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MASTER

UNIVERSAL CURVES FOR PREDICTING EQUILIBRIUM RADII OF RELATIVISTIC ELECTRON BEAMS INCLUDING THE EFFECTS OF ATTRACTIVE SELF FORCE, EXTERNAL B_z, INITIAL BEAM QUALITY AND SCATTERING IN THE GAS ENTRY FOIL

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The beam passes from vacuum into gas through a thin foil as shown in Fig. 1. The beam radius is independent of z in region 2 where the beam is space charge neutralized and at the surface of the foil in region 1 (vacuum). We incorporate the effect of foil scattering into a previous analysis.[1] At the surface of the foil in region 1 the average value of $p^2_\perp/p^2$ for beam particles is

$$<\theta^2_1> = \frac{1}{2}\frac{q^2_1}{a^2} \tag{1}$$

where q_1 is the beam quality before foil scattering. The beam energies from the foil with

$$<\theta^2_2> = \frac{1}{2}\frac{q^2_2}{a^2} \tag{2}$$

The vector addition of many individual small changes of $\bar{p}_\perp$ in random directions has the net result of cumulating the squares of the magnitudes of the individual vectors

$$<\theta^2_2> = <\theta^2_1> + <\theta^2_s> \tag{3}$$

In region 2 the adiabatic invarient is

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[1] E.P. Lee, Physics of Beam Compression Experiment, Memo of 26 Jan., 1973.

$$q_2 = \sqrt{\frac{k^2_c}{4} + k^2_\beta} \quad a^2 \tag{4}$$

(k_β is understood to have the subscript 2, but it is left off for simplicity.)

In Appendix I the envelope equation, which gives Eq. 4 for the special case of a independent of z, is derived using an idealized model. k_c and $k^2_\beta a^2$ are defined in Appendix I. Combining Eqs. 1, 2, 3 and 4 and defining $x = a^2/q_1^2$, results in the quadratic equation in x,

$$\left(\frac{q_1^2 k^2_c}{4}\right) x^2 + \left(k^2_\beta a^2 - 2\langle\theta^2_s\rangle\right)x - 1 = 0 \tag{5}$$

(This is Eq. 50 of ref. 2)

For $k^2_\beta \, a^2 > 0$ and $q^2_1 k^2_c = 0$, the solution is

$$x = \frac{1}{(k^2_\beta a^2 - 2\langle\theta^2_s\rangle)}$$

For $k^2 \; a^2 > 0$ and $q^2_1 k^2_c > 0$ the physically meaningful solution is

$$x = \frac{-(k^2_\beta a^2 - 2\langle\theta^2_s\rangle) + \sqrt{(k^2_\beta a^2 - 2\langle\theta^2_s\rangle)^2 + q^2_1 k^2_c}}{\left(\frac{q^2_1 k^2_c}{2}\right)} \tag{7}$$

B_z is the same in regions 1 and 2. In region 2 the self-force is directed radially inward; in region 1 it is directed radially outward and the magnitude is $(\beta\gamma)^{-2}$ times smaller. Therefore, in most cases of interest the beam is <u>not</u> in equilibrium in region 1, the net inward force being too weak if

$$2\langle\theta^2_s\rangle < k^2_{\beta 2} a^2 \; (1 + 1/\beta^2\gamma^2).$$

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(2) E.P. Lee, "Envelope Equation of a Charged Particle Beam" UCID 16490, April 15, 1974.

On Fig. 2, $\sqrt{x}$ is plotted vs $\langle\theta_s^2\rangle$ for parameters of interest in the Livermore beam experiments. Note that for $k_c = 0$, (a/q) diverges as $(k_\beta^2 a^2 - 2\langle\theta_s^2\rangle) \to 0$. Finite k_c removes the divergence.

In Appendix II numerical values of $\langle\theta_s^2\rangle$ are calculated for some common foils. In Appendix III numerical values of $\langle\theta_s^2\rangle$ are given for some common gases and the most important formulas for increase of beam radius with gas scattering are stated for completeness.

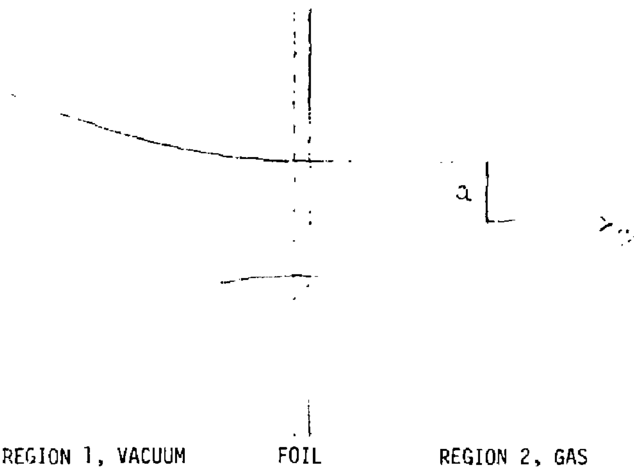

FIG. 1

Beam Passing from Vacuum into Gas Through a Thin Foil. In Gas the Beam has the Equilibrium Radius a.

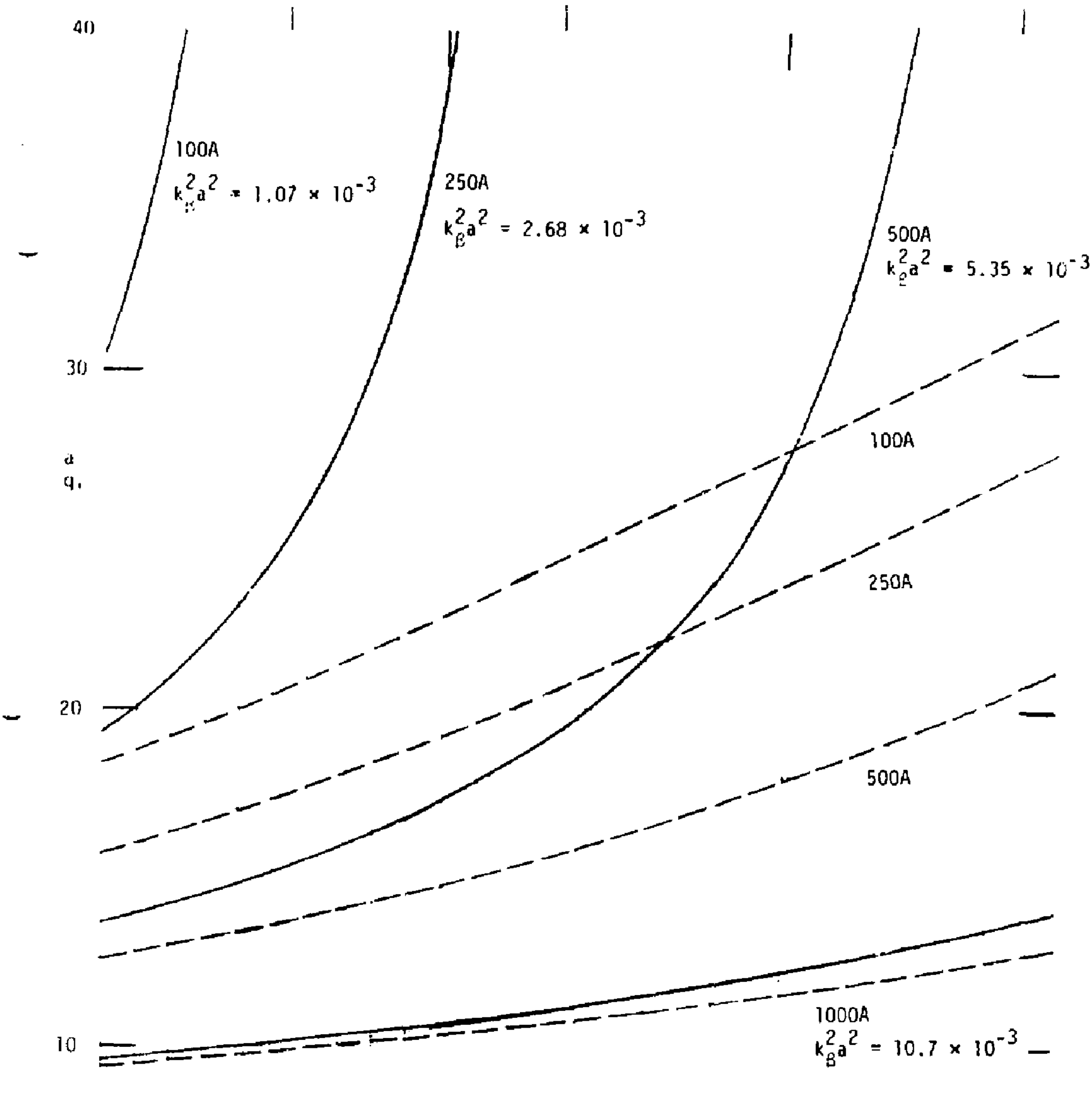

FIG. 2

Equilibrium radii divided by initial q vs foil mean squared scattering angle, , = 11

----- $q_1 k_c = 0$

------ q_1 = .01 radian cm, $B_z = 8.8 \times 10^3$ Gauss, k_c = 0.471 cm^{-1}

APPENDIX I

A.1 Derivation of the Equation of Motion for the Beam Radius

We use cylindrical coordinates (r,θ,z). We use an idealized model in which the fields are independent of θ, $v_\perp^2 = v_r^2 + v_\theta^2 \ll v^2$, where v is the particle total velocity, and there is no plasma current. The radial equation of motion of a beam electron is,

$$-e(E_r + \frac{v_\theta}{c} B_z - \frac{v_z}{c} B_\theta) = \gamma m_0 (\ddot{r} - \frac{v_\theta^2}{r}) \tag{8}$$

We use Gaussian units, e is the magnitude of electronic charge $v = \beta c$, $\gamma = (1-\beta^2)^{-1/2}$ and dot means time derivative .

$$B_\theta = - \frac{2(I/c)r}{a^2} \tag{9}$$

where (I/c) is the magnitude of beam current in emu, and $v_z > 0$. The current density is uniform inside the beam radius a, and a varies slowly with z.

$$E_r = - \frac{2(I/c)r}{a^2\beta} (1-f) \tag{10}$$

where f is the fraction of electrostatic field neutralized by plasma space charge. Substituting Eqs. 9 and 10, Eq. 8 can be written,

$$\frac{\ddot{r}}{\beta^2 c^2} = \left(\frac{v_\theta}{\beta c}\right)^2 \frac{1}{r} - k_c\left(\frac{v_\theta}{\beta c}\right) - k_\beta^2 a^2 \frac{r}{a^2} \tag{11}$$

where,

$$k_c = \frac{B_z}{\beta\gamma\left(\frac{m_0 c^2}{e}\right)} \tag{12}$$

(we neglest the contribution of the beam current to B_z) .

and,

$$k_\beta^2 a^2 = \frac{2(I/c)}{\beta\gamma\left(\frac{m_0 c^2}{e}\right)} \frac{(f-1/\gamma^2)}{\beta^2} \tag{13}$$

With axial symmetry, the canonical angular momentum P_θ is a constant. This gives a first integral of the azimuthal equation of motion.

$$P_\theta = \gamma m_0 r v_\theta - \frac{e}{c} r A_\theta = \gamma m_0 \beta c q \tag{14}$$

where $\beta c q$ is the value of $r v_\theta$ when $r A_\theta = 0$.

Since,

$$\oint \bar{A}\cdot d\bar{\ell} = \int (\bar{\nabla}\times\bar{A})\cdot d\bar{s}$$

$$2\pi r A_\theta = \pi r^2 B_z \tag{15}$$

Combining Eqs. 12, 14 and 15 gives,

$$\frac{v_\theta}{\beta c} = \frac{q}{r} + \frac{1}{2}k_c r \tag{16}$$

Substituting Eq. 16 into 11 gives,

$$\frac{\ddot{r}}{\beta^2 c^2} = \frac{q^2}{r^3} - \frac{k_c^2 r}{4} - (k_\beta^2 a^2)\,\frac{r}{a^2} \tag{17}$$

If the velocity distribution consists of particles with $\dot{r} \ll r\dot{\theta}$, then the beam surface follows the motion of a particle with $r = a$ and,

$$\frac{\ddot{a}}{\beta^2 c^2} = \frac{q^2}{a^3} - \frac{k_c^2}{4}\,a - (k_\beta^2 a^2)\frac{1}{a} \tag{18}$$

APPENDIX II

A.1 Foil Scattering

For electrons passing through $n_Z L$ atoms/cm^2 of atomic number Z, the cumulated mean squared scattering angle is,

$$\langle \theta_s^2 \rangle = \frac{8\pi e^4 \; Z(Z+1) n_Z L}{m_o^2 \gamma^2 \beta^4 c^4} \; \ell n \, \frac{\theta_{max}}{\theta_{min}} \qquad (19)$$

For $\gamma = 11$,

$$\langle \theta_s^2 \rangle = 1.68 \times 10^{-26} Z(Z+1) n_Z L \; \ell n \, \frac{\theta_{max}}{\theta_{min}}$$

For the "thin target" case (which is pertinant to the current beam experiment),

$$\frac{\theta_{max}}{\theta_{min}} = \frac{\hbar}{\sqrt{\pi} m_o c \beta} Z^{2/3} (n_Z L)^{1/2} \qquad (20)$$

For $\gamma = 11$,

$$\frac{\theta_{max}}{\theta_{min}} = 1.38 \times 10^{-10} Z^{2/3} (n_Z L)^{1/2}$$

Table I gives numerical values for several common foils.

Table I Coulomb Scattering of Electrons in Thin Foils, $\gamma=11$

	L	ρ	W	$\frac{\rho A}{W}$	Z	Z(Z+1)	$n_Z L$	$Z^{2/3}$	$\ln\frac{\theta_{max}}{\theta_{min}}$	$\sum n_Z L Z(Z+1) \ln\frac{\theta_{max}}{\theta_{min}}$	$\langle\theta_s^2\rangle$
	(.00254 cm)	(gm cm^{-3})	(gm)	($10^{23} cm^{-3}$)			($10^{20} cm^{-2}$)			($10^{22} cm^{-2}$)	(10^{-3})
Be	2	1.85	9.01	1.24	4	20	6.30	2.520	2.16	2.72	0.456
C	5	2.2	12.01	1.10	6	42	13.97	3.302	2.83	16.6	2.78
Al	2	2.702	26.98	0.6031	13	182	3.06	5.529	2.59	14.4	2.41
Ti	1	4.5	47.90	0.566	22	506	1.44	7.851	2.56	18.7	3.14
					6	42	1.045	3.302	1.535		
					1	2	0.238	1	---		
KAPTON ($C_{22}H_5O_{10}N_2$)	1	1.42	457.	.0187						1.15	0.193
					8	72	0.475	4	1.333		
					7	56	0.095	3.3659	0.438		
					6		3.13		2.08		
					1		0.712		0.15		
KAPTON	3									4.82	0.803
					8		1.425		1.88		
					7		0.285		0.99		

APPENDIX III

A.1 Gas Scattering

In the case of slow gas scattering where the beam is always near equilibrium, the most important special cases are[2]:

Case I, $k_c = o$, $k_\beta^2 a^2$ = constant

$$\frac{a}{a(o)} = \exp \frac{\langle\theta_s^2\rangle}{k_\beta^2 a^2}$$

Case II, $k_\beta^2 a^2 = o$, k_c = constant

$$a^2 - a^2(o) = 4 \frac{\langle\theta_s^2\rangle}{k_c^2}$$

In table II, numerical values are given which are particularly pertinent to Case I for a beam of a few hundred amperes.

Table II Coulomb Scattering of Electrons in Gases, $\gamma=11$

	L (m)	p (torr)	n ($10^{18}cm^{-3}$) 20°C	nL ($10^{20}cm^{-2}$)	Z	Z(Z+1)	$Z^{2/3}$	$\ell n\frac{\theta_{max}}{\theta_{min}}$	$<\theta_s^2>$ (10^{-3})
H_2	10	600	38.88	388.8	1	2	1	3.30	4.31
N_2	1	200	12.96	12.96	7	56	3.659	2.90	3.53
He	10	400	12.96	129.6	2	6	1.587	3.22	4.18
Ne	1	200	6.48	6.48	10	110	4.642	2.79	3.33
Ar	1	60	1.944	1.944	18	342	6.868	2.58	2.87
Kr	1	15	0.486	0.486	36	1332	10.90	2.35	2.55
Xe	1	7	0.2268	0.2268	54	2970	14.29	2.24	2.53