[illegible]
[illegible]

THE UNIVERSITY OF TEXAS AT AUSTIN

Center for Particle Theory

[illegible]

MASTER

ORO-3992-229

The Callan-Symanzik Equation in Gribov's Reggeon Field Theory and the Renormalized Pomeron Intercept

by

R. Acharya#

Institut fur Theoretische Physik, Freie Universitat
Berlin, West Berlin, Germany
and
Center for Particle Theory
University of Texas
Austin, Texas 78712*

*CPT visitor for July 1975.

Abstract

We utilize the Callan-Symanzik equation within the framework of Gribov's Reggeon field theory to prove that a <u>bare</u> Pomeron of unit intercept necessarily restricts the <u>renormalized</u> Pomeron to satisfy the same condition, i.e., $\alpha_{\text{renorm.}}(0) = 1$, <u>at</u> a nontrivial fixed point $\beta(g^*) = 0$ of the Callan-Symanzik equation. The proof of this assertion does not depend on the validity of the ε expansion ($\varepsilon = 4-D$).

In the Callan-Symanzik approach[1] to Gribov's Reggeon field theory,[2] it is ordinarily assumed that both the bare and the renormalized Pomeron have unit intercept: $\alpha_{bare}(0) = \alpha_{renorm}(0) = 1$. This assumption implies that a "dynamical" symmetry breakdown of the "zero mass" Pomeron field theory is impossible: If the bare Pomeron starts out "naked" with zero mass ("mass" = $\Delta_0 = 1 - \alpha_{bare}(0)$), then the interaction (triple Pomeron and higher) cannot produce a mass. In a completely different context, Gross and Neveu[3] have observed that infra-red stable theories cannot produce masses dynamically, with one possible exception: those theories in which the physical coupling constant is fixed at a nontrivial fixed point of the renormalization group, such as the Adler[4]-Johnson-Baker-Willey[5] electrodynamics. As a matter of fact, Schnitzer[6] has conjectured that a necessary condition for the dynamical breakdown of symmetry in a massless, relativistic field theory with just one dimensionless coupling constant, is the existence of a zero, $\beta(g^*) = 0$, in the Callan-Symanzik function.

In this brief note, we prove the following result: a massless bare Pomeron in Gribov's field theory with a nonvanishing bare, triple-Pomeron coupling, cannot acquire mass dynamically, at a fixed point of $\beta(g)$. This result may be of some interest in view of the recent work of Marchesini and Veneziano,[7] in which they raise the interesting point about possible constraints on the renormalized Pomeron intercept, in a theory satisfying

both 's' and 't' channel unitarity.

We follow closely the work of Abarbanel and Bronzon[8] in establishing our result and shall therefore omit details to which we refer to ref. 8. The canonical starting point of Gribov's Reggeon field theory is the specification of the Lagrangian in the space of the rapidity $\tau = \ln s$ and the impact parameter $\vec{x}$ respectively, with the "energy" $E = 1-J$ and the two-dimensional transverse momentum $\vec{k}$ playing the role of the corresponding conjugate variables. The simplest and the most obvious choice for L is a theory with a bare, linear trajectory, $\alpha_0(t) = \alpha_0 + \alpha_0{}'t$ $(t = -\vec{k}^2)$ and an interaction with a nonvanishing, bare triple-Pomeron coupling. According to an argument of Wilson,[9] higher order couplings and higher powers in $\vec{k}^2$ will not, in general, destroy the infra-red behaviour of the theory.

So, following ref. 2 and ref. 8, we take

$$L = L_0 + L_I$$

$$L_0 = \frac{i}{2}\,[\psi^+(\vec{x},\tau)\overleftrightarrow{\partial}_\tau\psi(\vec{x},\tau)] - \alpha_0{}'\nabla\psi^+(\vec{x},\tau)\nabla\psi(\vec{x},\tau) - (1-\alpha_0)\psi^+(\vec{x},\tau)\psi(\vec{x},\tau) \qquad (1)$$

and

$$L_I = -\frac{i\gamma_0}{2}\,[\psi^2\psi^+ + \psi\psi^{+2}] \qquad (2)$$

The purely imaginary bare, triple-Pomeron coupling is suggested by Gribov's signature analysis[2] and it is crucial to obtain infra-red stability, at least in perturbation theory. At the very outset, we deviate from Abanbanel and Bronzon: we require that although the bare Pomeron is massless, i.e., $\Delta_0 = 1 - \alpha_0 = 0$, the renormalized Pomeron is <u>not</u>: $\Delta = 1-\alpha \neq 0$. Consequently, we employ the following renormalization conditions on the renormalized Green's functions:

$$\Gamma_R^{(1,1)}(E,\vec{k}^2)\big|_{\substack{E=\Delta \\ \vec{k}^2=0}} = 0 \tag{3}$$

$$\frac{\partial}{\partial E}\,[i\Gamma_R^{(1,1)}(E,\vec{k}^2)]\big|_{\substack{E=-\Delta \\ \vec{k}^2=0}} = 1 \tag{4}$$

$$\frac{\partial}{\partial k^2}\,[i\Gamma_R^{(1,1)}(E,\vec{k}^2)]\big|_{\substack{E=-\Delta \\ \vec{k}^2=0}} = -\alpha'(\Delta) \tag{5}$$

and

$$\Gamma_R^{(1,2)}(E_1,\vec{k}_1,E_2,\vec{k}_2,E_3,\vec{k}_3)\big|_{\substack{E_1=2E_2=2E_3=-\Delta \\ \vec{k}_i\cdot\vec{k}_j=0}} = \frac{\gamma(\Delta)}{(2\pi)^{\frac{D+1}{2}}} \tag{6}$$

The renormalized intercept Δ, therefore, plays a dual role in the theory. As emphasized in ref. 8, Eq. (3) does <u>not</u> commit us to a renormalized Pomeron being a simple pole, but it only ensures that the singularity passes through $\alpha = 1-\Delta$ at $t = 0$

(rather than at $\alpha = 1$, as in ref. 8). Since there is no bare mass in the theory ($\Delta_0 = 0$), the Coleman[10] method of deriving the C-S equation gives

$$\left[\Delta\frac{\partial}{\partial\Delta} + \beta(g)\frac{\partial}{\partial g} + \zeta(g,\alpha')\frac{\partial}{\partial\alpha'} - \frac{(n+m)}{2}\gamma(g)\right]\Gamma_R^{(n,m)}(E_i,\vec{k}_i,g,\alpha',\Delta) = 0 \tag{7}$$

where

$$g(\Delta) = \frac{\gamma(\Delta)}{(\alpha'(\Delta))^{\frac{D}{4}}}\,\Delta^{\frac{D}{4}-1} \tag{8}$$

and the rest of the notation is standard.[8]

Now, suppose the renormalized coupling chooses to sit precisely at a fixed point 'g^*' satisfying $\beta(g^*) = 0$, $\beta'(g^*) > 0$. Then, we can derive the following Gribov-Migdal scaling law which is valid for <u>all</u> E and $\vec{k}^2$, <u>at</u> the fixed point:[8]

$$\Gamma_R^{(1,1)}(E,\vec{k}^2,g^*,\alpha',\Delta) = \Delta\left(-\frac{E}{\Delta}\right)^{1-\gamma(g^*)} \times \Phi_{1,1}\left[\left(-\frac{E}{\Delta}\right)^{-z(g^*)}\frac{\vec{k}^2\alpha'}{\Delta},\ g^*\right] \tag{9}$$

Eq. (3) and Eq. (9) demand that

$$\Phi_{1,1}(0,g^*) = 0 \tag{10}$$

since the renormalized intercept $\Delta \neq 0$, by assumption.

But then, Eq. (9) and Eq. (10) yield the unphysical result,

$$\Gamma_R^{(1,1)}(E, \vec{k}^2 = 0, g^*, \alpha', \Delta) = 0 , \quad \underline{\text{all}}\ E \qquad (11)$$

Therefore, we are forced to the conclusion that Δ must vanish to restore consistency.

At first sight, this result appears to contradict the result obtained by Sugar and White.[11] They have earlier arrived at the conclusion that the bare Pomeron pole must have an intercept greater than unity, in order for the renormalized Pomeron singularity to have unit intercept. The basis of their result is an integral representation for the intercept counter term derived in ref. 11:

$$\delta\Delta = \left[\frac{\gamma_0^2}{(\alpha_0')^{\frac{D}{2}}} \right]^{2/\epsilon} \int_0^\infty \frac{dx}{x^2} \left[1 - Z_3^{-1}(x) \right] \qquad (12)$$

As pointed out in ref. 11, the integral in Eq. (12) has a pole in ϵ at the physical value $\epsilon = D = 2$ so that $\delta\Delta$ is not well-defined unless an ultraviolet cut-off is introduced by hand. The Sugar-White argument is therefore cut-off dependent. In any event, in the limit of $g \to g^*$, the bare coupling $\gamma_0 \to \infty$[11] (for $E_N \neq 0$, where E_N is the subtraction point). As a result, the Sugar-White representation Eq. (12) is not useful at the fixed point $\beta(g^*) = 0$, at which our result holds.

The author is indebted to Professor A. Gleeson and Professor E.C.G. Sudarshan for their kind hospitality. It is a pleasure to acknowledge scores of critical and valuable discussions with Professor H. Kleinert on the subject of Gribov's Reggeon field theory.

References

1. C. G. Callan, Phys. Rev. D2, 1541 (1970).
   K. Symanzik, Commun. Math. Phys. 18, 227 (1970).
2. V. N. Gribov, Sov. Phys. J.E.T.P. 26, 414 (1968).
3. D. Gross and A. Neveu, Phys. Rev. D10, 3235 (1974).
4. S. L. Adler, Phys. Rev. D5, 3021 (1972).
5. K. Johnson and M. Baker, Phys. Rev. D8, 1110 (1973) and references therein.
6. H. J. Schnitzer, Phys. Rev. D10, 1800 (1974)
7. G. Marchesini and G. Veneziano, Phys. Lett. 56B, 271 (1975).
8. H. Abarbanel and J. Bronzon, Phys. Rev. D9, 2397 (1974); see also, M. Baker, Nuclear Physics B80, 61 (1974).
9. K. Wilson and J. Kogut, Phys. Reports. 12c (1974).
10. S. Coleman, lectures given at the 1973 International Summer School of Physics, "Ettore Majorana", Erica, Italy.
11. R. L. Sugar and A. R. White, Phys. Rev. D10, 4074 (1974). See also, W. R. Frazer and M. Moshe (to be published) and D. Amati and R. Jengo, Phys. Lett. 56B, 465 (1975).

#Supported in part by the DFG grant No. KL 25614 and U. S. Atomic Energy Commission, Contract No. AT(40-1)3992.