SIMILARITIES AND DIFFERENCES BETWEEN VOLUME-CHARGED (NUCLEAR) DROPS AND CHARGED CONDUCTING (RAIN) DROPS

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August 1974

ABSTRACT

The liquid drop model of the nucleus has been explored in the last forty years in the study of nuclear fission and, more generally, nuclear deformation energy surfaces. This model assumes the nucleus to be an incompressible drop with charges uniformly distributed throughout the volume. We have made a parallel study of a charged conducting drop with charges residing on the surface of the drop. This is the case of rain drops in an electrified cloud. Stable and unstable shapes of equilibrium has been calculated and their properties examined. Fundamental differences in the stability properties of the two types of drops are brought out and show up most significantly in their stability against reflection asymmetry. Certain similarities are also found, particularly in the ellipsoidal deformation of the drops, as well as their division into equal droplets. Both these similarities and differences are described and discussed.

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INTRODUCTION

The liquid drop model of nuclear fission was suggested (1) thirty five years ago. The model has been very useful for the understanding of nuclear fission data and has recently been found to be an important element in what has come to be known in nuclear physics as the Strutinsky method by which the predictions on the masses and stability of the yet-undiscovered superheavy nuclei are made (2).

There are two aspects of the model. The more difficult and less certain is the dynamical study of the liquid drop model. This involves assumptions regarding the fluid flow patterns, the viscosity and other properties to be assumed for the nucleus. Furthermore, the liquid drop may undertake a great variety of shapes, making the calculation very involved. In the last ten years, various attempts (3) have been made to tackle this problem and I believe that these have only been partially successful and there is still the basic question whether a nucleus is (dynamically) like a liquid drop at all. On the other hand, the other aspect of the model, the statics, has been fairly well established (4) and has demonstrated its value in nuclear fission in many ways. One studies the balancing of only two forces present in the deformable liquid drop, the Coulomb and surface tension forces. A recent work (5) includes also the centrifugal "force." No other properties of the liquid drop such as short range correlations and flow patterns need to be assumed in such a study. Indeed, it can be demonstrated (6) that the theory represents a more general system in which a liquid drop is a special example. This is what we call the leptodermous system, that is, a system with a thin surface region and a volume region of uniform density. In all these studies, the objective is to find the shapes of equilibrium of the system and their energies. These can be either a stable equilibrium point (a minimum) or an unstable equilibrium point (a saddle or a mountain top) in a multi-dimensional space with co-ordinates representing various deformation parameters. By looking at these equilibrium points a lot can be said about the system: whether the system tends to remain a sphere or undergo fission, whether the system

*Work performed under the auspices of the U. S. Atomic Energy Commission.

prefers to divide into two, three, or four droplets, and, with some generalization of the model, whether two droplets can coalesce into one.

With all that has been developed in the nuclear fission problem, it would be interesting to apply it to actual macroscopic rain drops which are electrically conducting and consider their shapes of equilibrium. This has the great advantage over the nuclear case that direct measurements in the laboratory on a drop can be made. Besides studying the rain drops on its own merit, a parallel theoretical and experimental study of the conducting drop may also throw light on the nuclear drop. Of course properties of the charged conducting drop is not a new area of study. In 1882, Lord Rayleigh (7) published a paper on the stability of a charged conducting drop under small oscillations. Other studies are made more recently (8). However, in the present work† we shall make a close comparative study of the nuclear drop and the rain drop using methods developed in the liquid drop theory of nuclear fission.

In the next section, some basic concepts of nuclear fission theory (9) will be described, before discussing, in the following section, simple similarities and differences between volume-charged and charged conducting drops. After that a method will be described to calculate the symmetric equilibrium shapes of the conducting drop and the results will be compared to those of a volume charged drop.

SOME BASIC CONCEPTS IN FISSION THEORY

For an incompressible volume charged drop, two forces are acting: a Coulomb force which tends to break up the drop and a surface tension which tends to keep it together. A quantity of importance is then the ratio of the Coulomb energy and the surface energy. One may define what is called the fissility parameter, x, as

$$x = \frac{E_c^{(o)}}{2E_s^{(o)}} \propto \frac{Q^2/R}{R^2} \propto \frac{Q^2}{V} ,$$

where $E_c^{(o)}$ and $E_s^{(o)}$ are Coulomb and surface energies of a sphere with charge Q, radius R, and volume V. For $x < 1$, the spherical drop is stable with respect to deformations and for $x > 1$, it turns out that the forces are such that the drop is in unstable equilibrium. The energy excess of a deformed drop over the original spherical drop may be written as

†This work was done in collaboration with W. J. Swiatecki.

$$E_s - E_s^{(o)} + E_c - E_c^{(o)} = E_s^{(o)}(B_s - 1) + E_c^{(o)}(B_c - 1)$$

$$= E_s^{(o)}\{(B_s - 1) + 2x\ (B_c - 1)\} \quad ,$$

where E_s and E_c are the surface and Coulomb energies of the drop and the superscript (o) implies that the quantity is evaluated for a sphere; also $B_s = E_s/E_s^{(o)}$ and $B_c = E_c/E_c^{(o)}$. If ξ is the energy excess in units of $E_s^{(o)}$, then

$$\xi = B_s - 1 + 2x(B_c - 1). \qquad (1)$$

In Fig. 1, we sketch the behavior of ξ as a function of deformation for a particular value of $x < 1$. The configuration at zero deformation, i.e., a sphere, is a potential energy minimum. The energy is increased as one deforms the drop until a point is reached where the disruptive Coulomb force is just balanced by the stabilizing surface tension. This point is called a saddle point. It is unstable with respect to the deformation leading to fission (but is stable with respect to other deformations). Obviously the curve will be different for different values of charge on the drop, i.e., different values of x (see Fig. 2). Thus for $x > 1$, the sphere is at a potential maximum.

Let ξ_R denote the difference in energy between the initial sphere and the final fragments at infinity in units of $E_s^{(o)}$. For division into two equal spheres which is illustrated in Figure 2, $\xi_R = 0$ at $x = 0.351$. For $x > 0.351$, $\xi_R < 0$, and for $x < 0.351$, $\xi_R > 0$. In the general case of division into n equal spheres, a general formula (4) may be written for ξ_R. The charge on each sphere is Q/n and its radius is $(R^3/n)^{1/3} = R\ n^{-1/3}$, so that the Coulomb energy of the n spheres is n multiplied by the Coulomb energy of each sphere:

$$E_c = n\ \frac{3}{5}\ \frac{(Q/n)^2}{R\ n^{-1/3}}$$

$$= \frac{3}{5}\frac{Q^2}{R}n^{-2/3}$$

thus

$$B_c = n^{-2/3} \quad .$$

Total surface energy of the n sphere is

$$E_s = \gamma\, n.\ 4\pi (R\, n^{-1/3})^2 = 4\pi R^2 \gamma n^{1/3}$$

thus

$$B_s = n^{1/3} \quad .$$

Hence the energy excess over the sphere in units of $E_s^{(o)}$ is

$$\xi_R = (n^{1/3} - 1) + 2x(n^{-2/3} - 1) \quad . \qquad (2)$$

For each value of n, this equation gives a straight line relation between ξ_R and x. By studying the system of straight lines for various values of n, the following can be deduced. For $x < 0.35$, the sphere has the lowest energy. For $0.35 < x < 0.61$, the division into two spheres gives the lowest energy. For $0.61 < x < 0.87$, the division into three equal spheres gives the lowest energy. Finally, for $0.87 < x < 1.12$, the division into four equal spheres gives the lowest energy.

In Fig. 3, we present the shapes of equilibrium of a volume charged drop as a function of the x values (10), so that we can compare them with the results we are going to obtain for a surface charged drop. The abscissa gives the fissility parameter x from 0 to 1. The ordinate gives R_{MIN}/R and R_{MAX}/R as a measure of the shape, where for an asymmetric shape radius R_{MIN} is the minimum radius of the neck of the drop and the two maximum radii R_{MAX} are the distances from the center of the neck (at its minimum radius) to the two ends of the drop. For a symmetric shape the two maxium radii are equal.

Along $R_{MAX}/R = 1$ is the sphere which is at a potential energy minimum for all $x < 1$. The rest of the curves represent a family of reflection symmetric equilibrium shapes and a family of reflection asymmetric equilibrium shapes. The two families cross each other at x = 0 396. Their shapes are schematically indicated in the figure. A point to notice is that along the symmetric family there is a fairly rapid change in the trend of R_{MAX}/R_o at x values around 0.7. It is found below that for a conducting drop a similar change occurs at a larger value of x. The notation (1) and (2) in the figure indicates whether the equilibrium shape is at (respectively) a saddle (unstable in only one direction) or a mountain top (unstable in two different directions).

COMPARISON OF A VOLUME CHARGED DROP AND A CHARGED CONDUCTING DROP

It is straightforward to apply the methods described in the last section to a charged conducting drop. Thus the fissility parameter x can be defined similarly as the ratio of the Coulomb energy to twice the surface energy. The equation (1) for the energy excess ξ over a spherical drop will be the same as for the volume charged drop case. Of course, the Coulomb energies will now be evaluated on the assumption that the drop is conducting.

Three simple similarities may be pointed out.

(a) For x = 0, there is no charge on the drop so that the equilibrium shapes are the same whether the drop is conducting or not. Also, it turns out nontrivially that as in the case of a volume charged drop (11), x = 1 represents the transition point where the spherical drop is stable for $x < 1$ and is unstable for $x > 1$.

(b) A second similarity is apparent if we look at the energy difference ξ_B from the initial to the final state when the drop is divided into equal spheres. We have described this in detail for a volume charged drop in reference to Eq. (2). When we make a similar study for a conducting drop, we get a completely identical equation and the corresponding discussions are applicable. The reason is that only spherical shapes are involved in both the initial and final states, and the Coulomb energy of a volume charged sphere (which is $\frac{3}{5} Q^2/R$) and that of a conducting sphere (which is $\frac{1}{2} Q^2/R$) differ by only a numerical factor, 6/5, that is the same for both states. Hence B_s and B_c are the same for both cases and the same energy Eq. (2) holds good.

(c) It also turns out that the Coulomb energy of a volume charged ellipsoid and that of a conducting ellipsoid differ also by the same numerical factor. Thus, the Coulomb energy of a conducting ellipsoid is given by (12)

$$E_c = \frac{1}{4} Q^2 \int_0^{\infty} \left[(a^2 + \lambda)\ (b^2 + \lambda)\ (c^2 + \lambda)\right]^{-\frac{1}{2}} d\lambda$$

so that

$$B_c = \frac{1}{2} R \int_0^{\infty} \left[(a^2 + \lambda)\ (b^2 + \lambda)\ (c^2 + \lambda)\right]^{-\frac{1}{2}} d\lambda$$

where a, b, and c are the lengths of the axes of an ellipsoid. This integral may be carried out analytically in the case of a spheriod where two of the axes are equal. B_c for a volume charged case is given (13) by exactly the same formula. Hence, if we make the drop to take on only ellipsoidal shapes, then any conclusions about the statics of the volume charged drop will be true for the conducting drop.

The first difference between the volume charged drop and a conducting drop can be found if we consider the division of the drop into two unequal spheres at an infinite distance apart, one with volume βV and the other with volume $(1 - \beta)V$. In Fig. 4 is plotted the energy change ξ_R between the initial and final states (14) as a function of β for various values of the fissility parameter x. For $\beta = 0$ and $\beta = 1$ we get a sphere with volume V which is just the initial state. For $\beta = 0.5$, we get two equal spheres. The energy change is zero at $x = 0.35$ for $\beta = 0.5$, as was pointed out above in connection with Fig. 2. For a conducting drop Fig. 5 is found (14). We note that here again the energy is zero at $x = 0.35$ for $\beta = 0.5$. However, except for the points at $\beta = 0$, 0.5, and 1.0 the curves in the two figures are very different. A potential minimum for a volume charged drop occurs at $\beta = 0.5$ for $x > 0.2$, but a potential maximum for a conducting drop occurs at $\beta = 0.5$ for all x values less than one. In the latter case minima occur at points where the fragments are unequal.

The major reason for the above differences is that the charge to mass ratio for a volume charged drop is a constant, but for a conducting drop it is not required to be a constant. This is also the underlying cause for the second difference that appears when we try to find the configuration with the absolute lowest energy for a drop with a given fissility parameter x. For a volume charged drop, this configuration is n equal droplets at infinity (4) and the number n depends on the x values of the drop [Eq. (2)]. One would at first expect that the same conclusion might hold for a conducting drop. But, as we shall show, for a conducting drop, the configuration at the lowest energy is one with all the charges Q on the drop taken off and distributed among many infinitesimal droplets at infinity. The total energy of the droplets may be made to vanish and only the surface energy of the original drop is left. The possibility of such a configuration is shown as follows. Let $\frac{1}{n}$ of the original drop of radius R be taken off carrying all the charge Q. This is then divided into m equal spheres, each with a charge Q/m. Thus for each sphere the sum of the Coulomb and surface energy is

$$4\pi R^2 \gamma \left(\frac{1}{nm}\right)^{2/3} + \frac{1}{2}\frac{Q^2}{R}(nm)^{1/3}\left(\frac{1}{m}\right)^2$$

Hence the total energy of the small spheres is m times this quantity:

$$4\pi R^2 \gamma\, m(nm)^{-2/3} + \frac{1}{2}\frac{Q^2}{R}(nm)^{1/3} m^{-2}\cdot m$$

$$= 4\pi R^2 \gamma\, m^{1/3} n^{-2/3} + \frac{1}{2}\frac{Q^2}{R} n^{1/3} m^{-2/3} \quad .$$

Now let us choose $m \propto n^s$. The energy of the droplets is now equal to

$$4\pi R^2 \gamma \, n^{-\frac{2}{3}+\frac{s}{3}} + \frac{1}{2}\frac{Q^2}{R} n^{\frac{1}{3}-\frac{2s}{3}} ,$$

which is zero when n goes to infinity provided

$$+2 > s > +\frac{1}{2}$$

and the proposed configuration is obtained. In other words, we have made the Coulomb energy of the given drop zero by dispersing the charge onto an infinite number of infinitesimal droplets without increasing the surface energy by a finite amount.

PARAMETERIZATION OF A CHARGED CONDUCTING DROP

In this part of the work we shall try to determine the equilibrium shapes of a charged conducting drop to be compared with those for a volume-charged drop (Fig. 3).

The calculation of the Coulomb energy of a conducting drop with an arbitrary shape is in general a difficult problem. However, it can be side-stepped by requiring the drop to assume a prescribed family of shapes,†† in fact, making the calculation of its Coulomb energy is a trivial matter. It is well-known from the theory of electrostatics that the electric potential due to a system of charges (total charge Q) at any point outside a given equipotential, is the same as that due to a charged conductor with the shape of this equipotential having a charge Q. Hence, if we require the drop to assume the shape of an equipotential of potential α, its Coulomb energy is just $\frac{1}{2}\alpha Q$. If R is the radius of a sphere that has the same volume as the drop and possesses the same amount of charge, its Coulomb energy is $\frac{1}{2} Q^2/R$. Hence we get

$$B_c = \alpha R/Q \quad .$$

The surface energy relative to that of the sphere, B_S, can simply be found by calculating its area numerically. Hence for a given fissility x the

††This is a common practiced procedure in the liquid drop model of fission. The true equilibrium points can be determined by looking at the convergence as one enlarges the family of shapes. An independent condition on equilibrium may also be used as illustrated below.

energy of the drop is calculated [Eq. 1], and equilibrium shapes, whose energies are stationary, are then determined.

For illustration, the shapes of equipotentials that enclose two equal points charges are shown in Fig. 6, where the volumes of the shapes have been normalized to the same value. We shall refer to these shapes as the symmetric $N = 2$ family, since they are generated with two point charges and are reflection symmetric. This figure displays a very restricted series of shapes. However, it is easy to increase the possible shapes by generating equipotentials of a larger number of point charges, which may be placed on a straight line so that the shapes are axially symmetric. The reflection symmetric $N = 3$ family is generated with two equal charges situated at equal distances on opposite sides of a third point charge. The shapes are shown in Fig. 7. They include the symmetric $N = 2$ family. Similarily, we can go on to $N = 4, 5, \ldots$ family of shapes.

In general, the N-family of axially symmetric shapes may be specified by giving the magnitudes of the N point charges and their positions as well as the value of the potential on the equipotential we are looking at. These are $2N + 1$ numbers. However, not all these numbers are required to specify a shape. Three numbers may be arbitrary: (1) The center of mass of all the point charges may be at any point in space; (2) The total charge may be fixed beforehand; (3) We can also present a scale by which the distances between the point charges are measured. Thus, we are left with $2N - 2$ parameters. (For reflection symmetric shapes, the distribution of point charges and their magnitudes are reflection symmetric with respect to the origin and we have only $N - 1$ parameters).

However, the shapes generated even by a large number of point charges are not general enough to represent an arbitrary shape. Thus, an oblate shape cannot be found in our scheme. This raises the question whether the equilibrium shapes we have determined are indeed true equilibrium configurations when the drop is free to take on any arbritrary shape. To answer this question a criterion can be developed to test a given shape for equilirbium. (A similar criterion exists for a volume charged drop (11).)

If the surface element dS is displaced normally by a small smount, δn, without affecting its local charge, σ, the Coulomb energy change is (12)

$$\delta E_C = -\int \frac{1}{2}\sigma \; \mathcal{E} \; \delta n \, dS \quad .$$

where $\mathcal{E}$ is the electric field at dS. The change in surface energy is

$$\delta E_S = \gamma \int \kappa \, \delta n \, dS$$

where γ is the surface tension coefficient, and κ is the curvature at dS. The total energy change is

$$\delta E = \delta E_c + \delta E_s \quad .$$

Subtracting $\delta n dS$ times a Lagrange multiplier k to ensure conservation of volume and equating the integrand to zero (for equilibrium shapes $\delta E = 0$ for any δn) gives

$$k = \gamma \kappa - \frac{1}{2} \sigma \mathcal{E} \quad .$$

By Gauss' Theorem,

$$\sigma = \frac{1}{4\pi} \mathcal{E}$$

$$k = \gamma \kappa - \frac{\mathcal{E}^2}{8\pi}$$

$$= \gamma \kappa_o \left(\frac{\kappa}{\kappa_o} - \frac{\mathcal{E}_o^2}{8\pi\gamma\kappa_o} \frac{\mathcal{E}^2}{\mathcal{E}_o^2} \right)$$

where κ_o is the curvature on a sphere with the same volume as the drop and $\mathcal{E}_o$ is the electric field on the sphere.

Since

$$x = \frac{E_c^{(o)}}{2E_s^{(o)}} = \frac{1}{\gamma\kappa_o} \frac{\mathcal{E}_o^2}{8\pi}$$

Thus

$$k = \gamma\kappa_o \left(\frac{\kappa}{\kappa_o} - x \frac{\mathcal{E}^2}{\mathcal{E}_c^2} \right) \quad .$$

Thus for an equilibrium shape, any point on its surface should satisfy $\Delta = 0$, where Δ is given by

$$\Delta = \frac{\frac{K}{K_o} - x\frac{\mathcal{E}^2}{\mathcal{E}_o^2}}{B_s - xB_c} - 1 \quad .$$

As a measure of the deviation from equilibrium we can define a root-mean-square value of Δ over the surface of the drop:

$$RMS = \left(\int |\Delta|^2 \, dS\right)^{\frac{1}{2}} \quad .$$

If RMS << 1, the drop is close to equilibrium. If RMS > 1, the shape is far from equilibrium. This quantity can be used as a measure of how close the shapes we obtain are to the true equilibrium.

SYMMETRIC EQUILIBRIUM SHAPES OF A CHARGED CONDUCTING DROP

Instead of going into mathematical details (6) we shall present here the results based on a family of shapes generated by two, three up to six point charges shown in Fig. 8. The figure should be compared to Fig. 3 for a volume charged drop. The series of curves with different N values are just successive orders of approximation of true equilibrium shapes. One hopes that for a high enough order of approximation, the results would be very close to the true ones, so that an even higher order will change the results very little. Typically, for successive orders the RMS values improve by a factor of two. For N = 6 parameterization, RMS $\sim$ 0.01 for x close to 1 and x < 0.8, but RMS $\sim$ 0.1 for x $\sim$ 0.9. This indicates that for x < 0.8 and x $\sim$ 1.0, the shapes we obtain are close to true equilibrium shapes, but for x $\sim$ 0.9, there are more uncertainties By studying the change of RMS values at x $\sim$ 0.9 for successive approximations, the RMS values are found to decrease very slowly, much less than factors of two. This indicates that our model of a conducting drop using the equipotential surfaces of point charges is probably not good enough in this region. A more general or more appropriate family of shapes appears to be in demand here. Hence, one should regard the calculated results in this region with great reservations.

Let us take the N = 6 curve at its face value and examine its main features. As the value of x goes from 1 toward small x values, the equilibrium shape elongates from a sphere, i.e., R_{MAX}/R increases with decreasing x in the region near x = 1. This is in constrast to cases of small x values ($x \lesssim 0.7$) where R_{MAX}/R is slowly decreasing with decreasing values of x. The shapes in the latter case are long and look like a dumb-bell. Similar to the volume charged drop case there exists a region where there is a rapid change of shape, but it occurs at $x \approx 0.9$ in the present case. Actually the curve for R_{MAX}/R even turns back at s = 0.887 and again at x = 0.906. However, it is in exactly this region that our results become unreliable and the double turn might be spurious (see Refs. 15 and 4 for a similar uncertainty which once existed in the volume charged case).

The nature of these equilibrium shapes may be found by looking at the signs of the second derivatives of their energy with respect to all the parameters. The following results are found when the shapes are restricted to only the degrees of freedom that allow reflection symmetric shapes. For $1 > x > 0.887$ the energy of the drop is a maximum in one degree of freedom, but a minimum in the other symmetric degrees of freedom. Between the bends, for $0.887 < x < 0.906$, the energy is a minimum. For values of x smaller than 0.906, it is again a maximum in one degree of freedom. With respect to the degrees of freedom that describe reflection asymmetric deformation, the energy of the drop is a minimum from x = 1 to x = 0.892. From x = 0.892 to x = 0.68, it is a maximum in one degree of freedom. Below x = 0.68 it appears to be a maximum in two degrees of freedom. Hence, the equilibrium point is a saddle from x = 1 to x = 0.892. From x = 0.892 to x = 0.887 it is a mountain top (unstable in more than one direction). Between the bends at x = 0.887 and x = 0.906 it is again a saddle. For x smaller than 0.906, it turns out to be a mountain top also. As discussed before the shapes close to x = 1 is fairly well determined, but at the bends the results are not reliable.

SUMMARY AND CONCLUSIONS

The static properties of a charged conducting drop are compared with those of a volume charged drop. Similarities as well as some of the differences are discussed. The symmetric equilibrium shapes of a conducting drop are determined with reasonable confidence for values of the fissility parameter x not in the neighborhood of 0.9. For x close to 0.9 a more general or more appropriate shape parameterisation than employed in this work has to be found so that equilibrium shapes at these values of x can be determined with greater reliability. This is important because it is in this region that we find possibilities of interesting stability features, such as the occurrence of a bend in the family of equilibrium shapes and of points at which there is a change in the number of degrees of freedom with respect to which the shape has a maximum energy.

It is interesting to note that even some ninety years after Lord Rayleigh's study of a charged conducting drop, the whole problem is still a very open subject. The present calculations have been able to determine the saddle points of a charged conducting drop for values of x from 0.892 to 1 where they are reflection symmetric. But for the region up to 0.892, one is still very ignorant of the saddle point shapes and energies of a charged conducting drop.

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FIGURE CAPTIONS

Fig. 1. Energy excess of a volume-charged liquid drop as a function of deformation.

Fig. 2. Energy excess of a volume-charged liquid drop deformation for different values of the fissility parameter x.

Fig. 3. The maximum and minimum radii of saddle point shapes of a volume-charged drop as a function of the fissility parameter x. The results for the symmetrical saddle point shapes are given by the solid curves, and the results for the asymmetric saddle point shapes by the dashed curves. *(Data taken from Ref. 10)*.

Fig. 4. The energy change in the division of a volume-charged drop into two spheres as a function of the fractional volume of one of the spheres for various values of x.

Fig. 5. Same as Fig. 4 for the case of a charged conducting drop.

Fig. 6. Shapes in the symmetry N = 2 family of equipotential surfaces.

Fig. 7. Shapes in the symmetry N = 3 family of equipotential surfaces.

Fig. 8. The maximum and minimum radii of the symmetric saddle point shapes of a charged conducting drop as a function of the fissility parameter x. Different curves correspond to the restriction to different families of shapes indicated by the values of N.

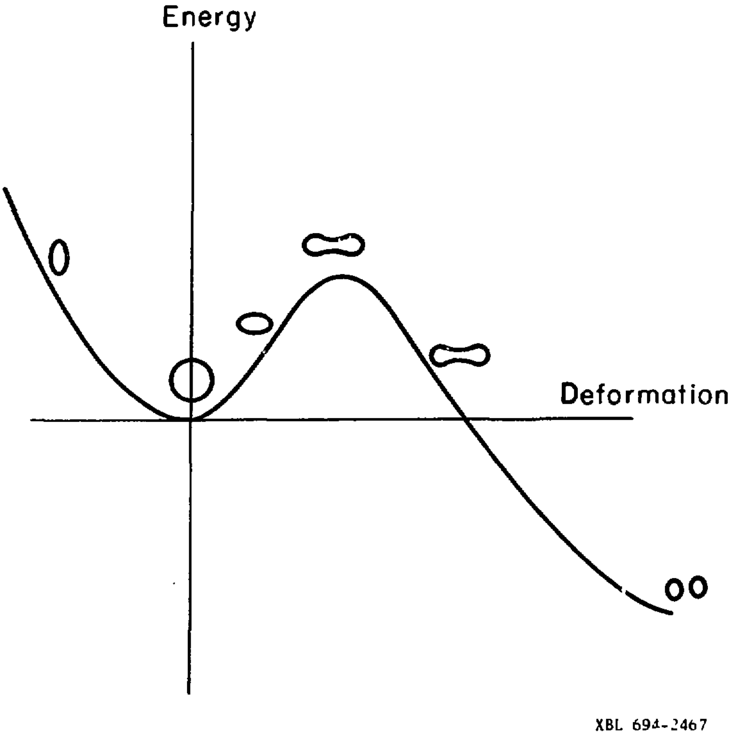


Fig. 1

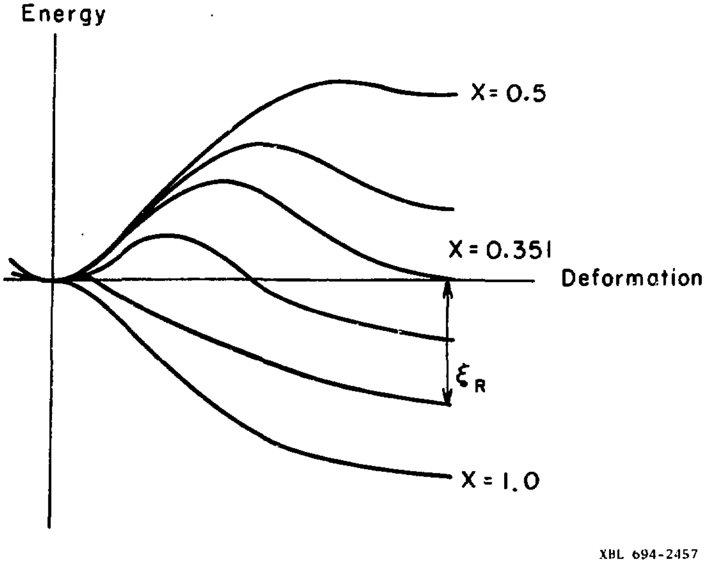
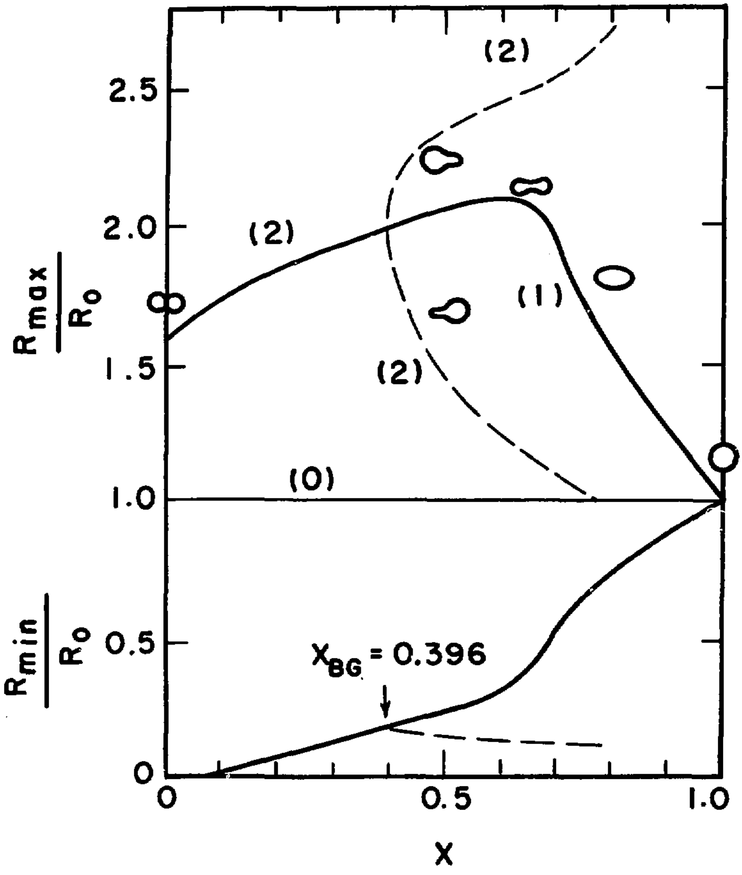
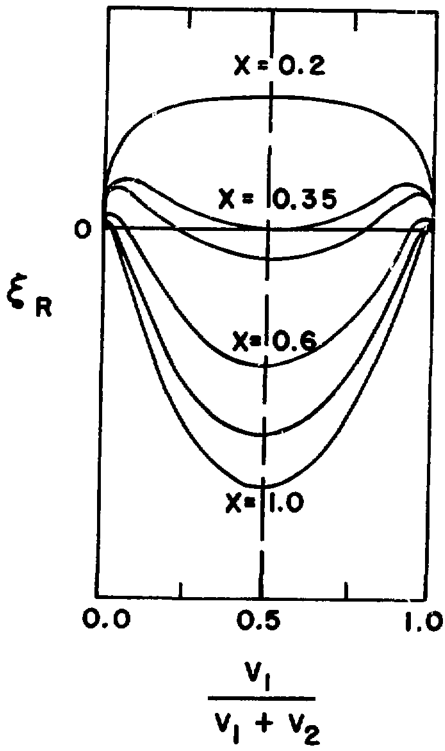


Fig. 2



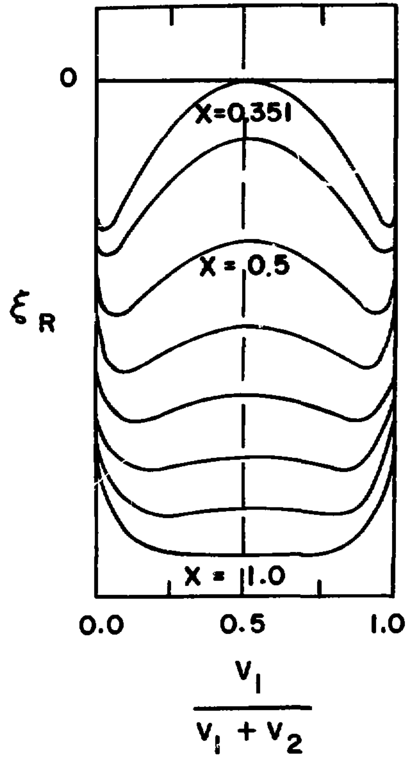
XBL 694-2461

Fig. 3



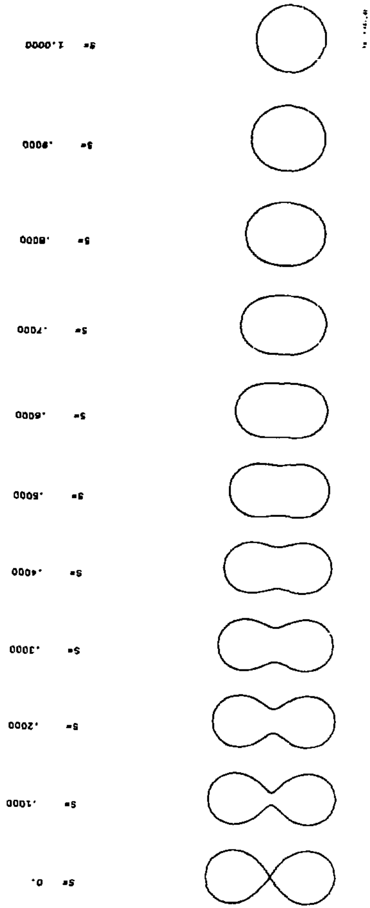
XBL 694-2458

Fig. 4



XBL 694-2459

Fig. 5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S= .8000
S= .7000
S= .6000
S= .5000
S= .4000
S= .3000
S= .2000
S= .1000
S= 0.

Fig. 6

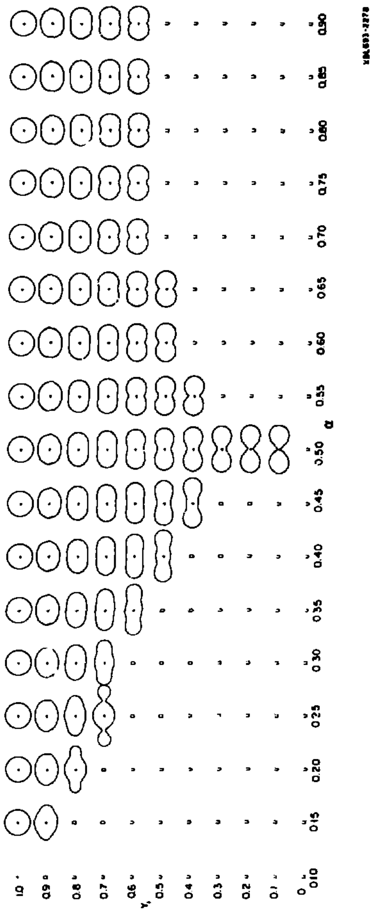
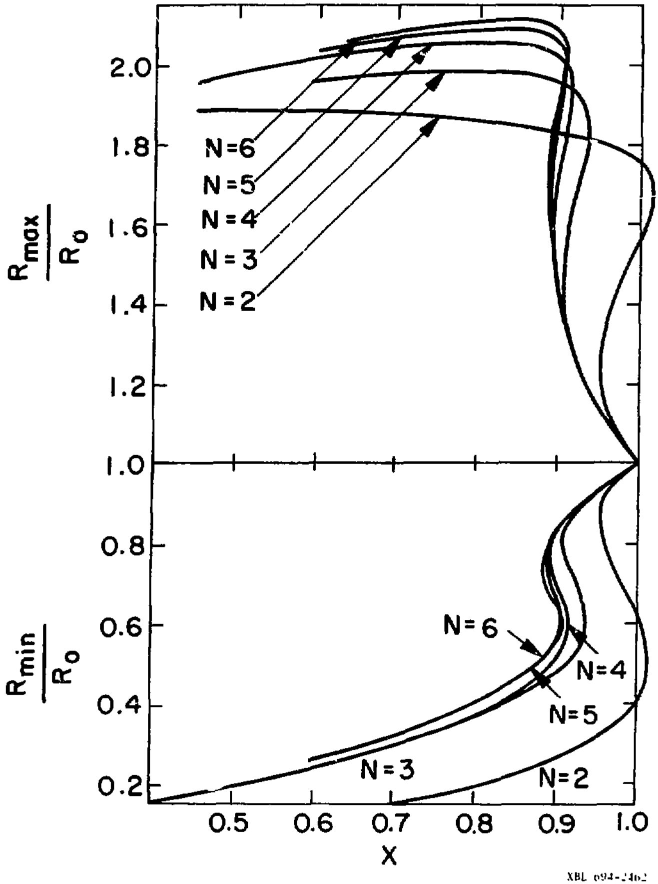

0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
α
γ,
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

Fig. 7



2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\frac{R_{max}}{R_o}$
$\frac{R_{min}}{R_o}$
N=6
N=5
N=4
N=3
N=2
N=6
N=4
N=5
N=3
N=2
0.5
0.6
0.7
0.8
0.9
1.0
X
XBL 694-2462

Fig. 8