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ELECTROMAGNETICALLY INDUCED PLASMA BACK CURRENT NEAR THE HEAD OF A RELATIVISTIC ELECTRON BEAM ENTERING GAS*

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*Work performed jointly under the auspices of the U.S. Atomic Energy Commission, and the Department of the Navy under the contract number NAonr 13-74.

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Introduction

The dynamics of a self-focused relativistic beam entering unionized gas are strongly dependent on the initial rate of generation of conductivity. This conductivity generation is important in determining the stability of the beam to transverse (hose) modes, and it also enters into the self-focusing of the beam by its influence on the generation of back plasma currents.

The purpose of the present report is to present a simple theoretical description of the influence of the conductivity generation near the beam head on the plasma back current. Diagnostic techniques for measuring the beam and plasma currents at different axial locations have been developed, and the present theoretical treatment can help in the interpretation of this data, including the deduction of conductivity from the back current measurements.

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Analysis

We use an idealized model with cylindrical coordinates (r, θ, z) and dependence only upon the radial coordinate r and the time, t . t is the local time after the head of the primary beam has passed the fixed observation point at axial position z. We assume that the primary beam has a total current $I_b(t)$ and uniform current density inside $r = r_b$ (see Fig. 1). The plasma conductivity $\sigma(t)$ is uniform inside $r = r_c = r_b$. There is a perfectly conducting wall at $r = r_w$. Between $r = r_c$ and $r = r_w$ the plasma conductivity is low enough so that the plasma current density is negligible. The conductivity is high enough in all regions so that electrostatic fields are negligible, i.e. $(4\pi\sigma)^{-1}$ is smaller than any time period of interest. Also the displacement current $(\partial E/\partial t)$ makes a negligible contribution to the magnetic field.

The z-component of time dependent plasma current is

$$I_p = \pi r_c^2 \sigma E \qquad (1)$$

where E is the z-component of time dependent electric field. E is given by

$$E = -\frac{1}{c}\frac{\partial \Phi}{\partial \tau} \qquad (2)$$

where Φ is the θ-component of magnetic flux per cm of z between $r = r_c$ and $r = r_w$. (We neglect the flux inside $r = r_c$, and so E and the plasma current density are uniform inside $r = r_c$). Φ is found by integrating $B_\theta(r)dr$ between $r = r_c$ and $r = r_w$;

$$\Phi = \frac{2}{c} \ln \frac{r_w}{r_c} (I_b + I_p) \quad . \tag{3}$$

We assume that the plasma conductivity is given by

$$\sigma = \frac{e^2 n_e}{m\, \nu_m} \tag{4}$$

where n_e is the time dependent plasma electron density and ν_m is the momentum transfer collision frequency with gas molecules. (We assume that the fractional contribution to the collision frequency of the reactive term (ω/ν_m) is negligible. Also, the gas molecule density is much larger than n_e so collisions with plasma ions are neglected and ν_m is constant for a particular gas molecule density.

Combining Eqs. 1-3 gives

$$I_p = -\tau_m \left(\frac{\partial I_b}{\partial t} + \frac{\partial I_p}{\partial t} \right) \tag{5}$$

where τ_m is the time constant for resistive decay of monopole plasma current distribution

$$\tau_m = \frac{2\pi\sigma r_c^2}{c^2} \ln \frac{r_w}{r_c} \tag{6}$$

<u>First Case</u>: Time rate of increase of n_e proportional to instantaneous beam current.

At relatively high gas pressures, the generation of plasma would be expected to be dominated by classical single-particle processes of ionization by the relativistic particles, hence we expect $\partial n_e/\partial t \sim J_b$ in the initial phases before recombination plays a role. Neglecting the dependence of ν_m on electron temperature we have $\tau_m \sim \sigma a^2 \sim n_e a^2$, and therefore

$$\frac{\partial \tau_m}{\partial t} = \frac{I_b(t)}{I_{b1}} \quad . \tag{7}$$

The constant I_{b1} can be calculated for a given gas; it has the physical significance of being the beam current for which $(\partial\tau_m/\partial t) = 1$. This form of normalization is used because τ_m is the same as the dipole time constant (τ_d), except for a geometrical factor, and $(\partial\tau_d/\partial t) \gtrsim 1$ is considered to be a condition for avoiding excessive growth of resistive hose instability near the beam bead.

We assume that the primary beam current increases linearly with time

$$I_b(t) = I_{b1}\,\frac{t}{\tau_b} \quad , \tag{8}$$

so (I_{b1}/τ_b) is the rate of increase.

Combining Eqs. 7 and 8, we have

$$\frac{\partial \tau_m}{\partial \tau} = \frac{t}{\tau_b} \quad . \tag{9}$$

Integrating Eq. 9 yields

$$\tau_m = \frac{1}{\tau_b} \int_{t'=0}^{t} t' dt' = \frac{t^2}{2\tau_b} \quad . \tag{10}$$

Combining Eqs. 5, (d/dt) of 8, and Eq. 10 we have

$$I_p = - \frac{t^2}{2\tau_b} \left(\frac{I_{b1}}{\tau_b} + \frac{dI_p}{dt}\right), \; 0 \le t < \infty \quad . \tag{11}$$

To eliminate dimensions in Eq. (11) we define

$$\frac{I_p}{I_{b1}} = I \quad , \tag{12}$$

$$\frac{t}{\tau_b} = \frac{2}{x} \quad . \tag{13}$$

Then Eq. (11) becomes

$$\frac{dI}{dx} = I + \frac{2}{x^2} \quad , \; \infty \ge x > 0 \quad . \tag{14}$$

Eq. (14) is readily solved using an integrating factor:

$$e^x \frac{d}{dx} \; Ie^{-x} = \frac{dI}{dx} - I = \frac{2}{x^2} \quad . \tag{15}$$

This yields

$$Ie^{-x} = - \int_{x'=x}^{\infty} dx' \frac{2}{x'^2} e^{-x'} + C \quad . \tag{16}$$

Integrating once by parts we have

$$Ie^{-x} = -\frac{2}{x}e^{-x} + \int_x^\infty dx' \frac{2e^{-x'}}{x'^2} + C$$

or

$$I = -\frac{2}{x} + 2e^{x}\varepsilon(x) + C\,e^{x} \quad , \tag{17}$$

where

$$\varepsilon(x) = \int_x^\infty \frac{dx'}{x'}\, e^{-x'} \tag{18}$$

is a form of the "Exponential Integral", a tabulated function.

Our initial condition is that at $t = 0$ $(x = \infty)$, $I = 0$, so $C = 0$. If we divide both sides of Eq. (18) by $(-2/x)$ we have the magnitude of the plasma current at any time divided by the beam current at that time.

$$-\left(\frac{I_p(t)}{I_{b1}\frac{t}{\tau_b}}\right) = 1 - xe^{x}\varepsilon(x) \quad . \tag{19}$$

For large x an asymptotic expansion of Eq. (19) is found by successive integrations by parts:

$$-\frac{I_p(t)}{I_{b1}\frac{t}{\tau_b}} \rightarrow 1 - xe^{x}\left[\frac{e^{-x'}}{-x'}\Bigg|_x^\infty - \frac{e^{-x'}}{-x'^2}\Bigg|_x^\infty + 2\frac{e^{-x'}}{-x'^3}\Bigg|_x^\infty - \ldots\right]$$

$$= \frac{1}{x} - \frac{2!}{x^2} + \frac{3!}{x^3} - \ldots \approx \frac{1}{x} = \frac{1}{2}\frac{t}{\tau_b} \quad .$$

And as $t \rightarrow \infty$,

$$-\left(\frac{I_p(t)}{I_{b1}\frac{t}{\tau_b}}\right) \rightarrow 1 \ .$$

Figure 2 presents a plot of $I_p(t)/I_B$ making use of Eq. (13) and the table of the Exponential Integral in Handbook of Mathematical Functions issued in 1964 by National Bureau of Standards.

Second Case: Time rate of increase of n_e proportional to square of the beam current.

In order to find the effect of a change in the assumed rate of increase of n_e with time, we now analyze the same situation as above except that we replace Eq. (7) with

$$\frac{\partial\tau_m}{\partial t} = \left(\frac{I_b(t)}{I_{b1}}\right)^2 \qquad (20)$$

This might model, for example, a case where breakdown of the gas by inductive E-field is increasing the rate of ionization over the classical rate. We still assume that the primary beam current increases linearly with time as

$$I_b(t) = I_{b1} \frac{t}{\tau_b} \quad . \tag{21}$$

Combining Eqs. (20) and (21) gives

$$\frac{\partial \tau_m}{\partial t} = \frac{t^2}{\tau_b^2} \quad . \tag{22}$$

Integrating Eq. (22)

$$\tau_m = \frac{1}{3} \frac{t^3}{\tau_b^2} \quad . \tag{23}$$

Combining Eqs. (5), (d/dt) of 8 and Eq. (23) we have

$$I_p = - \frac{t^3}{3\tau_b^2} \left(\frac{I_{b1}}{\tau_b} + \frac{dI_p}{dt} \right), \; 0 \leq t < \infty \; . \tag{24}$$

With the definitions

$$\frac{I_p}{I_{b1}} = 1 \; , \tag{25}$$

$$\frac{t}{\tau_b} = \frac{3}{x} \; , \tag{26}$$

eq. (24) becomes

$$\frac{dI}{dx} = \frac{xI}{3} + \frac{3}{x^2}, \quad \infty \geq x > 0 \quad . \tag{27}$$

The solution of Eq. (27) is found (using an integrating factor) to be

$$I = -e^{x^2/6} \int_x^\infty dx' \frac{3}{x'^2} e^{-x'^2/6} + Ce^{x^2/6} =$$

$$-\frac{3}{x} + e^{x^2/6} \int_x^\infty dx' \, e^{-x'^2/6} + Ce^{x^2/6} .$$

We may write this in the form

$$I = -\frac{3}{x} + e^{x^2/6} \sqrt{\frac{3\pi}{2}} \, \text{erfc}\left(\frac{x}{\sqrt{6}}\right) + Ce^{x^2/6},$$

where

$$\text{erfc}(y) = \frac{2}{\sqrt{\pi}} \int_y^\infty dx' \, e^{-x'^2}$$

is the complementary error function.

Since $I_p (t = \infty) = 0$ we must set $C = 0$, which leaves

$$I(x) = -\frac{3}{x} + e^{x^2/6} \sqrt{\frac{3\pi}{2}} \, \text{erfc}\left(\frac{x}{\sqrt{6}}\right) . \tag{28}$$

The ratio of the magnitude of plasma to beam current is

$$-\frac{I_p}{I_B} = -\frac{I}{t/\tau_b} = -\frac{xI}{3} = 1 - \frac{x}{3} e^{x^2/6} \sqrt{\frac{3\pi}{2}} \, \text{erfc}\left(\frac{x}{\sqrt{6}}\right) . \tag{29}$$

As $t \to \infty$ $(x \to 0)$

$$-\frac{I_p}{I_B} \to 1 - \sqrt{\frac{3\pi}{2}}\ \frac{x}{3} = 1 - \sqrt{\frac{3\pi}{2}}\ \frac{\tau_B}{t}$$

and as $x \to \infty$

$$I \to -\frac{9}{x^3} \quad ,$$

or

$$-\frac{I_p}{I_B} = -\frac{I}{t/\tau_b} = -\ \frac{I}{3/x} \to \frac{3}{x^2} = \frac{1}{3}\left(\frac{t}{\tau_b}\right)^2 \quad .$$

On Fig. 3 $(-I_p/I_b)$ is plotted vs (t/τ_b).

Discussion and Summary: The ratio of back plasma current to beam current is sketched as a function of time (t/τ_b) in Figs. 2 and 3 for the two different models of conductivity generation, assuming a linearly rising beam current. An alternate interpretation of these graphs is that they give the percentage return current at the end of the beam risetime, when the current has reached a final value I_f. The horizontal axis is then simply I_f/I_{b1}, where I_{b1} has the interpretation of the beam current magnitude for which $(\partial\tau_m/\partial t) = 1$.

An interesting aspect of these results is the fact that the percentage return current at $t = \tau_r$, when I_b has risen to I_f, is independent of the actual risetime ($\tau_r \equiv \frac{I_f}{I_{b1}} \tau_b$). The physical explanation for this result is that the conductivity (τ_m) at $t = \tau_r$ becomes smaller as the risetime (τ_r) is reduced, but the inductive electric field becomes greater (since $\partial I/\partial t$ increases) thus resulting in the same plasma current at $t = \tau_r$, when $I_b = I_f$, for all values of τ_r.

The results in Figures 2 and 3 show that significant plasma return currents (20 - 30%) should occur if the peak current rises to a value comparable to I_{b1}, thus approaching the regime $\partial\tau_m/\partial t \sim 1$. If the regime $\partial\tau_m/\partial t >> 1$ is achieved, a high degree of current neutralization ($I_p \sim -I_B$) will be obtained.

We have only discussed the plasma currents generated during the beam risetime phase thus far. If a beam current waveform like that in Fig. 4 is assumed, and the first model for conductivity generation is used, the solution for I_p with $t \geq \tau_r$ is

$$I_p = \frac{I_{po}}{[2t/\tau_r - 1]^{(I_{b1}/I_f)}}$$

with I_{po} the plasma current at $t = \tau_r$, as calculated above. The plasma current decays on a timescale governed by τ_r with an exponent determined by I_f/I_{b1}; smaller final currents decay faster, as expected, since the conductivity generation is weaker. Note that although I_{po} is independent of τ_r, as discussed above, the plasma current decays on a timescale set by τ_r; this is reasonable since $\tau_m(\tau = \tau_r) \to 0$ as $\tau_r \to 0$. The limit of a step-function $I_b(t)$, where $\tau_r \to 0$, would thus have $I_{Net} \equiv I_b$ and the plasma current of finite amplitude and zero duration. As expected, an infinitely fast risetime beam injected into a nonconducting gas would not induce any measurable plasma return current at the head.

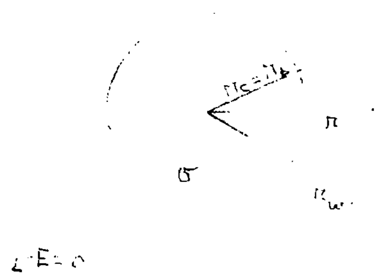

Fig. 1 Geometry for the idealized model.

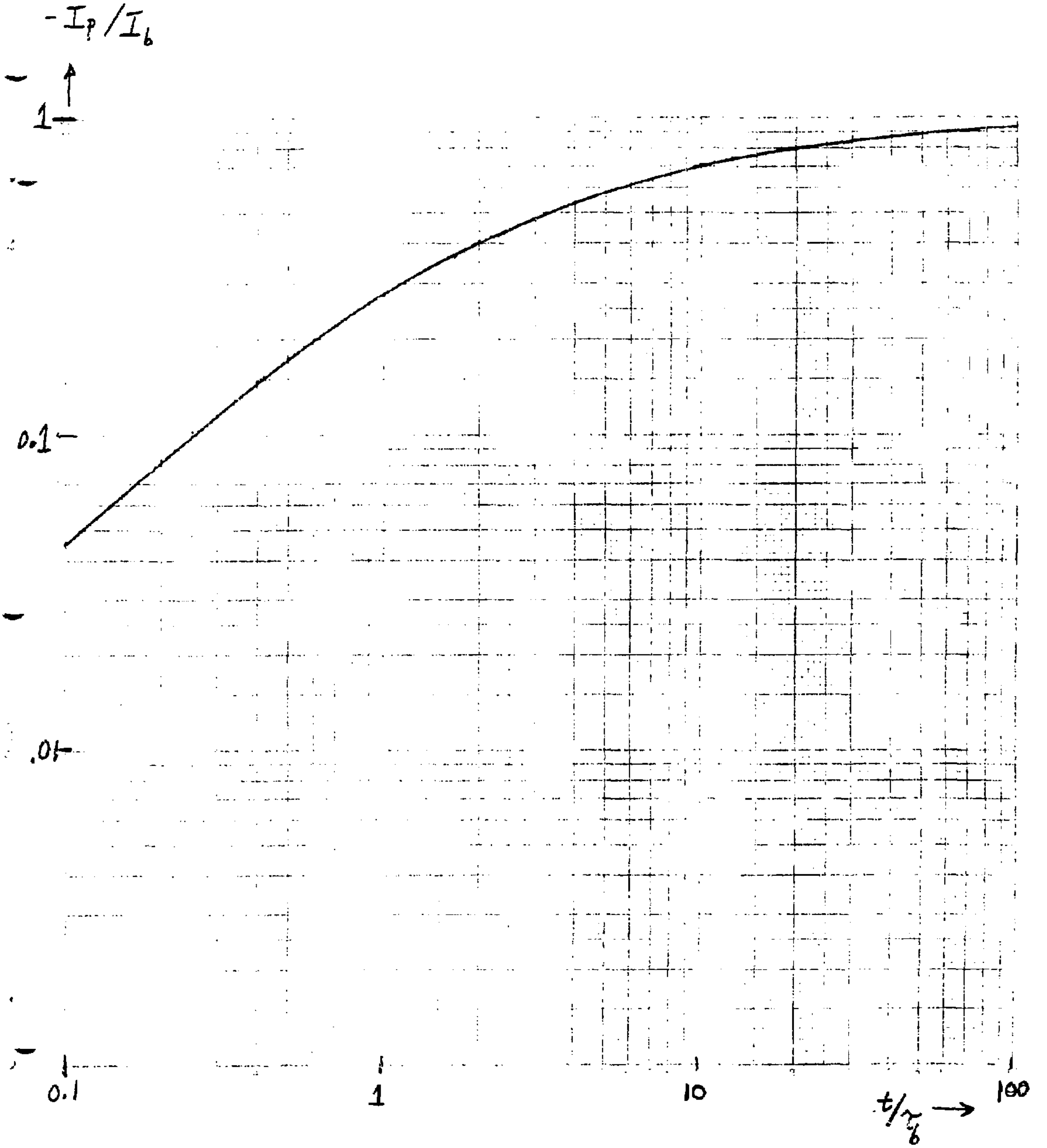

Fig 2 $[-I_p(t)/I_b(t)]$ vs $t/\tau_b = \partial\tau_m/\partial t = I_b/I_{b1}$.

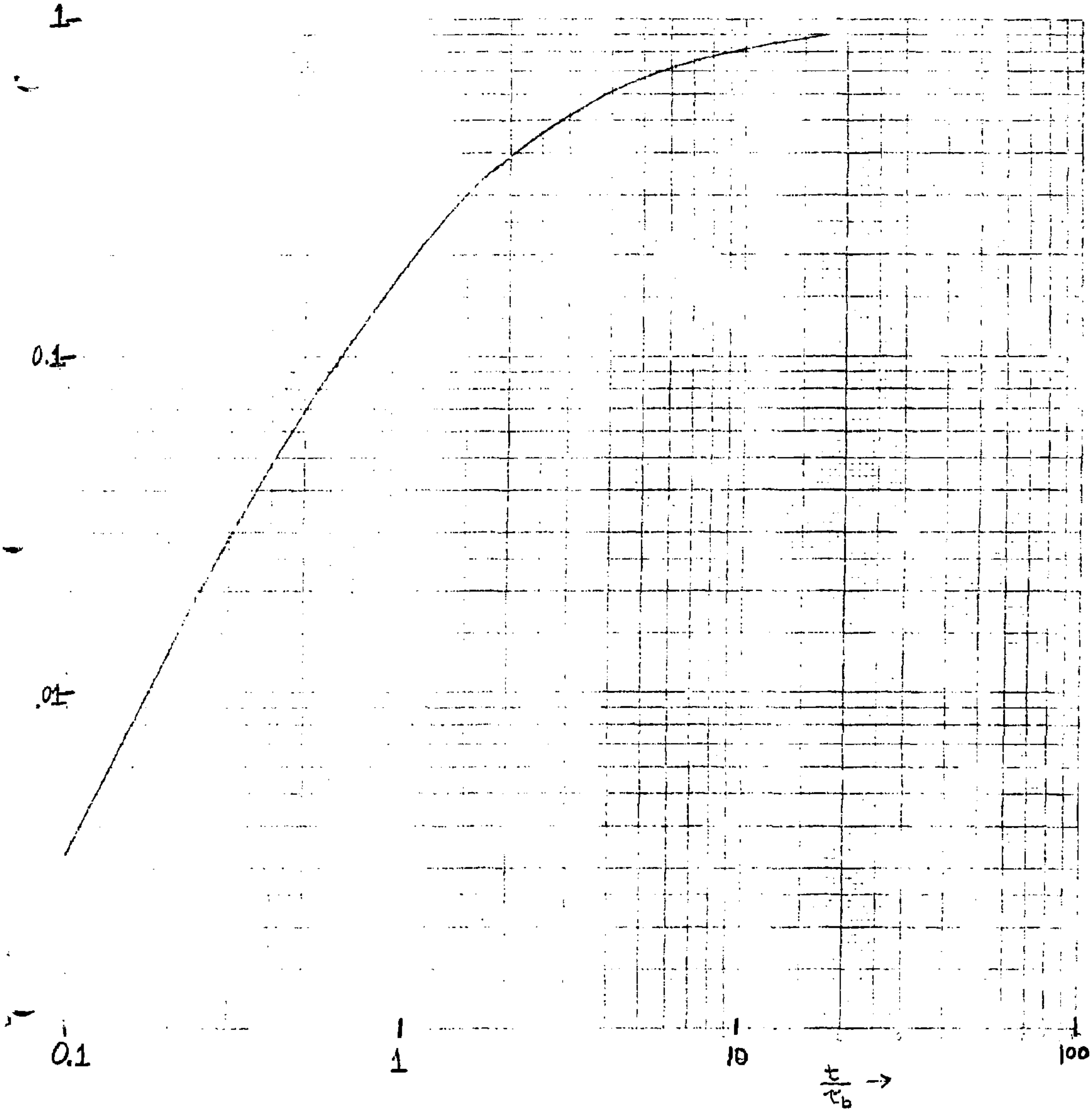

Fig. 3 $(-I_p/I_b)$ vs (t/τ_b) for $(\partial\tau_m/\partial t) = (\tau_b/I_{b_1})^2$ and $(I_b/I_{b_1}) = (t/\tau_b)$

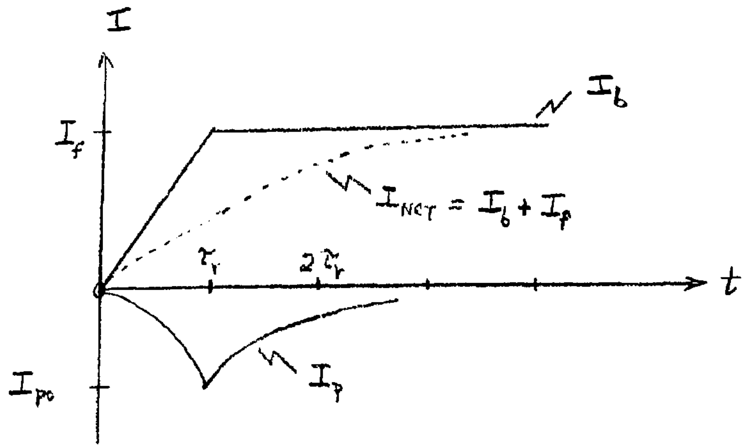

Fig 4. Qualitative nature of plasma current and net current with triangular beam shape and $\partial\tau_m/\partial\tau = I_b/I_{bf}$.