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PENETRATION OF HIGH-FREQUENCY WAVES INTO A WEAKLY INHOMOGENEOUS MAGNETIZED PLASMA AT OBLIQUE INCIDENCE AND THEIR TRANSFORMATION TO BERNSTEIN MODES

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A propagation of high-frequency electromagnetic waves in a weakly inhomogeneous magnetised plasma is investigated. We suppose the density gradient to be perpendicular to an external magnetic field and the waves to be incident obliquely upon the plasma from vacuum. We find that the transmission coefficient of the ordinary wave through the plasma resonance is approximately equal to one in a fairly wide range of angles of incidence γ near the value $\gamma_0 = \arcsin\sqrt{\omega_c/(\omega_c+\omega)}$. The transmitted ordinary wave is, at great densities, completely transformed into an extraordinary wave. Then it propagates back to the region of smaller density and is completely transformed into the Bernstein modes in the place of the hybrid resonance. Complications connected with the evanescent layer which arise when the high-frequency energy is transmitted into the plasma in the form of the extraordinary wave can thus be removed by using the ordinary wave with the angle of incidence chosen appropriately.

INTRODUCTION

The basic problems connected with the high-frequency plasma heating is the efficiency of the high-frequency energy transfer from vacuum to a plasma. A number of papers has been devoted to the linear theory of this problem most of them being quoted in the survey paper by Golant and Pilia (1971). Earlier papers concerning mostly wave propagation in a cold plasma are compiled in the monographs by Budden (1961) and by Ginzburg (1960). Most detailed studies have been devoted to the incidence of waves upon a plasma without a magnetic field. In these circumstances the most interesting situation arises if an electromagnetic wave is incident obliquely upon a plasma and if its vector of an electric field lies in the plane of incidence. Then this wave is partly transformed into a Langmuir wave in the region of the plasma resonance (Pilia 1966).

Two main problems have always attracted the attention of those who studied the wave propagation in the magnetised plasma. First, the linear transformation of waves near the hybrid resonance was investigated (Stix 1965; Pilia, Fedorov 1969). Secondly, the transmission of electromagnetic waves from vacuum to the plasma through the evanescent layer was studied. The latter problem was mostly solved only for the case when the density gradient of the plasma is perpendicular to a homogeneous magnetic field. If an ordinary wave is incident perpendicularly upon such a plasma it is reflected at the plasma resonance. At a normal incidence the extraordinary wave is partly reflected and partly transformed in the hybrid resonance to the Bernstein modes. But the transformation coefficient is approximately equal to one only for the wave the wavelength of which is comparable with the dimensions of the plasma (Kuehl 1967). The propagation of electromagnetic waves incident obliquely upon a cold slowly-varying, magnetized plasma was studied by Booker (1938, 1949). His papers show that the conditions for the transformation of the extraordinary wave get worse at an oblique incidence because the width of the evanescent layer between the place of reflection and the place of the hybrid resonance becomes bigger. If the ordinary wave is incident upon a slowly varying plasma at a certain angle the evanescent layer in the region of the plasma resonance disappears. This fact was pointed out by Mitjakov in 1959. One point of the matter, however, has not been clear so far. How effective is the transmission of energy of the ordinary wave through this region? Let us take, for instance, the component of the wave vector in the direction of the density gradient. In the WKB approximation it approaches zero in the place of plasma resonance, which is usually accompanied with wave reflection.

In our paper we deal with this problem i.e. we study a propagation of the ordinary wave of a small amplitude incident obliquely upon a collisionless magnetized plasma with the density gradient perpendicular to the magnetic field. We confine ourselves to the waves with frequencies ω bigger than the electron cyclotron frequency ω_c and we assume that the vacuum wavelength is essentially smaller than the characteristic length of the inhomogeneity of density $\varkappa^{-1}$ $(\varkappa = d \ln n/dy)$ i. e. we solve the problem using the WKB method. The penetration of the ordinary wave into the dense plasma can be investigated in the approximation of a cold plasma. In the last chapter we study the model of a hot plasma in connection with the transformation of electromagnetic waves into the electrostatic Bernstein modes in the region of the hybrid resonance.

STARTING EQUATIONS

We shall study the propagation of electromagnetic waves in a plane-stratified magnetized plasma. The coordinate system will be chosen in such a way that the gradient of plasma density and the external homogeneous magnetic field are parallel to the y- and z- axis respectively. We suppose that the electromagnetic wave of the form

(1) $$\vec{E} = \vec{E}_0 \exp\left\{-i\omega t + ik_z z + i\sqrt{k_v^2 - k_z^2}\, y\right\}$$

is incident from vacuum upon the plasma; k_z is the z-component of the wave vector and the length of the vacuum wave vector $k_v = \frac{\omega}{c}$. $\vec{E}_0$ is to be chosen in acordance with the condition $\operatorname{div} \vec{E} = 0$. The electric field within the plasma can then be supposed to have the form

(2) $$\vec{E} = \vec{E}_{k_z,\omega}(y) \exp\left\{-i\omega t + ik_z z\right\}$$

Using the Maxwell equations and the linearized equations of the cold magnetohydrodynamics we obtain for the Fourier component of the vector of the electric field $\vec{E}_{k_z,\omega}(y)$ the following set of equations

$$\frac{d^2E_x}{dy^2}+E_x\left(k_v^2\varepsilon_\perp-k_z^2\right)+iE_yk_v^2g=0 \tag{3a}$$

$$i\frac{dE_z}{dy}k_z+iE_xk_v^2g-E_y\left(k_v^2\varepsilon_\perp-k_z^2\right)=0 \tag{3b}$$

$$\frac{d^2E_z}{dy^2}+E_zk_v^2\varepsilon_\parallel-i\frac{dE_y}{dy}k_z=0 \tag{3c}$$

where $\varepsilon_\parallel=1-\frac{\omega_0^2(y)}{\omega^2}$, $\varepsilon_\perp=1-\frac{\omega_0^2(y)}{\omega^2-\omega_c^2}$, $g=\frac{\omega_0^2(y)\omega_c}{\omega(\omega^2-\omega_c^2)}$, ω_0 is the plasma frequency. In the set (3) the contributions from ion motion are neglected, which is justifiable in the high-frequency region. To make the notation simpler we have omitted the indices k_z and ω in the components of the electric field.

In the following discussion it is convenient to eliminate the components E_y and E_z from the set (3). The resulting equation for the component E_x is of the fourth order and has the following form:

$$\begin{aligned}&\varepsilon_\parallel\varepsilon_\perp E_x^{IV}-E_x'''\left(\varepsilon_\perp\varepsilon_\parallel'+2\varepsilon_\parallel\frac{g'}{g}\right)+E_x''\varepsilon_\parallel\left[\left(k_v^2\varepsilon_\perp-k_z^2\right)\left(\varepsilon_\perp+\varepsilon_\parallel\right)-\right.\\&\left.-k_v^2g^2\right]+E_x'\left\{2\varepsilon_\parallel\left[k_v^2\left(\varepsilon_\perp\varepsilon_\perp'-gg'\right)-\frac{g'}{g}\left(k_v^2\varepsilon_\perp-k_z^2\right)\right]-\right.\\&\left.-\varepsilon_\parallel'\left[\left(k_v^2\varepsilon_\perp-k_z^2\right)\varepsilon_\perp-k_v^2g^2\right]\right\}+E_x\varepsilon_\parallel^2\left[\left(k_v^2\varepsilon_\perp-k_z^2\right)^2-k_v^4g^2\right]=0.\end{aligned} \tag{4}$$

In the latter equation we neglected the terms proportional to the second derivative of density and the terms proportional to the square of the first derivative of density in the coefficients which appear at E_x'' and E_x. These terms could play some role only near the plasma resonance where $\varepsilon_{\parallel} = 0$. In a plasma without a magnetic field the terms of a similar character caused a singularity of the electric field of the electromagnetic waves incident obliquely on the plasma (Budden; Ginsburg). But near the point given by the condition $\varepsilon_{\parallel} = 0$ eq. (4) has four linearly independent analytical solutions. This can be shown (e.g. for the linear profile of density in the vicinity of the plasma resonance) by expanding the coefficients of eq. (4) (supplemented with the terms proportional to the square of the first derivative of density) into the series and by solving the resulting equation in terms of a power series. The exact form of the coefficients will not be needed for further considerations and therefore we confine ourselves to the investigation of the equation for the electric field of waves in the form (4).

ANALYSIS OF WAVE PROPAGATION BASED ON THE WKB APPROXIMATION

We solve eq. (4) only for waves whose vacuum wavelength is much smaller than the characteristic length of an inhomogeneity of plasma density, i.e. we suppose the inequality

$\varkappa / k_v \ll 1$ to hold. Then the WKB approximation can be used and a solution can be supposed to have the form

$$E_x = C(y) \exp \left\{ i \int^{y} k_y(y') dy' \right\} . \tag{5}$$

The amplitude of a wave $C(y)$ and the component of the wave vector $k_y(y)$ are assumed to be slowly varying functions of y. For these two quantities we get from eq. (4) the following expression

$$k^2_{y_{1,2}} = \frac{1}{2\varepsilon_\perp} \Big\{ -k_v^2 g^2 + (k_v^2 \varepsilon_\perp - k_z^2)(\varepsilon_\perp + \varepsilon_\parallel) \pm \tag{6}$$

$$\pm \sqrt{\left[(k_v^2 \varepsilon_\perp - k_z^2)(\varepsilon_\perp + \varepsilon_\parallel) - k_v^2 g^2\right]^2 - 4\varepsilon_\perp \varepsilon_\parallel \left[(k_v^2 \varepsilon_\perp - k_z^2)^2 - k_v^4 g^2\right]} \Big\},$$

$$C_{1,2}(y) = \sqrt{\frac{g(\sqrt{1+\beta\varepsilon_\parallel} \mp 1)}{k_{y_{1,2}} \left[k^2_{y_1} - k^2_{y_2}\right] \varepsilon_\perp}} \tag{7}$$

where $\beta = 4k_v^2 k_z^2 \omega^2 / \omega_c^2 (k_v^2 - k_z^2)^2$. After introducing dimensionless quantities $p = \omega_0^2/\omega_c^2$, $\alpha = \omega/\omega_c$ and $N_z = k_z/k_v$, expression (6) for k_y may be rewritten as follows:

$$\frac{k^2_{y_{1,2}}}{k_v^2} = \frac{1}{2\alpha^2(\alpha^2 - 1 - p)} \Big\{ 2p^2 - p\left[4\alpha^2 - N_z^2(2\alpha^2 - 1) - 1\right] + \tag{8}$$

$$+ 2\alpha^2(\alpha^2 - 1)(1 - N_z^2) \pm p\sqrt{(1-N_z^2)^2 + 4N_z^2(\alpha^2 - p)} \Big\} .$$

The dependence of the $k^2_{y_{1,2}}$ on the plasma density for $\alpha = 1{,}5$ and for various values of N_z^2 is plotted in Fig. 1.

From eqs. (5) to (7) we can see that the WKB method provides four independent solutions two of which, with index 1, correspond to the ordinary wave and two remaining, with index 2, correspond to the extraordinary wave. Conditions for the WKB approximation fail in the points where $k_{y_i} = 0$, $k_{y_i} = \infty$ or $k_{y_1} = k_{y_2}$. The interaction between waves (reflection or transformation) can occur only in these points. Moreover, in places where the wave vector approaches infinity eq. (4) cannot be used as it is. It must be supplemented with the terms connected with the thermal motion of electrons.

Let us first shortly analyse the situation occurring if the wave (1) enters the plasma. We suppose that both density and its first derivative are continuous on the plasma-vacuum boundary so that the wave reflection in this region can be neglected. Near the boundary, where the plasma density is small, the ordinary wave $\vec{E}_1$ has approximately the same wavelength as the extraordinary wave $\vec{E}_2$ ($k_{y_1} \simeq k_{y_2} \simeq \sqrt{k_v^2 - k_z^2}$), but their polarisations are different. From the set of equations (3) the following expressions for ratios of the components of electric vectors of the two waves are obtained:

$$(9) \quad E_{x1,2} = P_{1,2} E_{y1,2}\,, \qquad E_{y1,2} = -\frac{k_z}{\sqrt{k_v^2 - k_z^2}} E_{z1,2}\,,$$

where

$$(10) \qquad P_{1,2} = 2i\alpha \frac{1}{(1-N_z^2)(1 \pm \sqrt{1+\beta})}\,.$$

On entering the plasma the incident wave of the type (1) is generally split into an ordinary and an extraordinary wave whose amplitudes are given by

(11) $$E^{0}_{x1,2} = \mp \left(P_{2,1} E_{0y} - E_{0x} \right) \frac{P_{1,2}}{P_1 - P_2} \quad .$$

An appropriate choice of polarisation of the incident wave thus enables us to vary arbitrarily the ratio of energy flow in the ordinary wave to the energy flow in the extraordinary wave. If a linearly polarised wave is incident upon the plasma this ratio is greatest for the wave with the electric vector in the plane of incidence ($|E_1|^2/|E_2|^2 \doteq 1{,}86$ for $\alpha = 1{,}5$ and $N_z^2 = 0{,}4$) and smallest for the wave with the electric vector perpendicular to the plane of incidence.

In what follows we study the wave propagation in a plasma with a monotonically increasing density in the positive direction of the y-axis. Within the plasma, the extraordinary wave can propagate without obstacles from the boundary up to the place where $k_{y2} = 0$. The respective density is given by

(12) $$p = \alpha (\alpha - 1)(1 - N_z^2)$$

A qualitative analysis shows that in this place the wave is mostly reflected back to the plasma boundary and it leaves the plasma. A very small part of the wave energy is transmitted through the evanescent layer, bounded by the point (12) and the point of hybrid resonance ($p^2 = \alpha^2 - 1$). The wave vector of the extraordinary wave is infinite in the place of hybrid resonance. Considering the effect of thermal motion of particles we can show that the wave is transformed into the Bernstein modes here (Stix).

For small angles of incidence, i.e. for $N_z^2 < 1/(2\alpha+1)$, there exists still one point in the plasma in which $k_{y2} = 0$:

(13) $$p = \alpha(\alpha+1)(1-N_z^2).$$

If the angle of incidence is so big that $N_z^2 > 1/(2\alpha+1)$ there exists no other zero of the wave vector of the extraordinary wave but in the point y_T given by the condition

(14) $$p(y_T) = \frac{(1-N_z^2)^2 + 4\alpha^2 N_z^2}{4N_z^2},$$

the extraordinary and ordinary wave branches intersect ($k_{y1} = k_{y2}$). In such a point, transformation of waves can occur (Zaslavskii et al., 1964). In the following chapter we shall more fully investigate the solution of the equation (4) near the point (14) and deduce the transformation coefficient of the extraordinary wave into an ordinary one and vice versa.

The point (14) is the intersection point of branches also in the case of small angles of incidence but the wave vector components $k_{y1,2}$ are purely imaginary in this case. It follows from the qualitative analysis that as long as the conditions for the WKB approximation are satisfied a possible transformation of the extraordinary wave into the ordinary wave is very small and hence the extraordinary wave is practically completely reflected in the point (13) back to the lower densities. At small angles of incidence the ordinary wave propagates from the plasma boundary up to the point of plasma resonance

$$p = \alpha^2 \tag{15}$$

where it is also almost completely reflected.

For bigger angles of incidence when $N_z^2 > 1/(2\alpha + 1)$ the situation is more favourable for the penetration of the ordinary wave into a denser plasma. In this case there are two points where $k_{y1} = 0$. The corresponding plasma frequencios are determined by the following conditions

$$\begin{aligned} p &= \min\left\{\alpha^2, \ \alpha(\alpha+1)(1-N_z^2)\right\}, \\ p &= \max\left\{\alpha^2, \ \alpha(\alpha+1)(1-N_z^2)\right\}. \end{aligned} \tag{16}$$

The evanescent layer for the ordinary wave is situated between the points (16). The width of the layer depends on the value of N_z^2. In the following chapter we determine the reflection and transmission coefficients of the ordinary wave incident upon this layer and the dependence of these coefficients on N_z^2.

REFLECTION, TRANSMISSION AND TRANSFORMATION COEFFICIENTS OF THE ORDINARY WAVE

We shall study the behaviour of the solution of the equation (4) near the points (14) and (16), assuming that $N_z^2 > 1/(2\alpha+1)$. First we shall deduce the reflection and transmission coefficients of the ordinary wave incident upon the layer which is situated between the points given by (16). The y-component of the wave vector of the ordinary wave is small ($k_{y_1} \ll k_v$) within this plasma region and we can thus neglect, in eq. (4), the terms proportional to the third and fourth derivative of the electric field. Moreover, we shall suppose that the plasma density near the points given by (16) has a linear profile:

(17) $$p = \alpha^2 (1 + \varkappa y).$$

For simplicity we have put the origin of the coordinate system into the point of plasma resonance. On the basis of the above assumption we obtain from eq. (4) the following equation for the x-component of the electric field of the ordinary wave:

(18) $$\xi \frac{d^2 E_x}{d\xi^2} - \frac{dE_x}{d\xi} + \xi^2(\xi - a) b E_x = 0,$$

where

$$a = \frac{\alpha+1}{\alpha}\sqrt{\frac{k_v}{\varkappa}}\left(\frac{1}{\alpha+1} - N_z^2\right), \quad b = \frac{\alpha}{1-N_z^2}\left[1 + (\alpha-1)N_z^2\right], \quad \xi = y\sqrt{\varkappa k_v}.$$

Eq. (18) can be used only for the following values of the parameters a, b and the variable ξ :

$$(19) \qquad b \sim 1; \quad 1 < |\xi|, \ |a|, \ |\xi - a| < \sqrt{\frac{k_r}{\varkappa}} .$$

Solution of eq. (18) cannot generally be expressed in terms of known functions. We shall thus confine ourselves only to an analysis of the asymptotic solutions for $|\xi| \gg 1$, which are of the form

$$(20) \qquad E_{x1} = \sqrt[4]{\frac{\xi}{\xi - a}} \exp\left\{ \pm i \sqrt{b} \int_0^{\xi} \sqrt{\xi'(\xi' - a)}\, d\xi' \right\}$$

When the thickness of the evanescent layer equals zero eq. (18) has exact solutions

$$(21) \qquad E_{x1} = \exp\left\{ \pm i \frac{\sqrt{b}}{2} \xi^2 \right\} .$$

Keeping in the mind the restrictions (19) we cannot use the solution (20) or (21) near the points $\xi \simeq 0$, $\xi \simeq a$. In order to obtain a connection formula for the solutions in front of and behind the evanescent layer we must continue the solutions to the ξ - complex plane. (Heading 1962). In the following analysis we confine ourselves only to the case when $a > 0$. We look for a solution representing in the region $\xi > a$ a wave transferring energy to the region of a denser plasma i.e. the wave given by

$$(22) \qquad E_{x1} = H \sqrt[4]{\frac{\xi}{\xi - a}} \exp\left\{ -i \sqrt{b} \int_0^{\xi} \sqrt{\xi'(\xi' - a)}\, d\xi' \right\} .$$

It should be noted that a sign of the y-component of the group velocity of the ordinary wave is changed after transmission through the evanescent layer. The cuts and the Stokes lines in the ξ -complex plane related to the solution (20) are depicted in Fig. 2. Continuing the solution (22) into the lower complex plane we get the following combination of incident and reflected ordinary wave for $\xi < 0$

$$(23)\quad E_{x1} = -iH\sqrt[4]{\frac{\xi}{\xi-a}}\left\{\exp\left[i\sqrt{b}\int_0^{\xi}\sqrt{\xi'(\xi'-a)}\,d\xi'\right] + R\exp\left[-i\sqrt{b}\int_0^{\xi}\sqrt{\xi'(\xi'-a)}\,d\xi'\right]\right\}.$$

The absolute value of the Stokes constant R corresponding to the line N is determined by the conservation law of the energy flow. This law has the form

$$(24)\quad E_x^*\frac{dE_x}{d\xi} - E_x\frac{dE_x^*}{d\xi} = D\xi$$

D is a constant independent on ξ . From (22) to (24) we then get the following relation for $|R|$

$$(25)\quad 1-|R|^2 = \exp\left\{-2\sqrt{b}\left|\int_0^{a}\sqrt{\xi'(\xi'-a)}\,d'\xi\right|\right\}:$$

This method does not make it possible to find the phase of the constant R and therefore the phase of the reflected wave cannot be determined either. The amplitude of the reflection coefficient is equal to $|R|$. From eq. (25) we obtain this expression for .

(26) $$|R| = \sqrt{1 - \exp\left(-\frac{\pi\sqrt{b}\,a^2}{4}\right)}\,.$$

The amplitude of the transmission coefficient $|T|$ is

(27) $$|T| = \exp\left(-\frac{\pi\sqrt{b}\,a^2}{8}\right).$$

In an analogous way we obtain expression of the same form for the reflection and transmission coefficients also in the case when $a < 0$. It follows from (26) and (27) that the ordinary wave is fully transmitted through the place of plasma resonance if $a = 0$ ($R = 0$, $|T| = 1$).

The solutions (22) and (23) can be matched with the WKB solution in the region $|\xi| \gg 1$. Then the coefficient in the amplitude of the incident wave can be found. Using (6), (7), (11) and (17) we get

(28) $$H = E^0_{x1}\sqrt[4]{\frac{(1-N_z^2)\beta^2(1+\beta)}{4\alpha\left[1+(\alpha-1)N_z^2\right]\left(\sqrt{1+\beta}-1\right)^2}}\,.$$

After passing through the evanescent layer the ordinary wave propagates further into a denser plasma up to the place where the plasma density satisfy condition (14). In the vicinity of this point we must analyse the solution of eq. (4) in more detail. For this purpose let us expand the plasma density near the point y_T into the Taylor series:

(29) $$p = p(y_T)\left[1 + \varkappa(y - y_T) + \dots\right]$$

Using (29) we get the following approximate expressions for the y-components of the wave vectors (6) and amplitudes (7)

$$k_{y1,2} = k_y(y_T) \pm Q\sqrt{\tilde{\varkappa}(y_T - y)},$$

(30)

$$C_{1,2} = \sqrt{\frac{q(\sqrt{1+\beta\varepsilon_\parallel} \mp 1)}{4k_y^2\varepsilon_\perp Q}}\Bigg|_{y=y_T} \frac{1}{\sqrt[4]{\tilde{\varkappa}(y_T - y)}},$$

where

$$Q = \frac{k_v^2}{k_y} \frac{p^{3/2} N_z}{2\alpha^2(\alpha^2 - 1 - p)}\Bigg|_{y=y_T}$$

The general WKB solution (5) can then be written for $y < y_T$ as

$$E_x = \frac{1}{\sqrt[4]{y_T - y}} \Big\{ e^{ik_y(y_T)(y-y_T)} \Big[B_1^+ e^{-\frac{2}{3} i Q \sqrt{\tilde{\varkappa}} (y_T - y)^{3/2}} +$$

(31)

$$+ B_2^+ e^{\frac{2}{3} i Q \sqrt{\tilde{\varkappa}} (y_T - y)^{3/2}} \Big] + e^{-ik_y(y_T)(y-y_T)} \Big[B_1^- e^{-\frac{2}{3} i Q \sqrt{\tilde{\varkappa}} (y_T - y)^{3/2}} +$$

$$+ B_2^- e^{\frac{2}{3} i Q \sqrt{\tilde{\varkappa}} (y_T - y)^{3/2}} \Big] \Big\}.$$

In order to get a solution for $y > y_T$ we must continue the solution round the point 0 in the complex plane of $(y - y_T)$. A part of the solution proportional to $\exp(ik_y(y-y_T))$ and a part proportional to $\exp(-ik_y(y-y_T))$ do not affect each other (Moiseev 1966; Zaslavskij et al.). If we go with the solution round the origin of the coordinate system counter-clockwise the Stokes constants are equal to the imaginary unit i. The following relations between the wave amplitudes are obtained under the assumption that the solution for $y \gg y_T$ is finite

(32) $$B_1^+ = -iB_2^+, \quad B_1^- = -iB_2^-.$$

We can see that in the place of transformation y_T the backward ordinary wave is transformed fully into the forward extraordinary wave. This wave travels then back to lower densities and has the amplitude

$$(33)\quad |E_{x2}| = \left| T E^0_{x1} \sqrt{\frac{\sqrt{1+\beta}\,(1-N_z^2)k_\nu^3}{(\sqrt{1+\beta}-1)\alpha}}\; C_2(y)\right| .$$

The extra ordinary wave propagates without obstacles up to the region of hybrid resonance where the approximation of cold plasma fails.

THE EFFECT OF ELECTRON TEMPERATURE ON THE PROPAGATION OF WAVES

An equation for the electric field of waves in a hot inhomogeneous plasma can be derived in the linear approximation from the kinetic theory under the assumption that the mean Larmor radius of electrons ρ $\left(\rho = v_T/\omega_c = \sqrt{2T/m}/\omega_c\right)$ is much smaller than the characteristic length of inhomogeneity ($\varkappa\rho \ll 1$). The thermal corrections in eqs. (3) are important only for the extraordinary wave in the vicinity of the hybrid resonance. Using the general expression for the electric current in an inhomogeneous plasma with the Maxwell distribution function of electrons (Michailovskij 1967) we obtain eq. (3b) in the following form

$$i \frac{dE_z}{dy} k_z + i E_x k_v^2 g - E_y (k_v^2 \varepsilon_\perp - k_z^2) =$$

(34)

$$= \frac{3 v_T^2 k_v^2}{2(\omega^2 - \omega_c^2)(\omega^2 - 4\omega_c^2)} \frac{d}{dy} \left[\omega_0^2(y) \frac{dE_y}{dy} \right].$$

When deriving this equation we have retained only terms proportional to the small parameter $k_y^2 \rho^2$ at the component E_y. Other thermal corrections in eqs. (3)are unimportant for our problem. Further, in all equations we neglected the terms connected with collisionless damping thus supposing $|\omega - n\omega_c| >> |k_z v_T|$.

From eqs. (3a), (3c) and 34) we can derive for E_x a differential equation of the sixth order in the form

$$\varepsilon_\parallel A E_x^{VI} - (\varepsilon_\parallel A)' E_x^V + \varepsilon_\parallel \varepsilon_\perp E^{IV} + 2\varepsilon_\parallel \varepsilon_\perp' E_x^{III} -$$

(35)

$$- \varepsilon_\parallel (k_z^2 \varepsilon_\parallel + k_v^2 g^2) E_x'' = 0$$

where

$$A = 3 v_T^2 \omega_0^2(y) / 2(\omega^2 - \omega_c^2)(\omega^2 - 4\omega_c^2) .$$

In eq. (35) we neglected the terms proportional to E_x and E_x' because they represent, near the hybrid resonance, small corrections to the electric field of the extraordinary wave. The thermal terms neglected in eqs. (3a), (3c) and (33) would give rise, in eq.(35), to small thermal corrections of the coefficients at the fourth and the lower derivatives of E_x.

To clarify the physical meaning of the terms connected with thermal motion of electrons let us first analyse the WKB solution of the equation (35). We get following expression for the wave vectors

(36) $$k^2_{y_{2,3}} = \frac{1}{2A}\left(\varepsilon_\perp \pm \sqrt{\varepsilon_\perp^2 - 4A\,(k_z^2\varepsilon_\parallel + k_r^2 g^2)}\right).$$

If the waves are in the neighbourhood of the hybrid resonance and the temperature of the plasma is not too high (i.e. $|k_r^2 A| \ll \varepsilon_\perp^2 \ll 1$) the $k_{y_{2,3}}$ can be rewritten in a simple form:

(37) $$k^2_{y_2} = -\frac{1}{\varepsilon_\perp}\left(k_z^2\varepsilon_\parallel + k_r^2 g^2\right),$$

(38) $$k^2_{y_3} = \frac{\varepsilon_\perp}{A}.$$

It is clear now that the expression (37) represents a square of the wave vector of the extraordinary wave near the hybrid resonance. The expression (38) represents a square of the wave vector of the Bernstein mode the frequency of which lies within the interval (ω_c , $3\omega_c$) (Kopecký et al. 1969). The amplitude of the excraordinary wave is proportional to $\sqrt[4]{\varepsilon_\perp}$ in accordance with the limit $\varepsilon_\perp \to 0$ made in eq. (7) for $C_2(y)$. For the WKB amplitude of the x-component of the electric field of the Bernstein modes we get

(39) $$C_3(y) = G\sqrt[4]{\frac{A^7(y)\,k_r^4}{\varepsilon_\perp^5(y)}}$$

The WKB approximation cannot be used near the intersection of the branches ($k_{y_2} = k_{y_3}$) i.e. in the region where $|\varepsilon_\perp| \simeq |k_r \sqrt{A}|$; To determine the amplitude of the Bernstein modes G we must analyse behaviour of the solution in this region more closely.

In the following account we replace the E_x - component by the E_y -component using the approximate relation valid near the hybrid resonance

(40) $$E_x'' = i k_v^2 g E_y .$$

We make so, because the waves are electrostatic in this region and the electric field is practically parallel to the y-axis ($|E_x| \ll |E_y|$). We further suppose that the density has the linear profile in the vicinity of the point of hybrid resonance

(41) $$\varepsilon_\perp = - \varkappa(y_H)(y - y_H) \equiv s .$$

Using (35), (40) and (41) we get the following equation of the fourth order for E_y

(42) $$\frac{d^4 E_y}{ds^4} + K\left(s\frac{d^2E_y}{ds^2} + 2\frac{dE_y}{ds} + \bar{D}E_y \right) = 0$$

where

$$K = \frac{2(\omega^2 - 4\omega_c^2)}{3v_T^2 \varkappa^2(y_H)} , \qquad \bar{D} = -\frac{(k_z^2 + k_v^2)}{\alpha^2 \varkappa^2(y_H)} .$$

In the above equation we omitted the terms proportional to d^3E_y/ds^3 as their effect on the form of the solution is is negligible. If we express E_y in eq. (42) by means of the potential ϕ given by the relation $E_y = -\frac{d\phi}{ds}$ and if we integrate the resulting equation with respect to s we obtain

(43) $$\frac{d^4\phi}{ds^4} + K\left(s\frac{d^2\phi}{ds^2} + \frac{d\phi}{ds} + \bar{D}\phi\right) = 0.$$

The equation (43), only with another constant D, was studied by Kopecký et al. The results of that paper can be therefore applied to our problems. We consider, for example, the physical situation in which the extraordinary wave ($E_x \sim C_2 \exp\{-i\int k_{y2}\, dy\}$) propagates from a denser plasma into the region of the hybrid resonance. Supposing further that $\omega < 2\omega_c$, i.e. $K < 0$, and that the amplitude of the wave is decreasing for $y < y_H$ we obtain the following asymptotic expression for E_x ($E_x \sim \int \phi\, ds$) in the region $y > y_H$

(44) $$E_x = \bar{C}\left\{\frac{\sqrt[4]{\pi^2 \bar{D} s}}{\bar{D}} \exp\left[i2\sqrt{\bar{D}s} - i\frac{\pi}{4}\right] - i\sqrt[4]{\frac{\pi^2}{K^3 s^5}} \exp\left[\frac{2}{3}i\sqrt{Ks^3} - i\frac{\pi}{4}\right]\right\}$$

If we join the solution (44) to the WKB solution we can find that the incoming extraordinary wave is fully transformed into the Bernstein mode($E_x \sim C_3 \exp\{-i\int k_3\, dy\}$) transfering energy back to a denser plasma. From the joining conditions we can determine the amplitude of the waves $\bar{C}$ in the region of transformation and the amplitude G of the Bernstein mode:

(45) $$|\bar{C}| = |TE^0_{x1}|\sqrt{\frac{2}{\pi}\frac{k_y^2}{\varkappa^3}\frac{\sqrt{1+\beta}}{\alpha^2(\sqrt{1+\beta}-1)}}$$

$$(46) \qquad |G| \cdot |TE^{0}_{x1}| \frac{2}{3} \frac{|\alpha^2-4|}{\alpha^3} \frac{c^2}{v_T^2} \sqrt{\frac{2\sqrt{1+\beta}}{\sqrt{1+\beta}-1}}$$

Similarly, we could also use other results of the paper by Kopecký et al. and obtain a solution for the case $\omega > 2\omega_L$. The results of this paper concerning the collisionless Doppler damping of the Bernstein modes can be used as well.

CONCLUSION

The foregoing analysis shows how the high-frequency electromagnetic wave can penetrate into a slowly-varying magnetized plasma at an oblique incidence. On entering the plasma the incident wave is split into an ordinary and an extraordinary wave. The latter is mostly reflected back to the boundary of a plasma independently of the angle of incidence. The reflection point is given by the condition (12). As for the ordinary wave, we may distinguish three cases according to the magnitude of the angle of incidence. If this angle is small, so that $N_z^2 \lesssim 1/(2\alpha+1)$, the situation is the same as that at a normal incidence and the ordinary wave is reflected in the point of the plasma resonance. At great angles of incidence ($N_z^2 > \frac{1}{\alpha}$) the wave is reflected in the point determined by the first condition from (16).

The ordinary wave, however, can penetrate through an evanescent layer into a denser plasma provided that its angle of incidence lies in the vicinity of the value $\mu_0 = \arcsin \frac{1}{\sqrt{\alpha+1}}$. The thickness of the evanescent layer is given by the conditions (16) and depends strongly on If $N_z^2 = 1/(\alpha+1)$ (i.e. $\mu = \mu_0$) the layer disappears and the wave is fully transmitted through the region of the plasma resonance. The transmission of the ordinary wave through the evanescent layer is sufficiently intensive in the following interval of angles of incidence

(47) $$\Delta\mu \simeq 2 \sqrt{\frac{\varkappa}{k_v}} \sqrt[4]{\frac{2\alpha}{T^2}}$$

If the angle of incidence fulfils the given condition then the transmission coefficient of the wave T is greater than $1/e$. For instance, the interval (47) is approximately equal to 27° for the parameters $\alpha = 1,5$, $k_v/\varkappa = 10$

After the transmission of the evanescent layer the ordinary wave propagates up to the point y_T given by the condition (14) where it is fully transformed into an extraordinary wave. It travels then back into the region of a lower density. In the neighbourhood of the hybrid resonance this extraordinary wave is fully transformed into the Bernstein mode. - It is thus evident that in the case of oblique incidence there exists, at least for some angle of incidence, a possibility of full transformation of electromagnetic waves to Bernstein modes.

The transverse wavelength of the Bernstein modes is comparable with the electron Larmor radius and thus its Doppler damping is much greater than the damping of the incident waves. This fact can be important for the plasma heating. The damping of the Bernstein modes increases particularly intensively if their frequency lies near the harmonics of the electron cyclotron frequency.

The amplitude of the electric field of waves becomes greater in the region of transformation. From eqs. (36), (44) and (45) we can deduce the following expression for this amplitude

$$|E| \sim |E_0| \left(\frac{c}{v_T}\right)^{3/4} \tag{48}$$

Such an essential increase of the amplitude of electric field can lead to a further increase of the wave damping in the place of the hybrid resonance due to nonlinear mechanisms.

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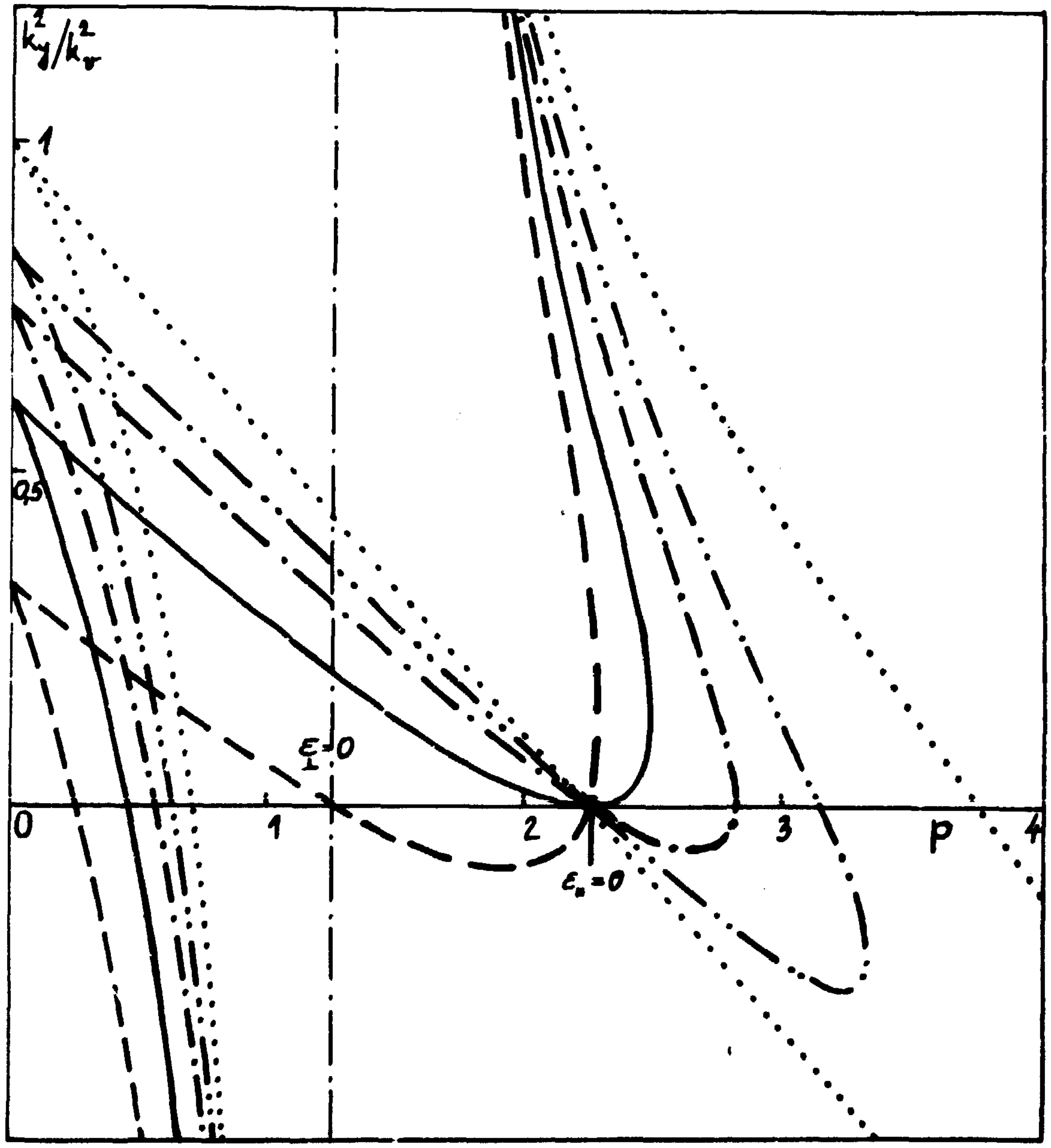

Fig.1. Square of the dimensionless wave vector component parallel to the density gradient $k_{y1,2}^2/k_v^2$ as a function of a dimensionless plasma density $\omega_0^2(y)/\omega_c^2$ for $\frac{\omega}{\omega_c} = 1,5$. The angle of incidence φ ($\sin\varphi = N_z$) is a parameter: $N_z^2 = 0$ - dotted line, $N_z^2 = 0,16$ - double dot-and-dashed line, $N_z^2 = 0,25$ - dot-and-dashed line, $N_z^2 = 0,4$ - full line, $N_z^2 = 0,66$ - dashed line.

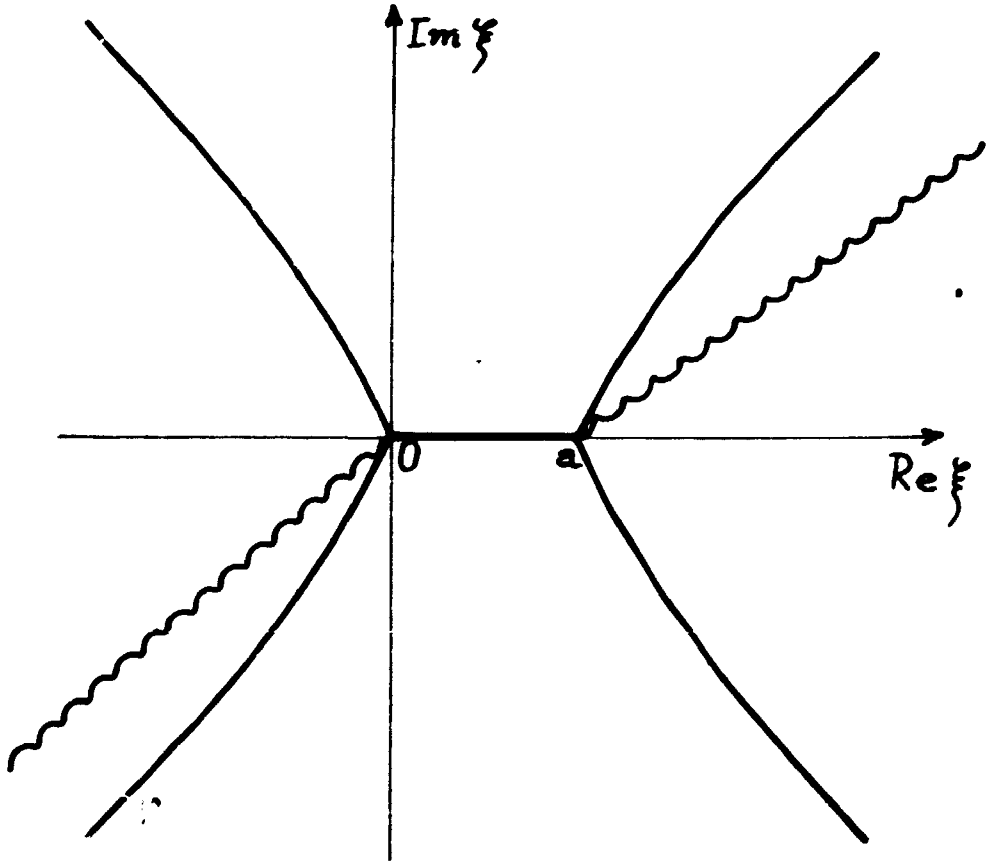

Fig. 2. The ξ -complex plane with cuts (wavy line) and Stokes line (full line) of the solution (20).