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DIFFUSION OF IMPURITIES IN TOKAMAKS IN THE PFIRSCH SCHLUTER REGIME

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DIFFUSION OF IMPURITIES IN TOKAMAKS

IN THE PFIRSCH-SCHLUTER REGIME.

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ABSTRACT.

Diffusion of impurities ions in the PFIRSCH-SCHLUTER regime is considered for the three possible diffusion regimes of hydrogen ions. It is found that in all cases, a strong thermal gradient could prevent the plasma contamination. Another decontamination method, which consists of giving to hydrogen ions a slow motion along the lines of force, is discussed.

I - INTRODUCTION.

The impurity ions are pumped inwards Tokamak configurations by a mechanism which results from the very principle of confinement in these configurations [1]. Assuming a constant temperature T, the hydrogen ions (charge $1 \times e$, density n_1) and the impurity ions (charge $Z \times e$, density n_Z) tends to thermodynamical equilibrium in a frame of reference rotating around the major axis where the electrostatic field $-\nabla\psi$ satisfies the condition :

$$n_1 \propto \exp - \frac{e\psi}{T} \quad ; \quad n_Z \propto \exp - \frac{Ze\psi}{T}$$

Therefore the density profiles n_1 and n_Z tends to equilibrium values such that $n_Z \propto n_1^Z$, i.e. the n_Z profile tends to concentrate inside the n_1 profile.

The flux of impurities ϕ_Z across a magnetic surface is caused by the friction forces experienced by the Z assembly, due to the difference between the diamagnetic velocities of species (1) and (Z). In the absence of a thermal gradient, these diamagnetic velocities are proportional to $\frac{c}{eB} T \frac{\nabla n_1}{n_1}$ and $\frac{c}{ZeB} T \frac{\nabla n_Z}{n_Z}$, respectively. A thermal gradient introduces further diamagnetic velocities proportional to $\frac{c}{eB} \nabla T$ and $\frac{c}{ZeB} \nabla T$ for particles of low and large energies. The flux ϕ_Z is therefore of the form (r is the minor radius of magnetic surfaces)

$$\phi_Z = A \left[\frac{dn_1}{n_1 dr} - \frac{1}{2} \frac{dn_Z}{n_Z dr} + \alpha \frac{dT}{T dr} \right]$$

where the quantities $\alpha = O(1)$ and A , for a given magnetic configuration, are functions of the temperature T, the densities n_1 and n_Z and Z. Of course these functions depend on the diffusion regime of ions (1) and (Z). In a realistic situation where $n_Z Z^2 > n_1$ the conditions of temperature and density for the various cases are the following

	Transition PFIRSCH SCHLUTER regime - Plateau regime	Transition Plateau regime - Banana regime
impurity ions (2)	$\frac{T_{ev}^2}{n_1} \simeq Z^2 qR \frac{n_Z Z^2}{n_1} \times 0{,}5\ 10^{-12}$	$\frac{T_{ev}^2}{n_1} \simeq Z^2 qR \left(\frac{R}{r}\right)^{3/2} \times \frac{n_Z Z^2}{n_1} \times 0{,}5\ 10^{-12}$
hydrogen ions (1)	$\frac{T_{ev}^2}{n_1} \simeq qR \frac{n_Z Z^2}{n_1} \times 0{,}5\ 10^{-12}$	$\frac{T_{ev}^2}{n_1} \simeq qR \left(\frac{R}{r}\right)^{3/2} \times \frac{n_Z Z^2}{n_1} \times 0{,}5\ 10^{-12}$

where T_{ev} is the temperature T in ev. Impurity ions with $Z \gg 1$ are typically in the PFIRSCH SCHLUTER regime even if hydrogen ions are in the plateau or banana regime. (For instance taking $Z = 2$, $R/r = 4$, $q = 2$, $n_e = 5\ 10^{13}$, $n_Z Z^2/n_1 = 5$, the hydrogen ions are in the banana regime for $T > 600$ ev and the Z ions are in the PFIRSCH-SCHLUTER regime for $T < 5$ KeV.

In this note, we calculate first the diffusion coefficients of Z ions, assuming that they are in the PFIRSCH SCHLUTER regime and

$$n_Z Z^2 \gg n_1 \quad ; \quad Z \gg 1 \quad ; \quad m_Z \gg m_1 \quad ; \quad q \gg 1 \quad ; \quad \frac{r}{R} \ll 1 \qquad (1)$$

for the various regime of hydrogen ions. We find that in all cases ϕ_Z is proportional to the quantity $\frac{d n_1}{n_1 dr} - \frac{1}{Z}\frac{d n_z}{n_z dr} - \frac{1}{2}\frac{dT}{T dr}$ and therefore that the inwards diffusion of impurities may be reversed if the thermal gradient $\frac{dT}{T dr}$ is strong enough. This generalises the result obtained by CONNOR [2] when both ions (1) and (Z) are in the banana regime. Actually, the creation of a strong thermal gradient, which could be obtained by heating ions at the plasma center and increasing neutral flux at the plasma edge, seems the more logical way of preventing contamination in Tokamaks. It is difficult to state, however, how strong the turbulence induced by such a thermal gradient will be and therefore which additional power will be necessary. Also, this turbulence could induce a strong inwards diffusion of impurities. In that case the whole concept of decontamination by a thermal gradient would be useless.

In view of these uncertainties, it seems interesting to explore other means of decontamination. Starting from the fact that the inwards flux ϕ_z of Z ions is primarily due to the friction forces experienced by these ions from hydrogen ions when the latter are in thermodynamical equilibrium in a frame of reference rotating around the major axis, as it is the case when only a density gradient dn_1/dr is present, two lines may be followed to reverse ϕ_Z : The first is to compensate the friction forces resulting from the density gradient dn_1/dr by additional forces (obtained for instance by R.F. pumping) acting selectively on Z ions. The second is to drive the hydrogen ion assembly out of the above mentioned thermodynamical equilibrium, so that the friction forces on the Z ions be altered. The creation of a thermal gradient is the best example of this strategy. In the Section (IV) we tentatively consider another possibility which essentially consists of giving to hydrogen ions a slow velocity $\overline{V_{\parallel 1}}$ along the lines of force. Inversion of ϕ_Z takes place if $\overline{V_{\parallel 1}}$ compensates the diamagnetic velocity $\frac{c}{eB_\theta} \frac{dn_1}{n_1 dr} T$ of the hydrogen assembly in the φ direction. (θ and φ correspond to poloidal and toroidal directions, respectively). The velocity $\overline{V_{\parallel 1}}$ might be induced by a reasonable magnetic pumping. The power to be injected in the plasma is much larger than the power which is necessary to maintain a thermal gradient $\nabla T/T > 2 \; \nabla n/n_1$ if the heat transport is neoclassical. However, by itself, a slow ordered motion of ions along the line of force does not induce instabilities. It is possible that this fact gives finally an advantage to the proposed decontamination method.

II – VALUE OF THE DIFFUSION COEFFICIENT OF IMPURITY IONS IN THE PFIRSCH-SCHLUTER REGIME.

Supposing the impurity ions in the PFIRSCH-SCHLUTER regime, and the conditions (1) verified, we obtain for the collisional flux ϕ_Z the following expression :

$$\frac{\phi_Z}{n_Z} = \frac{1}{\tau_1} q^2 Z \rho_{\mu_1}^2 \left[\left(\frac{dn_1}{n_1 dr} - \frac{1}{Z} \frac{dn_Z}{n_Z dr} \right) - \frac{1}{2} \frac{dT}{T dr} \right] \quad (2)$$

where

$$\rho_{th1} = \frac{V_{th1}}{\omega_{c1}} \quad ; \quad \omega_{c1} = \frac{eB}{m_1 c} \quad ; \quad V_{th1} = \left(\frac{2T}{m_1}\right)^{1/2}$$

$$\tau_1 = \frac{3 m_1^{1/2} \, T^{3/2}}{4 (2\pi)^{1/2} n_1 e^4 \, Log \Lambda} = 0{,}8 \; 10^6 \, \frac{A_1^{1/2} \, T_{ev}^{3/2}}{n_1} \qquad (2.)$$

$A_1 = m_1 / \text{proton mass}$

The diffusion is of course ambipolar, i.e. the flux ϕ_Z is associated with a flux $-Z\phi_Z$ of hydrogen ions.

The diffusion velocity ϕ_Z / n_Z is generally somewhat larger (by a factor $< Z$) than the heat diffusion velocity. . In the example considered above, taking $B = 30\ Kg$, $\frac{dn_1}{n_1 dr} = \frac{1}{10}\ cm^{-1}$ and $T = 500\ eV$, we have $\phi_Z / n_Z \simeq 200\ cm/sec$

III - JUSTIFICATION OF EQ.(2).

In the frame of reference $\mathcal{R}$ rotating around the major axis, where the electric field has an average radial component equal to 0, this electric field may be written (Cf. Fig.(1).

$$E = -\nabla u \quad ; \quad u = \tilde{\tilde{u}}(r) \sin\theta$$

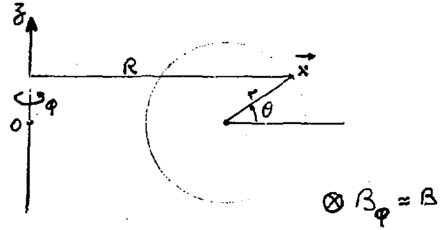

fig 1

The assembly of Z ions in the PFIRSCH-SCHLUTER regime behaves as a fluid with a pressure p_Z varying on each magnetic surface as :

$$p_z = \bar{p}_z(r) + \tilde{\tilde{p}}_z(r) \sin\theta$$

Neglecting the collisional transport of particles transverse to the magnetic field, the flux ϕ_z of Z ions across a magnetic surface is given by :

$$\phi_z = \frac{1}{2\pi R_o} \int_0^{2\pi} \left(R\, V_{dz}(\vec{x})\, n_z(\vec{x})\, s \quad \theta + c\, \frac{\partial U(\vec{x})}{r\,\partial\theta} n_z \; \frac{R}{B} \right) d\theta$$

$$R = R_o + r\cos\theta \qquad B = B_o \left(1 - \frac{r}{R_o}\cos\theta \right)$$

where V_{dz} is the average drift velocity of guiding centers of Z ions :

$$V_{Dz}(\vec{x}) = \frac{c}{Ze\,B_o} \; \frac{2\,T(x)}{R_o}$$

Therefore we have :

$$\phi_z = \frac{c}{Z\,e\,B_o\,R_o} \tilde{\tilde{p}}_z + \frac{c}{B_o} \frac{1}{R_o} \tilde{\tilde{u}}\, n_z \qquad (3)$$

The quantities p_z and u satisfy the equilibrium equation*:

$$-\nabla_\parallel p_z - n_z Z e \quad \nabla_\parallel u + \mathcal{F}_z = 0$$

where $\mathcal{F}_z(\vec{x})$ is the friction force (per cubic cm) along the lines of force experienced by the Z ions from the hydrogen ions. It results that on a magnetic surface :

$$\mathcal{F}_z(\vec{x}) = \tilde{\mathcal{F}}_z(r) \cos\theta$$

$$\left(\tilde{p}_z + n_z Z e \,\tilde{\tilde{u}}\right) \frac{1}{q R_0} = \tilde{\mathcal{F}}_z \qquad (4)$$

and using (3)

$$\phi_z = q \,\tilde{\mathcal{F}}_z \,\frac{c}{Z e B_0} \qquad (5)$$

The calculation of ϕ_z is equivalent to the calculation of the coefficient $\tilde{\mathcal{F}}_z$, which depends on the relative motion of the

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* The electric field $-\nabla_\parallel u$ is necessary to induce a perturbation of the electron density $\delta n_e/n_e \sim \frac{1}{T} e \,\tilde{\tilde{u}} \sin\theta$ insuring electrical neutrality : $\delta n_e = \delta n_1 + Z \delta n_z$
The equilibrium of the whole assembly of ions imposes roughly $T \nabla_\parallel (\delta n_1 + \delta n_z) \sim 0$ and therefore : $\delta n_z \sim \delta n_1$, $\delta n_e \sim Z \delta n_z$ and $e\tilde{\tilde{u}} \sim TZ \,\delta n_z/n_e$. The force $n_z Z e \nabla_\parallel u$ acting on Z ions is of the order of $(n_z Z^2/n_e) \nabla_\parallel p_z$ [3]

assembly (Z) and (1) along the lines of force.

If $Z \gg 1$, we may neglect the effect of the thermal gradient of the Z assembly, because this thermal gradient acts in (2) as $O\left(\frac{1}{Z}\ \nabla T\right)$ to be compared to $O\left(\nabla T\right)$.
The motion of the Z assembly (in the frame of reference $\mathfrak{R}$) is the superposition of two motions : 1) a solid motion around the major axis, which allows confinement, at an angular velocity

$$\Omega_Z = T \frac{d n_Z}{n_Z\, d\psi} \quad \frac{2\pi c}{Z e}$$

where ψ is the poloidal flux embraced by the magnetic surface, 2) a motion along the lines of force at a velocity w_Z , which must preserve the flux of Z ions across a flux tube, *i.e.* which must verify, on each magnetic surface :

$$w_Z / B = \text{constant}$$

$$w_Z = \overline{w}_Z - \widetilde{w}_Z \frac{r}{R} \cos\theta$$

Finally the Z assembly has in the φ direction (parallel direction) given by

$$V_Z = \overline{V}_Z + \widetilde{V}_Z \cos\theta$$

$$\overline{V}_Z = R_0 \qquad + w_Z$$

$$\widetilde{V}_Z = 2 r \Omega_Z - \overline{V}_Z \frac{r}{R_0}$$

$$\Omega_Z = T \frac{d n_Z}{n_Z\, d\psi} \quad \frac{2\pi c}{Z e} \tag{6}$$

1. Hydrogen ions in the banana regime.

The distribution function $F(\vec{x},\vec{v})$ of hydrogen ions may be taken a function of the constants of motion $\mathcal{E}$ (energy) and M_{z} (momentum around the major axis)

$$F(\vec{x},\vec{v}) = F_T(\vec{x},\vec{v}) = \frac{n_1(\psi')}{\left[2\pi \frac{T(\psi')}{m}\right]^{3/2}} \exp - \frac{\mathcal{E}}{T(\psi')} \qquad (7)$$

where

$$\psi'(\vec{x},\vec{v}) = \psi(\vec{x}) + \frac{R m V_\varphi}{e} 2\pi c = \frac{2\pi c}{e} M_z = \psi(\vec{x})\left(1 + O\left(\frac{q \rho_{\theta}}{R}\right)\right)$$

$\psi(\vec{x})$ is the poloidal flux for the magnetic surface passing through $\vec{x}$, and $n_1(\psi)$ and $T(\psi)$ are the density and the temperature on this surface. The distribution function $F(\vec{x},\vec{v}) = F_c(\vec{x},\vec{v})$ for the circulating particles is a function of $\mathcal{E}$, $M_z = \frac{e}{2\pi c}\psi'(\vec{x},\vec{v})$ and of another constant of motion allowing for free streeming along the lines of force. We take :

$$F_c(\vec{x},\vec{v}) = \frac{n_1(\psi')}{(2\pi T(\psi')/m)^{3/2}} \exp\left(-\frac{\mathcal{E}}{T(\psi')}\right)\left[1 + w(\vec{x},\vec{v})\right]$$

$$w(\vec{x},\vec{v}) = v_\parallel \; K(\alpha,\vec{x}) \quad g\left(\frac{\mathcal{E}}{T}\right) \qquad (8)$$

$$K(\alpha,\vec{x}) = \frac{1}{(\cos\alpha)} \quad h\left[1 - \frac{B_m(\vec{x})}{B(\vec{x})} \sin^2\alpha\right]$$

where α is the angle between B and V, $B_m(\vec{x})$ is the maximum value of β on the magnetic surface passing through x and $g(u)$ and $h(u)$ are unknown functions of u, with h going to 0 with u to insure continuity of $F(\vec{x},\vec{v})$ between trapped and circulating particles.

The force $\mathcal{F}_Z(\vec{x})$ in the φ direction ($\approx$ parallel direction) experienced by the Z assembly from hydrogen ions is given by (assuming $V_{th_Z} \ll V_{th_1}$)

$$\mathcal{F}_Z(\vec{x}) = \iiint n_Z m_1 \; F\left(\vec{x}, \vec{v}' + v_Z \frac{\vec{B}}{B}\right) V' \cos\alpha' \, A_D \frac{1}{2V'^3} \, d_3\vec{v}'$$

$$A_D = 8\pi e^4 Z^2 \, \mathcal{L}g\Lambda / m_1^2$$

where $\vec{V}'$ is the relative velocity of hydrogen ions with respect to the Z assembly :

$$V' = V - V_Z(\vec{x}) \frac{\vec{B}}{B} \qquad \alpha' = \text{angle}\left(\vec{V}', \vec{B}\right)$$

Using (7) and (8), developing $n(\psi')$ and $T(\psi')$ in the form $n_1(\psi') = n_1(\psi(x)) + \frac{d n_1}{d\psi}(\psi' - \psi)$, ... and performing the integral $\iiint d_3\vec{v}'$ as $\iint v'^2 dv' \, 2\pi \sin\alpha' d\alpha'$ we obtain

$$\mathcal{F}_Z(\vec{x}) = n_Z m_1 \frac{n_1(\psi)}{v_{th_1}} \qquad \frac{A_D}{3\pi^{1/2}}$$

$$\left[-\frac{2V_Z(\vec{x})}{V_{th_1}^2} + R\frac{2\pi c}{c} m_1 \left(\frac{dn_1}{n_1 d\psi} - \frac{1}{2}\frac{dT}{T d\psi}\right) + \frac{3}{2}A\left(\int_0^1 h(u)\,du\right)\frac{B(\vec{x})}{B_m(\vec{x})}\right] \tag{9}$$

$$A = \int_0^\infty g(u)\exp(-u)\,du$$

$$V_{th_1} = \left(\frac{2T(\psi)}{m_1}\right)^{1/2}$$

It results from (4) that the average value of $\mathcal{J}'_2(\vec{x})$ along the lines of force must be zero. Therefore we have

$$\frac{-2\,\overline{V_z}}{V\,th_1^{\,2}} + R_o \frac{2\pi c}{e}\, n_1 \left(\frac{d n_1}{n_1\, d\psi} - \frac{1}{2}\, \frac{dT}{T\, d\psi} \right) + \frac{3}{2} A \left(\int_0^1 h(u)\, du \right) \frac{B_o}{B_m(\vec{x})} = 0 \qquad (10)$$

where $\overline{V_z}$ is the average velocity of Z ions which appear in (6). We must also express the equilibrium of circulating hydrogen ions along the lines of force. For that we minimize the entropy production $\frac{ds}{dt}$ in the plasma due to collisions between ions.[4] If $n_z Z^2 \gg n_1$, and $V\,th_1 \gg V\,th\,z$ this production is only due to collisions between hydrogen and Z ions and has the value :

$$\frac{ds}{dt} = \frac{1}{2T^2} \iiint\!\!\iiint d_3\vec{x}\; d_3\vec{v}\,'\; \frac{\partial\, \delta u(\vec{x}, \vec{v}\,')}{\partial v'_\alpha}\; \frac{\partial\, \delta u(\vec{x}, \vec{v}\,')}{\partial v'_\beta}\, D_{\alpha\beta}\, F$$

where $D_{\alpha\beta}$ $(\alpha = 1, 2, 3)$ are the diffusion coefficients of hydrogen ions in velocity *space* $\vec{V}' = \vec{V} - V_2(\vec{x})\frac{\vec{B}}{B}$ and the variable $\delta u(\vec{x}, \vec{v}\,')$ is defined by the relation :

$$F\left(\vec{x}, \vec{v}\,' + V_2(\vec{x})\frac{\vec{B}}{B}\right) = \exp - \frac{\delta u(x, v')}{T}\; \frac{n}{T^{3/2}\left(\frac{2\pi}{m}\right)^{3/2}}\; \exp - \frac{m_1 v'^2}{2T}$$

$$n = n\left[\psi(\vec{x})\right], \quad T = T\left[\psi(\vec{x})\right]$$

The tensor $D_{\alpha\beta}$ has only components $D_{I\,I}$ and $D_{II\,II} = \frac{1}{2}\frac{A_D n_z}{V_1}$

in the two directions perpendicular to $\vec{V}'$, and therefore :

$$\frac{dS}{dt} = \frac{n_2}{2T^2} \iiint d_3 x \iint \frac{1}{\pi^{1/2}} \frac{1}{V'} \, dV' \sin \alpha' \, d\alpha' \quad A_D$$

$$\left(\frac{\partial \, \delta u}{\partial \alpha'}\right)^2 \quad \exp\left(-\frac{V'^2}{V_{th_1}^2}\right) \frac{n}{V_{th_1}^3}$$

$$\delta u(\vec{x}, \vec{V}') = \delta u(\vec{x}, V', \alpha')$$

Inserting the value of δu for circulating particles which can be derived from (8) :

$$\delta u(x, v', \alpha') = -T\left[-\frac{2V_z(\vec{x})}{V_{th_1}^2} + R\frac{2\pi c}{e} m_1\left(\frac{dn_1}{n_1 d\psi} + \left(\frac{v'^2}{V_{th_1}^2} - \frac{3}{2}\right)\frac{dT}{T d\psi}\right)\right.$$
$$\left. + g\left(\frac{v'^2}{V_{th_1}^2}\right) K(\alpha', \vec{x})\right] v' \cos\alpha'$$

we obtain $\frac{dS}{dt}$ as a functional of $g(u)$. Minimization with respect to $g(u)$ gives :

$$g\left(\frac{\mathcal{E}}{T}\right) \iiint_0 d_3\vec{x} \int_0^1 \left(\frac{\partial h}{\partial u}\right)^2 du(1-u) \; 2\left[1 - \frac{B(\vec{x})}{B_m(\vec{x})}(1-u)\right]^{1/2}$$
$$+\left(\int_0^1 h(u)du\right) \cdot \iiint d_3\vec{x} \left\{ -\frac{2V_z(\vec{x})}{V_{th_1}^2} + R\frac{2\pi c}{e} m_1\left[\frac{dn_1}{n_1 d\psi} + \left(\frac{\mathcal{E}}{T} - \frac{3}{2}\right)\frac{dT}{T d\psi}\right]\right\} \frac{B(\vec{x})}{B_m(\vec{x})}$$
$$= 0$$

where the integral $\iiint d_3 x$ is taken between two neighbouring magnetic surfaces. Multiplying by $\exp - \mathcal{E}/T$ and integrating,

we obtain :

$$\frac{-2\,\overline{\tilde{V}_z}}{V_{th_1}^2} + R_o\,\frac{2\pi c}{c}\;m_1\left(\frac{d\,n_1}{n_1\,d\psi} - \frac{1}{2}\,\frac{dT}{T\,d\psi}\right)$$

$$+ A\left[\int_0^1\left(\frac{\partial h}{\partial u}\right)^2(1-u)\overline{\left[1-\frac{B(\vec{x})}{B_n(\vec{x})}(1-u)\right]^{1/2}}\;2\,du\right]\left[\int_0^1 h(u)\,du\cdot\left(\frac{B_o}{B_m(\vec{x})}\right)\right]^{-1} = 0$$

where the bar means averaging along the lines of force.

Comparing this relation with (10) gives :

$A = 0$

$$-\frac{2\,\overline{\tilde{V}_z}}{V_{th_1}^2} + R_o\,\frac{2\pi c}{e}\;m_1\left(\frac{d\,n_1}{n_1\,d\psi} - \frac{1}{2}\,\frac{dT}{T\,d\psi}\right) = 0 \tag{11}$$

and it then results from (9) and (6)

$$\mathcal{F}_z(\vec{x}) = n_z\;m_1\;\frac{n_1}{V_{th_1}\;3\pi^{1/2}}\;A_D\left[-\frac{2\,\tilde{V}_z}{V_{th_1}^2} + r\,\frac{2\pi c}{e}\,m_1\left(\frac{dn_1}{n_1\,d\psi} - \frac{1}{2}\,\frac{dT}{T\,d\psi}\right)\right]\cos\theta \tag{12}$$

Using (11) and (6) to calculate $\tilde{V}_z$ and inserting in (12) we obtain :

$$\mathcal{F}_z(\vec{x}) = \mathcal{F}_z'\cos\theta = n_z\;m_1\;\frac{n_1}{3\pi^{1/2}\;V_{th_1}}\;A_D$$

$$\left[\frac{d\,n_z}{Z\,n_z\,d\psi} - \frac{d\,n_1}{n_1\,d\psi} + \frac{1}{2}\,\frac{dT}{T\,d\psi}\right]\frac{2\pi c}{e}\left(-m_1\,2\,r\right)\cos\theta$$

Inserting the value of $\tilde{\mathcal{J}}_z$ in (5) we obtain the expression (2) for ϕ_z .

2. Hydrogen ions in the Plateau regime.

We study the equilibrium of the assembly of hydrogen ions by considering first this equilibrium in the configuration $\mathcal{C}_o$ having the same magnetic surfaces as the real configuration but a constant value of the magnetic field on each magnetic surface. The equilibrium in the real configuration is obtained by the superimposing the parallel magnetic perturbation $\delta B = -B_o \frac{r}{R} \cos\theta$ to $\mathcal{C}_o$.

The distribution function $F_o(\vec{x}, \vec{v})$ of hydrogen ions in $\mathcal{C}_o$ is a function of the constants of motion $\mathcal{E}$, M_z and $V_{\parallel}$, or $\mathcal{E}$ and

$$\psi' = \frac{2\pi c}{e} M_z = \psi(\vec{x}) + \frac{2\pi c}{e} R\, m_1 V_\varphi$$

$$\psi'' = \psi' - \frac{2\pi c}{e} R_o\, m V_{\parallel} \simeq \psi(\vec{x}) + \frac{2\pi c}{e} m_1 r \cos\theta\, V_\varphi$$

We take :

$$F_o(\vec{x}, \vec{v}) = \frac{\nu(\psi', \psi'')}{\left(2\pi \frac{\tau(\psi', \psi'')}{m_1}\right)^{3/2}} \exp - \frac{\mathcal{E}}{\tau(\psi', \psi'')}$$

$\nu(\psi, \psi)$ and $\tau(\psi, \psi)$ being the density $n_1(\psi)$ and the temperature $T(\psi)$ of hydrogen ions on the magnetic surface embracing the poloidal flux ψ . We have

$$\frac{\partial \nu(\psi,\psi)}{\nu\,\partial\psi'} + \frac{\partial \nu(\psi,\psi)}{\nu\,\partial\psi''} = \frac{d n_1}{n_1\, d\psi}$$

$$\frac{\partial \tau(\psi,\psi)}{\tau\,\partial\psi'} + \frac{\partial \tau(\psi,\psi)}{\tau\,\partial\psi''} = \frac{d T}{T\, d\psi} \qquad (13)$$

Developing $\nu(\psi',\psi'')$ and $\tau(\psi',\psi'')$ at first order in $\psi' - \psi(\vec{x})$ and $\psi'' - \psi(\vec{x})$ we obtain :

$$F(\vec{x},\vec{v}) = \exp\left(-\frac{\mathcal{E}}{T}\right) \frac{n_1}{\left[2\pi\; T/m_1\right]^{3/2}}$$

$$\left[1 + \left(\frac{\partial\nu}{\nu\,\partial\psi'} + \left(\frac{\mathcal{E}}{T} - \frac{3}{2}\right)\frac{\partial\tau}{\tau\,\partial\psi'}\right)\frac{R\, m_1\, V_\varphi\, c\; 2\pi}{e} \right. \qquad (14)$$

$$\left. + \left(\frac{\partial\nu}{\nu\,\partial\psi''} + \left(\frac{\mathcal{E}}{T} - \frac{3}{2}\right)\frac{\partial\tau}{\tau\,\partial\psi''}\right)\frac{r\, m_1\, V_\varphi\;\; 2\pi c}{e}\;\cos\theta\right]$$

$$\psi' = \psi'' = \psi(x) \quad ; \; n_1 = n_1\left[\psi(\vec{x})\right] \; ; \; T = T\left[\psi(\vec{x})\right]$$

The average velocity around the major axis of the ions having an energy $\mathcal{E}$ is

$$T\,\frac{R\; 2\pi c}{e}\left(\frac{\partial\nu}{\nu\,\partial\psi'} + \left(\frac{\mathcal{E}}{T} - \frac{3}{2}\right)\frac{\partial\tau}{\tau\,\partial\psi'}\right)$$

It must be equal to the average velocity V_Z of Z ions, and therefore :

$$\frac{\partial\tau}{\partial\psi'} = 0 \;\; ; \;\; T\,\frac{\partial\nu}{\nu\,\partial\psi'}\; R_0\;\frac{2\pi c}{e} = \bar{V}_Z \qquad (15)$$

We may also express F as a function of the variables $\mathcal{E}, \psi', v_{\parallel}$. We obtain instead of (14) :

$$F = \exp -\frac{\mathcal{E}}{T} \frac{n}{(2\pi T/m)^{3/2}} \left\{ 1 + H(\psi', \mathcal{E}) - \left[\frac{\partial \nu}{\nu \partial \psi''} + \left(\frac{\mathcal{E}}{T} - \frac{3}{2} \right) \frac{\partial \tau}{\tau \partial \psi''} \right] \frac{R_0 \, m v_{\parallel} \, 2\pi c}{e} \right\} \qquad (16)$$

The term which is proportional to $v_{\parallel}$ corresponds to an average motion along the lines of force, and therefore around the magnetic axis, at an angular velocity ω_θ .For ions having the energy $\mathcal{E}$

$$\omega_\theta = -\left[\frac{\partial \nu}{\nu \partial \psi''} + \left(\frac{\mathcal{E}}{T} - \frac{3}{2} \right) \frac{\partial \tau}{\tau \partial \psi''} \right] \frac{2\pi c}{e} \, T \, \frac{1}{q}$$

The magnetic perturbation $\delta B = -B_0 \frac{r}{R_0} \cos\theta$ superimposed to the configuration $\mathcal{C}_0$ acts as a compressional magnetic pumping propagating around the magnetic axis at the velocity $-\omega_\theta$ with respect to the hydrogen ions having energy in the range $\mathcal{E}$, $\mathcal{E} + \delta\mathcal{E}$ .Using the Landau perturbation theory - applicable if hydrogen ions are in the plateau regime - we find the power per cubic cm which is provided in their rest frame to ions in this range :

$$\delta W = \int_{\mathcal{E}}^{\mathcal{E}+\delta\mathcal{E}} \frac{\pi}{2} \left(\frac{r}{R} \right)^2 \omega_\theta^2 \, \frac{n_i}{T} \left(m_i v_\perp^2 \right)^2 \frac{1}{(K_\parallel) v_{th_i}} \exp - \frac{\omega_\theta^2}{K_\parallel^2 v_{th_i}^2} \exp - \frac{v_\perp^2}{v_{th_i}} \frac{d v_\perp^2}{v_{th_i}^2} \qquad (17)$$

$$K_\parallel = \frac{1}{qR} \quad ; \quad \mathcal{E} = \frac{1}{2} m v_\perp^2$$

These ions experience a force in the Θ direction

$$\delta F_\Theta = -\frac{1}{r\omega_\Theta}\ \delta w$$

Using (17) and taking into account that $\omega_\Theta \ll K_\parallel V th_1$, we obtain the force F_Θ per cubic cm which applies to hydrogen ions

$$F_\theta = \int\left(-\frac{\delta w}{r\omega_\Theta}\right) = \frac{1}{r}\ \frac{\sqrt{\pi}}{2}\left(\frac{r}{R}\right)^2 n_1 T^2\ \frac{R}{V th_1}\quad \frac{2\pi c}{e}$$

$$\int_0^\infty \exp\left(-\frac{V_\perp^2}{V th_1^2}\right)\frac{dV_\perp^2}{V th_1^{\,2}}\left(\frac{V_\perp}{V th_1}\right)^4\left[\frac{\partial \nu}{\nu\,\partial\psi''} + \left(\frac{V_\perp^2}{V th_1^2} - \frac{3}{2}\right)\frac{\partial\tau}{\tau\,\partial\psi''}\right]$$

This force may be compensated only by the Laplace force $e\,\phi_1 B\varphi/c$ resulting from the radial flux $\phi_1 = Z\ \phi_Z$ of hydrogen ions across the magnetic surfaces. Using the expression (2) of ϕ_Z the relation $F_\theta \sim Z e\ \phi_Z\ \frac{B}{c}$

$$\frac{\partial V}{\nu\,\partial\psi''} + \frac{3}{2}\ \frac{\partial\tau}{\tau\,\partial\psi''} \sim x\left[\frac{dn_1}{n_1 d\psi} - \frac{1}{2}\ \frac{dT}{T d\psi}\right]$$

where $x = \frac{qR}{\tau_1 V th_1}\ \frac{n_Z Z^2}{n_1}$ is small when the hydrogen ions are in the plateau regime.

Taking also into account (13) and (15), we then obtain :

$$\frac{\partial\tau}{\tau\,\partial\psi'} = 0\ ;\ \frac{\partial\tau}{\tau\,\partial\psi''} = \frac{dT}{T d\psi}$$

$$\frac{\partial\nu}{\nu\,\partial\psi'} = \frac{dn_1}{n_1 d\psi} + \frac{3}{2}\ \frac{dT}{T d\psi}\ ;\ \frac{\partial\nu}{\nu\,\partial\psi''} = -\frac{3}{2}\ \frac{dT}{T d\psi} \qquad (18)$$

$$\overline{V}_Z = \left(\frac{dn_1}{n_1 d\psi} + \frac{3}{2}\ \frac{dT}{T d\psi}\right) T R\ \frac{2\pi c}{e}$$

Besides the irreversible effect calculated above, the magnetic perturbation $\delta B = -B_0 \frac{r}{R} \cos\theta$ induces at order $\frac{r}{R}$ a perturbation F of the equilibrium distribution function F_0. Putting : $\delta F = g(\mathcal{E}, M_z, V_{\parallel}) \cos\theta$

We obtain from the VLASOV equation $d(F + \delta F)/dt = 0$ using the laws of motion in the presence of the perturbation δB

$$\frac{d\mathcal{E}}{dt} = \frac{dM_z}{dt} = 0 ; \qquad \frac{dV_{\parallel}}{dt} = -\frac{\mu}{m_1} \nabla_{\parallel} \delta B = -\frac{\mu}{m} \frac{1}{qR_0} \sin\theta \frac{r}{R_0} B_0$$

$$\mu = \tfrac{1}{2} m_1 V_{\perp}^2 / B_0$$

and the expression (16) of F :

$$\exp\left(-\frac{\mathcal{E}}{T}\right) \frac{n_1}{\left(\frac{2\pi T}{m_1}\right)^{3/2}} \left(\frac{\partial \mathcal{V}}{\mathcal{V}\partial \psi''} + \left(\frac{\mathcal{E}}{T} - \frac{3}{2}\right) \frac{\partial \tau}{\tau \partial \psi''}\right) \frac{2\pi c}{e} \mu \frac{1}{q} \sin\theta B_0 \frac{r}{R_0}$$

$$- g \sin\theta \frac{V_{\parallel}}{qR_0}$$

It results :

$$\delta F = g \cos\theta = \exp\left(-\frac{\mathcal{E}}{T}\right) \frac{n_1}{\left(\frac{2\pi T}{m_1}\right)^{3/2}} \cdot$$

$$\left(\frac{\partial \mathcal{V}}{\mathcal{V}\partial \psi''} + \left(\frac{\mathcal{E}}{T} - \frac{3}{2}\right) \frac{\partial \tau}{\tau \partial \psi''}\right) \frac{2\pi c}{e} \frac{1}{2} \frac{m_1 V_{\perp}^2}{V_{\parallel}} r \cos\theta \quad (19)$$

Finally, using (14) (18) and (19), we obtain the distribution function $F(\vec{x}, \vec{v})$ of hydrogen ions in the real configuration

$$F = F_0 + \delta F = \exp\left(-\frac{\mathcal{E}}{T}\right) \frac{n_1}{(2\pi\ T/m_1)^{3/2}}$$

$$\left\{ 1 + \left(\frac{dn}{n\,d\psi} + \frac{3}{2}\ \frac{dT}{T\,d\psi}\right) R\, m_1\, V\varphi \frac{c}{e}\ 2\pi\ + \right. \tag{20}$$

$$\left. \left(\frac{\mathcal{E}}{T} - \frac{3}{2}\right) \frac{dT}{T\,d\psi}\ r\, m_1 V\varphi\ \frac{2\pi c}{e} \left(1 + \frac{1}{2}\ \frac{V_\perp^2}{V_\parallel^{\ 2}}\right) \cos\theta \right\}$$

The friction force experienced by the Z assembly at point $\vec{x}$ is given by

$$\mathcal{F}_2(\vec{x}) = \iiint n_2\, m_1\ \ F\left(\vec{x}\,,\ \vec{V}' + V_2(\vec{x})\ \frac{\vec{B}}{B}\right) V' \cos\alpha'\ \frac{1}{2\,V'^3}\ A_D\ d_3\vec{V}'$$

$$= \frac{n_1\, n_2\, m_1}{(2\pi\ \ T(\psi)/m_1)^{3/2}} \quad \frac{A_D\ V_{th_1}^{\ 2}}{4}$$

$$\left\{ \left[\frac{-\,2\,V_2(\vec{x})}{V_{th_1}^{\ 2}} + R\ \frac{2\pi c}{e}\quad m_1\left(\frac{dn_1}{n_1\,d\psi} + \frac{3}{2}\quad \frac{dT}{T\,d\psi}\right)\right] \frac{4\pi}{3} \right. \tag{21}$$

$$\left. -\,2\ \frac{dT}{T\,d\psi}\ \ r\ \cos\theta\ \ m_1\ \ \frac{2\pi c}{e}\quad \frac{8\pi}{3} \right\}$$

Using (18) and (6) to calculate $\overline{V_Z}$ and $\widetilde{V_Z}$, and replacing $V_2(\vec{x})$ by $\overline{V_2} + \widetilde{V_2}\ \cos\theta$ and R by $R + r\cos\theta$ in (21), this relation becomes

$$\mathcal{F}_z(\vec{x}) = \tilde{\mathcal{F}}_2 \cos\theta = \frac{n_z n_1 m_1}{\left(2\pi T(\psi)/m_1\right)^{3/2}} \left[-\frac{d n_z}{Z n_z d\psi} + \frac{d n_1}{n_1 d\psi} - \frac{1}{2}\frac{dT}{T d\psi} \right]$$

$$\frac{A_D V_{th_1}^2}{4} \quad \frac{4\pi}{3} \quad \frac{2\pi c}{e} \quad 2r \quad m_1 \cos\theta$$

and results in (2) through (5).

3. Hydrogen ions in the PFIRSCH-SCHLUTER Regime .

Now, the assembly of hydrogen ions behaves as a fluid. The flux of particles and energy due to guiding center motion are given by

$$\phi_{gP} = n_1 T \frac{2c}{e B_0 R_0} \quad \vec{o_z}$$

$$\phi_{gE} = \frac{5}{2} T \phi_{gP}$$

The quantities div ϕ_{gP} and div ϕ_{gE} must be cancelled by the divergences div ϕ_P and div ϕ_E where $\phi_P = \varphi_P \vec{B}$ and $\phi_E = \varphi_E \vec{B}$ are parallel fluxes of particles and energy along the lines of force . We have on a magnetic surface

$$B \nabla_\parallel \varphi_P = -\,\mathrm{div}\,\phi_{gP} = -\frac{2c}{eB_0 R_0} \frac{d n_1 T}{dr} \sin\theta$$

$$\phi_P(\vec{x}) = \bar{\phi}_P + \tilde{\phi}_P \cos\theta$$

$$\tilde{\phi}_P = -\bar{\phi}_P \frac{r}{R} + \frac{2\pi c}{e} \frac{d n_1 T}{d\psi} \, 2r \qquad (22)$$

and similarily

$$\phi_E(\vec{x}) = \overline{\phi}_E + \widetilde{\phi}_E \cos\theta$$

$$\widetilde{\phi}_E = -\overline{\phi}_E \frac{r}{R} + \frac{2\pi c}{e} \frac{d(n_1 T^2)}{d\psi} 2r \qquad (23)$$

It results from the general expression for ϕ_E [5] :

$$\phi_E = \frac{5}{2} T \phi_P + \phi_{E\,irr}$$

where $\phi_{E\,irr}$ is the energy flux associated with irreversible mechanisms, in the present case a linear combination of $\nabla_{\parallel} T$, $\nabla_{\parallel} n_z$, $\nabla_{\parallel} n_1$ and $\nabla_{\parallel} u$, that

$$\overline{\phi}_E = \frac{5}{2} T \overline{\phi}_P \qquad (24)$$

All the information upon the departure from thermodynamical equilibrium of the hydrogen assembly is contained in the values of ϕ_P and ϕ_E, and we do not need to calculate $\nabla_{\parallel} T$, ... to obtain the friction force that this assembly applies to the Z assembly. We take the distribution function $F(\vec{x},\vec{v})$ of hydrogen ions at point $\vec{x}$ of the form :

$$F(\vec{x}\,\vec{v}) = \frac{n[\psi(\vec{x})]}{(2\pi T[\psi(\vec{x})]/m_1)^{3/2}} \exp -\frac{\varepsilon}{T} \left[1 + \left(X(\vec{x}) + Y(\vec{x}) \frac{V^2}{V_{th_1}^2}\right)\frac{V\cos\alpha}{V_{th_1}}\right]$$

From the equations :

$$\phi_P(\vec{x}) = \int_0^{\pi}\int_0^{\infty} F(\vec{x},\vec{v})\, V\cos\alpha\; 2\pi V^2 dV \sin\alpha\, d\alpha$$

$$\phi_E(\vec{x}) = \int_0^{\pi}\int_0^{\infty} F(\vec{x},\vec{v})\, V\cos\alpha \left(\frac{1}{2} m_1 V^2\right) 2\pi V^2 dV \quad \alpha\, d\alpha$$

we obtain,

$$\left(x + \frac{5}{2}\; y\right) \frac{1}{2}\; n_1\; V_{th_1} \;=\; \phi_P$$

$$\left(\frac{5}{2}\; x + \frac{35}{4}\, y\right)\frac{1}{2}\; n_1\; V_{th_1}\; T \;=\; \phi_E$$

It then results from (22) (23) and (24)

$$x(\vec{x}) = \bar{x} \;+\; \tilde{x}\; \cos\theta \qquad (25)$$
$$y(\vec{x}) = \bar{y} \;+\; \tilde{y}\; \cos\theta$$

where

$$\bar{x} \;=\; \frac{2\,\bar{\Phi}_P}{n_1\, V_{th_1}} \quad ; \quad \bar{y} = 0$$

$$\tilde{x} = -\,\bar{x}\,\frac{r}{R} \;+\; \frac{2}{n_1 V_{th_1}}\;\frac{2\pi c}{e}\; 2r\;\; n_1 T\left[\frac{dn_1}{n_1\, d\psi} - \frac{3}{2}\,\frac{dT}{T\, d\psi}\right] \qquad (26)$$

$$\tilde{y} = \frac{2}{n_1 V_{th_1}}\;\; \frac{2\pi c}{e}\;\; 2r\;\; n_1 T\;\; \frac{dT}{T\,d\psi}\,.$$

The force $\mathcal{F}_Z(\vec{x})$ experienced per cubic cm by the Z ions from hydrogen ions is equal to :

$$\mathcal{F}_z(\vec{x}) = \iiint \left[\; F\left(\vec{x}_1\;\; v' + V_z(\vec{x})\,\frac{\vec{B}}{B}\right) n_z\; m_1\, v' \cos\alpha\;\; A_D\, \frac{1}{2}\, v'^3\;\; d_3 v'\right.$$

$$= n_z\, m_1 \quad \frac{n}{\left(2\pi\; T/m_1\right)^{3/2}}\;\; \frac{A_D}{4}\;\; V_{th_1}^2 \qquad (27)$$

$$\left[-\,2\, V_z(\vec{x})/V_{th_1}^{\,2} \;\; + \left(x(\vec{x}) + y(\vec{x})\right)/V_{th_1}\right]\;\; \frac{4\pi}{3}$$

Using (6) and (26) and expressing that the average value of $\mathcal{F}_2(\vec{x})$ along the lines of force is zero, we obtain

$$\bar{V}_z = (\bar{x} + \bar{y}) \frac{V\mathcal{H}}{2}$$

Then using again (6) and (26)

$$\mathcal{F}_2(\vec{x}) = \tilde{\mathcal{F}}_2 \cos\theta = n_2 \frac{m_1 \, n_1}{(2\pi \, T/m_1)^{3/2}} \quad \frac{A_D}{4} \, V\mathcal{H}_1^2$$

$$\left(\frac{4r}{V\mathcal{H}_1^2} T \frac{2\pi c}{e}\right)\left[-\frac{d\, n_2}{Z\, n_2\, d\Psi} + \frac{d\, n_1}{n_1\, d\Psi} - \frac{1}{2}\frac{d\,T}{T\,d\Psi}\right]\frac{4\pi}{3} \cos\theta$$

which gives eq.(2) through (5)

IV. INVERSION OF THE FLUX OF IMPURITIES CAUSED BY AN ORDERED MOTION OF HYDROGEN IONS.

1. Necessary ordered velocities.

We now assume that there is no thermal gradient but that the hydrogen ions have an imposed motion around the magnetic axis at an angular velocity Ω_θ. Their average velocity in the φ direction (in the frame of reference $\mathfrak{R}$ rotating around the major axis where the average electric field is zero) is then given by

$$V_1 = \Omega_1 R + V_{\parallel 1}'$$

$$V_{\parallel 1} = \bar{V}_{\parallel 1}\left(1 - \frac{r}{R}\cos\theta\right)$$

$$\Omega_1 = r \frac{dn_1}{n_1\, d\Psi} \quad \frac{2\pi c}{e} \qquad (28)$$

$$\Omega_\theta = \frac{\overline{V_{\parallel 1}}}{q R}$$

The term $\Omega_1 R$ in V_1 is the diamagnetic motion around the major axis which justifies the confinement, on which is superimposed the motion along the lines of force (preserving the particle flux across the flux tubes) which gives rise to the angular velocity Ω_θ around the magnetic axis.

We assume again that the Z ions are in the PFIRSCH-SCHLUTER regime, but to simplify, we assume from now that $n_Z Z^2 < n_1$. We introduce the possibility that the whole plasma (i.e. the frame of reference $\mathcal{R}$) rotates around the major axis at an angular velocity Ω_φ. We must of course have $R\Omega_\varphi \ll v_{th_1}$, for the equilibrium and stability not to be altered. However if $m_Z \gg m_1$, we may have $R\Omega_\varphi \; v_{th_Z}$. In these conditions, due to the centrifugal effect,* the Z ions concentrate on the outer parts of the magnetic surfaces :

$$n_Z = \bar{n}_Z + \tilde{n}_Z \cos\theta + \tilde{\tilde{n}}_Z \sin\theta$$

$$\tilde{n}_Z = \bar{n}_Z \quad \frac{m_Z \Omega_\varphi^2 R r}{T} \qquad (29)$$

The velocity V_Z of the Z ions in the φ direction (in the frame $\mathcal{R}$) is again the superposition of a diamagnetic motion in the B_θ field and a parallel motion along the lines of force:

$$V_Z = \Omega_Z R + V_{\parallel Z}$$

$$\Omega_Z = T \frac{dn_Z}{n_Z \, d\psi} \; \frac{2\pi c}{Z e} \qquad (30)$$

The quantity $V_{\parallel Z} n_Z / B$ is a constant on each magnetic surface

* If $n_Z Z^2 < n_1$, we may neglect the θ dependent component of the electrostatic potential.

and therefore :

$$V_{\parallel z}\, n_z = K(r) - K(r)\frac{r}{R}\cos\theta \qquad (31)$$

The relative velocity along the lines of force of the hydrogen assembly with respect to the z assembly is obtained from (28), (30) and (31)

$$\Delta V = \Omega' R_0\left(1+\frac{r}{R_0}\cos\theta\right) + \overline{V_{\parallel 1}}\left(1-\frac{r}{R_0}\cos\theta\right) - \frac{K}{n_z}\left(1-\frac{r}{R_0}\cos\theta\right) \qquad (32)$$

where

$$\Omega' = \Omega_1 - \Omega_z \simeq \frac{c}{e B_\theta}\, T\, \frac{dn_1}{n_1\, dr}\, \frac{1}{R} \qquad (33)$$

if $\left|\frac{dn_1}{n_1\, dr}\right| \gg \frac{1}{z}\left|\frac{dn_z}{n_z\, dr}\right|$

The equilibrium of the z assembly along the lines of force writes :

$$-T\,\nabla_\parallel n_z + n_z\,\nabla_\parallel\left(m_z\,\Omega_\varphi^2\, R\, r\cos\theta\right) + f\, n_z\, \Delta V = 0 \qquad (34)$$

Where f is the friction coefficient between the assemblies (z) and (1) :

$$f \simeq \frac{m_1}{\tau_1}\, z^2$$

where τ_1 , is defined by (2a).

It results from (34) and (32)

$$n_2 = \overline{n_2} + \widetilde{n_2} \cos\theta + \widetilde{\widetilde{n_2}} \sin\theta$$

$$\widetilde{n_2} = \overline{n_2} \quad \frac{m_2 \Omega_\varphi^2 R r}{T}$$

$$- \widetilde{\widetilde{n_2}} \frac{1}{qA} m_2 \Omega_\varphi^2 R_0 r \frac{1}{2} + f \Big[\overline{n_2} \left(\Omega' R_0 + \overline{V}_{\parallel 1} \right) - K$$

$$+ \frac{1}{2} \widetilde{n_2} \left(r\Omega' - \overline{V}_{\parallel 1} \frac{r}{R_0} \right) \Big] = 0$$

$$- T \frac{\widetilde{\widetilde{n_2}}}{q R_0} + f \overline{n_2} \left(\Omega' r - \overline{V}_{\parallel 1} \frac{r}{R_0} \right) + f K \frac{r}{R_0} + f \widetilde{n_2} \left(\Omega' R_0 + \overline{V}_{\parallel 1} \right) = 0$$

These relations determine K and $\widetilde{\widetilde{n_2}}$. In particular :

$$\widetilde{\widetilde{n_2}} / f = n_2 \tau_1 / m_1 z^2 = \frac{qA}{T} \left[\overline{n_2}\, 2\, \Omega' r + \widetilde{n_2} \left(\Omega' R + \overline{V}_{\parallel 1} \right) \right] \tag{35}$$

The flux ϕ_z of z ions across a magnetic surface is then given by :

$$\phi_z = \frac{1}{2\pi} \int_0^{2\pi} d\theta \; V_{d2} \sin\theta \; n_2 = \frac{1}{2} V_{d2} \widetilde{\widetilde{n_2}} \tag{36}$$

where V_{d2} is the guiding center drift velocity

$$V_{d2} = \frac{c}{Z e B} m_2 \frac{V_{\parallel z}^2 + \Omega^2 R^2}{R}$$

In the absence of the rotation Ω_φ, $\widetilde{n_2} = 0$ (Cf. Eq.29), and the flux ϕ_2 given by (36) has the value (2). It is directed inwards if $d n_1 / n_1 dr < d n_2 / Z n_2 dr$. The presence of the rotations

Ω_φ , Ω_Θ result in the inversion of ϕ_z if the term proportional to $\tilde{n}_z$ in (35) changes the sign of $\tilde{\tilde{n}}_z$. Assuming that

$\frac{\tilde{n}_z}{n_z} > \frac{2r}{R}$ i.e., taking into account (29)

$$\frac{m_z \Omega_\varphi^2 R r}{T} > \frac{2r}{R} \tag{37}$$

this circumstance occurs when $(\bar{V}_{\parallel 1} + \Omega' R)\Omega' < 0$, i.e., taking into account (33)

$$\left|\bar{V}_{\parallel 1}\right| > \left|\Omega_1 R\right| = C_{H_1}\left|\frac{dn_1}{n_1 dr}\right| \frac{qR}{r} \frac{V_{H_1}}{2}$$
$$\bar{V}_{\parallel 1} \cdot \Omega_1 < 0 \tag{38}$$

If the z ions are in the banana regime (as would be the case for the He^{2+} ions) the inversion of ϕ_z occurs without rotation of the whole plasma around the major axis, if the condition (38) is satisfied. This result is nearly obvious (again with $n_z z^2 < n_1$)if we consider the radial diffusion of the assembly as being due to the friction in the φ direction experienced by the trapped z ions from hydrogen ions. This friction changes its sign if the circulating hydrogen ions have an angular velocity around the major axis $\approx \Omega_z$. Taking into account (28) such a situation corresponds to a velocity $\bar{V}_{\parallel 1}$ equal to $-(\Omega_1 - \Omega_z)R \approx -\Omega_1 R$ which is the threshold value given by (39). (Actually the rotation around the major axis at the frequency Ω_φ makes the z ions in the PFIRSCH-SCHLUTER regime to behave like trapped particles.)

2. Realization by a magnetic pumping.

The plasma rotation around the magnetic axis at the angular velocity Ω_φ verifying (37) (if necessary) does not seem difficult to achieve, because of the small value to the viscosity coefficient of the magnetized plasma. The rotation around the magnetic axis at the angular velocity $\Omega_\theta = \frac{\bar{V}_{\parallel 1}}{qR}$, while $\bar{V}_{\parallel 1} \ll V_{th_1}$, is more difficult to induce, because this motion takes place through the modulation $-B_0 \frac{r}{R} \cos\theta$ of the static field.

Let us assume that the hydrogen ions are in the banana regime. The rotation at frequency Ω_θ of circulating hydrogen ions is restrained by the friction force F_θ in the θ direction that these particles experience from trapped ions (z ions play no role if $n_z z^2 < n_1$). A force F_θ' acting in the direction θ on circulating particles must cancell the force F_θ . This force (per particle) is given by

$$F_\theta = \frac{m_1}{\tau_1} \left(\frac{2r}{R}\right)^{1/2} \bar{V}_{\parallel 1} \quad \frac{qR}{r}$$

Using the threshold value given by (38)

$$F_\theta = \frac{m_1}{\tau_1} \quad e\,\mathcal{H}_1 \quad \frac{d n_1}{n_1 dr} \left(\frac{2r}{R}\right)^{1/2} \frac{1}{2} V_{th_1} \left(\frac{qR}{r}\right)^2 \qquad (39)$$

The force F_θ' may be provided for instance by a magnetic perturbation of the torsional type [6] induced by coils as shown on fig. 2, oscillating in the frame of reference $\mathcal{R}$ of the plasma at the frequency ω :

$$\delta B_\varphi = 0$$

$$\delta B_r = -\,\varepsilon\, b \cos(\ell\theta + M\varphi + \omega t) \left(\frac{r}{r_p}\right)^{|\ell|-1}$$

$$\delta B_\theta = b \sin(\ell\theta + M\varphi + \omega t) \left(\frac{r}{r_p}\right)^{|\ell|-1}$$

$$M/R \ll \ell/r \qquad\qquad \varepsilon = \frac{\ell}{|\ell|}$$

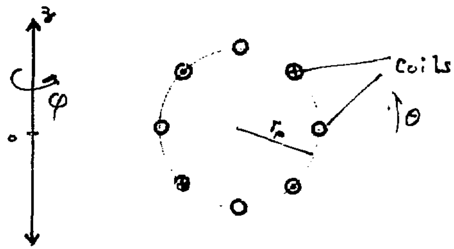

fig. 2

Such a magnetic pumping propagates in the plasma as in vacuum if

$$\omega \ll K_{\parallel} C_A \quad ; \quad K_{\parallel} = \frac{1}{R}\left(M + \frac{\ell}{q}\right)$$

where C_A is the Alfven velocity. Its irreversible action on the particules is the same as that of the fictitious potential

$$\delta \mathcal{H} = \frac{1}{M+\frac{\ell}{q}} \; \frac{b}{B}\left(\frac{r}{r_p}\right)^{|\ell|-1} \sin\left[\left(\ell - \varepsilon\right)\theta + M\varphi + \omega t\right] m_i\left(V_{\parallel}^2 + \frac{V_{\perp}^2}{2}\right)$$

By resonance with the circulating particles having in the frame $\mathcal{R}'$ a parallel velocity $V_R = -\omega R/\left[M + q^{-1}(\ell - \varepsilon)\right]$, it provides forces F'_{θ} and F'_{φ} in the θ and φ directions and a power W' (in the laboratory frame) related by :

$$\frac{F'_{\theta}\, r}{\ell - \varepsilon} = \frac{F'_{\varphi}\, R}{M} = -\frac{W'}{\omega_0} \qquad (40)$$

where ω_0 is the angular frequency in the laboratory frame : $\omega_0 = \omega - M\Omega\varphi$. The force F'_{θ} in the LANDAU regime - which would be generally effective in the present case - is given by :

$$F'_{\theta} = \frac{\sqrt{\pi}}{2}\left(\frac{b}{B}\right)^2 \frac{1}{K_{\parallel}^2}\left(V_R^4 + V_R^2\, V_{th_i}^2 + \frac{1}{2} V_{th_i}^2\right)$$

$$\frac{m_i^2}{R^2}\left(\frac{r}{r_p}\right)^{2(|\ell|-1)} \exp - \frac{V_R^2}{V_{th_i}^2} \quad \frac{V_R}{T\, V_{th_i}}\left(\frac{\ell - \varepsilon}{r}\right)$$

$$V_R = \frac{\omega R}{M + \frac{\ell - \varepsilon}{q}}$$

Choosing $V_R = 1{,}5\ V_{th_1}$, we obtain :

$$F_\theta' = -3\left(\frac{b}{B}\right)^2 \frac{1}{\left(M+\frac{\ell}{q}\right)^2}\left(\frac{r}{r_p}\right)^{2(|\ell|-1)} V_{th_1}^2\, m_1\ \frac{\ell-\varepsilon}{r}\ \frac{w}{|w|}$$

$$w = -1{,}5\ \frac{V_{th_1}}{R}\left(M+\frac{\ell-\varepsilon}{q}\right)$$

$$w_o = w - M\Omega\varphi$$

The condition $F_\theta' = -F_\theta$, where F_θ is given by (39), then imposes :

$$\frac{b}{B} \simeq 0{,}5\left(M+\frac{\ell}{q}\right)\frac{r_p}{R}\left(\frac{r_p}{r}\right)^{|\ell|-2}\frac{1}{(\ell-\ )^{1/2}}$$

$$\left(\frac{qR}{r}\right)^{1/2}\left(\rho_{th_1}\frac{dn_1}{n_1 dr}\right)^{-1/2}\left[\frac{1}{V_{th_1}}\ \frac{qR}{\tau_1}\left(\frac{R}{r}\right)^{3/2}\right]^{-1/2}$$

For instance, taking $M = 1$ and $\ell = -2$, and assuming that $q = 3$ at the plasma edge, the magnetic pumping propagates up to the magnetic surface $q = 2$, (where the ALFVEN resonance takes place). We have in the pumped region $M+\frac{\ell}{q} \sim 0{,}15$. Taking T = 1 Kev, $n_1 = 3\ 10^{13}\,cm^{-3}$ $R/r = 3$, we obtain the necessary value $b/B \sim \frac{3}{1000}$ oscillating at the frequency $w/2\pi \sim 20\ kHz$.

The force F_φ' induced by the pumping could be easily cancelled by a φ modulation of the static field, which would provide momentum to the plasma in the φ direction only. The power W' which is received per particle from the pumping, and which is the minimal cost to be paid to maintain the departure from thermal equilibrium allowing decontamination, is given by (40), where $F_\Theta' = -F_\Theta$. Using (39) and assuming that $\omega_0 \lesssim \omega$ and $(Mq + \ell) < \mathcal{E}$ we find the power $n_1 W'$ per cubic cm :

$$n_1 W' = 1{,}5 \; \frac{n_1}{\tau_1} \left(\frac{2r}{R}\right)^{1/2} \rho_{th_1} \left|\frac{dn_1}{n_1 dr}\right| T \left|\frac{\mathcal{E}}{|H| - \mathcal{E}}\right|$$

In the case considered above : $n_1 W' = 1/10$ watts/cm^3. While it is not very important, it is however much larger than the neo-classical diffusion losses W_D which scales as (per cubic cm):

$$W_D \sim \frac{n_1 T}{\tau_1} \left(\frac{R}{r}\right)^{3/2} q^2 \left(\frac{dT}{T dr} \; \rho_{th_1}\right) \frac{\rho_{th_1}}{r}$$

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