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ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE

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POWER LAWS IN PARTICLE PHYSICS

D. Amati and S. Fubini

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POWER LAWS IN PARTICLE PHYSICS*)

D. Amati and S. Fubini

G E N E V A

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*) Based on lectures given in 1974 at the 9th Finnish Summer School in Theoretical Physics, Ekenäs (Finland), and in the Academic Training Programme of CERN.

ABSTRACT

These lecture notes present a theoretical survey of the different scaling asymptotic behaviours found in experimental data, and their connection with the underlying hadron dynamics. As an initial classification of the phenomena, purely hadronic processes, and processes including weak or electromagnetic interactions, are considered, both in inclusive and exclusive reactions. The transition between different power laws is investigated. The main properties of the different scaling behaviours are treated, in the light of the parton and the multiperipheral models. Finally the presence of logarithmic asymptotic behaviours and the dynamical origin of scales are discussed.

FOREWORD

This report is based on lectures given by one of us (D.A.) at the 9th Finnish Summer School in Theoretical Physics (Ekenäs, 20-28 June 1974) and by the other (S.F.) in the Academic Training Programme of CERN.

We dedicate this modest work to the memory of our friend and colleague, Antonio Stanghellini, on the tenth anniversary of his death. We remember with emotion the wonderful friendship and collaboration in those years in which all three of us were at the Theory Division of CERN. The spirit we developed in that collaboration is at the very root of the ideas developed in this report.

CONTENTS

1. INTRODUCTION

Although important progress has been made in the study of hadrodynamics, we still lack a satisfactory theory of strong interactions. The idea that hadrons are composite objects is playing a more and more important role; however the fact that the constituents (quarks) have never been observed represents a challenging problem.

In order to progress in our search for the general laws governing hadron physics, we are greatly helped when simple empirical trends are revealed in phenomena having to deal with the structure and interactions of hadrons.

An important development in this direction is the successful use of power laws in order to correlate a large number of experimental asymptotic data.

In these lectures we shall discuss different scaling phenomena and the connection they have with the underlying hadron dynamics. We will present a way of looking at and correlating these scaling phenomena that we understand better, and then make an adventurous jump to a new facet of hadrodynamics which, in our view, has been revealed by recent e^+e^- annihilation data. We shall not follow experimental data too closely so that no fits will be shown in these lectures. We prefer to take some qualitative trends and suggestions from the data.

Scaling reflects the existence of a basic parameter (scale) with dimensions (let us say energies), so that when a kinematical variable in a phenomenon is much larger than it, we find simple asymptotic power developments in such a kinematical variable.

This implies that we do not find -- for the time being -- other basic high-energy scales. There is no basic reason for this and it could well happen that we will encounter a new fundamental scale (100 GeV, for instance), which would then define a new scaling régime. What can be said is that, up to now, we can understand asymptotic properties of phenomena without the need of introducing new scales besides the typical one -- of the order of some hundreds of MeV -- which appears in phenomena involving hadrons. This scale is the scale of masses, Regge trajectory slopes, average transverse momenta, etc. Let us stress that the absence of large fundamental (energy) scales does not imply that high-energy physics has no new features, or that it is nothing more than a tail of low-energy processes. Indeed, the way in which the basic scale will enter into high-energy processes is governed by dynamics and this dynamics is just what we want to understand. This means that we want to understand what can be the forces among particles that can lead to those asymptotic power behaviours. And when we say forces we do not want to imply a phenomenological description of them but -- as generally done in any field theoretical inspired approach -- as determined by the same particles we are trying to describe.

After a general survey of the different scaling properties revealed by present experimental evidence, we shall discuss the main phenomenological relations between different power behaviours.

The second part of these lectures will be dedicated to specific models leading to scaling in well-defined kinds of physical processes. We shall discuss in detail both parton and multiperipheral models and their interconnections.

We shall then refer briefly to dual models, especially in connection with the scaling properties in e^+e^- annihilation.

We shall finally survey the modern theoretical ideas which have led us beyond simple scaling laws and which suggest the presence of logarithmic asymptotic behaviours.

2. SURVEY OF THE DIFFERENT KINDS OF SCALING

We know by now several phenomena which show asymptotic power behaviours.

The general trends of such scaling laws sometimes differ greatly from process to process, together with the nature of their theoretical justification. Sometimes the same physical process shows very different properties in different kinematical configurations.

In order to introduce some order into the different kinds of scaling laws, we shall use two criteria:

a) whether the process is a purely hadronic one or includes weak or electromagnetic interactions;

b) whether the process exhibiting scaling is an inclusive or exclusive one.

A classification according to these criteria is given in Table 1. We anticipate that such a classification is not sufficient to characterize completely a scaling behaviour. Indeed, in the next section we shall introduce a new criterion, that of the "dimensionality" of the asymptotic process.

Table 1

Classification of different power laws in particle physics

	Purely hadronic processes	Processes including weak or electromagnetic interactions
Inclusive amplitude	a) Scaling in one-particle inclusive distribution (at small transverse momentum) b) Scaling in one-particle inclusive distributions (at large transverse momentum)	a) Scaling in deep inelastic electron and neutrino scattering (Bjorken scaling) b) High-energy e^+e^- annihilation
Exclusive amplitude	a) Asymptotic behaviour of elastic scattering at fixed momentum transfer (Regge behaviour) b) Asymptotic behaviour of elastic scattering at fixed angle	Asymptotic behaviour of hadron electromagnetic form factors

Let us now discuss in some detail the processes listed in Table 1.

2.1 Strong inclusive

Total cross-sections as a function of energy are almost constant; the small logarithmic increase will be discussed in Section 6.

The main features of multiple production are:

i) Transverse momentum k_T is strongly limited (the average k_T is of the order of 300 MeV, independently of the initial energy).

ii) Secondaries are homogeneously distributed in longitudinal rapidity.

iii) Multiplicities increase with the logarithm of the initial energy.

Those features which were theoretically predicted in 1962, on the basis of the multiperipheral model, can be described by the scaling property of the one-particle inclusive spectrum:

$$\frac{d\sigma}{d^3k/k_0} \approx s^{-\frac{A}{2}} F(x_0, k_T^2) \tag{2.1}$$

where

$$x_0 = \frac{2k_0}{\sqrt{s}} = \frac{2(k\,p_A)}{(p_A+p_B)^2}$$

and the kinematics is that represented in the graph of Fig. 1. F is a function which

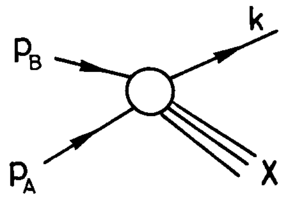

Fig. 1

decreases rapidly with increasing k_T^2 and which is regular for $x_0 \sim 0$.

Experimentally $A \sim 0$, which is in agreement with the Regge analysis ($-A/2 = \alpha_P(0) - 1$).

As said before, the function $F(x_0, k_T^2)$ strongly damps large values of k_T. If, however, we insist on observing rare events with large k_T, then a new power régime holds, given by

$$\frac{d\sigma}{d^3k/k_0} \approx s^{-\frac{B}{2}} G(x_0, x_T) \tag{2.2}$$

where

$$x_0 = \frac{2k_0}{\sqrt{s}} \quad , \quad x_T = \frac{2k_T}{\sqrt{s}}$$

and where present experimental evidence seems to indicate that $B \approx 8$.

The transition between the two power régimes in Eqs. (2.1) and (2.2), which seems to be smooth, will be discussed in detail later in this section.

2.2 Strong exclusive

We shall concentrate our attention on high-energy elastic scattering.

It is well known that the differential cross-section exhibits a narrow forward peak, whose width varies slowly with energy.

The usual description of the scattering cross-section for large s and small t is given by the Regge formula

$$\frac{d\sigma}{dt} \simeq H(t)\ s^{2\alpha(t)-2} . \tag{2.3}$$

In the case in which no quantum numbers are exchanged, $\alpha(0) \approx 1$ (Pomeron exchange). On the other hand, for exchange of non-exotic quantum numbers we have $\alpha(0) \cong \frac{1}{2}$. For not too large values of t the linear form of the trajectory

$$\alpha(t) = \alpha(0) + \beta t \tag{2.4}$$

works very nicely with the "universal slope"

$$\beta \simeq 1\ (\mathrm{GeV})^{-2} \tag{2.5}$$

for all trajectories; except the Pomeron that seems to have a smaller slope.

We have seen that the largest part of elastic events takes place for small values of t. If we insist on observing the rare events with large values of both s and t, a new fixed-angled scaling law sets in.

Introducing the scattering angle as a dimensionless variable $t/s \sim -\frac{1}{2}(1 - \cos\theta)$ we have

$$\frac{d\sigma}{dt} \simeq s^{-c}\ K(\theta) \tag{2.6}$$

where the exponent c takes the values

$$\begin{aligned} c &\simeq 10 \quad \text{for } NN \text{ and } N\bar{N} \text{ scattering} \\ c &\simeq 8 \quad \text{for } \pi N \text{ scattering} \\ c &\simeq 7 \quad \text{for } \pi \text{ photoproduction .} \end{aligned} \tag{2.7}$$

2.3 <u>Inclusive reactions induced by weak or electromagnetic interactions</u>

The most celebrated example is <u>deep inelastic lepton scattering</u>:

lepton + nucleon → lepton + anything,

which, in the one-photon (intermediate boson) exchange approximation, is represented by the diagram of Fig. 2.

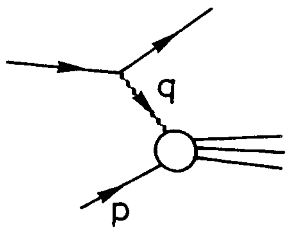

Fig. 2

This means that (for example, in the electron case) the reaction can be schematized as

$$(\gamma) + N \to \text{anything},$$

where we study <u>the total cross-section</u> of a virtual space-like photon of momentum q_μ on a nucleon target of momentum p.

In this way the experimental findings can be schematized by means of the two total cross-sections $\sigma_T(s,q^2)$, $\sigma_L(s,q^2)$, corresponding to transversal and longitudinal photons, respectively, as a function of the squared c.m. energy $s = (p + q)^2$ and of the mass q^2 of the impinging "photon".

The celebrated Bjorken scaling law states that for large values of q^2 and fixed $\omega = 1 - (s/q^2)$ the two cross-sections σ_T and σ_L have the asymptotic form

$$\sigma(s, q^2) \approx |q^2|^{-\frac{d}{2}} \varphi(\omega) \quad . \tag{2.8}$$

Experimentally $d \approx 2$, which is in agreement with canonical scaling (i.e. no appearance of the scale in the expression for the cross-section, see next section).

Another inclusive reaction induced by electromagnetic currents, which has called for a lot of attention because of its puzzling features <u>is high-energy e^+e^- annihilation</u>.

The fundamental experimental feature is that charged secondaries are produced with low momentum ($\sim$ 500 MeV). The average momentum is roughly independent of the initial energy. The asymptotic régime has a form which depends on whether we are interested in the numerous events in which the secondary c.m. energy p_0 is of the order of 500 MeV or whether we look for the rare events in which p_0 is of the order of magnitude of the initial c.m. energy q_0.

In the e^+e^- c.m.s.

$$\begin{aligned} s = (q-p)^2 &= q_0^2 - 2q_0 p_0 + M^2 \\ q^2 &= q_0^2 \quad . \end{aligned} \tag{2.9}$$

Let us introduce the single inclusive amplitude $d\sigma/(d^3p/p_0)$. The total cross-section is given in terms of the inclusive distribution by the energy sum rule

$$\sigma_{e^+e^-} = \frac{1}{q_0} \int \frac{d\sigma}{d^3p/p_0} \, d^3p \quad . \tag{2.10}$$

We can now study the asymptotic forms of the inclusive amplitude.

i) <u>Large q_0, small p_0</u>

$$\frac{d\sigma}{d^3p/p_0} \approx |q^2|^{-\frac{a}{2}} f(p_0) \tag{2.11}$$

where $f(p_0)$ decreases with increasing p_0. The dominant contribution to the integral (2.10) for the total cross-section depends essentially on the values of p_0 for which Eq. (2.11) is valid, so that we get

$$\sigma_{e^+e^-} \approx A \, q_0^{-a-1} = A \, q^{2\left(\frac{-a-1}{2}\right)} \quad . \tag{2.12}$$

Present experiments seem to indicate that $a \approx -1$.

ii) Large q_0 and p_0

If $f(p_0)$ decreases with increasing p_0, we expect few events with large p_0. This implies that if we insist, nevertheless, on looking at these events, we will find a new scaling law

$$\frac{d\sigma}{d^3p/p_0} \approx |q^2|^{-\frac{b}{2}} g(\omega) \tag{2.13}$$

where, again

$$\omega = 1 - \frac{s}{q^2} \simeq \frac{2p_0}{q_0} . \tag{2.14}$$

We see that, kinematically, the one-particle inclusive e^+e^- annihilation cross-section looks similar to the total deep inelastic lepton cross-section. The scaling variable ω ranges over:

$1 < \omega < \infty$ for deep inelastic lepton scattering

$0 < \omega < 1$ for e^+e^- annihilation .

Indeed, theoretically, they are given by the absorptive part of the same amplitude (forward virtual Compton scattering on a hadron) in a different kinematical region. Let us remark, besides, that the canonical value $b = 4$ in Eq. (2.13) is not in contradiction with experimental data.

We shall discuss later the relation between the scaling behaviours (2.11) and (2.13).

We now come to the last topic on our list.

2.4 Weak-electromagnetic exclusive

The only well-known case is that of electromagnetic form factors of hadrons, which again exhibit asymptotic power behaviour approximately equal to $(q^2)^{-\delta}$ for large q^2.

The experimental value of δ in correspondence with both nucleon form factors seems to be $\delta \approx 2$.

2.5 Transition between different asymptotic limits

In the previous discussion we have seen that the same amplitude can exhibit in different kinematic situations different scaling limits. We wish now to take up the interesting question of the transition between those different limits.

The important question is "can we interchange the different limits?" Now it is reasonable that in a model leading to a unified description of different scalings this should be possible. As we shall see later this happens, for example, in the framework of the multiperipheral model.

Let us consider the example of a one-particle inclusive amplitude in hadron-hadron collisions.

In order to study in some detail the interchange of the two limits, let us write the inclusive amplitude in the general form:

$$\frac{d\sigma}{d^3k/k_0} = s^{-\frac{B}{2}} \varphi\left(x_0, x_T, \frac{\mu}{k_T}\right) \tag{2.15}$$

where μ is the fundamental scale of the strong hadron phenomena.

The large k_T scaling law follows from the existence of the limit

$$\lim_{\mu/k_T \to 0} \varphi\left(x_0, x_T, \frac{\mu}{k_T}\right) \longrightarrow G(x_0, x_T) \,. \tag{2.16}$$

On the other hand, the small k_T scaling law implies:

$$\lim_{x_T \to 0} x_T^{\gamma} \varphi\left(x_0, x_T, \frac{\mu}{k_T}\right) = (2k_T)^{-\gamma} F(x_0, k_T) \tag{2.17}$$

where

$$\gamma = B - A \,. \tag{2.18}$$

If we now take the limit $\mu/k_T \to 0$ (i.e. $k_T \to \infty$) on both sides of Eq. (2.17), we finally get

$$\lim_{x_T \to 0} x_T^{\gamma} G(x_0, x_T) = \lim_{k_T \to \infty} (2k_T)^{\gamma} F(x_0, k_T) \,. \tag{2.19}$$

Equation (2.19) expresses the consistency between the two scaling laws since, experimentally, $B > A$, i.e. $\gamma > 0$, it tells us that the structure function $G(x_0, x_T)$ is singular at the point $x_T = 0$.

Another interesting example of transition between different power laws -- expected to hold if they have the same dynamical origin -- is that of deep inelastic scattering and e^+e^- annihilation. As said before, the total cross-section for the first process and the one-particle inclusive cross-section for the second, are given in terms of the forward absorptive part of the same virtual Compton amplitude in different kinematical regions ($1 < \omega < \infty$ for the first, $0 < \omega < 1$ for the second). In any reasonable dynamical model (describable with Feynman diagrams, for instance) the same scaling power in (2.8) and in (2.13) (i.e. $b = d + 2$) is expected. Independently of this fact, let us discuss some specific kinematical configurations:

a) <u>$\omega \to \infty$, called Regge region for electroproduction</u>

Here $s \gg -q^2 \gg \mu^2$ (the energy scale). If limits can be interchanged, we could reach this region by making first $s/-q^2 \to \infty$ at fixed q^2 and then q^2 large. In the first limit, we would expect a Regge behaviour, i.e.

$$\sigma \simeq s^{\alpha(0)-1} h(q^2) \,. \tag{2.20}$$

The second limit explores the high q^2 behaviour of the wave function $h(q^2)$. The interchangeability of limits (i.e. the same dynamical origin) would imply, by comparing (2.8) with (2.20)

$$\lim_{\omega\to\infty} \omega^{1-\alpha(0)} \varphi(\omega) = \lim_{q^2\to\infty} |q^2|^{\alpha(0)-1+\frac{d}{2}} h(q^2) . \tag{2.21}$$

b) $\omega \sim 1$. Form factor region

Here $s = (1 - \omega)\, q^2 >> q^2$ and, in particular, s remains fixed if $\omega = 1 + O(\mu^2/q^2)$.

We could obtain the same region setting first $q^2 \to \infty$ by leaving s (the squared mass of the excited state in "γ" + p) fixed. We would therefore measure the form factor of the excitation of the state with $(\text{mass})^2 = s$ from the proton. If this form factor drops as a power $(1/q^2)^\delta$, we expect, by comparing with Eq. (2.8) the Drell-Yan rule

$$\varphi(\omega) \xrightarrow[\omega\to 1]{} |1-\omega|^{2\delta-\frac{d}{2}} . \tag{2.22}$$

Let us remark that on rather general grounds we expect the functions $\varphi(\omega)$ and $g(\omega)$, defined on different sides of $\omega = 1$, to have similar behaviours around that point.

c) $\omega \sim 0$. Low-energy secondaries in e^+e^- annihilation

Indeed the fixed p_0 situation studied above corresponds to $\omega \sim 0$ (as $p_0/\sqrt{q^2}$). If the limits can be exchanged, the comparison of expression (2.11) with expression (2.13) leads to

$$\lim_{\omega\to 0} \omega^{b-a} g(\omega) = \lim_{p_0\to\infty} (2p_0)^{b-a} f(p_0) . \tag{2.23}$$

2.6 Dimensional scaling

The classification of the different scaling laws made in the last section is based on the most obvious principles; however, it is not specific enough in order to characterize completely the different behaviours.

Indeed, we have seen that the same reaction shows very different features in different kinematic configurations. For example:

i) In the case of elastic scattering, the scaling laws at fixed t and at fixed angle are completely different.

ii) In the strong inclusive one-particle distributions, again, the scaling laws for large and small k_T exhibit different powers.

iii) Similarly, in the e^+e^- case, we have two different power laws depending on whether the secondary has a small or large momentum.

Now the analogy between the empirical situation in the three cases mentioned above is even closer. In (i) and (ii) events with large transverse momenta are very rare.

In the e^+e^- events [referred to in (iii)] there is no privileged direction in the c.m. frame of reference; all space directions should be considered as "transverse". From this point of view the fact that secondaries are mainly slow particles can again be interpreted by saying that also here events with large "transverse" momentum are rare.

We are now in a position to infer from the previous discussion some kind of empirical rule. There seems to be a fundamental scale which we call μ (of the order of a fraction of a GeV) in processes involving hadrons. Let us consider the set of kinematical variables which are large as compared with the fundamental scale μ. We shall define the "dimensionality" of a process as the dimensionality of the (space time) subspace spanned by the "large" kinematical variables. According to this criterion the processes discussed in the previous sections can be classified as follows:

a) Three-dimensional processes: High-energy fixed-angle elastic scattering; large k_T one-particle inclusive reactions.

b) Two-dimensional processes: Small k_T one-particle inclusive reactions; deep inelastic electron scattering; e^+e^- annihilation with production of a fast secondary.

c) One-dimensional processes: e^+e^- annihilation with production of slow secondaries.

The following empirical rule seems to emerge: In a given reaction the most probable events are those with the smallest dimensionality.

Let us see how this rule applies in the case of inclusive reactions.

i) Strong inclusive reactions

The initial particles have large momenta spanning two dimensions: energy and longitudinal momentum. Therefore the smallest dimensionality of an event is two.

According to our rule, the dominant processes should be the two-dimensional ones in which all secondaries have arbitrary energy and longitudinal momentum, but small transverse momenta k_T. If we look for one large k_T, we expect to find cross-sections with a faster decrease rate with the total energy. We also expect the events to be mainly three-dimensional, i.e. with all momenta having small components perpendicular to the plane defined by the observed k_T and the incident direction.

ii) e^+e^- annihilation

In the same way we can understand the different scaling behaviours in high-energy e^+e^- annihilation and deep inelastic scattering. Only one dimension (time-like) is excited by the virtual photon and, according to our empirical rule, the most probable events will be those in which no large space momentum will be excited. We expect, therefore, the majority of the events to have low-energy secondaries leading therefore to a scaling law such as that of Eq. (2.11). If we wish to look for the rare events with fast secondaries, we should expect mostly two-dimensional events, i.e. with mostly aligned secondaries.

The two-dimensional scaling power is probably the same as that appearing in deep inelastic lepton scattering, whose minimum (and dominant in our view) dimensionality is two.

Three-dimensional events (two large momentum components in e^+e^- and large k_T in electroproduction) should be less probable by an order of magnitude. We want to stress again that from our point of view the main features of e^+e^- annihilation (one-dimensional) have little to do with deep inelastic scattering. If an analogy should be drawn, this should be limited to the rare events where at least one energetic secondary is produced.

* * *

General surveys on experimental data discussed in this section can be found in Proc. 17th Internat. Conf. on High Energy Physics, London, 1974 (Rutherford Lab., Chilton, 1974). In particular, the rapporteurs' talks of A.N. Diddens (p. I-41), K. Zalewski (p. I-93). V. Barger (p. I-193), B. Richter (p. IV-37), and F.J. Gilman (p. IV-149).

For dimensional scaling, see D. Amati and S. Fubini, Phys. Letters 69B, 293 (1974).

3. PERFECT GAS, OLD AND NEW: THE PARTON MODEL

Power laws usually are valid when a problem is invariant with respect to some kind of dilation invariance.

We shall first illustrate this point by recalling the celebrated Boyle law for perfect gases. We shall then see that the same dimensional argument leads to the Bjorken scaling law. We shall finally discuss how other scaling laws can be obtained on the basis of the parton model, which is the modern version of the perfect gas.

We first discuss the Boyle isothermal equation of the state of a perfect gas. Since this gas contains point-like non-interacting molecules, the equation of state can depend only on two physical quantities: the mass and the kinetic energy (i.e. the velocity) of the molecules. The equation of state relating pressure with volume should be invariant with respect to a dilation leaving mass and velocity invariant. We shall thus have invariance with respect to the following transform:

$$\begin{bmatrix}\text{length} \to \alpha \text{ length,} \\ \text{time} \to \alpha \text{ time,}\end{bmatrix} \text{ which implies } \begin{bmatrix}\text{volume} \to \alpha^3 \text{ volume} \\ \text{pressure} \to 1/\alpha^3 \text{ pressure}\end{bmatrix}$$

Our equation of state will thus be invariant with respect to

$$\begin{aligned} V &\to \alpha^3 V \\ p &\to \alpha^{-3} p \end{aligned} \tag{3.1}$$

and will thus have the well-known form

$$pV = \text{const.} \tag{3.2}$$

Let us now go back to particle physics. It is easy to see that Bjorken scaling can be obtained by a very similar argument. Deep inelastic scattering is induced by electromagnetic interaction, whose coupling constant is dimensionless.

If we assume that at large values of s and q^2 all masses in the problem become negligible, then our problem becomes invariant with respect to the dilation

$$\begin{aligned} s &\to \gamma s \\ q^2 &\to \gamma q^2 . \end{aligned} \tag{3.3}$$

If we consider the cross-sections σ_T and σ_L, the dimensionless functions

$$\varphi(s, q^2) = q^2 \sigma(s, q^2) \tag{3.4}$$

should be invariant with respect to the transformation (3.3). From this it follows that $\varphi(s,q^2)$ depends only on their ratio:

$$\varphi(s, q^2) = \varphi\left(1 - \frac{s}{q^2}\right) = \varphi(\omega) \tag{3.5}$$

which is the Bjorken scaling law. This simple derivation can be visualized by introducing the elementary particle analogue of the perfect gas: the parton model.

Deep inelastic scattering is viewed as the incoherent superposition of elastic scattering (see Fig. 3) on point-like -- non-interacting -- constituents of the nucleon: the partons.

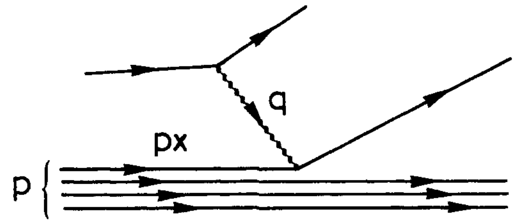

Fig. 3

The function $\phi_{L,T}(x)$ (where $x = 1/\omega$) is simply related to the parton distribution inside the hadron. If we consider the target hadron in the infinite momentum frame, the parameter x $(0 < x < 1)$ represents the fraction of the total momentum possessed by the single parton.

There are other power laws which can be obtained on the basis of the parton model. They involve more detailed properties of the partons and of their possible interactions.

To consider partons as nearly free objects inside the hadrons is, of course, a rough approximation which is perhaps reasonable for the deep inelastic lepton scattering. Indeed, the parton is ejected by the virtual photon and the subsequent interactions can be hopefully neglected.

Let us ask ourselves which other processes can be safely approximated by requiring strong interactions to appear in a minimal way. This could be the case for large k_T inclusive one-particle distributions. Indeed, we can think that two partons -- one from each incoming hadron -- undergo hard scattering which communicates to them large transverse momentum, as depicted in Fig. 4.

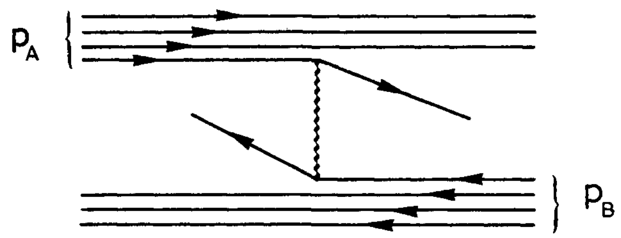

Fig. 4

The final configuration would look much like a doubling of the deep inelastic configuration. Indeed, as we shall show later, the amplitude for large k_T can be reduced to a convolution of two deep inelastic structure functions. If the single hard parton-parton strong interaction should not involve any dimensional constant, the same counting of dimensions would give a one-particle inclusive cross-section behaving as k_T^{-4}, i.e $B = 4$ in Eq. (2.2).

As discussed in Section 2, experimental data at the ISR indicate B $\sim$ 8, while NAL results would favour even larger values of B. This implies that dimensional quantities do indeed appear in strong interactions allowing, however, power scaling laws.

In the parton framework, this change of power from the canonical one is rather artificial. It could be obtained by allowing the hard strong interaction to introduce a dimensional coupling constant. Another point of view is to assign to present energies a transient scaling behaviour hoping to find finally the desired k_T^{-4} at very high energies. This temporary dominance of non-leading terms could be provided by specific forms of the structure functions.

Hard collisions in exclusive reactions could also reveal parton structure, although these depend on more specific theoretical details. Examples of this kind are asymptotic properties of form factors and fixed-angle two-body processes at high energies. These two processes (see Table 1) are related to the two inclusive processes described before, with the extra requirement that the outgoing parton should be recombined with the remaining ones to re-form the initial hadron. This recombination process involves therefore the number of partons inside the hadron and the interaction among them. The leading diagrams are those of Fig. 5, in which again the strong interactions act a minimum number of times.

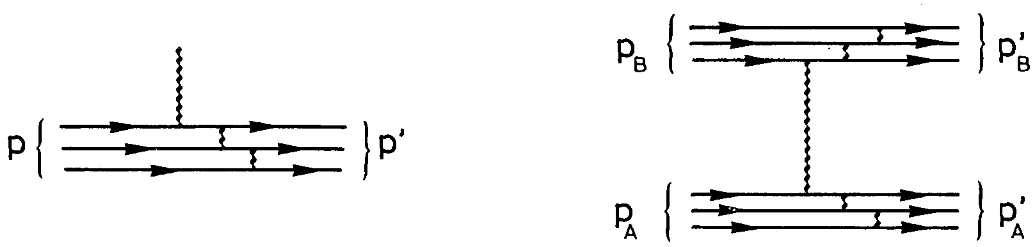

Fig. 5

If the strong interaction responsible for recombination is again assumed to be a dimensionless one (i.e. vector-gluon exchange) then it is easy to find that the power of those asymptotic behaviours depends linearly on the number of partons in the hadrons. If we take the point of view that partons are simply the quarks of the naïve quark model (qqq for baryons, $q\bar{q}$ for mesons) then we obtain the fixed-angle behaviour of Eq. (2.6), with the exponents of Eqs. (2.7) which are indeed those suggested by experiment. In addition we obtain $(q^2)^{-\delta}$ power laws for the form factors where δ = 1 for mesons and δ = 2 for baryons.

Looking at the previous discussion we see that we have been led to introduce more and more specific assumptions about partons and about their role in the hadron structure. In particular:

a) Scaling in deep inelastic lepton scattering is based on the electromagnetic interaction of a point parton.

b) Scaling in large k_T inclusive distribution involves also the form of strong interaction between partons.

c) Form factors and large-angle elastic scattering depend strongly on how many "active" quarks are contained in the hadron.

If we want to proceed further, we have to make use of even more detailed knowledge of the hadron structure and the parton model loses much of its beauty and simplicity.

Indeed the parton model in its most elementary form does not lead naturally to the strong limitation in transverse momenta. This requires the introduction of much more sophisticated versions, inspired, for example, by the multiperipheral model. At this point we feel it is appropriate to deal directly with the original multiperipheral model which is still considered at present as one of the starting points for understanding scaling in hadron physics.

* * *

For reviews on the parton model, see for instance
R.R. Feynman, Photon-hadron interactions (Benjamin, New York, 1972),
J. Kogut and L. Susskind, Phys. Rep. 8C, 75 (1973), and
P. Landshoff, Proc. 17th Internat. Conf. on High Energy Physics, London, 1974 (Rutherford Lab., Chilton, 1974), p. V-57.

For recent applications to large k_T, see
J.F. Gunion, Proc. 17th Internat. Conf. on High Energy Physics, London, 1974 (Rutherford Lab., Chilton, 1974), p. I-125, and
S.J. Brodsky, Phys. Rep. (to be published).

4. MULTIPERIPHERAL MODEL

The main purpose of this section is to discuss the physical phenomena which are directly connected with the strong limitation of transverse momenta in hadronic processes. One of the models leading naturally to such limitation is the multiperipheral model. This is due to the typical form of the multiperipheral graphs (see Fig. 6) in which the basic input is a low-energy phenomenon

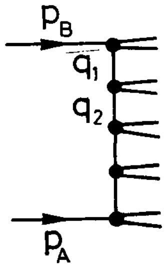

Fig. 6

It is easily seen that the requirement that the propagators $P(q^2)$ of the exchanged virtual particles should be large requires all transverse momenta of secondaries to be small. In other words, the k_T of the secondaries should mainly lie within the fundamental scale which enters the model through the propagator $P(q^2)$.

If we concentrate our attention on inclusive processes, we shall see that there are two groups of phenomena, both explained in terms of multiperipheral dynamics, which show completely different features:

i) Two-dimensional hadronic phenomena, like power behaviour of cross-sections and low k_T scaling, which depend only on the structure of the multiperipheral chain and on the requirement that the propagator $P(q^2)$ cuts off sufficiently strongly large values of k_T.

ii) "Parton properties", like Bjorken scaling and large k_T phenomena, which depend on "how" the propagator cuts off large k_T events. The "parton" power laws are obtained when the propagator, as suggested by field theory, behaves like a power for large values of q^2.

It is well known that the whole multiperipheral dynamics is contained in a physical amplitude, which obeys a fundamental integral equation that follows from a summation of all multiperipheral diagrams.

4.1 The multiperipheral amplitude

The fundamental property of the multiperipheral model is the uniformity of the multiperipheral chain. This allows one to express all properties of inclusive reactions in terms of the fundamental (forward) amplitude $A(p,q)$ which, as shown in Fig. 7, gives the sum of all multiperipheral contributions to the (suitably normalized) total cross-section $A(p,q)$ for off-mass shell particles p and q.

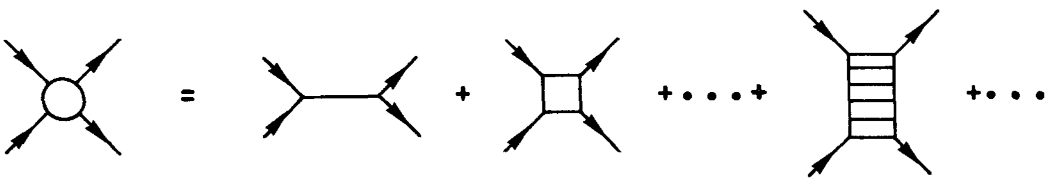

Fig. 7

For simplicity we shall work in the framework of a $g\phi^3$ model. All particles will be spinless, so that our amplitude will depend only on invariant quantities:

$$A(p,q) = A(p^2, q^2, s) \qquad s = (p+q)^2 .$$

The different inclusive quantities can be expressed in terms of the amplitude A.

i) Total cross-section

One simply computes $A(p^2,q^2,s)$ on the mass shell, i.e. for $p^2 = q^2 = m^2$.

ii) Deep inelastic structure functions

As shown in the graph in Fig. 8, the structure functions $T_{\mu\nu}$ can be simply obtained by the convolution

$$T_{\mu\nu}(p,q) = \int A(p,q')\, V_\mu\, V_\nu\, \delta[(q-q')^2-\mu^2]\, d^4q' \tag{4.1}$$

where V_μ is the vertex operator coupling the photon to the particle on the multiperipheral chain.

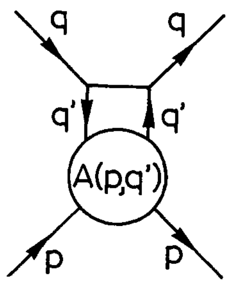

Fig. 8

It will be seen that under appropriate asymptotic conditions $T_{\mu\nu}$ obeys the Bjorken limit. If one compares the multiperipheral picture of deep inelastic scattering with the parton picture, one can state that the multiperipheral amplitude "describes" the parton distribution inside the hadron.

iii) One-particle inclusive amplitude

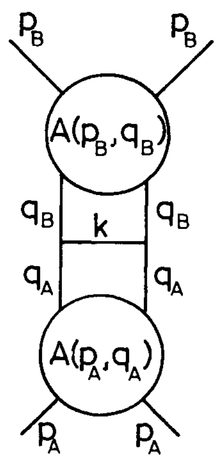

Fig. 9

As shown in Fig. 9, the one-particle inclusive amplitude $B(p_A,k,p_B)$ is given by the following quadratic expression in A:

$$B(p_A, k, p_B) = \iint A(p_B, q_B)\, A(p_A, q_A)\, P^2(q_B^2)\, P^2(q_A^2) \times \delta^4(q_A - q_B - k)\, d^4q_B\, d^4q_A \,. \tag{4.2}$$

It would of course be easy to generalize Eq. (4.2) and obtain the expression for the n-particle inclusive amplitude as an integral containing n + 1 times the amplitude A.

We are now ready to write the fundamental integral equation for the multiperipheral amplitude which can be easily obtained by summing the contributions of all multiperipheral processes

$$A(p, q) = A_0(p, q) + \int P^2(q'^2)\, A(p, q')\, \delta[(q'-q)^2 - \mu^2]\, d^4q' \tag{4.3}$$

where A_0 is the low-energy amplitude (see Fig. 10).

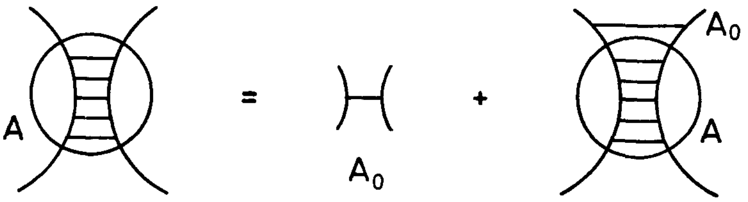

Fig. 10

Equation (4.3) is closely analogous to a Bethe-Salpeter equation; it can be viewed as the dynamical equation for the "parton wave function" inside hadrons.

4.2 <u>Small k_T inclusive processes</u>

We wish now to study the amplitude $A(s,q^2)$ for large s and small q^2. In this situation, if the propagator $P^2(q'^2)$ cuts large values of q'^2, we can write Eq. (4.3) in a greatly simplified form. One can neglect the inhomogeneous term A_0 and perform a relativistic approximation in the kinematics of Eq. (4.3). Expressed in the invariant variables

$$s = (p + q)^2, \quad u = -q^2$$

the multiperipheral equation takes the form

$$A(s, u) = \int A(s', u')\, K\left(\frac{s'}{s}, u, u'\right) \frac{ds'}{s'} \,. \tag{4.4}$$

The kernel $K(s'/s,u,u')$ <u>depends only on the ratios s'/s</u>. This fundamental property is the origin of all small k_T scaling properties.

Equation (4.4) is invariant for

$$s \to \gamma s \qquad\qquad s' \to \gamma s' \,. \tag{4.5}$$

This implies that A(s,u) has the form

$$A(s,u) = s^{\alpha}\,\phi(u) \tag{4.6}$$

where α and B(u) are eigenvalues and eigenfunctions of the integral equation

$$\phi(u) = \int \phi(u')\; H_{\alpha}(u,u')\, du' \tag{4.7}$$

$$H_{\alpha}(u,u') = \int_0^1 x^{\alpha}\, K(x,u,u')\, dx \;. \tag{4.8}$$

The form (4.6) leads immediately to a power behaviour of the total cross-section. The exponent α is a model-dependent parameter; we have no simple explanation for the experimental value $\alpha = 1$ of the Pomeron intercept.

It is now easy to see how the multiperipheral scaling law works in one-particle inclusive distributions. Let us refer to the amplitude $B(p_A,k,p_B)$ defined in Eq. (4.2) and depicted in Fig. 9. We introduce the kinematical variables

$$(p_A + k)^2 = s_A \qquad\qquad (p_A + p_B)^2 = s$$

$$(p_B + k)^2 = s_B \qquad\qquad k^2 = \mu^2 \;.$$

The transverse component k_T is related to the other kinematical variables by the relation

$$\begin{vmatrix} p_B^2 & \dfrac{s_B - p_B^2 - \mu^2}{2} & \dfrac{s - p_A^2 - p_B^2}{2} \\ \dfrac{s_B - p_B^2 - \mu^2}{2} & \mu^2 + k_T^2 & \dfrac{s_A - p_A^2 - \mu^2}{2} \\ \dfrac{s - p_A^2 - p_B^2}{2} & \dfrac{s_A - p_A^2 - \mu^2}{2} & p_A^2 \end{vmatrix} = 0 \;. \tag{4.9}$$

It is convenient to consider as independent variables the transverse momentum k_T^2, and the partial energies s_A and s_B; the total energy s will then be given by Eq. (4.9). When s_A is large Eq. (4.9) reduces to

$$s_B = p_B^2\,\frac{s_A}{s} + (\mu^2 + k_T^2)\,\frac{s}{s_B} \;. \tag{4.10}$$

When both s_A and s_B are large we simply have

$$\mu^2 + k_T^2 = \frac{s_A s_B}{s} \;. \tag{4.11}$$

It is now easy to recognize the scaling properties of the amplitude $B(s_A,s_B,k_T^2)$. This can be done by introducing the asymptotic form (4.6) into the expression (4.2). More directly, by adding one more rung either on the top or at the bottom of the diagram in Fig. 9, one can recognize that $B(s_A,s_B,k_T^2)$ obeys two multiperipheral equations, one in the variable

p_A, the other in the variable p_B, so that simple scaling properties in the variables s_A and s_B can be immediately inferred.

So we arrive at the following conclusions:

$$B(s_A, s_B, k_T^2) \to s_A^{\alpha}\, \beta_1(s_B, k_T^2) \quad \text{when } s_A \to \infty$$
$$B(s_A, s_B, k_T^2) \to s_B^{\alpha}\, \beta_2(s_A, k_T^2) \quad \text{when } s_B \to \infty \tag{4.12}$$

$$B(s_A, s_B, k_T^2) \to (s_A s_B)^{\alpha}\, b(k_T^2) ; \text{ie } \beta_1(s_B, k_T^2) \to s_B^{\alpha}\, b(k_T^2)$$
$$\text{when both } \begin{cases} s_A \to \infty \\ s_B \to \infty . \end{cases} \tag{4.13}$$

Equations (4.12) and (4.13) can be expressed in a more fashionable form if one introduces the Feynman variable

$$x_0 = \frac{s_A}{s} \tag{4.14}$$

and uses s, x_0, and k_T^2 as independent variables. Using Eq. (4.14) and rewriting Eq. (4.10) in the form

$$s_B = p_B^2\, x_0 + (\mu^2 + k_T^2)\frac{1}{x_0} \tag{4.15}$$

the first of Eqs. (4.12) reads

$$\begin{aligned} B(s, x_0, k_T^2) &\to s^{\alpha}\left\{x_0^{\alpha}\, \beta\left[s_B(x_0), k_T^2\right]\right\} = \\ &= s^{\alpha}\, F(x_0, k_T^2) . \end{aligned} \tag{4.16}$$

From Eq. (4.15) we see that the asymptotic condition $s_A \to \infty$, $s_B \to \infty$ is reached for $s \to \infty$, $x_0 \to 0$. Using Eq. (4.11) we can rewrite Eq. (4.13) in the form

$$\lim_{x_0 \to 0} F(x_0, k_T^2) = (\mu^2 + k_T^2)^{\alpha}\, b(k_T^2) . \tag{4.17}$$

Equation (4.17) states that there should be a constant plateau in the pionization region and it implies that the multiplicity increases with log s.

4.3 "Parton properties"

We wish now to derive in the framework of the multiperipheral model some typical parton results, like Bjorken scaling and large k_T power laws. We shall use again the multiperipheral kinematics discussed in Section 4.1 and investigate asymptotic limits which are completely different from the ones used in small k_T events. In this situation, the form of A(s,u) for large values both of s and u will play a fundamental role. This will imply that our results will be sensitive to the form of the propagator P(u) or, in other words, the specific nature of particles exchanged and produced in multiperipheral diagrams.

In what follows, we shall consider all particles to be scalar or, analogously, we shall work in a $g\phi^3$ theory. The introduction of spin is not a problem as regards kinematics, but is a delicate matter dynamically. Indeed, if one considers Lagrangian theories with elementary spinning particles, the presence of ultraviolet divergences is reflected -- in the multiperipheral model -- by a lack of damping of large virtual masses q^2 in the multiperipheral equation. In other words, $P(q^2)$ ceases to be a strong damping function and, as a consequence, Reggeization is lost. This is not the case if spinning particles are not elementary. This led to the multi-Regge generalization of the multiperipheral model and to the dual model. We shall come back to this point; for the time being let us remain with the $g\phi^3$ theory which has the advantage of great simplicity.

In order to discuss the deep inelastic scattering, let us first discuss the already-mentioned interchangeability of Regge and Bjorken limits in the multiperipheral model.

From an analysis of the kernel of the integral Eq. (4.7) for $\phi_\alpha(u)$ one can find

$$\phi(u) \rightarrow \frac{c}{u^{1+\alpha}} \quad \text{for} \quad u \rightarrow \infty . \tag{4.18}$$

In so doing we have, of course, implied $s \gg u \gg \mu^2$, i.e.

$$\omega = \frac{s}{u} + 1 \rightarrow \infty .$$

Let us now discuss the behaviour of $A(s,u)$ at large values of u. From a purely kinematical calculation one sees that even if $u = -q^2$ is large in the multiperipheral amplitude represented by Fig. 8, $u' = -q'^2$ can still be small if $s' < u'/(\omega - 1)$ *). This is reflected by the integral Eq. (4.4) by

$$A(s,u) \xrightarrow[\substack{s\rightarrow\infty \\ \omega\ \text{fixed}}]{} \frac{1}{u}\,\psi(\omega) \tag{4.19}$$

where the structure function $\psi(\omega)$ is given by

$$\psi(\omega) = \frac{1}{\omega-1}\int du' \int^{u'(\omega-1)} ds' \;\; P^2(u')\, A(u's') . \tag{4.20}$$

For large ω

$$\psi(\omega) \rightarrow c\,\omega^{\alpha} \qquad \text{for} \quad \omega \rightarrow \infty \tag{4.21}$$

where c is indeed the same constant appearing in Eq. (4.18).

It is a simple matter to realize that the scaling (canonical) behaviour (4.19) is intimately related to the scaling of deep inelastic lepton scattering if the particles in the ladder have the elementary character we have supposed. The current would couple to them without form factors. Indeed, the process is given by the diagram in Fig. 11, in which everything -- but the first link -- is the same as before. If the current couples without form factor, the deep inelastic cross-section is obtained from $A(s',u')$ in the same way (apart from spin details) as $A(s,u)$ is obtained from it through Eq. (4.4). Therefore a scaling behaviour analogous to (4.19) to (4.21) also follows.

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*) In modern words, this implies a large rapidity gap between the first and the second particle of the multiperipheral chain.

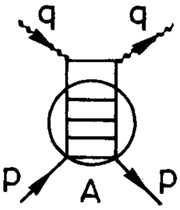

Fig. 11

Let us now discuss the large k_T behaviour. If k_T is much larger than the scale, the distribution function $F(x_0,k_T)$ has a simple inverse power behaviour in k_T. This power is controlled by the propagators $P(q^2)$. We can define as two independent scaling parameters $x_A = s_A/s$, $x_B = s_B/s$ or

$$x_T = 2\sqrt{x_A\, x_B} \sim \frac{2k_T}{\sqrt{s}} \quad ; \quad \eta = \sqrt{\frac{x_A}{x_B}} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

θ being the c.m. angle of $\vec{k}$ with the incident direction. When k_T is large in the form (4.2) for the inclusive spectrum, the phase space forces one of the links, on either side of the observed particle in the ladder, to have a large transverse momentum and, therefore, a large q^2. Besides getting a power drop due to one of the $P(q^2)$ in Eq. (4.2), we are again sensitive to the behaviour of one of the $A(s,q^2)$ for large values of q^2.

For k_T^2 much larger than μ^2, the kernel of Eq. (4.2) becomes very simple and we finally obtain

$$\frac{d\sigma}{d^3k/k_0} \simeq s^{-4}\, \mathcal{G}(x_T, \eta). \qquad (4.22)$$

The structure function G is given by a simple integral bilinear in the function χ defined by

$$\chi(\omega) = \frac{\omega^{-\alpha}}{c}\, \psi(\omega) \qquad (4.23)$$

$\psi(\omega)$ being given by Eqs. (4.20) and (4.21). The calculation described before, which computes the diagram of Fig. 9, does not consider the contribution to the inclusive spectrum of the first or last particle of the multiperipheral chain. Their inclusion gives rise to an additional contribution to G which is now linear in χ.

As discussed before, the function $\chi(\omega)$ is related to the structure function measured at SLAC and therefore we could be ready to compare our results with large k_T data. This could be done if we assimilate the elementary particles of the multiperipheral model to real hadrons. We shall come back to this very fundamental point showing why this can be safely done in some cases (strong scaling), less safely in others (large k_T and deep inelastic lepton scattering), and not at all in some other processes (e^+e^- annihilation).

We expect, however, the transition from elementary particle to real hadron (quark recombination) to affect large k_T and deep inelastic lepton scattering in a similar way. They are related processes in the multiperipheral or parton framework. Indeed, whatever the quark recombination mechanism should be, if it does not ruin scaling it will give rise to an inclusive distribution in deep inelastic lepton scattering $\gamma(q^2) + p \to k + X$ of the form $f(\omega,R)$ where $R = k\cdot p/q\cdot p$. The large k_T structure function (4.22) is then given by

$$G(x_T,\eta) = \left(\frac{x_T}{2}\right)^{-8} \int_0^1 dR\, R^{-8} \int_{\frac{x_T\eta}{2R}}^{1-\frac{x_T}{2\eta R}} dy\, y(1-y)[y^2+(1-y)^2]\, f\!\left(\frac{2yR}{x_T}, R\right) \chi\!\left(\frac{2(1-y)R\eta}{x_T}\right). \quad (4.24)$$

Unfortunately, we do not know $f(\omega,R)$ from deep inelastic scattering. Reasonable forms for that function give rise to good fits with large k_T data. Let us recognize, by the way, that the same expression (the multiperipheral one) was used for large and small k_T. As discussed in Section 2, the different scaling for these two situations is reflected by the divergence of the structure function $G(x_T,\eta)$ for $x_T \sim 0$.

4.4 Exclusive amplitude

The imaginary part of the elastic scattering amplitude is seen in the framework of the multiperipheral model as the shadow of multiple production and it is illustrated in Fig. 12.

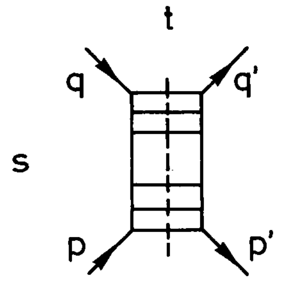

Fig. 12

In the "non-forward multiperipheral amplitude" of Fig. 12 there is a mismatch between the initial and the final chain. It is clear that as soon as the momentum transfer t increases, the virtual particle momenta are forced further and further away from the mass shell. This gives rise to a strong diffraction peak at $t = 0$. The suppression of large t becomes, of course, stronger when the number of virtual propagators (i.e. the multiplicity) increases. Since multiplicity increases with the logarithm of energy, we expect a logarithmic shrinkage of the diffraction peak.

The study of the amplitude in Fig. 12 involves a simple generalization of the forward equation discussed in the previous section. The result is the famous asymptotic Regge formula

$$A(s,t) \simeq \beta(t)\, s^{\alpha(t)} \quad (4.25)$$

where again the Regge trajectory $\alpha(t)$ is the eigenvalue of an appropriate non-forward integral equation.

It is important to notice that, whereas the general asymptotic formula (4.25) is independent of the detailed form of the propagator $P(q^2)$ (provided of course that large q^2 are sufficiently cut off!), the form of the trajectory $\alpha(t)$ is indeed strongly model dependent.

In particular, in the $g\phi^3$ theory (i.e. spinless elementary particles)

$$\lim_{t \to -\infty} \alpha(t) = -1 . \tag{4.26}$$

This result is not surprising; it just means that for both large s and t all higher multiperipheral contributions are suppressed and the limit $s \to \infty$, $t \to -\infty$ is simply obtained in the Born approximation. We see that the value of $\alpha(t)$, when $t \to -\infty$ is connected to the point where Regge asymptotism and fixed-angle asymptotism overlap. It can indeed be seen directly that the multiperipheral model leads to a power law for fixed-angle scattering and that there is a smooth transition between fixed t and fixed $\cos\theta$ asymptopia.

Another power law, which appears naturally in the multiperipheral model, is that of the electromagnetic or weak form factors of hadrons lying on Regge trajectories. These hadrons are generated by ladders and the form factors are given by the diagram of Fig. 13.

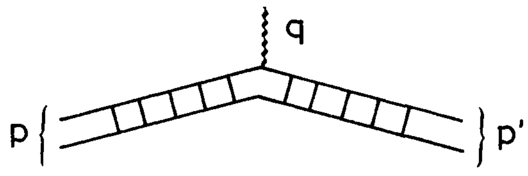

Fig. 13

The behaviour of the form factor for large q^2 is again determined by the asymptotic behaviour of $P(q^2)$. In the $g\phi^3$ theory, the form factor of a spin J particle behaves asymptotically as $1/(q^2)^{2J+2}$.

* * *

For a review on the multiperipheral model results, see for instance
W. Frazer, L. Ingber, C.H. Mehta, C.H. Poon, D. Silverman, K. Stowe, P.D. Ting and H.J. Yesian, Rev. Mod. Phys. 44, 284 (1972), and
M.L. Goldberger, Proc. Internat. School of Physics "Enrico Fermi", Course LIV: Developments in High-Energy Physics, Varenna, 1971 (Academic Press, New York and London, 1972), p. 1.

For parton multiperipheral properties, see
D. Amati, L. Caneschi and M. Testa, Phys. Letters 43B, 186 (1973), and
D. Amati and L. Caneschi, Phys. Letters 50B, 373 (1974).

5. ELEMENTARY AND COMPOSITE PARTICLES

The multiperipheral model, as every model that Reggeizes, gives rise to hadrons which are composite objects. However, elementary constituents have been used -- both as exchanged and produced particles. We know, however, that all observed hadrons lie on Regge trajectories and are not therefore elementary objects. It is therefore legitimate to ask how the various results are modified when the exchanged and produced particles are also composite (Reggeons).

This is an empty statement if we do not provide a dynamical model for the Reggeon, due to the fact that we do not need only a Reggeon propagator but also a Reggeon wave function that tells us how this entity behaves in terms of the q^2 of a particle coupled to it. In order to be more specific, let us take again a ladder for a Reggeon and ask ourselves what happens if in the preceding discussion all the intermediate particles in the multiperipheral model are replaced by ladders.

Nothing essential happens to Reggeization and strong scaling. The asymptotic value of $\alpha(t)$ for $t \to -\infty$ decreases. The exclusive processes, such as fixed-angle scattering and form factors, get a more rapid power decrease. The increase in the power is proportional to the degree of compositeness. We can easily understand, in this sense, the quark counting rate for fixed-angle scattering discussed in Section 3. In the limit of infinite compositeness, we would obtain exponential behaviours for form factors and fixed-angle scattering.

The inclusive processes such as deep inelastic scattering and large k_T do not change at all <u>if</u> the elementary constituent (quark) is ejected, while they scale with a higher power if what is ejected is a bound state. This fact, well-known for deep inelastic scattering and which takes place in a similar way in large k_T phenomena, is one of the puzzles of particle theory. Particles are able to absorb large q^2 (or $k_T{}^2$), due to the presence of hard structures. But <u>these hard structures should come out, and therefore be observed</u>. Simple dynamical models for quark re-interaction are unable to avoid the quarks (or states with their quantum numbers) leaving without spoiling canonical scaling, i.e. changing the scaling power. We are lacking a clear dynamical mechanism for quark imprisonment, which would explain to us why scaling is not affected and which other processes would be instead.

We have of course, in our theoretical arsenal, approaches in which all hadrons are Reggeons (i.e. non-elementary). The most consistent example in that direction is the dual model. This model easily gives rise to Reggeization and strong scaling. However, large k_T spectra and fixed-angle scattering drop exponentially. If amplitudes containing weak or electromagnetic currents are introduced in the model in a way suggested by factorization of purely hadronic amplitudes, then one also finds lack of deep inelastic scaling and/or exponentially falling form factors. These results are not surprising for a model which gives a typical soft description of hadrons. The lack of hard constituents (or, equivalently, the infinite number of them) of the dual approach*) can be better visualized by

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*) Dual theorists have good arguments indicating that hard properties will be obtained should dual loops be incorporated (an infinite number of them). This shows the complementarity between the elementary particle approach (Feynman diagrams) and dual theories. A Born approximation in the dual model Reggeizes and has strong scaling. In order to obtain this, we must sum over an infinite number of Feynman diagrams. Properties which instead appear at the Born level of Feynman diagrams (such as the point-like scaling described above) imply the sum over an infinite number of dual diagrams.

the string or fishnet approaches. In the following, we shall call point-like properties the scaling behaviours in deep inelastic, large k_T, fixed-angle scattering and form factors in the sense that in our way of interpreting hadron phenomena, they are revelatory of the presence of hard structures.

Let us now ask which form a dynamical model should have in order to give rise also to a one-dimensional scaling. For two-, three- (and four-) dimensional scaling we needed the presence of point-like structures, quarks, and the dilemma remained as to how the quark imprisonment mechanism should work in order not to change the aforementioned scaling properties. But let us now ask: whatever that mechanism could be, can it still not be essential in a dynamics with one-dimensional scaling? The answer is obviously no. Indeed, the current, which in e^+e^- annihilation carries a lot of energy and no momentum, would give rise to a quark pair having a large momentum in opposite directions. If the quark should propagate, the event will contain a preferential direction and be, therefore, typically two-dimensional. In order to find a dynamics which gives one-dimensional behaviour, it is therefore fundamental to take into account dynamically the quark imprisonment. Another better known example can be given. One-dimensional dynamics will be at work in the calculation of large masses (again only energy in the rest system) or -- in other words -- of asymptotic properties of Regge trajectories. We are thus led to the well-known relation between the linear rising spectrum and the quark imprisonment scheme (or quark algebra without quarks!).

Summarizing the preceding discussion, we see that if we seek for a dynamical model exhibiting one-dimensional scaling, this model should have the following necessary conditions:

i) It should have point-like characteristics. This implies power behaved scalings as discussed before.

ii) All physical hadrons should be Regge-like, i.e. composite. This implies imprisonment of point-like constituents.

Unfortunately, we do not have any theoretical model endowed with both those properties. The multiperipheral or the parton approach have (i) but not (ii), while the dual model has (ii) but not (i).

A first attempt to construct a model that, even if it shows some unsatisfactory aspects, contains both properties has recently been made*). It is a dual model for currents. The starting point is a specific form for a scattering amplitude of an arbitrary number of virtual photons and hadrons. This amplitude has nothing else but poles in all channels including the photon's channels. All poles lie on linear Regge trajectories and in this sense the model -- as every dual model -- meets the second property discussed before. The first one is instead revealed by the presence of fixed poles which are the dual counterparts of (non-existing) elementary structures. The fixed poles are indeed there in order to avoid poles in undesired variables, which -- if there -- could be associated with channels containing quarks.

Having a specific form for current amplitudes, it is straightforward to compute the absorptive part of the forward absorptive Compton amplitude which, as discussed before,

*) D. Amati, S. Ellis and J. Weis, Nuclear Phys. B84, 141 (1975).

describes the cross-sections for processes "γ" + p $\to$ everything for $1 < \omega < \infty$ and "γ" $\to$ p + everything for $0 < \omega < 1$. The absorptive amplitude in question turns out to be given by $(q^2)^{-d/2}$ times a structure function $\varphi(\omega)$ behaving as

$$\varphi(\omega) \sim \begin{cases} \omega^{\alpha(0)} & \text{for} \quad \omega \sim \infty \\ |1-\omega|^{2\delta - d/2} & \text{for} \quad \omega \sim 1 \\ \omega^{-\Delta} & \text{for} \quad \omega \sim 0 \end{cases}$$

where δ is given by the asymptotic behaviour of form factors $[F(q^2) = (q^2)^{-\delta}$ for scalar particles$]$ and $\Delta = 2\delta + \alpha(0) - 1$. The scaling power d is a free parameter of the model; it can therefore be set -- rather arbitrarily -- to its canonical value. The behaviours of $\varphi(\omega)$ for $\omega \sim 1$ and $\omega \sim \infty$ are the ones expected on the basis of Eqs. (2.21) and (2.22). The divergence of $\varphi(\omega)$ for $\omega \sim 0$ gives the transition to a one-dimensional configuration. Let us remark that in that model the <u>same</u> parameter Δ controls the two- to three-dimensional transitions. This means, for instance, that the smallness of large k_T events in deep inelastic scattering (or e^+e^- two-particle inclusive annihilation) is correlated with the increase of R ($R = \sigma_{e^+e^-\to\text{hadrons}}/\sigma_{e^+e^-\to\mu^+\mu^-}$) and the abundance of low-energy secondaries in e^+e^- annihilation. The model discussed above is, of course, a very crude one. It is, however, hard to find in the accepted folklore a more reasonable one.

All our language, models and approaches have been triggered by small k_T: it is our point of view that limitation of secondary's momenta in e^+e^- annihilation can be of comparable weight towards the understanding of hadron dynamics and that this cannot be uncorrelated with the understanding of the quark imprisonment.

6. BEYOND SCALING

In the Introduction we have pointed out that hadron phenomena do not need -- for the time being -- any basically new high-energy scale. We were able to recognize in different phenomena the single basic hadronic scale (of the order of half a GeV). Asymptotic behaviour of a power type was recognizable when a variable became much larger than this. A typical example was the Regge behaviour of two-particle amplitudes at large energies. The dominant Regge pole -- the Pomeron -- should have intercept one $[\alpha_P(0) = 1]$ in order to explain the constancy of total cross-sections.

We know, however, that total cross-sections show a slow increase (of a logarithmic type) in the 100-1000 GeV region. This could be attributed to the presence of a new high-energy scale (of the order of 100 GeV), which governs the beginning of a new non-scaling asymptotic behaviour. In this section we want to argue that in the light of our present understanding -- described in the preceding sections -- the 100 GeV scale is not a new entity, but is dynamically generated by our low-energy parameters. The logarithmic behaviour appears as an infrared phenomenon and can be interpreted as the first step towards a super asymptotic behaviour which, at least theoretically, can be understood as a critical phenomenon.

Let us go back to the multiperipheral model and let us ask if it could give rise to a consistent picture for all high energies. The answer could be yes if the resulting Regge trajectories, i.e. the powers of s in the overlap function generated by multiperipheral events (Fig. 6) turn out to be smaller than one. This would imply vanishing asymptotic cross-sections.

There are two reasons for that. First, there is no difficulty in considering the exchanged particles in the multiperipheral chain to be the output Regge poles as long as $\alpha(0) < 1$. Secondly, if cross-sections do not vanish for increasing energies, there is no justification to have neglected the direct re-interaction of final particles which are not neighbouring in the multiperipheral chain, i.e. with large rapidity gaps. This would ruin the multiperipheral model results, which are based on short-range correlations.

Now, in the 5-100 GeV region, cross-sections seem to be quite flat and satisfy reasonably well the Pomeranchuk theorem. More precisely, the Regge analysis of that "high-energy" region indicates a Pomeron (call it $\mathbb{P}$) of intercept one and non-dominant Reggeons with intercept near $\frac{1}{2}$.

The fact that, notwithstanding $\alpha_{\mathbb{P}}(0) = 1$, data seem to agree with multiperipheral model results in the 5-100 GeV region, suggests that $\mathbb{P}$ is perhaps not so strongly coupled. The cuts generated by its re-interaction could then be harmless. This suggestion is supported by:

a) the anomalously small ratio of elastic to inelastic cross-sections (of the order of 1/5 for pp scattering, as compared to 1 for a black disk);

b) the success of weak absorptive corrections to Regge analysis of two-body and quasi-two-body scattering;

c) the small triple Pomeron coupling g as measured from diffractive production of large masses.

Multipomeron corrections, which seem to be small in the 5-100 GeV region, cannot, however, remain small at too high energy. Indeed, inelastic amplitudes in which $\mathbb{P}$ is exchanged (such as those of Fig. 14), even if small, due to the smallness of g, will finally

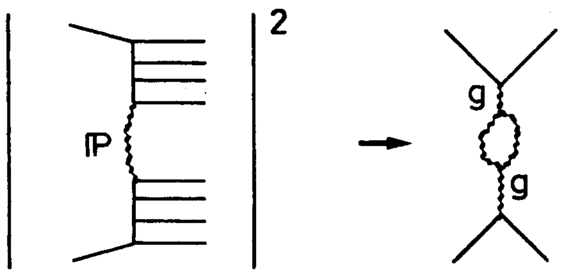

Fig. 14

dominate the single $\mathbb{P}$ exchange, due to the fact that they increase logarithmically with s. Responsible for this fact is the circumstance that for $\alpha(0) = 1$, Regge poles and cuts coming from pole iterations, coincide. Indeed, the quantity $\eta = 1 - \alpha(0)$ acts as a mass*) in the sense that an exchange of n poles gives rise to a cut with $\eta_n = n_\eta$. For $\eta = 0$ we have a zero mass problem in the sense that the pole and all branch points coincide. This is the origin of the infrared phenomenon mentioned before.

It is clear, therefore, that for sufficiently high energies (which imply large distances in the language in which η is a mass) we cannot avoid the summing over the coinciding thresholds. This nuisance of a zero mass situation is, however, partly compensated by the generation of a calculable long range phenomenon. The renormalization group equations are the technical expression of this loss of scale and make it possible to obtain the behaviour of the amplitude for $1 - J \sim 0$ (low "energies") and therefore*) for $\log s \sim \infty$.

The solution of the renormalization group equations implies the calculation of critical exponents. It uses the techniques introduced by K. Wilson for critical phenomena, in particular the ε expansion. Indeed, the Pomeron iterations look similar to the perturbative expansion in a non-relativistic field theory in one-time and two-space dimensions. The triple Pomeron coupling would be dimensionless if the number of space dimensions were four. The ε expansion allows one to compute the critical exponents as a series in $\varepsilon = 4 - d$ (d should be set equal to 2 at the end). The leading behaviour of the elastic amplitude obtained that way is

$$A(s,t) \sim s\,(\log s)^{\gamma}\,F[t(\log s)^{z}] \tag{6.1}$$

with $\gamma \sim 1/6$, $z \sim 13/12$. The average multiplicity of secondaries grows as $\langle n \rangle = (\log s)^{7/6}$.

*) In the language in which η is a mass, $1 - J$ appears as the energy and log s is the conjugate variable in the Mellin transform of the amplitude. This means that the high log s behaviour is given by the low-"energy" one $(J - 1 \sim 0)$ which is crucially dependent on the zero mass $[1 - \alpha(0) = 0]$ phenomenon.

The results we have discussed are expected to hold for infinite energies ($\log s \to \infty$). It is therefore legitimate to ask whether they are relevant for our present -- and future -- laboratory energies. In order to answer this question, it is necessary to analyse how the critical infrared phenomenon is approached. In so doing one finds a new energy scale s_2 so that if $s > s_2$ the expression (6.1) is the leading term of a convergent development.

In order to estimate s_2, let us first define a reference energy s_1 at which the Pomeron parameters -- slope and triple-Pomeron coupling g -- are defined. Then s_2 is roughly given

$$\log s_2 \simeq g_*^2/g^2 \cdot \log s_1 \tag{6.2}$$

where g_* is the triple-Pomeron coupling at infinite energies, i.e. the critical value of g. A very rough estimate shows that an order of magnitude for s_2 of $\sim 10^4$-10^6 GeV^2 is wise.

As said before, for $s > s_2$ we expect the expression (6.1) to be the leading term in an expansion in $(\log s/\log s_2)^{-n}$. For $s < s_2$ we expect instead to find a convergent expansion in terms of a renormalized Pomeron with unit intercept and its associate cuts (Gribov calculus). This leads to an expansion in $(\log s/\log s_2)^n$.

The fact that s_2 is estimated to be so much larger than laboratory energies shows that the last mentioned expansion should be rapidly convergent. Or, in other words, at machine energies we expect the onset of the first few corrections to the Pomeron pole. These should be responsible for the slight growth observed in total cross-sections.

Results such as that of Eq. (6.1), even if asymptotically valid, seem to be beyond our present experimental reach.

Let us notice the similarities between the dynamical generations of the different energy scales.

In the approach we pictured, the basic scale is seen for instance in $\langle k_T^2 \rangle$ and the Regge slope. How quickly the Regge régime is reached depends also on the coupling in the multiperipheral chain, related to the factor of log s in the average multiplicity. This is what leads to the scale s_0 ($\sim$ 1 GeV^2) in the $(s/s_0)^\alpha$ expansion.

To recognize clearly the Pomeron we need, however, energies larger than 1 GeV. We know that a rapidity gap ($\log s/s_0$) of at least 2 is needed. This implies $s \gtrsim s_1 \sim 7$ GeV^2. This value is determined by the ratios of the Pomeron couplings as compared to couplings of the non-leading Reggeons.

Energies s_1 are those in which the Pomeron can be well identified and its parameters determined. Those parameters are then the ones that determine the onsetting of the infrared phenomena and therefore to the new scale s_2 of Eq. (6.2).

We see how this dynamical succession of scales -- in which no *ad hoc* new scale is introduced -- is not only natural but also necessary for the consistency of the theoretical picture. Indeed if power laws are not just invented but interpreted in terms of a dynamical mechanism (exchanges for instance), dynamics itself will suggest how this mechanism will be iterated or repeated in terms of parameters which are understandable and measurable. This will show how a simple picture can cease to be appropriate and how a new one becomes more suitable. The region in which this happens represents of course a dynamically obtained scale.

* * *

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