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Experimental determination of material buckling in fast reactors

Programme BUCKLING

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A B S T R A C T

The developed method for determination of material buckling in fast reactor systems has been analyzed and some improvements made. The non-cylindrical shape of the reactor core is taken into account and the linear correlation between experimental and theoretical reaction rate traverses has been thoroughly studied. The corresponding computer programme BUCKLING is written and applied to measurements in the fast reactor assembly SNEAK - 7A in Karlsruhe. The evaluation of measurements was performed with the latest cross-section sets available at the Kernforschungszentrum in Karlsruhe.

C O N T E N T S

1. INTRODUCTION

Material buckling characterizes media of a nuclear rector, independently of their shape and surroundings. Its experimental determination could provide a suitable test of the cross-section data being used, since accurate calculational methods for evaluations of parameters of a fast reactor are available (material buckling is the eigenvalue of the space independent multigroup diffusion equation). The value of material buckling is determined by the fundamental mode of neutron spectrum. However, the interaction between the core and reflector in the fast reactor facility is very marked, so that spectral and harmonic effects are large even at the center of the core. That excludes the possibility to measure the fundamental mode of neutron flux in the reactor medium directly.

The method of determining material buckling experimentally, was developed by the Masurca-group in Cadarache, France (1). It is based on synthesizing the fundamental mode of neutron flux from measured fission rate distributions, of suitably chosen detectors, along the principal axes of the core. The basic theory of the method is described briefly in the next section. Further investigations and some improvements of the method have been made by the SNEAK-group in Karlsruhe, Germany (2,6).

The present report shows the influence of the cylindrization effect on determination of the spatial fundamental mode of neutron flux. The way how it can be accounted for, and the analysis of the linear correlation between experimental and calculated reaction rate traverses is included also. The programme BUCKLING for determining axial and radial bucklings of the cylindrical core, written in the FORTRAN language for the CDC-3600 computer, is listed and its features explained. It is based on the developed theory and includes the corrections given in the report.

Experiments for material buckling determination in the assembly SNEAK-7A are analyzed and the results obtained by the programme BUCKLING are given and discussed. The necessary theoretical calculations of various parameters and reaction rate traverees, have been done in Kernforschungszentrum, Karlsruhe using in multigroup calculations the most recent cross-section sets available there.

2. THEORY OF THE EXPERIMENT

The diffusion equation for the given multiplying region with a constant composition, using common notations for cross-sections, etc., can be written in the operator form as:

$$D(E)\nabla^2\Phi(\vec{r},E) + H\Phi(\vec{r},E) = 0, \qquad (2.1)$$

where:

$$H \equiv -\sigma_t(E) + \int_E \sigma_s(E' \to E)dE' + k\,\chi(E)\int_E \nu\sigma_f(E')dE'$$

$$k \equiv 1/k_{eff}$$

Symbols σ stand here for macroscopic cross-sections. The continuous energy notation is used throughout the paper instead of a multigroup notation for easier readability. The scalar neutron flux can be expressed through the eigenvalues B_i^2 and the corresponding eigenvectors $\varphi_i(E)$ of the diffusion matrix H, and the eigenfunctions $f_i(\vec{r})$ of the Laplacian:

$$\Phi(\vec{r}, E) = \sum_i A_i f_i(\vec{r})\varphi_i(E), \qquad (2.2)$$

with:

$$\nabla^2 f_i(\vec{r}) + B_i^2 f_i(\vec{r}) = 0, \qquad (2.3)$$

where A_i are the proportionality constants.

The largest eigenvalue, B_i^2, is by definition the material buckling of the medium. This eigenvalue is positive for a medium with $k \geq 1$, and the associated eigenvector $\varphi_i(E)$

is the asymptotic spectrum (or fundamental mode spectrum). To the material buckling is associated the so called fundamental spatial mode of the eigenfunction $f_i(\vec{r})$. Omitting the index 1 for the fundamental mode and the asymptotic spectrum, and supposing that the neutron flux is normalized by the condition $f(o) = 1$, Eq. (2.2) can be written as:

$$\phi(\vec{r},E) = f(\vec{r})\,\phi(E) + E(\vec{r}, E) \qquad (2.4)$$

The first term represents the fundamental mode, and the second is the contribution of transient modes.

According to the Eq. (2.3), the material buckling is given by:

$$B_m^2 = -\frac{\nabla^2 f(\vec{r})}{f(\vec{r})} \qquad (2.5)$$

In order to determine experimentally the quantities on the right hand-side of this equation, one should measure the spatial distribution of the fundamental mode in the medium. As already mentioned, the asymptotic neutron spectrum is practically never established in any part of the reactor core. Nevertheless, it is possible to define a "buckling" which is spatially and energetically dependent, by the following relation.

$$B^2(\vec{r}, E) = -\frac{\nabla^2 \phi(\vec{r}, E)}{\phi(\vec{r}, E)}$$

The measurement of a neutron flux distribution consists in fact of measuring the reaction rate distribution of a detector with the macroscopic cross-section $\sigma_d(E)$:

$$R_d(E) = \int_E \sigma_d(E)\,\phi(\vec{r}, E)\,dE \qquad (2.6)$$

Using the relation (2.5) it is possible to define "buckling" associated to the detector d as:

$$B_d^2(r) = -\frac{\nabla^2 R_d(r)}{R_d(r)} = -\frac{\nabla^2 \int_E \sigma_d(E)\Phi(\vec{r}, E)\, dE}{\int_E \sigma_d(E)\Phi(\vec{r},E)\, dE} \qquad (2.7)$$

Assuming that the neutron flux can be separated in several principal directions, e.g. in axial and radial direction by the core of the cylindrical type, the total buckling can then be expressed by the sum of an axial buckling, $\alpha_d^2(\vec{r})$ and a radial one $\beta_d^2(\vec{r})$:

$$B_d^2(\vec{r}) = \alpha_d^2(\vec{r}) + \beta_d^2(\vec{r}) \qquad (2.8)$$

Inserting expression (2.4) for the neutron flux into Eq. (2.7), one obtains:

$$B_d^2(r) = -\frac{\nabla^2 f(\vec{r}) \int_E \sigma_d(E)\Psi(E)dE + \nabla^2 \int_E \sigma_d(E)\, E(\vec{r},E)dE}{\int_E \sigma_d(E)\,[f(\vec{r})\Psi(E) + E(\vec{r}, E)]\, dE}$$

The following question can be raised now: What properties should detector d have, in order that B_d^2 be equal to material buckling, B_m^2 ? Recalling that the eigenvectors $\Psi_i(E)$ are orthogonal to the adjoint asymptotic spectrum with diffusion coefficient as a weight function /1/:

$$\int_E D(E)\Psi^*(E)\Psi_i(E) = 0, \text{ for } i>1 \qquad (2.9)$$

it can be seen that a detector with a cross-section proportional to $D(E)\Psi^*(E)$,

$$\sigma_d(E) \sim D(E)\Psi^*(E) \qquad (2.10)$$

would fulfill the requirement stated above. Substituting expression (2.10) into the relation for $B_d^2(\vec{r})$, one easily obtains:

$$B^2_{D\varphi^*}(\vec{r}) \equiv B_m^2,$$

i.e., the buckling associated to a detector of a cross-section prcportional to $D(E)\varphi^*(E)$ is identical to the material buckling. However, such a direct measurement of the fundamental mode $f(\vec{r})$ is not possible, since there exist no detector with an effective cross-section of Eq. (2.10).

2.1. Determination of the fundamental mode traverse

In its original form, the procedure for determining geometrical bucklings and hence material buckling (by these measurements one deals always with a reactor in a critical state) was the following /1/:

From the definition of B_d^2, Eq. (2.7), it is obvious that the discrepancy between B_d^2 and B_m^2 depends on how well does $\sigma(E)$ fit to represent $D(E)\varphi^*(E)$. A careful study of various fission rate detectors has shown that the value of material buckling can be enclosed between the "buckling" values obtained by threshold and non-threshold detectors. The procedure, adopted as a standard procedure fot the experimental determination of geometrical bucklings in nuclear centers in Cadarache and Karlsruhe, consists in measuring and calculating the fission rate traverses of four detectors along the principal axes of the reactor core. The four detectors (from practical reasons the measurements are performed with fission chambers) include two non-threshold detectors: U235 and Pu239, and two threshold ones: U 238 and Np237. Each traverse gives the corresponding "buckling". The correlation between calculated and experimental

"buckling" values is assumed to be linear, so that the value of experimental buckling can be deduced, as shown in Fig. 1.

Further investigations have shown that such a procedure might not be reliable enough in cases of small reactor cores /2/. On account of very strong influences of a surrounding reflector, the values of B_d^2 ($\vec{r}$) are not constant even in a quite small region near the center of the core. In such cases one should not expect to obtain good single values for "bucklings", out of the measured fission rete traverses. Their combination to obtain an experimental value for buckling would thus be improper, too.

The way out of this difficulty was found in determining the fundamental mode traverse, rather than bucklings directly. Instead of representing buckling values, as shown in Fig. 1, the same procedure is adopted to determine the fundamental mode traverse according to measured and calculated values of fission rate traverses, as shown in Fig. 2. This is performed along the whole core, step by step, at suitably chosen distances. The value of buckling is then obtained by fitting this traverse to the fundamental spatial mode (sine of Bessel-J_o function for a cylindrical core).

Let us now examine more thoroughly the assumed linear correlation of measured and calculated values of traverses along the principal axex of the core. Many calculations have shown that there were no systematic discrepancies between calculated and experimental values of B_d^2 for threshold and non-threshold detectors. The same was true with comparison of traverses themselves. In each case the correlation coefficients were close to one justifying thus the assumed linear relationship. Instead of merely applying the regression analysis to the given set of four pointes (see Fig.2), it might be interesting to know how really well the fundamental mode distribution can be approximated by the linear combination fo several fission rate detectors.

The question of how good the approximation:

$$f(\vec{r}) \approx \sum_{j=1}^{K} \alpha_j R_j (\vec{r}),$$

is, according to the theory given above, may be written as:

$$D(E)\Phi^{*}(E) \approx \sum_{j=1}^{K} \alpha_j \sigma_j (E) \qquad (2.11)$$

The problem is to determine the coefficients α_j, for K detectors, so that the right hand-side of the equation be as close as possible to the left one. The best approximation is obtained by applying the least squares method. Such minimization procedure always includes the choice of weight factors. It seems logical to apply the neutron flux in this case, or more precisely its fundamental mode, but other choices are possible too. The minimization condition has then the following form:

$$\int_E W(E)\left[D(E)\Phi^{*}(E) - \sum_{j=1}^{K} \alpha_j \sigma_j (E)\right]^2 dE \rightarrow \min \qquad (2.12)$$

with:

$$W(E) = \frac{\Psi(E)}{\int_E \Psi(E)dE} \qquad (2.13)$$

The calculation itself is done applying the multigroup procedure and is explained fully in Section 3.

2.2. Cylindrization correction

The reactor core is usually of the cylindrical form. While in the axial direction requirements of the diffusion theory are practically always fulfilled, i.e. the fundamental mode distribution of neutron flux obeys cosine function, in the radial direction the Jo-Bessel function hardly justifies its distribution. Namely, the cross-section of the core is a circle only in a rough approximation (see Fig. 3). According to the cylindrization correction already applied for k_{eff} calculations, it seems reasondable to apply such a corection in calculating the fundamental mode of neutron flux in the radial direction. This correction can be obtained by comparing a one-dimensional diffusion calculation with a two-dimensional in XY-geometry, as:

$$cor\ (x) = \frac{f_r(x)}{f_{xy}(x)} \qquad (2.14)$$

In both calculations the same axial bucklings are to be used. The experimental fundamental mode, calculated by the procedures described in Section 2.1, is to be multiplied with this correction function.

3. COMPUTER PROGRAMME "BUCKLING"

The programme calculates sets of values of axial or radial buckling for different core regions, from the measured data obrained by fission rate detectors. It is based on the theary given in prerions sections. The input descripton of the programme is given in Section 3.1.

First of all, programme reads input data cards and classifies calculated and measured traverses of fission rate detectors. Sine measurements and calculations are, in general, performed in different points along the axial or radial direction of the core, it is necessary to make an interpolation and to express them for equal distances. It is commonly used to make a polynomial fit up to the eighth order of the measured traverse points, and then find the values corresponding to the distances of calculated traverses. Each of the four previously mentioned traverses (U235, U238, Np237 and Pu239) is fitted to the polynomial consisting of terms with only even exponents (their distribution resembles the cosine or J_o curve):

$$T(x) = a_o + \sum_{i=1}^{n} a_{2i} (x - b)^{2i} \qquad (3.1.)$$

The upper limit of summation takes on the values 1, 2, 3, and 4, succesively. Ater each least square fit, the sum of squared residuals is calculated and the polynomial with the smallest sum of square i.e., the one that best represents the measured traverse, has been chosen. This part of the calculation, as well as the linear regression analysis is performed by the subroutine POLFIT, specially written for these purposes.

All of the four pairs of traverses, and the calculated fundamental mode, are then normalized to unity in the center of the core (its position has been determined by the parameter b in Eq. (3.1) for measured traverses).

The next step in the programme is the determination of the fundamental mode of neutron flux in the core. Two alternatives are possible in the programme:

1. The linear regression analysis is papplied to the four pairs of fission rate traverses, as explained in Section 2.1 (see Fig. 2). At suitable points along the axis of the core (they are taken from the calculation of fission rate traverses) the results of the measured traverses are plotted against the corresponding calculated ones, as illustrated in Fig. 2. The least squares fit of a straight line is made to the four resulting points. From the value of calculated "$D\phi^*$" detector, one obtains the experimental $D\phi^*$ value. The procedure is repeated for all calculated points along the axis.

2. The procedure according to Eq. (2.11) is used. The cofefficients a_j have to be known and given as input data.

In this report, when evaluating experiments in SNEAK-7A, the special programme named PARADET was written in order to compute these coefficients. The calculation was done applying the multigroup procedure, so that expression (2.12) was used in the form:

$$\sum_{g=1}^{G} w^g \left[D^g \varphi^{*g} - \sum_{j=1}^{K} a_j \sigma_j^g \right]^2 \rightarrow \min. \qquad (3.2)$$

with:

$$w^g = \frac{\varphi^g \Delta E^g}{\sum_{g=1}^{G} \varphi^g \Delta E^g} .$$

Here g denotes the number of energy group (totally G groups), and ΔE^g is the corresponding energy range.

After the experimental distribution of the fundamental mode has been computed by one of these alternatives, the final step in the programme is performed. It consists in fitting the eigenfunciton cos- or J_o - to that mode. Before this fitting takes place, if radial buckling is to be evaluated, the cylindrization correction can be applied. (it's an option in the programme). This correction is given by Eq. (2.14) and it must be known previously and given in the input. The spacial programme, named CYLCOR was written for this purpose. Since, as for traverses, the one-and two-dimensional calculations connot be generally performed in the same points, the fundamental mode obtained by one-dimensional calculation is first fitted to polynomial of the eighth order, and the values corresponding to XY-distances are then calculated.

The obtained values for the fundamental mode are then fitted to the expression:

$$f(x) = a_1 \Psi(a_2 x), \qquad (3.3)$$

where Ψ takes on cosine or J_o - Bessel function form, depending on the problem case. Parameter a_2 represents the value of axial or radial buckling, respectively. This procedure can be repeated for different ranges in the core, as stated by the input conditions (record K1 in Section 2.1).

The errors, in the form of standard errors, the bucklings are given with, can be evaluated in several ways.

Firstly, starting with errors of the measurements given as input data for experimental traverses, each fitting procedure uses weight factors, based on the errors obtained from the previous one. Variances of points taken into the polynomial and linear regression analysis are calculated as /7/:

$$\mathrm{var}\,(Y_i) = \sigma^2 \left[\frac{1}{\sum_i \delta_i} + (F_a^T M^{-1} F_a)\right],$$

where the matrix F_a is given as:

$$F_a = \begin{bmatrix} 1 \\ x \\ x^2 \\ \cdot \\ \cdot \\ \cdot \\ x^k \end{bmatrix}$$

M is the variance-covariance matrix, and δ_i are the weight factors expressed by standard errors of the points. The final fit to the fundamental mode function is performed with the special subroutine CURFIT, which is an abreviated version of the subroutine CURVEFIT * /7/.

Secondly, it is allowed that weight factors of the determined fundamental mode be independent of the previous regression analysis. In this case they can be taken as being constant or proportional to the amplitude of the fundamental mode traverse.

A complete list of the programme BUCKLING is given in Appendix A. This is a standard FORTRAN programme, and no special operating instructions are needed. The

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* The correlation of errors is not allowed in this abreviated version.

subroutines: CURFIT, POLFIT and FUNCT, needed by the programme are listed in Appendix B. FUNCT is a special subroutine which calculates linear function, polynomial, cosine function and J_o function, together with their derivatives. The programme also needs the subroutines MINV and DGELG from the IBM Scientific Subroutines Package.

3.1. Input description of the programme BUCKLING

The following list shows the way of writing the input data, The letter K, followed by the required format statements, denotes one record. The letter S explains the choice of various possibilities that are allowed in the programme, or conditions governed by it. The dimensions of physical quantities have to be expressed in the CGS-system of units.

K1 (format 6I5)

NCAS - If NCAS=1, the radial buckling is determined (i.e., the radial traverses are fitted to J_o function), and for NCAS=2, the axial buckling is determined (cosine fitting);

IG - (IG-1) represents the maximum number of rejected points in the polynomial fitting. IG is limited to 10;

IS - Rejection step. The ratio (IG-1)/IS must be an integer;

ISIG - This parameter determines weight factors in the fitting procedure to cosine or J_o function:

1: weight factor are constant,
2: weight factor are proportional to the amplitude of the fundamental mode traverse,
3: weight factors are determined according to the previous regression analysis;

NVER - For NVER = 1, the $D\varphi^*$ traverse is calculated according to the alternative 1, and for NVER=2, according to the alternative 2 (Section 3);

NKORR - If NKORR=1, the cylindrization correction is to be taken into account. If no such corrections, NKORR = 0.

K2 (format 2E13.6)

$(A_{oo}(I),I=1,2)$ - Initial values of parameters (Eq.3.3.) by the fitting procedure to the fundamental mode function. $A_{oo}(1)$ stands for the amplitude and $A_{oo}(2)$ for buckling.

K3 (format 2E13.6)

XN - Multiplication factor for expressing measured coordinates of the traverses in cm;

YN - Multiplication factor for expressing calculated coordinates of traverses in cm.

S4 IF NVER=2, then K4, otherwise K6.

K4 (format I5)

NAL - Number of detectors in traverse measurements. Usually, four detectors are used.

K5 (format 8F10.0)

(AL(I), I=1,NAL) - Values of coefficients α_j for each detector, according to Fq.(3.2).

K6 (format I5)

NN - Number of calculated points of reaction rate distributions (maximum 40).

K7 (format 8F10.0)

(Y(I,1),I=1,NN) - Coordinates of calculated reaction rates distributions.

S8 If NKORR =1, Then K8, otherwise S9.

K8 (format 8F10.0)

(CORR(I),I=1,NN) - Cylindrization correction given for each coordinate of the calculated reaction rate distributions.

S9 For each detector in the sequence: U235, U238, Np237, Pu239 and "$D\varphi^*$", records K9 to K10.

K9 (format I5)

KENN - Atomic number of the detector isotope (arbitrary for $D\varphi^*$).

K10 (format 8F10.0)

(Y(I,2),I=1,NN) - Values of the calculated reaction rates of detectors, according to the points given in K7.

S11 For each detector in the sequence: U235, U238, Np237 and Pu239, records K11 to K12.

K11 (format 2I5)

KENN - Atomic number of the detector isotope,

N - Number of measured points of reaction rate distributions (maximum 120).

K12 (format 3E13.5)
(X(I,2), SIGMX(I,2),X(I,1),I =1,N)
- Values of the measured reaction rates of detectors, their standard deviations and cocrdinates.

END

4. ANALYSIS OF MEASUREMENTS IN THE ASSEMBLY SNEAK-7A

The fast critical facility SNEAK /8/ (Schnelle Null-Energie-Anordnung Karlsruhe) was utilized to perform physics experiments on a clean plutonium-fueled fast assembly 7A. This assembly was a simple one-zone core with PuO_2 - $-UO_2$ fuel and reflected by depleted uranium. The matrix plane of the assembly is shown in Fig. 3, together with positions of control rods. The cross-section of the core cylindrized for an RZ two-dimensional dscription is given in Fig. 4. The fuel elements are suspended vertically with a lattice pitch of 5.44 cm. The core cell consisted of two platelets: a PuO_2-UO_2 platelet and a graphite platelet of 0,626 cm and 0. 3126 cm thicknesses, respectively. The homogenized compositions of the core and blanket are given in Table 1. The control rods were loaded with an enriched uranium cell /8/. For fomogeneous calculations they were smeared over an outer ring of the core zone (denoted as the outer core zone in Fig. 4.). The assembly and its features are described in more detail in Ref. 6.

The fission rate traverses of four detectors (U235, U238, Np237 and Pu239) were measured in the axial and radial directions with fission chambers hiving a 6 mm outer diameter and 25 mm active length. The axial traverses were performed along vertically made channels (marked positions "x" in Fig. 3), while the radial traverses were performed along the horizontally made channel (the dotted line in Fig. 3). Axial traverse measurements were performed in the central and edge elements of the core to check the separability of the neutron flux. Both measurements yielded the same form of the distribution, justifying thus taht the neutron flux was separable in axial and radial components. The data of these measurements are given in Ref. 3.

The calculated traverses were obtained using two-dimensional multigroup diffusion calculations with MOXTOT cross-section set /4/, and the newest KFKINR set /5/. The use of two different sets was aimed to check their influence on the results, and therefore the validity of the proposed method for material buckling determination. The axial traverses were calculated in RZ-geometry, and the radial in XY-geometry where the control rods were considered in their respective positions (i.e., not smeared over the core). Since, corrections for heterogeneity and transport effects are throughout small, they were neglected.

The cross-section data for fission detectors, used in calculations, are given in Table 2. The obtained results for diffusion coefficients, fundamental modes of neutron and adjoint fluxes, used in computing the experimental fundamental mode (Eq. 3.2), are shown in Table 3. The calculations were performed using standard 26-group codes in Kernforschungszentrum Karlsruhe. The three last groups are omitted because of the small values of neutron flux in them.

Calculations have shown that experimental buckling values did not depend on the choice of cross-section sets used in calculating radial and axial traverses. The results obtained from radial and axial bucklings are given in Tables 4 and 5, respectively. The values obtained for axial bucklings when changing the region of fit, are practically constant, so that only one representative value is given. The determination of radial buckling is always a crucial point in the analysis, since control rods cause flux perturbations and the core itself has an irregular cross-section (see Fig. 3).

The influence of control rods demanded that only a small inner portion of radial traverses could be used in evaluations. Therefore, the representative values of the radial buckling are taken from approximately r ≈ 15 cm. Alternatives 1 and 2 refer to two different

ways of calculating the fundamental mode traverse, as explained in Section 2.1. An example of the output list according to the alternative 2 is given in Appendix C.

5. CONCLUSIONS

The material buckling is calculated as the sum of axial and radial bucklings and is given in Table 6 (the assembly was always critical when performing traverse measurements). The calculated material buckling was obtained by a zero-dimensional homogeneous multigroup calculation. In these calculations material buckling was the eigenvalue of the diffusion equation with k_{eff} set equal to 1. The experimental value of material buckling when taking no cylindrization correction into account is by no means too small. This correction, as expected, is important by small cores. The results obtained with linear regression analysis, compared to those obtained by alternative 2, are so to say more "experimental", since the second alternative tries to approach the experimental fundanetal mode to the calculated one. By varying the weight factors in the analysis one can govern this trend. However the introduction of that alternative in this report was merely aimed to show that the linear regression analysis applied can be analytically justified.

It is seen that the experimental material buckling is overestimated by calculations. The overestimation ranges from about 3% to 5% depending on the cross-sections being used. The standard error of the experimental values is lower than 0.5%. Results obtained for SNEAK-7A, together with earlier experiences in material buckling determination, show that the flux mapping technique combined with the developed method of interpretation delivers very satisfactory results. The influence of the theory, which is present in the method, is certainly less than the experimental error, so that accuracy better than 1% in determining the material buckling can be obtained.

6. REFERENCES

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TABLE 1.

Material composition of the assembly SNEAK-7A.
(atom densities $10^{-24}/cm^3$)

Isotope	Inner core zone	Outer core zone	Blanket
Al	.0000080	.0011906	
C	.0260987	.0255387	.0000135
Cr	.0022423	.0022390	.0011080
Fe	.0079713	.0079824	.0039549
Mn	.0001109	.0001178	.0000875
Mo	.0000165	.0000145	.0000100
Nb	.0000089	.0000077	.0000085
Ni	.0011664	.0011818	.0009845
O	.0218462	.0211909	
^{239}Pu	.0026374	.0023434	
^{240}Pu	.0002369	.0002105	
^{241}Pu	.0000215	.0000191	
^{242}Pu	.0000011	.0000010	
Si	.0000933	.0000932	.0000453
^{235}U	.0000586	.0002958	.0001624
^{238}U	.0079604	.0080456	.0399401

TABLE 2.

Fission cross-sections for detector materials

(values given in barns)

Group	Energy range ΔE	σ_f(U235)	σ_f(U239)	σ_f(Np237)	σ_f(U238)
1	6.5-10.4Mev	1.532	2.094	2.08	0.934
2	4.0-6.5Mev	1.070	1.724	1.45	0.540
3	2.5-4.0Mev	1.169	1.865	1.57	0.520
4	1.4-2.5Mev	1.290	1.954	1.64	0.472
5	0.8-1.4Mev	1.219	1.744	1.42	0.0272
6	0.4-0.8Mev	1.17	1.608	0.66	$8.45*10^{-4}$
7	0.2-0.4Mev	1.30	1.552	0.076	$3.2*10^{-6}$
8	0.1-0.2Mev	1.50	1.530	0.025	
9	46.5-100kev	1.78	1.578	0.014	
10	21.5-46.5kev	2.20	1.688	0.012	
11	10.0-21.5kev	2.65	1.85	0.012	
12	4.65-10.0kev	3.60	2.34	0.012	
13	2.15-4.65kev	5.16	3.27	0.012	
14	1.0-2.15kev	7.20	4.70	0.012	
15	465-1000 ev	11.30	7.25	0.012	
16	215-465ev	16.35	12.25	0.012	
17	100-215kv	20.19	18.17	0.012	
18	46.5-100ev	30.18	58.90	0.012	
19	21.5-46.5ev	38.84	17.85	0.0 2	
20	10.0-21.5ev	49.0	85.36	0.012	
21	4.65-10.0ev	50.9	33.9	0.012	
22	2.15-4.65ev	17.1	10.7	0.012	
23	1.0-2.15ev	31.9	22.6	0.012	

TABLE 3.

Diffusion coefficients and fundamental modes of neutron flux and neutron importance of the core SNEAK-7A

The normalization is performed with: $\int_E \nu\sigma_f(E)\phi(E)dE=1$ for neutron flux; $\int_E X(E)\phi^*(E)dE=1$ for adjoint flux

Energy group	Difusion coefficient D^g	Neutron flux φ^g	Adjoint flux φ^{*g}
1	3.322	0.292	4.786
2	2.728	1.899	4.127
3	2.157	4.206	4.217
4	2.122	8.251	4.027
5	1.542	7.914	3.770
6	1.344	12.774	3.757
7	1.081	16.087	3.790
8	0.971	15.746	3.756
9	0.880	14.204	3.716
10	0.840	11.994	3.688
11	0.786	7.909	3.778
12	0.645	4.936	3.891
13	0.732	3.171	3.990
14	0.749	1.586	4.345
15	0.721	0.735	4.897
16	0.703	0.276	5.334
17	0.672	0.0904	5.104
18	0.613	0.0159	5.302
19	0.634	0.00392	3.159
20	0.584	0.00046	5.034
21	0.666	0.000069	3.393
22	0.741	0.000031	5.838
23	0.631	0.000004	3.074

TABLE 4.

Measured radial bucklings in SNEAK-7A.

$\beta(m^{-1})$ R = 28.63 cm

Region of fit (r/R)	alternative 1		alternative 2 with cylindri. (case 3)
	no cylindriz. (case 1)	with cylindr. (case 2)	
0.51		6.129±0.007	6.273±0.012
0.56	6.075±0.005	6.149±0.008	6.300±0.011
0.63	6.070±0.005	6.170±0.008	6.322±0.010
0.72	6.047±0.005	6.178±0.005	6.346±0.008
0.76	6.032±0.007	6.180±0.005	6.354±0.008
0.86		6.189±0.004	6.384±0.010
0.89	6.008±0.006	6.190±0.003	6.390±0.009

TABLE 5.

Measured axial buckling in SNEAK-7A.

$\alpha(m^{-1})$ H= 22.02 cm

Region of fit (h/H)	alternative 1
0.73	4.742±0.003

TABLE 6.

Material buckling in SNEAK-7A.

B^2 (m^{-2})

	Case 1	Case 2	Case 3
Experiment	59.39±0.14	60.05±0.18	61.84±0.30
MOXTOT		62.06	
KFKINR		62.91	

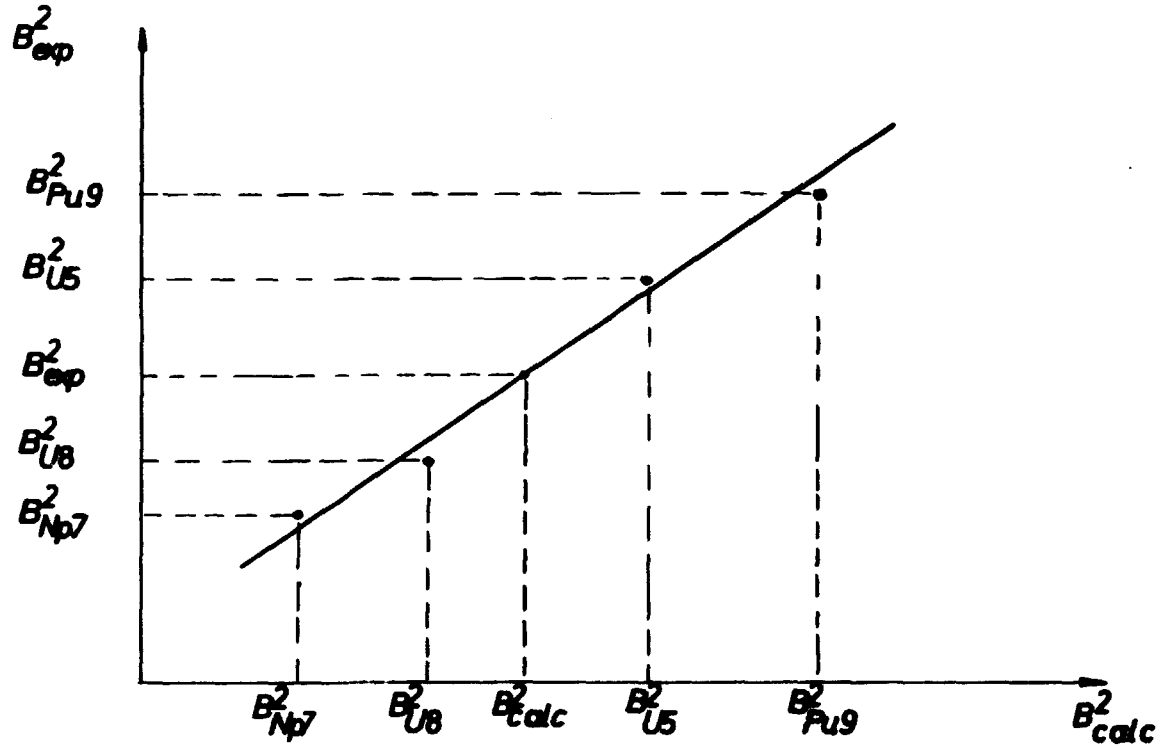

Fig.1. Determination of the „experimental" buckling value

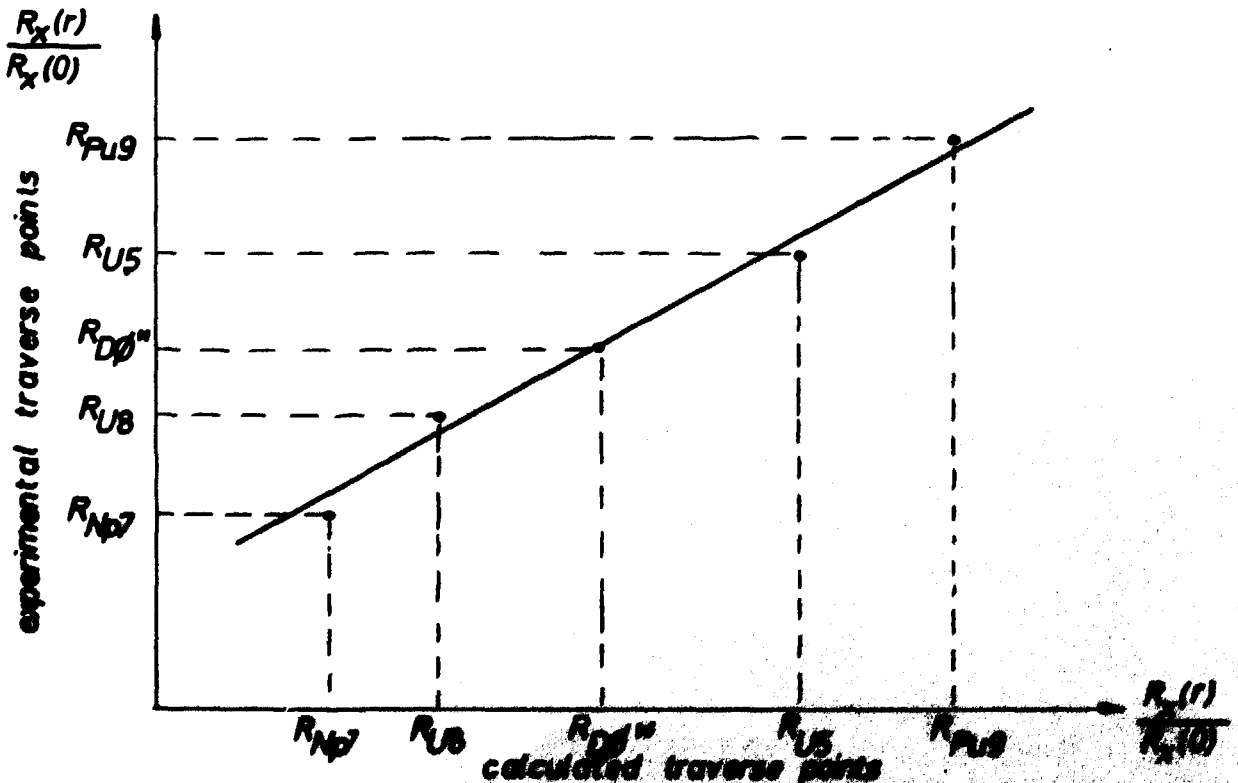

Fig.2. Determination of the „experimental" traverse of a „Dø" detector

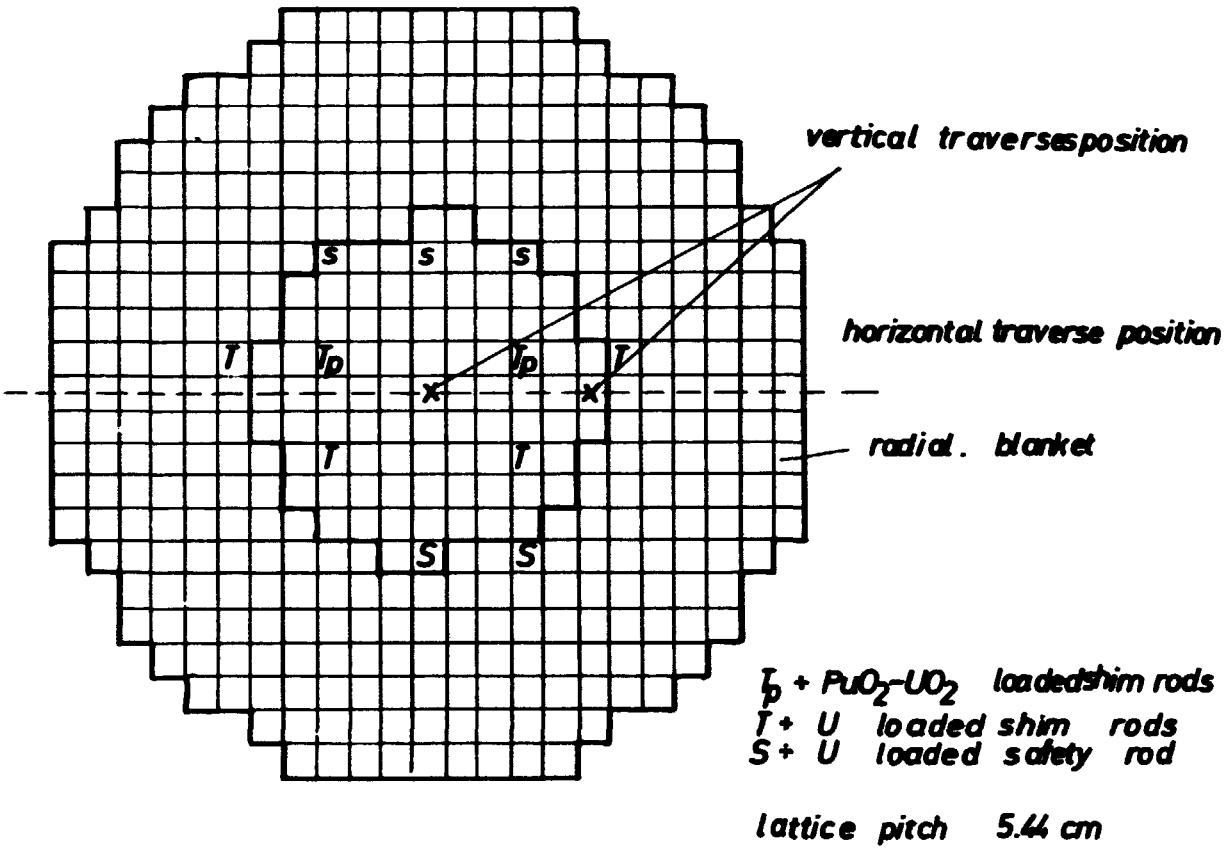

Fig.3. Core map of SNEAK-7A assembly

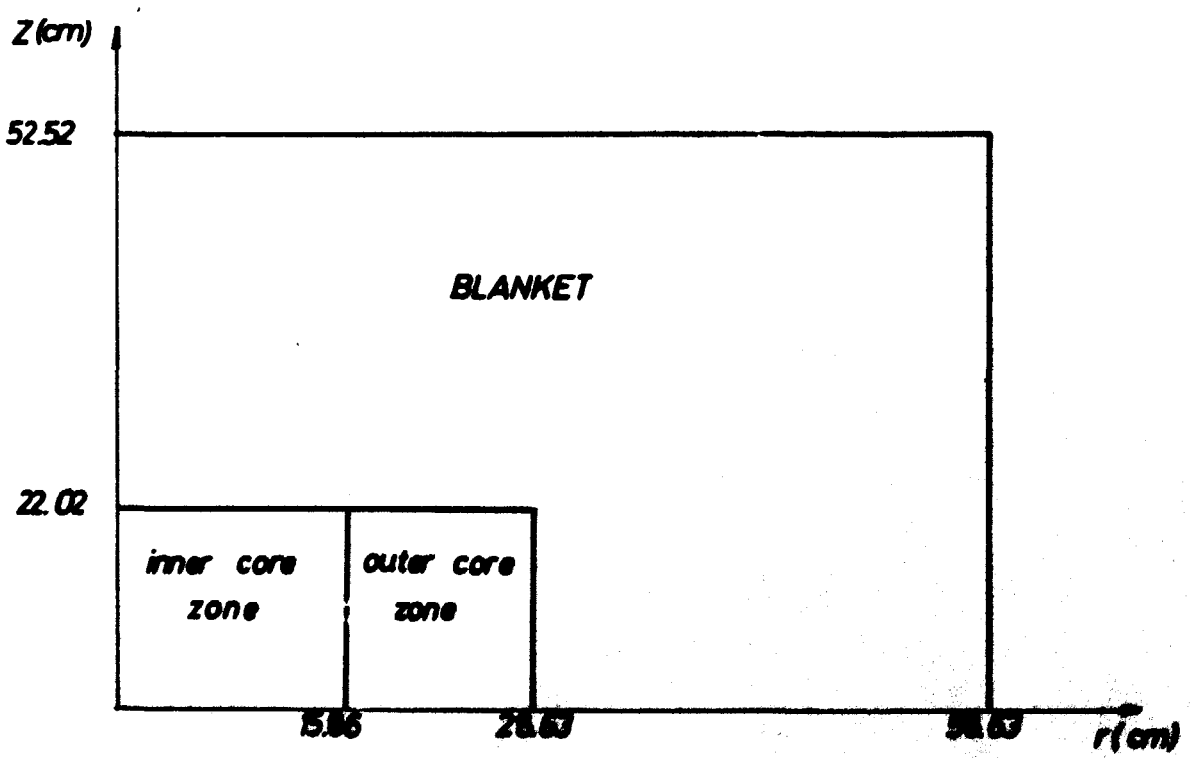

Fig.4. RZ-model for the assembly SNEAK-7A

APPENDIX A.

LIST OF THE PROGRAMME BUCKLING

```
      PROGRAM BUCKLING
C
C     BUCKLING DETERMINATION IN FAST REACTORS
C
      COMMON NFUNCT,NCAS,ICH,KP
      DIMENSION IPO(10,10),APP(10,4),AUX1P(120,4),R(40,10),Y(120,2)
      DIMENSION SUMMA(40),D1(40,10),B(40,10),DE(40,10),XX(120,2)
      DIMENSION CORR(40),SIGMX(120,2),A00(10),D(40,10),D2(40,10),AL(10)
      DIMENSION AUX1(120),X(120,2),F(120)
      DIMENSION FX(120,2),FA(120,10),A(10),DA(50,10),SIGMAX(120,2)
      DIMENSION SU(40,9,10),RR(40,9,10)
      DOUBLE PRECISION A0(10),AUX2(100)
C
  100 FORMAT (1H1,3X,*POLYNOMIAL FITTING FROM *,F9.4,2X,*UNTIL*,F9.4,
     **(CM)*)
  101 FORMAT (5X,*POLYNOMIAL ORDER*,I3)
  102 FORMAT (I4)
  104 FORMAT (3E13.5)
  105 FORMAT (4I6)
  107 FORMAT (1H0,4X,*NO CONVERGENCE  ,NER=*,I2,2X,*CASE=*,I2)
  120 FORMAT (1H0,5X,*COOR*,4X,*U235*,18X,*U238*,18X,*NP237*,17X,
     **PU239*,17X,*DPHI*,6X,*TRAV*,6X,*SIG(TRAV)*/6X,*(CM)*)
  121 FORMAT (2X,F9.4,10F11.6,1X,E11.4)
  125 FORMAT (1H0,3X,*AMPLITUDE*,14X,*BUCKLING*,16X,*STANDARD*,
     *6X,*REGION OF FIT*,/29X,*(CM-1)*,18X,*ERROR*,11X,*(CM)*/)
  126 FORMAT (4X,F8.5,12X,E13.6,12X,E11.4,5X,F10.4)
  130 FORMAT (1H1,50X,*P R O G R A M  *//47X,*B U C K L I N G*///
     */4X,*I N P U T  V A L U E S*//8X,*PROGRAM VERSION*,30X,I4/8X,
     **TYPE OF DISTRIBUTION*,25X,I4/8X,*MULTIPLICATION FACTORS FOR COORD
     *INATES*/15X,*EXPERIMENTAL VALUES*,19X,F10.5/15X,*CALCULATED VALUES
     **,21X,F10.5/8X,*MAXIMUM NUMBER OF REJECTED POINTS*,12X,I4/8X,*STEP
     * OF REJECTION*,28X,I4/8X,*TYPE OF WEIGHTS FACTORS*,22X,I4/8X,
     **INITIAL VALUES FOR THE FITTING PROCEDURE*/15X,*AMPLITUDE*,31X,
     *E13.6/15X,*ARGUMENT*,32X,E13.6/8X,*CYLINDRIZATION CORRECTION*,20X
     *,I4///)
  150 FORMAT (6I5)
  151 FORMAT (2E13.6)
  153 FORMAT (8F10.0)
C
C     READ INPUT DATA
C
      READ (60,150) NCAS,IG,IS,ISIG,NVER,NKORR
      READ (60,151) (A00(I),I=1,2)
      READ (60,151) XN,YN
      IGG=IG-1
      PRINT 130,NVER,NCAS,XN,YN,IGG,IS,ISIG,A00(1),A00(2),NKORR
      IF(NVER.EQ.1) GO TO 250
      READ (60,150) NAL
      READ (60,153) (AL(I),I=1,NAL)
  250 ITMAX=50
      NDIM=120
      M=2
      MAXLS=20
      EPS=1.E-5
      EPS1=1.E-3
      SUPRE=1.E300
```

```
      READ (60,150) NN
      READ (60,153) (Y(I,1),I=1,NN)
      IF(NNORR.EQ.0) GO TO 500
      READ (60,153) (CORR(I),I=1,NN)
  500 DO 70 I=1,NN
   70 Y(I,1)=Y(I,1)*YN
      NCA=0
    1 NCA=NCA+1
      IF(NCA.GT.5) GO TO 3
      READ (60,150) KEN
      READ (60,153) (Y(I,2),I=1,NN)
      DO 2 I=1,NN
    2 R(I,NCA)=Y(I,2)/Y(1,2)
      IF(NCA.LE.5) GO TO 1
    3 READ (60,150) KENN,N
      READ (60,104) (X(I,2),SIGMX(I,2),X(I,1),I=1,N)
C
C     POLYNOMIAL FIT OF EXPERIMENTAL TRAVERSES
C
      KM=N/2
      DO 4 I=1,N
      X(I,1)=X(I,1)*XN
      XX(I,1)=X(I,1)
      SIGMX(I,2)=SIGMX(I,2)*X(I,2)
    4 SIGMAX(I,2)=SIGMX(I,2)
      NO=N
      NFUNCT=3
C     REDUCTION OF EXPERIMENTAL POINTS
      DO 60 II=1,IG,IS
      I1=II
      I2=N-II+1
      SUPP=SUPRE
      IC=0
C     CHANGING THE ORDER OF POLYNOMIAL FUNCTION
  300 IC=IC+1
      SUPR=SUPRE
      ICM=IC
      K=ICM+1
      KP=K
C     FIND THE COORDINATE DISPLACEMENT
      G=0.
      Q1=0.5
      LA=0
      LB=0
      GO TO 9
    6 LB=1
    7 Q1=0.5*Q1
    8 G=G+Q1
    9 DO 10 I=I1,I2
   10 X(I,1)=XX(I,1)-G
      DO 11 I=1,K
   11 A(I)=1.
      CALL POLFIT (K,NDIM,X,F,FX,FA,SIGMAX,A0,A,AUX1,AUX2,NER,I1,I2,SUPR
      IF(NER.EQ.0) GO TO 12
      PRINT 107,NER,NCA
   12 IF(ABS(Q1).LE.EPS1) GO TO 15
```

```
      IF(SUP.GE.SUPR) GO TO 13
      SUPR=SUP
      IF(LA.GE.1) GO TO 6
      GO TO 8
   13 LA=LA+1
      LB=LB+1
      G=G-W1
      IF(LB.GT.1) GO TO 14
      GO TO 7
   14 Q1=-W1
      LB=0
      GO TO 8
C     CHOOSE THE ORDER OF POLYNOMIAL
   15 IF(SUP.GT.SUPP) GO TO 302
      SUPP=SUP
      DO 301 I=1,K
  301 APP(I,IC)=A0(I)
      KK=K*K
      DO 310 I=1,KK
  310 AUX1P(I,IC)=AUX1(I)
      KIC=IC
  302 IF(IC.LT.4) GO TO 300
      K=KIC+1
      IC=KIC
      ICM=IC
  303 SUMA=0
      DO 16 I=I1,I2
   16 SUMA=SUMA+1./(SIGMX(I,2)**2)
      SUMA=1./SUMA
      DO 18 I=1,K
   18 A(I)=APP(I,IC)
      KK=K*K
      DO 320 I=1,KK
  320 AUX1(I)=AUX1P(I,IC)
      CALL FUNCT(K,NDIM,A,Y,F,FX,FA,1,NN,1)
      DO 17 I=1,NN
   17 RR(I,NCA,II)=F(I)/A(1)
      SUP=SUPP
      DO 21 I=2,NN
      SUMMA(I)=0.
      DO 20 J=1,K
      SUM=0.
      DO 19 L=1,K
   19 SUM=SUM+FA(I,L)*AUX1(L+K*(J-1))
   20 SUMMA(I)=SUMMA(I)+SUM*FA(I,J)
   21 SU(I,NCA,II)=(SUMMA(I)+SUMA)*SUP/(I2-I1+1-K)
      IPO(II,NCA)=IC
   69 CONTINUE
C     REPEAT THE PROCEDURE FOR EACH EXPERIMENTAL TRAVERSE
      IF(NCA.LT.9) GO TO 1
C
C     FIND THE D*PHI* TRAVERSE
C
      N=4
      K1=NN/2
      K=2
```

```
      KP=0
      DO 61 II=1,IG,IS
C     CHOICE OF THE VERSION
      IF(NVER.EQ.1) GO TO 350
C     LINEAR FIT OF CROSS-SECTIONS
      DO 26 IB=1,NN
      D(IB,II)=0.
      DE(IB,II)=0.
      DO 96 JJ=1,NAL
      D(IB,II)=D(IB,II)+AL(JJ)*RR(IB,JJ+5,II)
   96 DE(IB,II)=DE(IB,II)+AL(JJ)*SU(IB,JJ+5,II)*AL(JJ)
      IF(IB.GT.1) GO TO 999
      DDD=U(1,II)
  999 D(IB,II)=D(IB,II)/DDD
      DE(IB,II)=SQRT(DE(IB,II))/DDD
      GO TO 26
C     LINEAR FIT OF TRAVERSES
  350 DO 26 I=2,NN
      NFUNCT=2
      DO 22 L=1,N
      X(L,1)=R(I,L)
      X(L,2)=RR(I,L+5,II)
   22 SIGMAX(L,2)=SQRT(SU(I,L+5,II))
      X(1,1)=0.9*X(1,1)
      X(2,1)=1.1*X(2,1)
      X(3,1)=1.1*X(3,1)
      X(4,1)=0.9*X(4,1)
      X(1,2)=0.9*X(1,2)
      X(2,2)=1.1*X(2,2)
      X(3,2)=1.1*X(3,2)
      X(4,2)=0.9*X(4,2)
      XAV=0.
      SUMA=0.
      DO 80 L=1,N
      XAV=XAV+X(L,1)/(SIGMAX(L,2)**2)
   80 SUMA=SUMA+1./(SIGMAX(L,2)**2)
      XAV=XAV/SUMA
      DO 23 J=1,K
   23 A(J)=1.
      CALL POLFIT (K,NDIM,X,F,FX,FA,SIGMAX,A0,A,AUX1,AUX2,NER,1,4,SUP)
      IF(NER.EQ.0) GO TO 24
      PRINT 107,NER,N
   24 D(I,II)=A0(1)+A0(2)*R(I,5)
      Q=0.
      Z=0.
      DO 25 J=1,N
      Z=Z+X(J,1)**2
   25 Q=Q+(X(J,1)-XAV)**2
      A0(2)=0.
      DO 90 L=1,N
   90 A0(2)=A0(2)+((X(L,1)-XAV)**2)/(SIGMAX(L,2)**2)
      A0(1)=0.
      DO 91 L=1,N
   91 A0(1)=A0(1)+X(L,1)/(SIGMAX(L,2)**2)
      A0(2)=DSQRT(SUP/(N-K)*1./A0(2))
      A0(1)=DSQRT(A0(1)/SUMA)*A0(2)
```

```
      DE(I,II)=DSQRT(Q/Z*A0(1)**2+((R(I,5)-XAV)**2)*A0(2)**2)
   26 CONTINUE
C
C     WRITE EACH TRAVERSE INCLUDING THE D*PHI* ONE AND ITS STANDARD ERROR
C
      PRINT 100,(XX(II,1),XX(NO-II+1,1))
      DO 400 NCA=6,9
  400 PRINT 101,IPO(II,NCA)
      PRINT 120
      D(1,II)=1.
      DE(1,II)=0.
      DO 27 I=1,NN
   27 PRINT 121,(Y(I,1),R(I,1),RR(I,6,II),R(I,2),RR(I,7,II),R(I,3),
     *RR(I,8,II),R(I,4),RR(I,9,II),R(I,5),D(I,II),DE(I,II))
C
C     FIT THE D*PHI* TRAVERSE TO THE FUNDAMENTAL MODE OF THE FLUX
C     DISTRIBUTION IN THE REACTOR CORE
C
      NFUNCT=1
      II=2
      DO 40 I=2,NN
   40 Y(I,2)=D(I,II)
      IF(NKORR.EQ.0) GO TO 502
      DO 501 I=2,NN
  501 Y(I,2)=Y(I,2)*CORR(I)
  502 IF(II.GE.5) GO TO 50
      I22=NN
      GO TO 51
   50 I22=NN-II+5
   51 DO 31 I2=K1,I22
      MAXIT=ITMAX
      SUP=SUPRE
      DO 28 I=1,K
   28 A0(I)=A00(I)
      DO 29 I=I1,I2
      SIGMAX(I,1)=0.
      GO TO (203,202,201),ISIG
  201 SIGMAX(I,2)=DE(I,II)
      GO TO 29
  202 SIGMAX(I,2)=Y(I,2)
      GO TO 29
  203 SIGMAX(I,2)=1.
   29 CONTINUE
      CALL CURFIT (K,M,NDIM,MAXIT,MAXLS,EPS,SUP,Y,F,FX,FA,
     *             SIGMAX,A0,A,DA,AUX1,AUX2,NER,I1,I2)
      IF(NER.EQ.0) GO TO 30
      PRINT 107,NER,I2
   30 D1(I2,II)=A(1)
      D2(I2,II)=A(2)
      B(I2,II)=A0(2)
   31 CONTINUE
C
C     WRITE THE OBTAINED BUCKLING VALUES
C
      PRINT 125
      DO 32 I=K1,I22
   32 PRINT 126,(D1(I,II),D2(I,II),B(I,II),Y(I,1))
   61 CONTINUE
C
      END
```

APPENDIX B.
LIST OF THE SUBROUTINES CURFIT, POLFIT AND FUNCT

```
      SUBROUTINE CURFIT (K,M,NDIM,MAXIT,MAXLS,EPS,SUP,X,F,FX,FA,
     *                   SIGMAX,A0,A,DA,AUX1,AUX2,NER,I1,I2)

      COMMON NFUNCT,NCAS,ICH,KP
      DIMENSION X(NDIM,1),SIGMAX(NDIM,1),F(1),FX(NDIM,1),FA(NDIM,1)
      DIMENSION A(1),DA(MAXIT,1),AUX1(1)
      DOUBLE PRECISION A0(1),AUX2(1)

      DO 7 J=1,K
    7 A(J)=A0(J)
      IL=0
      IT=1
    8 LS1=0
      LS2=0
      I=1
    9 LS2=LS2+1
   10 LS1=LS1+1
      CALL FUNCT (K,NDIM,A,X,F,FX,FA,I1,I2,1)
      DO 17 I=I1,I2
      AUX1(I)=0.
      DO 16 L=1,M
   16 AUX1(I)=AUX1(I)+(FX(I,L)*SIGMAX(I,L))**2
   17 AUX1(I)=1./AUX1(I)
      SUM=0.
      DO 22 J=I1,I2
      SUM=SUM+AUX1(J)*F(J)**2
   22 CONTINUE
      IF(SUM.GE.SUP) GO TO 38
      SUP=SUM
   23 CALL FUNCT (K,NDIM,A,X,F,FX,FA,I1,I2,2)
      DO 29 J=1,K
      DO 29 J1=1,K
      AUX2(J+K*(J1-1))=0.
      DO 27 I=I1,I2
   27 AUX2(J+K*(J1-1))=AUX2(J+K*(J1-1))+FA(I,J)*AUX1(I)*FA(I,J1)
   29 CONTINUE
      IF(IL.GT.0) GO TO 40
      DO 33 J=1,K
      A0(J)=0.
      DO 32 I=I1,I2
   32 A0(J)=A0(J)+FA(I,J)*AUX1(I)*F(I)
   33 CONTINUE
      CALL DGELG (A0,AUX2,K,1,0.E-10,IER)
      IF(IER.EQ.-1) GO TO 63
      DO 34 J=1,K
      DA(IT,J)=A0(J)
   34 A(J)=A(J)-DA(IT,J)
      DO 35 J=1,K
      IF(ABS(DA(IT,J)).GT.EPS*ABS(A(J))) GO TO 36
   35 CONTINUE
      GO TO 37
   36 IT=IT+1
      DO 37 J=1,K
   37 A0(J)=A(J)
      IF(IT.GT.MAXIT) GO TO 64
```

```
      GO TO 8
   38 Q=Q*0.5
      IF(L-1.GT.MAXLS)  GO TO 40
      DO 39 J=1,K
   39 A(J)=A0(J)-Q*DA(IT-1,J)
      GO TO 10
   40 IF(L-2.GT.MAXLS) GO TO 45
      IF(L-2.GT.1) GO TO 41
      Q=0.5
   41 IF(II.NE.2) GO TO 43
      DO 42 J=1,K
   42 A(J)=A0(J)+Q*DA(IT-1,J)
      GO TO 9
   43 DO 44 J=1,K
   44 A(J)=A0(J)+Q*DA(IT-2,J)
      GO TO 9
   45 IF(II.EQ.2) GO TO 65
      DO 46 J=1,K
   46 A(J)=A0(J)
   47 IL=IL+1
      GO TO 23
   48 KK=K*K
      DO 59 I=1,KK
   59 AUX1(I)=AUX2(I)
      CALL MINV(AUX1,K,D,F,SIGMAX)
      DO 61 J=1,K
      DO 60 I=1,K
   60 AUX2(I+K*(J-1))=SUM/(I2-I1+1-K)*AUX1(I+K*(J-1))
   61 A0(J)=DSQRT(AUX2(J+K*(J-1)))
      MAXIT=IT-1
      NER=0
      RETURN
   63 NER=1
      RETURN
   64 NER=2
      RETURN
   65 NER=3
      RETURN
      END
```

```
      SUBROUTINE POLFIT (K,NDIM,X,F,FX,FA,SIGMAX,A0,
     *                   A,AUX1,AUX2,NER,I1,I2,SUP)
C
C
      COMMON NFUNCT,NCAS,ICH,KP
      DIMENSION X(NDIM,1),F(1),FX(NDIM,1),FA(NDIM,1),AUX1(1)
      DIMENSION A(1),SIGMAX(NDIM,1)
      DOUBLE PRECISION A0(1),AUX2(1)
C
      IT=0
   30 CALL FUNCT(K,NDIM,A,X,F,FX,FA,I1,I2,1)
      DO 3 J=1,K
      DO 3 J1=1,K
      AUX2(J+K*(J1-1))=0.
      DO 2 I=I1,I2
    2 AUX2(J+K*(J1-1))=AUX2(J+K*(J1-1))+FA(I,J)*1./(SIGMAX(I,2)**2)*
     *FA(I,J1)
    3 CONTINUE
      IF(I1.EQ.1) GO TO 31
      DO 6 J=1,K
      A0(J)=0.
      DO 5 I=I1,I2
    5 A0(J)=A0(J)+FA(I,J)*1./(SIGMAX(I,2)**2)*X(I,2)
    6 CONTINUE
      CALL DGELG (A0,AUX2,K,1,1.E-10,IER)
      IF(KP.EQ.2) GO TO 60
      IF(K.EQ.2) GO TO 10
    6 DO 7 I=I1,I2
      GO TO (70,80,90,100),ICH
   70 F(I)=A0(1)+(X(I,1)**2)*A0(2)
      GO TO 7
   80 F(I)=A0(1)+(X(I,1)**2)*A0(2)+(X(I,1)**4)*A0(3)
      GO TO 7
   90 F(I)=A0(1)+(X(I,1)**2)*A0(2)+(X(I,1)**4)*A0(3)+(X(I,1)**6)*A0(4)
      GO TO 7
  100 F(I)=A0(1)+(X(I,1)**2)*A0(2)+(X(I,1)**4)*A0(3)+
     *(X(I,1)**6)*A0(4)+(X(I,1)**8)*A0(5)
    7 CONTINUE
      GO TO 12
   10 DO 11 I=I1,I2
   11 F(I)=A0(1)+A0(2)*X(I,1)
   12 SUP=0.
      DO 1 I=I1,I2
    1 SUP=SUP+((X(I,2)-F(I))**2)/(SIGMAX(I,2)**2)
      IT=1
      DO 4  I=1,K
    4 A(I)=A0(I)
      GO TO 30
   31 IF(KP.EQ.2) GO TO 61
      IF(K.EQ.2) GO TO 50
   61 KK=K*K
      DO 4 I=1,KK
    4 AUX1(I)=AUX2(I)
      CALL MINV(AUX1,K,D,F,FX)
    5 IF(IER.EQ.-1) GO TO 8
      NER=0
      RETURN
    8 NER=1
      RETURN
      END
```

```
      SUBROUTINE FUNCT (K,NDIM,A,X,F,FX,FA,I1,I2,NF)
C
C
      COMMON NFUNCT,NCAS,ICH,KP
      DIMENSION A(1),X(NDIM,1),F(1),FX(NDIM,1),FA(NDIM,1)
C
      GO TO (10,20,30),NFUNCT
   10 GO TO (11,12),NCAS
C
C     BESSEL FUNCTION
C
   11 GO TO (16,17),NF
   16 DO 5 I=I1,I2
      X1=ABS(X(I,1)*A(2))
      IF(X1.EQ.0.) GO TO 170
      CALL BESJ (X1,0,B0,1.E-6,IER)
      CALL BESJ (X1,1,B1,1.E-6,IER)
      GO TO 171
  170 B0=1.
      B1=0.
  171 F(I)=X(I,2)-A(1)*B0
      FX(I,1)=A(1)*A(2)*B1
    5 FX(I,2)=1.
      RETURN
C
   17 DO 18 I=I1,I2
      X1=ABS(X(I,1)*A(2))
      IF(X1.EQ.0.) GO TO 180
      CALL BESJ (X1,0,B0,1.E-6,IER)
      CALL BESJ (X1,1,B1,1.E-6,IER)
      GO TO 181
  180 B0=1.
      B1=0.
  181 FA(I,1)=-B0
   18 FA(I,2)=A(1)*ABS(X(I,1))*B1
      RETURN
C
C     SIN FUNCTION
C
   12 GO TO (23,24),NF
   23 DO 8 I=I1,I2
      F(I)=X(I,2)-A(1)*COS(A(2)*X(I,1))
      FX(I,1)=A(1)*A(2)*SIN(A(2)*X(I,1))
    8 FX(I,2)=1.
      RETURN
C
   24 DO 25 I=I1,I2
      FA(I,1)=-COS(A(2)*X(I,1))
   25 FA(I,2)=A(1)*X(I,1)*SIN(A(2)*X(I,1))
      RETURN
C
C     STRAIGHT LINE
C
   20 DO 40 I=I1,I2
      F(I)=A(1)+A(2)*X(I,1)
      FA(I,1)=1.
```

```
   40 FA(I,2)=A(I,1)
      RETURN
C
C     POLYNOMIAL FUNCTION
C
   33 DO 33 I=I1,I2
      GO TO (50,60,70,80),ICH
C     SECOND ORDER
   50 F(I)=A(1)+(X(I,1)**2)*A(2)
      FA(I,1)=1.
      FA(I,2)=(X(I,1)**2)
      GO TO 33
C     FOURTH ORDER
   60 F(I)=A(1)+(X(I,1)**2)*A(2)+(X(I,1)**4)*A(3)
      FA(I,1)=1.
      FA(I,2)=(X(I,1)**2)
      FA(I,3)=(X(I,1)**4)
      GO TO 33
C     SIXTH ORDER
   70 F(I)=A(1)+(X(I,1)**2)*A(2)+(X(I,1)**4)*A(3)+(X(I,1)**6)*A(4)
      FA(I,1)=1.
      FA(I,2)=(X(I,1)**2)
      FA(I,3)=(X(I,1)**4)
      FA(I,4)=(X(I,1)**6)
      GO TO 33
C     EIGHTH ORDER
   80 F(I)=A(1)+(X(I,1)**2)*A(2)+(X(I,1)**4)*A(3)+(X(I,1)**6)*A(4)+
     *(X(I,1)**8)*A(5)
      FA(I,1)=1.
      FA(I,2)=(X(I,1)**2)
      FA(I,3)=(X(I,1)**4)
      FA(I,4)=(X(I,1)**6)
      FA(I,5)=(X(I,1)**8)
   33 CONTINUE
      RETURN
      END
```

APPENDIX C.
INPUT - OUTPUT LIST (example)

PROGRAM
BUCKLING

INPUT VALUES

PROGRAM VERSION	2
TYPE OF DISTRIBUTION	1
MULTIPLICATION FACTORS FOR COORDINATES	
EXPERIMENTAL VALUES	0.10000
CALCULATED VALUES	1.00000
MAXIMUM NUMBER OF REJECTED POINTS	8
STEP OF REJECTION	4
TYPE OF WEIGHTS FACTORS	1
INITIAL VALUES FOR THE FITTING PROCEDURE	
AMPLITUDE	9.500000-001
ARGUMENT	5.500000-002
CYLINDRIZATION CORRECTION	1

POLYNOMIAL FITTING FROM -24.2900 UNTIL 24.1600(CM)
POLYNOMIAL ORDER 8
POLYNOMIAL ORDER 8
POLYNOMIAL ORDER 8
POLYNOMIAL ORDER 8

COOH (CM)	U235		U238		NP237		PU239		DPHI+	TRAV	SIG(TRAV)
0.0000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0.0000+000
0.9067	0.999208	0.999299	0.999140	0.999213	0.999147	0.999194	0.999198	0.999286	0.999165	0.999228	8.3606-004
1.8133	0.996834	0.997196	0.996592	0.996851	0.996596	0.996777	0.996785	0.997143	0.996724	0.996911	7.9198-004
2.7200	0.992685	0.993685	0.992271	0.992912	0.992342	0.992753	0.992774	0.993572	0.992644	0.993047	7.2724-004
3.6267	0.987371	0.988759	0.986273	0.987391	0.986405	0.987126	0.987176	0.988573	0.986926	0.987636	6.5530-004
4.5334	0.980336	0.982459	0.978578	0.980286	0.978790	0.979907	0.979999	0.982147	0.979602	0.980675	5.9470-004
5.4400	0.971709	0.974028	0.969198	0.971594	0.969517	0.971108	0.971270	0.974299	0.970704	0.972167	5.6443-004
6.3467	0.961600	0.965478	0.958148	0.961315	0.958598	0.960745	0.961004	0.965032	0.960233	0.962113	5.7316-004
7.2534	0.950018	0.954751	0.945448	0.949456	0.946064	0.948840	0.949233	0.954359	0.948251	0.950522	6.1168-004
8.1600	0.936969	0.942063	0.931124	0.936033	0.931938	0.935424	0.935996	0.942294	0.934760	0.937412	6.6017-004
9.0667	0.922517	0.929158	0.915204	0.921064	0.916261	0.920528	0.921327	0.928857	0.919791	0.922801	7.0014-004
9.9734	0.906698	0.914265	0.897726	0.904586	0.899076	0.904196	0.905272	0.914076	0.903408	0.906725	7.1996-004
10.8800	0.889570	0.898126	0.878751	0.886645	0.880435	0.886479	0.887891	0.897989	0.885677	0.889227	7.1601-004
11.7867	0.871202	0.880495	0.858345	0.867299	0.860411	0.867433	0.869260	0.880636	0.866692	0.870359	6.9257-004
12.6934	0.851675	0.861742	0.836594	0.846620	0.839098	0.847125	0.849460	0.862074	0.846520	0.850188	6.6126-004
13.6000	0.831096	0.841856	0.813639	0.824698	0.816612	0.825629	0.828598	0.842364	0.825255	0.828794	6.3801-004
14.5067	0.809580	0.820928	0.789631	0.801622	0.793091	0.803018	0.806799	0.821572	0.803026	0.806257	6.3538-004
15.4134	0.787208	0.799165	0.764679	0.777496	0.768632	0.779374	0.784147	0.799775	0.779962	0.782670	6.5308-004
16.3200	0.764042	0.776578	0.738859	0.752425	0.743294	0.754780	0.760706	0.777052	0.756127	0.758130	6.7671-004
17.2267	0.740114	0.752962	0.712197	0.726500	0.717114	0.729304	0.736512	0.753475	0.731522	0.732718	6.8736-004
18.1334	0.715448	0.728904	0.684687	0.699800	0.690106	0.703012	0.711593	0.729120	0.706209	0.706512	6.7372-004
19.0400	0.690037	0.704259	0.656279	0.672376	0.662250	0.675953	0.685949	0.704046	0.680190	0.679566	6.4215-004
19.9467	0.663875	0.679027	0.626903	0.644222	0.633518	0.648137	0.659579	0.678290	0.653432	0.651887	6.2078-004
20.8534	0.636985	0.653143	0.596513	0.615274	0.603927	0.619547	0.632518	0.651861	0.626000	0.623440	6.3917-004
21.7600	0.609409	0.626449	0.565102	0.585377	0.573512	0.590115	0.604816	0.624731	0.597957	0.594116	6.8045-004
22.6668	0.581200	0.598043	0.532684	0.554244	0.542341	0.559693	0.576546	0.596803	0.569368	0.563696	6.8646-004
23.5734	0.552418	0.569285	0.499275	0.521457	0.510478	0.528073	0.547770	0.567926	0.540330	0.531865	7.4445-004
24.4800	0.523120	0.537707	0.464905	0.486386	0.477998	0.494920	0.518568	0.537842	0.510873	0.498130	1.4266-003
25.3868	0.493362	0.502983	0.429578	0.448161	0.444978	0.459772	0.489011	0.506174	0.481128	0.461810	3.1867-003
26.2934	0.463195	0.463901	0.393289	0.405648	0.411492	0.422032	0.459167	0.472429	0.451158	0.422011	6.3432-003
27.2001	0.432654	0.418842	0.355981	0.357336	0.377601	0.380891	0.429101	0.435917	0.421059	0.377537	1.1416-002
28.1068	0.401758	0.365758	0.317506	0.301319	0.343358	0.335336	0.398865	0.395761	0.390864	0.326870	1.9110-002
29.0135	0.370508	0.302069	0.277809	0.235199	0.308799	0.284089	0.368491	0.350842	0.360669	0.268103	3.0321-002

AMPLITUDE	BUCKLING (CM-1)	STANDARD ERROR	REGION OF FIT (CM)
1.00082	6.258203-002	1.1995-004	13.6000
1.00103	6.273377-002	1.1942-004	14.5067
1.00124	6.287405-002	1.1741-004	15.4134
1.00146	6.300155-002	1.1407-004	16.3200
1.00167	6.311575-002	1.0968-004	17.2267
1.00188	6.321691-002	1.0453-004	18.1334
1.00209	6.330630-002	9.9033-005	19.0400
1.00229	6.338638-002	9.3637-005	19.9467
1.00249	6.346092-002	8.8892-005	20.8534
1.00271	6.353524-002	8.5509-005	21.7600
1.00296	6.361639-002	8.4539-005	22.6668
1.00329	6.371326-002	8.7616-005	23.5734
1.00375	6.383675-002	9.7080-005	24.4800
1.00439	6.399988-002	1.1560-004	25.3868