hole; so the particle-phonon vertices connecting single-particle states 1 and 2 are simply modified by adding the factors $(U_1U_2 - V_1V_2)$ for each scattering vertex and $(U_1V_2 + U_2V_1)$ for each vertex of creation or annihilation of the quasiparticle pair.[1)]

In the recent high-precision (d,p) experiments on even Sn isotopes, rather weak $\ell = 3$ transfers populating $7/2_1^-$ states in neighbouring odd Sn isotopes have been identified and the corresponding spectroscopic factors determined.[3)-5)]

In the framework of the quasiparticle-vibration coupling picture, the lowest negative-parity state in odd Sn isotopes can be described as predominantly one $h_{11/2}$ quasiparticle. In the zeroth-order approximation, the next group of negative-parity states is the one-phonon multiplet $|\tilde{h}_{11/2},12;\ I = 7/2^-_1, 9/2^-_1, 11/2^-_2, 13/2^-_1, 15/2^-_1\rangle$ at the energy $\tilde{\varepsilon}_{h_{11/2}} + \hbar\omega$. The phonon energy $\hbar\omega$ can be taken as the energy of the first excited 2^+ state in the corresponding even Sn isotopes which play the role of basic vibrators.

The spectroscopic factor of the $7/2^-_1$ multiplet state in the leading order is given by the diagrams in Fig. 1. In the "scattering process" (a) the transferred neutron is added to the $1f_{7/2}$ configuration of the higher empty shell (82-126), and after first-order scattering on the phonon field (phonon emission) goes into the multiplet state $|h_{11/2},12;7/2\rangle$. In the "ground-state correlation process"(b) the particle-vibration interaction first creates (in even Sn isotope) the $(h_{11/2}, 0f^{-1}_{7/2})2$ quasiparticle-hole pair and a phonon, and the transferred neutron is added to the hole-state $0f^{-1}_{7/2}$ (below the valence shell), i.e. the hole is destroyed and the final state is the $|h_{11/2},12;7/2^-\rangle$ multiplet state in the corresponding odd Sn isotope.

The contribution from diagrams (a) and (b) is

$$S(7/2^-_1) = \frac{(4\pi)^2 \langle\kappa\rangle^2 B(E_2)(2^+_1 \to 0^+_1)_\nu}{9 Z^2\ 1.2^4\ A^{4/3}} \cdot \frac{\langle h_{11/2} \| Y_2 \| f_{7/2}\rangle^2}{2\cdot\frac{7}{2}+1} \cdot (U_{h_{11/2}})^2 \left[\frac{1}{\tilde{\varepsilon}_{h_{11/2}} - \varepsilon_{1f_{7/2}} + \hbar\omega} + \frac{1}{-\tilde{\varepsilon}_{h_{11/2}} - \varepsilon^{-1}_{0f_{7/2}} - \hbar\omega}\right]^2 \tag{1}$$

Here, the $U_{h_{11/2}}$ factor arises from the particle-vibration coupling vertices $U_{h_{11/2}} U_{1f_{7/2}} - V_{h_{11/2}} V_{1f_{7/2}} = U_{h_{11/2}}$ (because $U_{1f_{7/2}} = 1$) and $U_{h_{11/2}} V_{0f_{7/2}} + V_{h_{11/2}} U_{0f_{7/2}} = U_{h_{11/2}}$ (because $V_{0f_{7/2}} = 1$), for diagrams (a) and (b), respectively. The only difference between diagrams (a) and (b) appears in the energy denominators, because they are topologically equivalent. Since both energy denominators are comparable in magnitude and negative, the "scattering process" and "ground-state correlation process" will compete and add coherently.

In $S(7/2^-_1)$ calculated from formula (1), no free parameters appear. The experimental $B(E2)(2^+_1 \to 0^+_1)_\nu$ are taken from the corresponding Sn isotopes,[6] and the interaction strength $\langle k\rangle \approx 40$ MeV corresponds to the estimate from Ref. 1. The single-particle positions $\varepsilon_{1f_{7/2}} - \varepsilon_{0h_{11/2}} = 5.2$ MeV and

$\varepsilon_{0h_{11/2}} - \varepsilon_{0f_{7/2}} \approx 13.3$ MeV are deduced from Nilsson diagrams in Ref. 1.

The spectroscopic factors are calculated for two sets of BCS solutions for $U_{h_{11/2}}$ and quasiparticle energies $\tilde{\varepsilon}_{h_{11/2}}$:(I) solutions for pairing force[7] and (II) solutions for the Yale-Shakin potential. [8]

The calculated and experimental spectroscopic factors $S(7/2_1^-)$ are compared in Table I. The ground-state correlations obviously play an important role: they contribute about 30% to the spectroscopic amplitude and increase $S(7/2_1^-)$ by about a factor of two. The relative contribution of the "ground state correlation process" is somewhat larger for heavier than for lighter Sn isotopes. The results are similar for both types of BCS solutions, (I) and (II).

The calculated values, including ground-state correlations, are in very good agreement with experiment. This is expected once the characteristic coupling strength

$$\xi_2 = \frac{\langle k \rangle \sqrt{\frac{\hbar \omega_2}{2C_2}}}{\bar{E}}$$

(Ref. 1), where $\bar{E}$ represents the average energy denominator, is approximately 0.1. The experimental situation in ^{113}Sn is not completely clear. [3] Extending the analogy with ^{117,119,121,123}Sn one would expect the $7/2_1^-$ state at about 1.6 MeV for this nucleus. However, in that region, peaks of several contaminations appear (which might mask some states) and the only observed state with sizeable maximal cross-section (0.228 mb/sr) and with no definite ℓ-value assigned is the 1.54 MeV state. [3] The tentative $\ell = 4$ assignment for that state would violate rather strongly the analogy with other Sn isotopes and the agreement with BCS predictions, but for $\ell = 3$ it would correspond to a magnitude similar to that predicted in Table I.

Only the "scattering process" contributes to the population of the $9/2_1^-$, $13/2_1^-$ and $15/2_1^-$ members of the multiplet because there are no shell-model configurations of the same spin in or below the valence-shell. The corresponding single-particle positions in the higher shells are $\varepsilon(0h_{9/2}) - \varepsilon(0h_{11/2}) = 5.8$ MeV, $\varepsilon(0j_{13/2}) - \varepsilon(0h_{11/2}) \approx 25$ MeV, and $\varepsilon(0j_{15/2}) - \varepsilon(0h_{11/2}) = 13.3$ MeV as determined from the Nilsson diagrams in Ref. 1, respectively. The calculated results, using formula (1) with the change $1f_{7/2} \to 0h_{9/2}, 0j_{13/2}, 0j_{15/2}$ and without the second energy denominator, are listed in Table II. The expressions for $S(9/2_1^-)$ and $S(13/2_1^-)$ involve spin-flip matrix elements $\langle h_{11/2} \| Y_2 \| h_{9/2} \rangle$ and $\langle h_{11/2} \| Y_2 \| j_{13/2} \rangle$, respectively. The spin-flip matrix elements are generally small (in the classical limit of large j, spin-flip matrix elements vanish) and therefore the corresponding transfer reactions are almost forbidden. For $S(13/2_1^-)$ this effect is even more pronounced because of appreciably higher $0j_{13/2}$ single-particle position. The $9/2_1^-$ and $13/2^-$ states are expected to be very weakly, if at all, excited in (d,p) reactions. The $S(15/2_1^-)$ does not involve a spin-flip matrix element, but the $0j_{15/2}$ single-particle state is rather high and the ground-state correlations are absent, so it is more than an order of magnitude smaller than $S(7/2_1^-)$.

While the $7/2_1^-, 9/2_1^-, 13/2_1^-$ and $15/2_1^-$ states are populated by the first-order processes which involve configurations outside the valence shell, the $11/2_2^-$ multiplet state is populated by a process involving only $h_{11/2}$ valence-shell configuration:

$$S(11/2_2^-) = \frac{(4\pi)^2 B(E_2)(2_1^+ \to 0_1^+)_\nu}{9Z^2 \cdot 1 \cdot 2^4 \cdot A^{4/3}} \cdot \frac{\langle h_{11/2} \| Y_2 \| h_{11/2} \rangle^2}{2 \cdot \frac{11}{2} + 1} \cdot \langle K \rangle^2 \cdot \left[\frac{U_{h_{11/2}}(U^2_{h_{11/2}} - V^2_{h_{11/2}})}{\hbar\omega} + \frac{V_{h_{11/2}}(2U_{h_{11/2}}V_{h_{11/2}})}{-\hbar\omega - 2\tilde{E}_{h_{11/2}}} \right]^2 \quad (2)$$

For $A \lessapprox 123$ there is $U^2_{h_{11/2}} > V^2_{h_{11/2}}$, and therefore the second part of the spectroscopic amplitude ("ground state correlation process") is out of phase with the first ("scattering process"). Obviously the role of ground-state correlations substantially increases as $U_{h_{11/2}}$ comes closer to $V_{h_{11/2}}$. The quantitative result is therefore very sensitive to details of BCS solutions. The characteristic coupling strength f_2 for the valence-shell process is now larger than in the previous cases ($\bar{E}$ is smaller because it does not involve configurations outside the valence-shell), so the role of higher-order terms is expected to become more important. The sensitivity of the $S(11/2_2^-)$ to the BCS solutions is reflected in its faster change from isotope to isotope (Table II). Specifically, $S(11/2_2^-) \gtrapprox 0.1$ in ^{113}Sn is predicted, while for heavier Sn isotopes, $S(11/2_2^-)$ is smaller.

In the experimental data for ^{113}Sn ,[3] there exists indeed a state of unassigned ℓ-value with a maximal cross-section which for $\ell = 5$ would correspond to $S(11/2^-_2) = 0.15$; in agreement with the theoretical prediction of Table II. For heavier Sn isotopes ($^{119, 121, 123}Sn$) the non-assigned states around $7/2^-_1$ do not reveal the state with such a sizeable maximal cross-section as in ^{113}Sn . The available maximal cross-sections of the $^{119,121,123}Sn$ states in this region, in the case of $\ell = 5$, would give spectroscopic factors of the same order of magnitude as predicted in Table II.

ACKNOWLEDGMENTS

The authors wish to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. One of the authors (S.B.) expresses her gratitude to Conselho Nacional de Pesquisas, and Coordenação do Aperfeiçoamento de Pessoal de Nível Superior. (T.B.L.) wishes to thank the Fundação de Amparo à Pesquisa do Estado de São Paulo and Instituto de Física, Universidade de São Paulo, São Paulo, for financial support. (M.J.B.) is grateful to the Instituto de Física, Universidade de São Paulo, São Paulo for financial support.

REFERENCES

1) A. Bohr and B.R. Mottelson, Nuclear Structure, Vol. 2, to be published.

2) G. Alaga, F. Krmpotić, V. Lopac, V. Paar, and L. Šips, to be published.

3) T. Borello, E. Frotta-Pessoa, C.Q. Orsini, O. Dietzsch and E.W. Hamburger, Rev. Bras. Fis. 2, 157 (1972), and to be published;
M.J. Bechara, M.Sc. Thesis, Universidade de São Paulo, São Paulo 1973.

4) L. Goldman and B.L. Cohen, private communication as quoted in Ref. 3.

5) M.J. Bechara, T. Borello-Lewin, O.Dietzsch, E.W. Hamburger, H. Miyake and A.F.R.T. Piza, in Proceedings of the International Conference on Nuclear Physics, Munich 1973, Eds. J. de Boer and H.J. Mang, p. 224.

6) C.K. Ross and R.K. Bhaduri, Nucl. Phys. A196, 369 (1972).

7) L.S. Kisslinger and R.A. Sorensen, Rev. Mod. Phys. 35, 853 (1963).

8) A. Rimini, private communication;
Y.K. Gambhir, A. Rimini and T. Weber, Phys. Rev. C3, 1965 (1971).

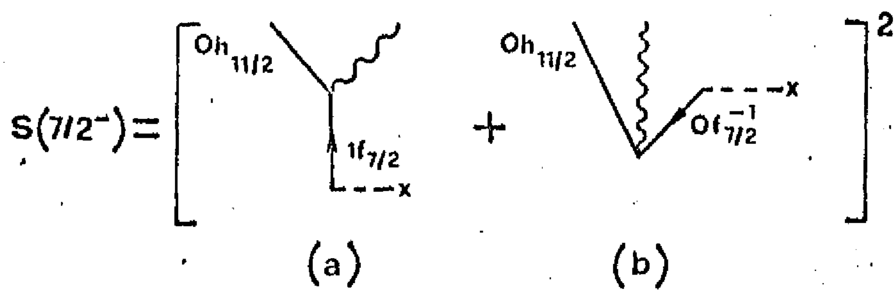

Fig. 1 First-order processes contributing to the $S(7/2_1^-)$. (a) is the "scattering process" and (b) the "ground-state correlation process".

TABLE I *)

Experimental and theoretical (formula (1) without and with contribution from the ground-state correlations) spectroscopic factors of the $7/2^-_1$ states in odd Sn isotopes.

A	WITHOUT G.S.C.		WITH G.S.C.		EXP.
	I	II	I	II	
113	0.038	0.031	0.068	0.060	
115	0.029	0.023	0.052	0.046	
117	0.021	0.018	0.039	0.037	0.042 **)
119	0.018	0.015	0.035	0.031	0.034***)
121	0.014	0.011	0.027	0.024	0.024***)
123	0.012	0.009	0.023	0.019	0.020 †)
125	0.006	0.005	0.013	0.012	

*) In the second and third columns ground-state correlations (second energy denominator in formula (1)) are neglected, and in the fourth and fifth columns they are included. The spectroscopic factors I and II are calculated by using BCS solutions from Refs. 7 and 8, respectively.

**) Ref. 4

***) Ref. 5

†) Ref. 3

TABLE II

Theoretical spectroscopic factors (formula (1)) of $9/2^-_1$, $13/2^-_1$, $15/2^-_1$ and $11/2^-_2$ multiplet states in odd Sn isotopes.

A	$9/2^-_1$		$13/2^-_1$		$15/2^-_1$		$11/2^-_2$			
							Without G.S.C.		With G.S.C.	
	I	II	I	II	I	II	I	II	I	II
113	0.0009	0.0008	0.00003	0.00002	0.0036	0.0034	0.152	0.176	0.126	0.154
115	0.0007	0.0006	0.00002	0.00002	0.0028	0.0026	0.081	0.095	0.054	0.072
117	0.0005	0.0005	0.00002	0.00002	0.0022	0.0021	0.027	0.048	0.007	0.026
119	0.0005	0.0004	0.00002	0.00002	0.0020	0.0018	0.020	0.020	0.003	0.004
121	0.0003	0.0003	0.00001	0.00001	0.0015	0.0014	0.003	0.003	0.003	0.001
123	0.0003	0.0002	0.00001	0.00001	0.0013	0.0011	0.001	0.0003	0.010	0.016
125	0.0002	0.0001	0.00001	0.00001	0.0007	0.0007	0.006	0.005	0.043	0.033