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ASPECTS OF THE NEUTRON NOISE ANALYSIS IN VIEW OF DETERMINING THE KINETIC PARAMETERS OF THE NUCLEAR REACTORS WITH THERMAL NEUTRONS AT IFA - BUCHAREST

I. METHOD OF THE REDUCED VARIANCE

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I. METHOD OF THE REDUCED VARIANCE

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Abstract : In the first part of this paper, a succint introduction is made to the fundamental concepts specific of the study of the stochastic processes. The non-Poissonian character is emphasized of the probability distribution of the number of neutrons recorded by a chain of counters within a given time interval, i.e. of the number representing the physically observable that is holding informations concerning the fluctuations of the neutron populations in the reactor. The difficult problem of collecting the experimental data in view of determining the reduced variance of the physically observable is solved by the authors by means of an electronic equipment which is sorting these experimental data in such a way that the relative recording frequencies of a given number of neutrons in Δt interval is directly determined. In the second part of the paper the fundamental diagram is described of the analysis chain after which a minute description is given of the fundamental diagram as well as of the signal diagram supplied by the stochastic multi-channel analyser achieved by printed circuits in the form of a NIM module.

I. INTRODUCTION

The conditions under which the neutron population within a nuclear reactor can be kept constant as a function of time (steady operating conditions of the neutron population) result from the general evolution equation of the neutron number within the active region of the reactor; this equation can be expressed as follows / 1 / :

$$\hat{P}N - \hat{D}N + S = \frac{\partial}{\partial t}N \qquad (1)$$

where, in symbolical writing, the rates of appearance and disappearance of the neutrons are $R_p = \hat{P}N$, respectively $R_D = \hat{D}N$ and where appears moreover a term expressing the contribution of an excitation source of intensity S.

The production rate of the neutrons is proportional to the fission rate of the fuel nuclei.

The neutron population within the reactor is constant as a function of time ($\frac{\partial}{\partial t} N = 0$), in the absence of an excitation (S = 0), only as long as an equilibrium is achieved between rates R_d and R_p. The state of the reactor when these conditions are fulfilled is called critical state.

If, however, for achieving the steady operating conditions of the neutron population, the presence is necessary of an additional neutron source, the reactor is said to be in a subcritical state. The intensity of this neutron source is a function of want of balance between rates R_p and R_d and of the level of the neutron populating within the environment. For a given level of the population, source S is a measure of the subcriticallity. The intensity of the source is expressed by

$$S = R_1 - R_p \tag{2}$$

Actually, even under operating conditions that at a macroscopic scale are called steady, the neutron population in the reactor displays certain fluctuations due to the stochastic character of the microscopic interaction processes between neutrons and nu - clei within the environment. For instance, the probability for a neutron to provoke the fission of a fuel nucleous depends on se- veral factors such as:

- the position within the reactor active area of the respective fuel nucleous;
- the composition of the active zone around the res - pective spot;
- the microscopic cross-sections of the neutrons-nu- clei interactions depending on the energy of the neutrons.

On the other hand, the function of the neutron spec - tral distribution is strongly influenced by the form and the geo- metrical dimensions of the active area as well as by the relative concentrations of the materials forming it.

Owing to these factors, the fissions of the fuel nu - clei do not occur at regular time intervals. The existence of a dispersion of the time intervals between the fissions offers a partial explanations of the fluctuations existing in the neutron population within the reactor. As a result of the fission of a fuel nucleus provoked by a neutron existing in the reactor, two fission fragments and a number ν_p of neutrons (prompt neutrons) are simultaneously produced. The fission fragments are radioactive nuclei in an excited state, the atomic numbers of which are related by :

(3) $Z_1 + Z_2 = Z$ (Law of conservation of the electric charge)

It is experimentally found that the nuclei of a fissile material that splits in apparently identical conditions (with neutrons of the same energy) do not emit every time the same pair of fission fragments, nor to they emit the same number ν_p of prompt neutrons.

These relative frequencies represent as a matter of fact estimations of the probability that as the result of a fission the nucleus of a given species would appear, respectivelly of the probability that the fission would release a given number of prompt neutrons.

When these probabilities are known, it is possible to determine the statistical mean value of the prompt neutrons produced by the fission of a fuel nucleus.

(4)

$$\tilde{\nu}_p = \sum_{\nu_p=1}^{\infty} \nu_p \cdot p_{\nu_p}$$

Quantity $\tilde{\nu}_p$ is a constant of the material, very interesting as concerns the self maintaining of the chain (fission) reactions within the active region of the reactor, i.e. of the reactor working. Beside the prompt neutrons, the fission of a fuel nucleus releases also neutrons due to the radioactive disintegration of the fission fragments (delayed neutrons).

A fission fragment releases a single neutron by disintegration. It is a random phenomenon since there is a dispersion in the time interval between the disintegration moment of the nucleus and the appearance of the neutron owing to the fission . This dispersion is characteristic of the phenomenon of radioactive disintegration.

The above mentioned phenomena emphasize the stochastic character of the neutrons interactions with the surrounding nuclei and chiefly of the fuel nuclei fission.

The stochastic character of the microscopic processes peculiar to the neutron multiplication within a reactor explains at the macroscopic scale, the presence of the fluctuations of the neutron population in the reactor. This kind of fluctuations takes the name of neutron noise / 2 /.

There results that the neutron population N(t), existing at moment t considered with respect to an arbitrarily chosen origin, within a critical or subcritical reactor, where the statistical balance of the neutron population is nevertheless achieved presents fluctuations around a mean value $\widetilde{N(t)}$ expressed by

$$(5) \qquad \widetilde{N(t)} = \hat{M}\{N(t)\} = m_N$$

where $\hat{M}$ is the operator of the statistic mean

$$(6) \qquad \hat{M}\{N(t)\} = \int_0^\infty x \, d\, F_N(x)$$

and $F_N(x)$ is the distribution function of the random variable $N(t)$.

The neutron population $N(t)$ within the reactor at time t should therefore not be interpreted in a deterministic sense. Since it is a random variable, the reactor state is determined if its moments of any order are known. To characterize the evolution as a function of time of the neutron population, it is necessary to use a set of such functions depending on time and on an infinity of random parameters. Such a family of functions defines a stochastic process / 3 /.

Under conditions easily fulfilled in experiments, the study of a stochastic process reduces itself to the study of a single sample function of the process. With reference to a single sample function, time mean values can be defined such as :

(7) $$\langle N(t) \rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} N(t)\,dt$$ (time mean value)

(8) $$\langle N^2(t) \rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} N^2(t)\,dt$$ (mean square value called also total mean power)

(9) $$R_N(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} N(t)N(t+\tau)\,dt$$ (auto correlation function)

A stochastic process is called ergodic when all the time means exist and coincide , independently of the used sample, with the corresponding statistical means. If the process is for instance Gaussian, it is easy to show that the condition :

(1o) $$\int_{-\infty}^{\infty} |R_N(\tau) - m_N|^2\,d\tau < \infty$$

involves ergodicity.

The ergodicity property is very important since it reduces the evaluation of the statistical means of a stochastic process to the estimation of time means by taking into account a single sample of the process. Such an experiment is only valuable if the process is ergodic and stationary or at least stationary in a restricted sense.

A stochastic process is stationary in a restricted sense if all its statistical properties are invariant with regard to translations occuring along the time axis; in particular when distribution function $F_N(x)$ and statistical mean m_N do not depend on time while the selfcorrelation function

$$(11) \qquad \hat{M}\left\{N(t_1)\ .\ N(t_2)\right\} = R_N(t_1,\ t_2)$$

depends only on difference $t_2 - t_1 = \tau$, the process is stationary of the second order.

The fluctuations of the neutron population within a reactor around the mean value are strongly influenced by certain factors depending both on the composition of the active zone and on the geometrical size of the latter. It could be expected that the analysis of the neutron noise would permit to determine certain global characteristics of the active zone which would prove usefull for describing the transient behaviour of the reactor as a deterministic physical system. These characteristics that can be deduced from moments of the I and II order of the neutron population are usually called kinetic parameters.

The state variable in regard to which equation of evolution (1) is written is therefore the statistical mean of the neutron population.

The solution of the equation of evolution is influenced by the values the following parameters :

α_p - the decrease constant of the prompt neutron population;

l - the mean life of the neutrons

β_{ef}- the effective fraction of the delayed neutrons

k_p- the effective multiplication factor of the prompt neutrons, or, depending on these, the subcritical prompt reactivity of the reactor.

(12)
$$\rho_p = \frac{1 - k_p}{k_p}$$

The mathematical theory of the neutron noise in a reactor makes use of the usual methods and concepts for the study of any stochastic processes. As any mathematical theory for the study of physi - cal phenomena, it is based on hypotheses liable to simplify the mathematic relations between the state variables occuring in the description of the respective system. The theoretical relations are however valuable only as long as they are verified by experiments. Therefore, a mathematic theory appears as useful as concerns the development and the diversifying of the methods for the measurements of the parameters of a physical system.

Therefore, a mathematical theory should operate with state variables of the physical system, susceptible to be physical observables.

When studying the neutron noise, a physical observable (which is also a random variable) can be the number of neutron recorded during a time interval Δt by a counting device provided with a neutron detector and placed within the active area. Let $z(\Delta t)$ be this observable. The study of the fluctuations of the neutron population is thus reduced to the study of the statistical properties of random variable $z(\Delta t)$ / 3 /.

Should the neutron detector be placed in the neighbourhood of a radioactive source, the probability distribution function of the random variable $z(\Delta t)$ would be of the Poisson type, i. e.

$$\text{(13)} \qquad \text{Prob}\left\{ z(\Delta t) = c(\Delta t) \right\} = \frac{(\tilde{\lambda})^c e^{-\tilde{\lambda}}}{c!} = p_c(\Delta t)$$

Poisson's distribution function is characterised by a single parameter ($\tilde{\lambda}$), which is the statistic mean of the random variable.

Therefore $\tilde{\lambda}$ is the mean value of the number of neutrons $z(\Delta t)$, recorded by the counting device during the interval Δt.

The moments of higher order of variable $z(\Delta t)$ can in turn be expressed as a function of only parameter $\tilde{\lambda}$ of Poisson's distribution :

$$\text{(14)} \qquad p_c(\Delta t) = \frac{(\tilde{v}\Delta t)^c}{c!} \exp(-\tilde{v}\Delta t)$$

where $\tilde{v}$ the mean counting rate.

For instance, the variance of the random variable defined by

$$\text{(15)} \qquad \hat{M}\left\{ [z(\Delta t) - \hat{M}\, z(\Delta t)]^2 \right\} = \hat{V}\left\{ z(\Delta t) \right\} = \sigma^2_{z(\Delta t)}$$

is equal to its mean value, respectively

(16) $$\hat{M}\left\{\left[z(\Delta t) - \widetilde{z(\Delta t)}\right]^2\right\} = \hat{M}\left\{z(\Delta t)\right\}$$

or

$$\sigma^2_{z(\Delta t)} = m_{z(\Delta t)}$$

If the neutron detector of the counting device is placed within the active zone of a reactor, the probability distribution of random variable $z(\Delta t)$ will evince a deviation from Poisson's distribution due to the chain fission reactions within the active zone.

In other words, the neutron detector can only record during the interval Δt neutrons belonging to the same chain of fissions. The hypothesis of the independence of the events (an events means here the recording of a neutron), characteristic of a Poisson's stochastic process, can no more be accepted as true. Because of the correlated events, relation

(18) $$\frac{\sigma^2_{z(\Delta t)}}{m_{z(\Delta t)}} = 1$$

is no verified any more. In such a situation, relation (18) should become / 4, 2, 5, 6, 11 /.

(19) $$\frac{\sigma^2_{z(\Delta t)}}{m_{z(\Delta t)}} = 1 + \psi(\Delta t)$$

Term $\psi(\Delta t)$ represents the deviation of the probability distribution of variable $z(\Delta t)$ from Poisson's distribution and becomes increasingly important as the weight of the correlated events increases. In this paper, we present a method of analysis of the neutron noise within a nuclear reactor that has the attention of the Laboratory of Reactor Physics at IFA, with a view to be used for the experimental studies in the lattices of reactors with thermal neutrons.

It is the matter with the method of the reduced variance of the number of neutrons $z(\Delta t)$ recorded by a counting device during a time interval Δt.

In the first part of the paper, the theoretical bases of the method are expounded. The second part is devoted to describing a chain of analysis of the fluctuations and especially of the stochastic multichannel selector, an achievement of the Laboratory of Nuclear Electronics co-operating with that of Reactor Physics.

It should be pointed out that experimentally neither the probability distribution of random variable $z(\Delta t)$ not its statistic moments are accesible to direct measurements.

Only estimations of these statistic characteristics can be determined. As it will be shown further in this paper, the dependence of the reduced variance on the length of the recording time interval Δt, takes the form / 7, 8 /.

$$\text{(2o)} \qquad \frac{\frac{1}{n}\sum_{i=1}^{n} c_i^2(\Delta t) - \left(\frac{1}{n}\sum_{i=1}^{n} c_i(\Delta t)\right)^2}{\frac{1}{n}\sum_{i=1}^{n} c_i(\Delta t)} = 1 + \gamma(\Delta t)$$

Relation (2o) is only valid for small values of interval Δt, for which the contribution of the delayed neutrons can be disregarded.

In this assumption function $\psi(\Delta t)$ previously introduced is expressed by :

$$(21) \qquad \psi(\Delta t) = \epsilon D_\nu \left(\frac{k_p}{1-k_p} \right)^2 \left(1 - \frac{1-\exp(-\alpha_p \Delta t)}{\alpha_p \Delta t} \right)$$

Except the recording interval Δt, function $\psi(\Delta t)$ depends on several constants including the reactor kinetic parameters α_p and ρ_p which can be determined by suitable fitting procedures. For determining the kinetic parameters, the experimental measurement is required of the reduced variance of the number of recorded neutrons for various values of length Δt of the recording interval.

This method is only apparently simple. The relative error of measurement of function $\psi(\Delta t)$ and the total number n of intervals Δt are related by / 9 /

$$(22) \qquad \frac{\Delta(1+\psi(\Delta t))}{1+\psi(\Delta t)} = \sqrt{\frac{2}{n}}$$

If for $\psi(\Delta t)$ = o.ol we accept a measuring error of lo% there results from (22) that the necessary number of recording intervals should satisfy relation

$$n \geq 10^6$$

The achievement of the recording device to which, as suggested by (2o), this experiment is leading, raises particularly intricate tebhnical problems / 2, lo /. This is the reason that directed as towards the achieving of an electronic equipment (fig.1) capable of permitting the direct determination of the probabilility distribution of the random variable $z(\Delta t)$ along the variation range of the recording interval Δt .

This equipment achieved in the IPA Laboratory of Nuclear Electronics is the stochastic multichannel analyser. It represents the key-element of an analysis chain of the fluctuations of the neutron population (fig. 2) made in fact with the help of elements available on the market.

We shall describe succintly the working of the equipment that permits a rational enough use of relation (2o) when determining $\phi(\Delta t)$.

Let $t = 0$ be the time when the START knob is pushed that releases the equipment for which the possibility is provided of stopping it either manually or controlled by a time T which can be selected by switch PRESELECTION fixed on the stochastic selector panel. The time interval $(0,T)$ of a recording can be considered as divided into a number of an adjacent subintervals of equal duration Δt.

(23)
$$\Delta t = \frac{T}{n}$$

Every Δt subinterval is separately analysed by the equipment. The number of neutrons that can be detected within such a subinterval has a higher limit imposed by the very method of processing the experimental data. According to the informations supplied by the literature, this method becomes inapplicable when the number of the neutrons detected within a Δt interval grows higher than 2^7.

If, for instance, in the first time subinterval Δt, neutrons are recorded ($c \leqslant 2^7$), the content of the adress of the memory device ВЗ95В grows by a unity. At the end of the experiment (after time T) the memory addresses bear the following distribution:

$$\left\{ q_c(\Delta t) \ , \ c = 0, 1, \ldots, 127 \right\} \qquad (24)$$

where $q_c(\Delta t)$ is the number of intervals - from the total n- when the detector recorded c neutrons.

When distribution $\{q_c(\Delta t)\}$ is known, the relative frequencies can be determined of the intervals where c neutrons were recorded . These frequencies, supplied by :

$$f_c(\Delta t) = \frac{q_c(\Delta t)}{n} \qquad (25)$$

are estimations of the probability that the number of neutrons detected in a Δt time interval would be c.

Therefore, the working of the recording device is only correct as long as quantities $q_c(\Delta t)$ verify relation:

$$\sum_{c=o}^{127} q_c(\Delta t) = n \qquad (26)$$

By using the recorded distribution $\{ q_c(\Delta t)\}$, we shall be able to compute the empiric moments of the I and II orders of random variable $z(\Delta t)$; which are unbiased estimations of the statistical mean value and of the mean-square value, respectively,

$$\mu_z = \sum_{c=o}^{127} c \ f_c(\Delta t) \qquad (27)$$

(28) $$S_z^2 = \sum_{c=0}^{127} c^2 \, f_c(\Delta t)$$

These quantities permit to determine function (Δt) by means of

(29) $$\phi(\Delta t) = \frac{S_z^2 - \mu_z^2}{\mu_z} - 1$$

So as to allow for additional tests concerning the correctness of working of the equipment, it is moreover necessary to record separately the total number N_D of the recorded neutrons during the whole time T of an experiment.

The average number of neutrons corresponding to a single time interval can therefore be equally deduced from ratio $\frac{n_D}{n}$, that should coincide with the empirical mean value

(3o) $$\frac{n_D}{n} = \mu_z$$

Lastly, the recorded data permit moreover to verify relation

(31) $$n_D = \sum_{c=1}^{127} c \, q_c(\Delta t)$$

The experimentally, obtained data permit therefore to determine function $\phi(\Delta t)$ corresponding to a given value Δt of the argument.

However, the determination of the kinetic parameters of the reactor active zone can only be made if the dependence of function $\phi(\Delta t)$ on duration Δt is known.

The experiment must therefore be repeated for all the argument values $\Delta t_1, \Delta t_2, \ldots, \Delta t_j$ to which, through relation (29) , correspond the quantities: $\phi(\Delta t_1)$, $\phi(\Delta t_2)$, ..., $\phi(\Delta t_j)$.

II. METHOD OF THE VARIANCE OF DETECTED NEUTRONS NUMBERS / 7 /

Let dt_1 and dt_2 be two disjoint time intervals.

We propose to deduce an expresion of the average number of neutrons detected within these intervals by a neutron detector while taking into account the neutron multiplication due to the chain fission reactions. To do this, we shall introduce the following probabilities:

- $p(m, dt_1)$ = the probability for m neutrons to be recorded in dt_1
- $p(m, dt_1; n, dt_2)$ = the joint probability for m neutrons to be recorded in dt_1, and for n neutrons to be recorded in dt_2
- $p(n, dt_2/m, dt_1)$ = the conditional probability for n neutrons to be recorded in dt_2, if m neutrons were recorded in dt_1.

Let $r(t)$ be the instantaneous count rate at time t (random variable). The number of recordings in dt is then $r(t)dt$ while the statistic mean value of the product of the number of recordings in dt_1 by the number of recordings in dt_2 will be

$$(32) \qquad \langle r(t_1)dt_1, r(t_2)dt_2 \rangle = \sum_{m,n} m\, n\, p(m, dt_1; n, dt_2)$$

When the time intervals are small, this mean value is approximated by the probability of a single recording, whence

$$(33) \qquad \langle n(t_1)\, n(t_2) dt_1\, dt_2 \rangle \simeq p(1,\ dt_1;\ 1,\ dt_2)$$

We shall express this probability as the sum of probabilities of two disjoint events. The first term represents the probability of a recording in dt_2 whether a neutron was recorded or not in dt_1. The second term expresses the contribution to the mean value of the events defined by a recording in dt_2 after having been recorded in dt_1. The pairs of recordings in dt_1 and dt_2 can therefore be either accidental or correlated.

The uncorrelated (accidental) pairs of recordings are produced by neutrons with no common ancestor , meaning that they do not belong to the same fission chain.

On the other hand, the correlated pairs originate in recordings of neutrons yielded by the same fission reaction. This manner of interpreting the pairs of recordings will be better explained later on.

We can write :

$$(34) \qquad p(1,dt_1;\ 1,dt_2) = p_a(1,dt_1;\ 1,dt_2) + p_c(1,dt_1;\ 1,dt_2)$$

where

$$(35) \qquad p_c(1,dt_1;\ 1,dt_2) = p(1,dt_1)\ .\ p(1,dt_2/1,dt_1)$$

represents the probability of the recording of a correlated pair of neutrons within the intervals dt_1 and dt_2.

Uncorrelated pairs

The uncorrelated pairs of recordings are due to detected neutrons that do not belong to the same fission chain.

So as to be able to deduce the probability expression $p_a(1, dt_1; 1, dt_2)$ we shall define the following quantities:

F = the mean fission rate within the active zone of the reactor

ε = the efficiency of the neutron detector

by relations / 1 /

(36) $$F = \int_{(E)} \int_{(V_R)} dE\, dV\, \Sigma_f(\vec{R},E) . \nu(E)\, n(\vec{R},E)$$

where

$dE . n(\vec{R},E)$ - is the mean value of the number of neutrons per unit volume around the point the position vector of which is $\vec{R}$ the neutron energy being of E to E + dE,

$\Sigma_f(\vec{R}, E)$ - is the probability with regard to the unit path that a neutron of E energy at space point $\vec{R}$ would produce the fission of a fuel nucleus.

(37) $$\varepsilon = \frac{\int_{(E)} dE \int_{(V_R)} \Sigma_D(\vec{R},E)\, \nu(E) n(\vec{R},E)\, dV}{\int_{(E)} dE \int_{(V_R)} \Sigma_f(\vec{R},E)\, \nu(E)\, n(\vec{R},E) dV}$$

The detector efficiency is therefore equal to the ratio between the mean rate of the detector recording and the mean rate of the reactor fissions. The probability of a recording in dt is

(38) $$p_d(1, dt) = \varepsilon F dt$$

so that the probability of a uncorrelated pair recording in dt_1 and dt_2 is supplied by

(39) $$p_a(1,dt_1;\ 1,dt_2) = \varepsilon^2 F^2 dt_1 dt_2$$

<u>Correlated pairs</u>

Let $\Pi(t_k,t)$ be the probability with regard to the unit time that a neutron produced within the reactor at time t as the result of a fission would have a descendent (itself being one of them) within the system at time t_k.

The probability of one count in dt_1 from a progeny of a fission which emits ν_p prompt neutrons at time t is :

(4o) $$\nu_p F dt\ \Pi(t_1,t)\ \frac{dt_1 \varepsilon}{\bar{\nu}_p}$$

The probability of a recording in dt_1 to be followed by another one in dt_2 from a progeny of the same fission at time t is :

(41) $$(\nu_p-1)\ \Pi(t_2,t)\ \frac{\varepsilon dt_2}{\bar{\nu}_p}$$

The probability of the correlated pairs is therefore :

(42) $$\frac{\nu_p(\nu_p-1)}{(\bar{\nu}_p)^2}\ F\varepsilon^2\ dt\ dt_1\ dt_2\ \Pi(t_2,t)\ \Pi(t_1,t)$$

The fission reaction of a fuel nucleus at time t has released ν_p neutrons but with probability p_{ν_p}, so that to the above expression we must apply the operator of the "statistic mean" whence

$$(43) \qquad \frac{\widetilde{\nu_p(\nu_p-1)}}{(\tilde{\nu}_p)^2} \; F \varepsilon^2 \, dt \cdot dt_1 \, dt_2 \; N(t_2,t) \; N(t_1,t)$$

The probability of a recording in dt_1 and of another one in dt_2 due to neutrons released by earlied fissions of nuclei is supplied by integration

$$(44) \qquad p_c(1,dt_1;\; 1,dt_2) = F \varepsilon^2 D_\nu dt_1, dt_2 \int_{-\infty}^{t_1} N(t_1,t) \; N(t_2,t) \; dt$$

The statistic mean value of the number of both uncorrelated and correlated pairs recorded in dt_1 and dt_2 is therefore

$$(45) \quad \langle n(t_1) n(t_2) dt_1 dt_2 \rangle = \varepsilon^2 F^2 dt_1 dt_2 + F\varepsilon^2 D_\nu dt_1 dt_2 \int_{-\infty}^{t_1} N(t_1,t) N(t_2,t) dt$$

Probabilities $N(t_1,t)$ and $N(t_2,t)$ are solutions of the following kinetic equations / 5, 8 / :

$$(46) \qquad \frac{dN}{dt} = \frac{\rho - \beta}{\Lambda} N + \sum_{(i)} \lambda_i C_i + \delta(t)$$

$$(47) \qquad \frac{dC_i}{dt} = - \lambda_i C_i + \frac{\beta_i}{\Lambda} N$$

In the case of a small t, the "source" of delayed neutrons can be assumed as approximately constant so that the solution of the above system of equations is

(48) $$\Pi(t,0) = e^{-\alpha_o t} \quad , \text{ where } \quad \alpha_o = \frac{\rho - \beta}{\Lambda}$$

The general solution of the equations takes the form

(49) $$\Pi(t,0) = \Lambda \sum_{(k)} A_k \exp(-\alpha_k t)$$

where A_k and α_k are constants resulting from the transfer function of the reactor

(5o) $$T(s) = \frac{1 - \sum\limits_{(i)} \frac{s\,\beta_i}{s+\lambda_i}}{s(\ell + \sum\limits_{(i)} \frac{\beta_i}{s+\lambda_i}) - \rho} = \sum_{(k)} \frac{A_k}{s + \alpha_k}.$$

With the former notations, α_o represents the decrease constant of the population of prompt neutrons

(51) $$\alpha_p \equiv \alpha_o$$

When t is small the general solution coincides with the parti - cular one if $A_o = 1$. We have then

(52) $$\Pi(t_1,t) = \sum_{k=o}^{6} \beta_k \exp\left[-\alpha_k (t_1 - t)\right]$$

(53) $$\Pi(t_2,t) = \sum_{k=o}^{6} \beta_k \exp\left[-\alpha_k (t_2 - t)\right]$$

Relation (45) becomes

(54) $\langle \mathcal{R}(t_1)\mathcal{R}(t_2)dt_1dt_2\rangle = \varepsilon^2F^2dt_1dt_2+F\varepsilon^2D_v dt_1dt_2 \sum_{i,j}\frac{B_iB_j}{\alpha_i\alpha_j}\exp[-\alpha_i(t_2-t_1)]$

Introducing notation

$$Q_i = \sum_{j=1}^{n+1} \frac{B_j}{\alpha_i+\alpha_j}$$

we shall obtain

(55) $\langle \mathcal{R}(t_1)\mathcal{R}(t_2)dt_1dt_2\rangle = F^2\varepsilon^2dt_1dt_2+F\varepsilon^2D_v dt_1dt_2 \sum_{(i)} A_iQ_i\exp[-\alpha_i(t_2-t_1)]$

Let us denote by Δt a time interval including interval (t_1,t_2) and let $c(\Delta t)$ be the number of recordings within this interval. The number of recording pairs will be

$$C^{(2)}_{c(\Delta t)} = \frac{c(\Delta t).[c(\Delta t)-1]}{2}$$

and the mean number of pairs will be :

(56) $$\left\langle \frac{c(\Delta t).[c(\Delta t)-1]}{2}\right\rangle = \int_{t_2=0}^{\Delta t} dt_2 \int_{t_1=0}^{\Delta t_2} dt_1\langle \mathcal{R}(t_1).\mathcal{R}(t_2)\rangle$$

(57) $$\frac{1}{2}\langle c(\Delta t).[c(\Delta t)-1]\rangle = \frac{\varepsilon^2F^2\Delta t^2}{2} + F\varepsilon^2D_v\Delta t \sum_{k=1}^{n+1} \frac{A_kQ_k}{\alpha_k}\left(1-\frac{1-\exp(-\alpha_k\Delta t}{\alpha_k\Delta t}\right)$$

But $\varepsilon F\Delta t$ represents the mean number of recordings within interval Δt, i.e.

(58) $$\langle c(\Delta t)\rangle = \varepsilon F\Delta t$$

and therefore

(59) $$\frac{\langle c^2(\Delta t)\rangle - \langle c(\Delta t)\rangle^2}{\langle c(\Delta t)\rangle} = 1+2\varepsilon D_\nu \sum_{k=1}^{n+1} \frac{A_k Q_k}{a_k}\left(1 - \frac{1-\exp(a_k\Delta t)}{a_k\Delta t}\right)$$

or

(6o) $$\frac{\langle c^2(\Delta t)\rangle - \langle c(\Delta t)\rangle^2}{\langle c(\Delta t)\rangle} = 1+ \sum_{k=1}^{n+1} Y_k\left(1 - \frac{1-\exp(-a_k\Delta t}{a_k\Delta t}\right)$$

which was deduced by E.N.Courant, R.L.Wallace, D.R Frisch, D.J. Liffler and R.N.Albrech in the form

(61) $$\frac{\langle c^2(\Delta t)\rangle - \langle c(\Delta t)\rangle^2}{\langle c(\Delta t)\rangle} = 1 + \sum_{(k)} \phi_k(\Delta t)$$

Structure and performances of the system

For data accumulation, the memory register BM 96-B (4o96 addresses) is used which in an accumulation regime has a capacity of 10^6/adress.

The complete structure of the system (fig. 1) is formed by :

- the analyser itself, conceived as module NIM (fig. 2)
- the memory device BM96-B

- the neutron detector D with a o,5 μs time resolution
- the time-mark generator G, with a period Δt_G = 5o μs - 1oo ms
- the n_D counter with a $1o^9$ capacity for recording the total number of analysed pulses (detector counts).

The analyser preselects the information accumulation received from the detector in recording intervals T of $1o^3$, $1o^4$, $1o^5$ or $1o^6$ or as case may be any time interval.

The analysis chain can accept up to 127 pulses from the detector during an interval Δt_G (for a number higher than 127 pulses, the 127-th address will increase by 1 its content for each recording interval.

Operation

During the feeding of the installation, the START-STOP (SP) switch is in position 1 (STOP) so that the binary counter with seven bits (NB) and flip-flops FF1, FF2 and FF3 are held at the zero state (fig. 2). The equipment is reset (decimal counter n) with the help of knob ZERO. At the same time, counter n_D is separately reset too. For data accumulation, swith SP is put on position 2 (START).

When the first pulse comes from time-mark generator G (SP being on 2), FF1 passes on the back wave front, on "1" by opening gate P_2 when the first pulse from G achieve its end (fig. 3). It is necessary that the first pulse should not pass so as to ensure the equality of all the access periods of the pulses from detector D.

The second pulse from G pass through P_2 where it undergoes a delay t_1 = 2oo ns; its wave front releases mono-flip-flop MS (the duration of the pulse produced by MS is Δ_{MS}=1 μs). The back wave of the MS shapes with the help of delay t_2 a needle pulse of 1oo ns, the back wave of which passes FF2 and FF3 in "1". From this moment forward, gate P_1 is opened and the pulses from D are recorded on n_D and NB.

When the third pulse comes from G, the sequence of events produced by the second pulse is repeated. On the other hand, FF2 is reset and blocks gate P_1 and since FF3 is in state "1", all impulses coming from G, beginning by the third one are recorded on NZ. At the same time, MS releases on the first front the memory cycle (IM). During this cycle (of 16 ± 2 μs) the NB contents is transferred to the address register RA of memory device BM 96-B, the RA address content passes into information register RI; this content increased by 1 is returned to the specified address in RA.

The negative impulse from reset NB, while its back wave brings again in "1" FF2 which opens again gate P_1.

Delay t_1 is necessary for the transient processes in NB to be at an end before the memory cycle is released.

The flip-flop FF3 avoids the releasing of the memory cycle when the second pulse arrives from G which would have resulted in the recording of an event at address zero. On the other hand, the second pulse is not recorded on NZ either.

The duration of the recording period is $\Delta t = \Delta t_G - \Delta_{MS}$.

If in a Δt_G interval, detector D supplied to the output more than 127 pulses, at the 127-th pulse gate P_3 opens and blocks the passage of the pulses through P_1 so that the event will be registered on address 127. The frequent inaccurate recording of events on address 127 results in the lighting of lapm OVERLOAD. In this case, the frequency of time-mark generator G should be increased.

When the contents of (n) reach lo^3, lo^4, lo^5 or lo^6 (as a function of the position of swith PRESELECTION), PS becomes "1", the signalling lamp of the self-stopping (SA) and FF1 and FF2 are therefore reseted zero; the last memory cycle is released , after the recording the NB content is reset and FF3 is reset to (which avoids the possibility of an accidental release of the memory cycle as well as of the altering of the accumulated data). The experiment, is thus "automatically" stopped.

For a new experiment, SP is again set on 1, the equipment is reset after wich SP is passed in position 2.

The information accumulation can also be stopped " manually" at any time by putting SPon pposition 1. The interrupted experiment can however be continued by simply putting SP again on position 2.

Characteristics of the analyzer input/output pulses

- output from time-mark generator and detector
  - polarity - positive
  - amplitude - 2 ÷ 3o V
  - duration (at 1.5 Volts) > 0,2 μs at the generator and 0,02 μs at the detector
- output towards the counter
  - polarity - positive
  - amplitude - 3,5 V

- duration - the same as that of the pulses from the detector

- towards memory device BM96-B
  - polarity - positive
  - amplitude - 7,4 V ± 10%
  - rise time - 0.3 μs
  - fall time - 4o ns
  - duration - 1 μs.

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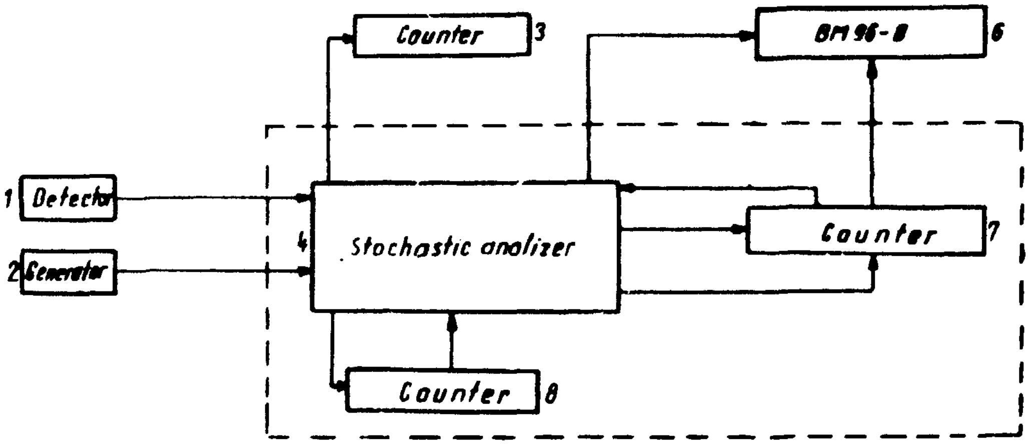

Fig. 1

1 - Detector ($T' \geq 0{,}5$ µs ; 2 - Time-mark-generator ($\Delta t_G = 10^2$ µs - 10^2 ms); 3 - Counter 10^9 (n_D) ;
4 - Stochastic analyser ; 5 - Decimal counter 10^6;
6 - BM 96 B ; 7 - Binary counter (NB) 2^7.

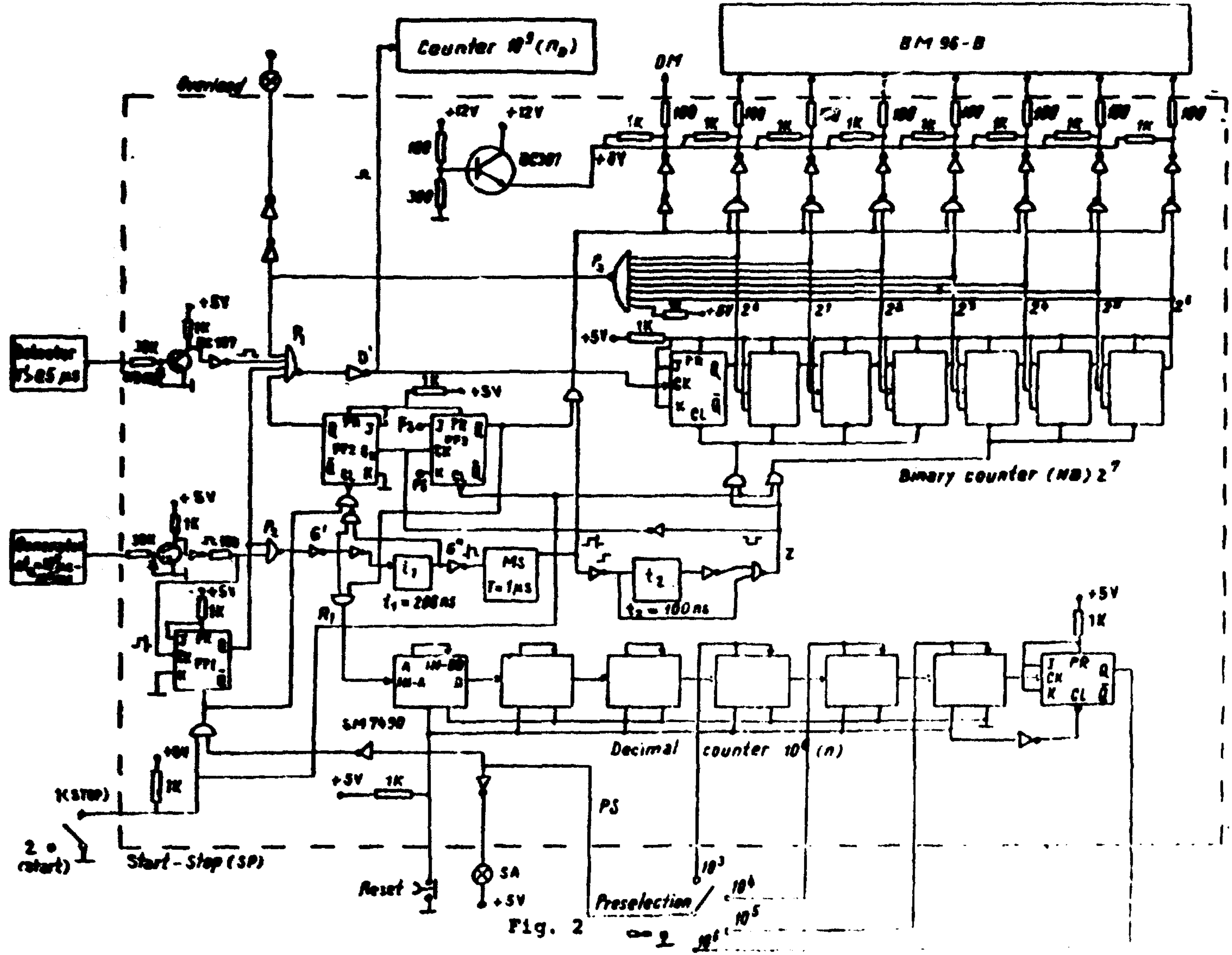

Fig. 2

Fundamental diagram of the stochastic analyser

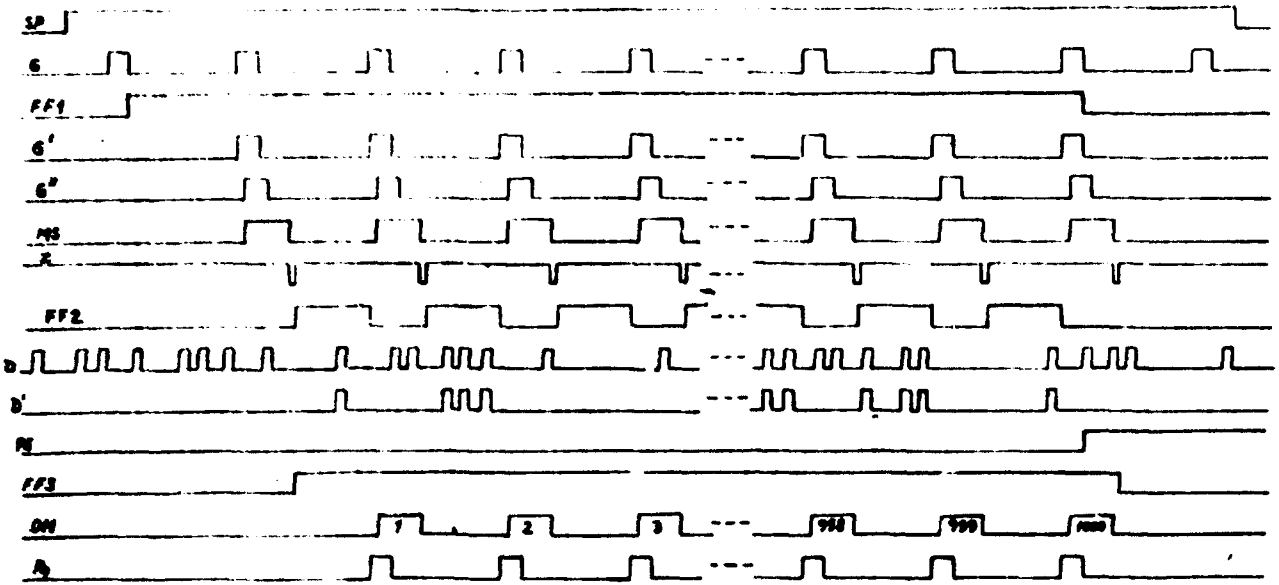

Fig. 3

The signal diagram in the stochastic analyser.