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GENERATION OF AN INTENSE STATIONARY WAVE IN MODULATED BEAM-PLASMA SYSTEMS

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ABSTRACT

Basic equations and numerical results, describing the nonlinear interaction of a weak modulated electron beam with a single stationary one-dimensional wave, excited in a cold magnetic field free plasma, are presented and discussed. The effect of all possible irreversible processes (e.g. plasma turbulence) accompanying this interaction is simulated by a constant effective collision frequency ν_{eff} of plasma electrons. Starting from the nonlinear Poisson equation, expressions for the amplitude and phase of the beam-excited wave are derived and then solved numerically, together with the equations of the beam electrons motion. The results are compared with those of the time model. Remarkable differences detectable experimentally are established.

1. INTRODUCTION

Both theoretical and experimental activities, devoted to beam-plasma systems, exhibit remarkable revivification in the last few years. This is connected with the discovered role of the strongly nonlinear effect of the beam particles trapping into the potential wells of a single quasimonochromatic wave, excited by an initially cold electron beam (e.g. O'Neil and Winfrey (1972); Shapiro and Shevchenko (1971); Piffl et al. (1971)). For simplicity reasons theoreticians prefer time to spatial models. Although these two models often have a lot of common features, we shall demonstrate that there may be also remarkable differences between them. The spatial model is obviously the appropriate one for the description and explanation of experimental data measured on facilities with a permanent injection of a pre-modulated electron beam into the interaction region. These experiments are not so frequent as experiments in which waves are spontaneously excited from the thermal noise level by an unmodulated beam. They have, however, at least one significant advantage from the physical point of view, by providing more precise and more reproducible experimental results. It is therefore worth discussing the specific features of the spatial problem more in detail.

Limiting our attention only to the simplest model able to perform this task we study an one-dimensional wave ($k_\perp \approx 0$) with a stationary amplitude $E_0(x)$ and fixed frequency ω_M (different in general from the electron plasma frequency ω_p), excited in a magnetic field free, cold, but generally "collisional" plasma. (To explain the total loss of energy of the electron beam leaving the interaction region, observed as a rule in real experimental facilities, the wave absorption has to be respected in general). The corresponding collision frequency ν_{eff}, of course, need not be necessarily determined by binary collisions only. The time development of the instability in a dense plasma with dominant binary collisions $\gamma_L << \nu$ (γ_L is the linear growth rate has been studied recently by Ivanov et al. (1972). Also if binary collisions are negligible, however, some weakly nonlinear processes (like parametric instabilities) may produce a turbulent state of plasma and can be thus responsible for the absorption of the beam-generated wave energy in a stationary state. For times shorter then the characteristic time of an effective beam-excited wave energy transformation due to weakly nonlinear interactions, the spontaneous excitation of the wave is practically non-dissipative ($\nu_{eff} \approx 0$). Since in this case plasma eigen-modes have a zero group velocity, solutions of the time model have no relation to the problem under investigation. If, however, a stationary regime is established in a system with a modulated beam, the time duration of the interaction from the point of view of plasma electrons is significantly longer and various weakly nonlinear processes may be fully developed. The absorption of the beam-excited wave is then hardly negligible in general.

There exists another difference between time and spatial models also discussed in the present paper. The excitation of a wave with the frequency ω_M different from that of the most unstable wave ($\omega \approx \omega_p$) can be forced upon the system by an appropriate pre-modulation of the beam. Direct influence of a finite temperature of plasma electrons resulting in a non-zero group velocity of the excite plasma eigen-modes, is neglected, on the other hand, for the following reasons :

a) It has been demonstrated from the collisionless magnetic-field free hot plasma model by Shapiro, Shevchenko (1972) and O'Neil, Winfrey (1972) that the same system of dimensionless equations describes the excitation of the most unstable modes both in the time and spatial model, the actual physical quantities differing only by a constant scaling factor.
b) The temperature of plasma electrons measured by e.g. Šunka and Jungwirth (1974) is significantly lower than it would be necessary for the group velocity of the excited wave to become comparable with the beam velocity v_b , the observed value of the linear spatial growth rate to be explicable by the plasma temperature effect only. Also other relations between the measured characteristics of the interaction differ significantly from those predicted from the collisionless hot plasma model.

2. BASIC EQUATIONS

The intensity of the wave electric field, $E(x,t) = -\dfrac{\partial \Psi}{\partial x}$ having a stationary amplitude $E_0(x)$, may be written as

$$E(x,t) = Re\, E_0(x)\, e^{i\left(\int^{x} k_0(x')dx' - \omega_M t\right)} \tag{2.1}$$

in the region, where a single strong one-dimensional wave is excited by the beam pre-modulated at a frequency ω_M . $k_0(x)$ is the real part of the wave vector $k(x)$ and

$$E_0(x) = E_0(0)\, e^{-\int^{x} \varkappa(x')\,dx'} \;; \qquad \varkappa(x) = Im\, k(x). \tag{2.2}$$

(Without any loss of generality we further choose e.g. $i\,E_0(x) < 0$).

Our aim is to determine $k(z)$, and the development of the distribution function of beam electrons. Assuming that the unperturbed density of the initially cold electron beam n_b is sufficiently low if compared with the plasma density n_p, that the unperturbed beam velocity v_b exceeds sufficiently the thermal velocity of plasma electrons v_T and simulating all possible weakly nonlinear processes by introducing a constant ν_{eff}, the Poisson equation may be written as

$$(2.3) \quad \varepsilon_p(\omega_M) E_0(0) \frac{\partial}{\partial z} e^{i\left(\frac{\omega_M}{v_b} z - \int_0^z \delta k(z')dz'\right)} = 4e\omega_M \int_0^{\frac{2\pi}{\omega_M}} e^{i\omega_M t} \rho_b(z,t)\,dt$$

where

$$(2.4) \qquad \delta k(z) \equiv k(z) - \frac{\omega_M}{v_b} \; ; \qquad \varepsilon_p(\omega) \equiv 1 - \frac{\omega_p^2}{\omega(\omega + i\nu_{eff})}$$

is the linear plasma dielectric constant and $\rho_b(z,t)$ the actual beam density.

For the numerical calculation it is convenient to substitute a set of equal beam-sheets, succeeding each other after the same time interval $2\pi/N\omega_0$, for the unperturbed beam. If, further, the equation of continuity

$$(2.5) \qquad \frac{\partial \rho_b}{\partial t} + \frac{\partial}{\partial z}\left(v_b(z,t)\,\rho_b(z,t)\right) = 0$$

is respected and

$$\delta\omega \equiv \omega - \omega_p \ll \omega_p \; ; \qquad \nu_{eff} \ll \omega_p$$

is assumed to be valid, the eq. (2.3) gets the form

$$(2.6) \quad \frac{2\delta\omega + i\nu_{eff}}{\omega_p} \left|E_0(z)\right| = \frac{8\pi|e|n_b v_b}{N\omega_M} \sum_{j=1}^{N} e^{i\left(\varphi_j - Re\int_0^z \delta k(z')dz'\right)}$$

where

$$(2.7) \qquad \varphi_j \equiv \omega_M \left(t_j - \frac{z}{v_b}\right),$$

$t_j(z)$ being the transit time of the j-th sheet through a given z-plane. The summation on the right-hand side of eq. (2.6) is performed over all N clusters that enter the interaction region during the same period of the excited wave.

By introducing the dimensionless variable

$$(2.8) \qquad \eta \equiv -\varkappa_L z \equiv A \frac{\omega_0 z}{v_b}$$

and the dimensionless phase and amplitude of the excited wave

$$(2.9) \qquad \mu(\eta) \equiv -\frac{\delta k(z)}{\varkappa_L} \equiv -\frac{\delta k(z) v_b}{A\omega_M}.$$

$$(2.10) \qquad F(\eta) \equiv \left| \frac{e E_o(x) \omega_M}{m \varkappa_L^2 v_b^3} \right| \equiv \left| \frac{e E_o(x)}{m \omega_M v_b A^2} \right|$$

the real and imaginary part of eq. (2.3) lead to the equations

$$(2.11) \qquad \frac{\varkappa_L^2}{\varkappa_\nu^2} F(\eta) = - \frac{2}{N} \sum_{j=1}^{N} \cos \left(\varphi_j(\eta) - \beta(\eta) \right)$$

$$(2.12) \qquad \frac{\varkappa_L^2}{\varkappa_{\delta\omega}^2} F(\eta) = \frac{4}{N} \sum_{j=1}^{N} \sin \left(\varphi_j(\eta) - \beta(\eta) \right)$$

where

$$(2.13) \qquad \varkappa_\nu^2 = \frac{\omega_{pb}^2 \omega_p}{2 |\delta\omega| v_b^2} ; \qquad \varkappa_{\delta\omega}^2 = \frac{\omega_{pb}^2 \omega_p}{2 \nu_{eff} v_b^2} ; \qquad \beta(\eta) = \int_0^\eta \mu(\eta') d\eta' .$$

The quantities $\varkappa_L$, $\varkappa_\nu$ and $\varkappa_{\delta\omega}$ (mutually dependent, of course) have the following physical meaning : $\varkappa_L$ is the actual spatial growth rate of the beam-excited wave; $\varkappa_\nu$ is the hypotetical spatial growth rate of the same wave, but in a collisionless plasma and $\varkappa_{\delta\omega}$ is the spatial growth rate of a wave with $\delta\omega = 0$, excited in the studied system with the common ν_{eff} . It is worth mentioning that in the limits

$$(2.14a) \qquad \delta\omega \to 0 \;\Rightarrow\; \varkappa_L^2 \to \varkappa_{\delta\omega}^2 ; \qquad \varkappa_\nu^2 \to +\infty$$

$$(2.14b) \qquad \nu_{eff} \to +0 \;\Rightarrow\; \varkappa_L^2 \to \varkappa_\nu^2 ; \qquad \varkappa_{\delta\omega}^2 \to +\infty$$

eqs. (2.11) and (2.12) are completely independent on the beam and plasma parameters. In general, however, the relations between $\varkappa_L$, $\varkappa_\nu$ and $\varkappa_{\delta\omega}$ are rather complicated. We use, therefore, another approach to retain only the independent parameters. (It proves to be only a single such parameter μ_L the linear value of $\mu(\eta)$) in equations for $F(\eta)$ and $\mu(\eta)$.

By using the eqs. (2.13), (2.14a), it can be easily proved that the equation of motion of the j-th sheet

$$(2.15) \qquad \frac{d^2 z_j}{dt^2} = \frac{|e E_o(z_j)|}{m} \sin \left(\int_0^{z_j} k(z') dz' - \omega_M t \right)$$

may be transformed into the following dimensionless form

$$(2.16) \qquad \frac{d^2 \varphi_j}{d\eta^2} = F(\eta) \left(1 + A \frac{d\varphi_j}{d\eta} \right)^3 \sin \left(\varphi_j(\eta) - \beta(\eta) \right)$$

Such a transformation is in fact identical with that performed by O'Neil and Winfrey (1972). We have retained, however, also the term proportional to $3A\,d\varphi_j/d\eta$ and its higher powers on the right-hand side of the eq. (2.16). Since we were not pressed to omit such terms even in the equations (2.11) and (2.12), we have thus conserved the principial possibility to check up their actual significance.

The eqs. (2.11), (2.12) and (2.16) form a closed set that, however, can be integrated analytically in the linear region $\left|\varphi_j - \frac{2\pi_j}{N}\right| \ll 1$ only. To get the corresponding non-trivial solution

(2.17)
$$F_L(\eta) = F(0)\,e^{\eta} \quad ; \qquad \mu_L(\eta) = \mu_L(0)$$
$$\varphi_{jL}(\eta) = \frac{2\pi_j}{N} + \frac{F(0)}{(1+\mu_L^2)^2}\left[(1-\mu_L^2)\sin\left(\frac{2\pi_j}{N} - \mu_L\eta\right) + 2\mu_L\cos\left(\frac{2\pi_j}{N} - \mu_L\eta\right)\right]$$

the following equations have to be valid. (Their relation to the linear dispersion equation is obvious).

(2.18a)
$$\frac{\varkappa_L^2}{\varkappa_\nu^2} = \frac{1-\mu_L^2}{(1+\mu_L^2)^2}$$

(2.18b)
$$\frac{\varkappa_L^2}{4\varkappa_{\delta\omega}^2} = \frac{\mu_L}{(1+\mu_L^2)^2}$$

It follows from the eqs. (2.13) and (2.18a,b) that $\mu_L \in (0,1)$ is a monotonic function of $\delta\omega/\nu_{eff}$, being unity for $\delta\omega = 0$ and zero for $\nu_{eff} = 0$.

The expression for $\varphi_{jL}(\eta)$ determines the boundary conditions at $\eta = 0$ for the numerical calculations, whereas the relations (2.18a,b) enable us to exclude the excess parameters from eqs. (2.11) and (2.12). We thus get

(2.19)
$$\frac{1-\mu_L^2}{(1+\mu_L^2)^2}\,F(\eta) = -\frac{2}{N}\sum_{j=1}^{N}\cos\left(\varphi_j(\eta) - \beta(\eta)\right)$$

(2.20)
$$\frac{\mu_L}{(1+\mu_L^2)^2}\,F(\eta) = \frac{1}{N}\sum_{j=1}^{N}\sin\left(\varphi_j(\eta) - \beta(\eta)\right) .$$

These equations are identical with those derived under less restrictive assumptions from the energy momentum conservation laws (Jungwirth, 1973).

By excluding $F(\eta)$ from the last two equations we get the implicit equation for $\beta(\eta)$

(2.21)
$$\frac{1}{N}\sum_{j=1}^{N}\left\{(1-\mu_L^2)\sin\left(\varphi_j(\eta) - \beta(\eta)\right) + 2\mu_L\cos\left(\varphi_j(\eta) - \beta(\eta)\right)\right\} = 0$$

Multiplying eq. (2.19) by $(1-\mu_L^2)$ and eq. (2.20) by $2\mu_L$ and summing the results, we get

$$(2.22)\quad F(\eta)=\frac{2(1+\mu_L^2)^2}{(1+\mu_L^4)N}\sum_{j=1}^{N}\left\{\mu_L \sin(\varphi_j(\eta)-\beta(\eta))-(1-\mu_L^2)\cos(\varphi_j(\eta)-\beta(\eta))\right\}$$

Differentiating the eq. (2.21) with respect to η and respecting eqs. (2.21), (2.22), we can derive a similar expression for $\mu(\eta)$, too

$$(2.23)\quad \mu(\eta)=\frac{2}{NF(\eta)}\sum_{j=1}^{N}\frac{d\varphi_j}{d\eta}\left[2\mu_L \sin(\varphi_j(\eta)-\beta(\eta))-(1-\mu_L^2)\cos(\varphi_j(\eta)-\beta(\eta))\right]$$

Some preliminary numerical results derived from eqs. (2.16), (2.21-3) have been reported by Jungwirth, Krlín (1973). In what follows more complete and detailed data are presented and discussed.

NUMERICAL RESULTS

Trajectories of 500 beam-particles per one wave-length have been calculated with the step $\Delta\eta = 0{,}05$. Numerical calculations have been performed for $\mu(0) = 0$; $F(0) = 1/4$ (Figs. a) which corresponds to a dominant effect of the negative plasma dielectric constant - the off-resonance case, for $\mu(0) = 1/\sqrt{3}$; $F(0) = 1/6$ (Figs. b) which corresponds to the maximum value of the linear spatial growth rate $\varkappa_L = \varkappa_{L\,max}$ calculated formally for a given constant ν_{eff} and for $\mu(0) = 1$; $F(0) = 1/4$ (Figs. c) which corresponds to a dominant absorption. Solving the eq. (2.16) the value $A = 0{,}04$ of the scaling factor (relating the dimensionless and actual parameters of the system) has been used. By this choice the numerical results are very well applicable at least to systems with $A \in (0; 0{,}08)$, which is the typical region of operation of beam-plasma systems. To be able to compare the dynamics of the interaction with that of the time model, also some results obtained by Jungwirth, (1974) (Fig. 1d) have been included. In the latter case

$$(3.1)\quad \tau \equiv \mu_L t\,; \qquad F(\tau)\equiv\left|\frac{e k_0 E(t)}{m\mu_L^2}\right|; \qquad \mu(\tau)\equiv\frac{k_0 v_0 - \omega_0(t)}{\mu_L}\quad .$$

From Figs. 1a-d the oscillating character of the squared wave amplitude F^2 (solid line) is obvious. These oscillations are, however, far less regular in the spatial models than in the temporal one. If the fundamental wave absorption plays an important role in the interaction, the first and partially the second peaks of $F^2(\eta)$ are dominant(Figs. 1b,c). This can be seen also from the evolution of the integral amount of energy transferred from beam to the excited wave $\Delta W_b = \frac{W_b - W_b(z)}{\Delta W_b}$ (dotted line). The absorption of the fundamental wave results in the proportionality of $F^2(\eta)$ to the first derivative of ΔW_b. In the time problem the oscillations of $F^2(\eta)$ are almost regular Fig. 1d. The corresponding beam energy losses are proportional to $F^2(\eta)\,(F^2(\eta) - \frac{16}{9\sqrt{3}}\,\frac{W_b - W_b(z)}{W_b\,\frac{k_b}{\omega}})$ and are not, therefore, indicated in the figure.

In the off-resonance case with negligible absorption ($\mu_L = 0$) the averaged beam energy remains almost unchanged in the stationary state. Thus, the interaction between the beam and the wave is limited to the exchange of momentum only. This proves to be the sufficient reason for the wave amplitude and phase to be spatially dependent functions. To check the numerical accuracy, we have calculated separately the averaged beam energy also in the case $\mu = 0$. The constancy of this value better than   percents of the unperturbed beam energy has been determined within the whole interval of investigated $\eta \in (0, 9,2)$. Unlike the first maximum of $F^2(\eta)$ reached for $\eta = 3,3$, the second maximum at $\eta = 6,3$ is so little expressive that we may consider $F^2(\eta)$ to be almost constant inside the interval $4,9 < \eta < 7,2$. For $\eta > 7,5$ the squared amplitude starts to grow again, the third maximum being comparable with the first one.

By completing the manuscript of this paper we have become acquinted with the fact that Rogashkova et al. (1973) have solved a special example of the wave excitation which should be involved in our off-resonance model. Their dimensionless coordinate ξ is related to η simply as $\xi = \eta/\Lambda$. The following special choice of the wave, beam and plasma frequencies $\frac{\omega_p^2}{\omega^2} = 1,6$; $\frac{\omega_{pb}^2}{\omega^2} = 0,01$ indicates that $\Lambda = 0,13$ has to be chosen in our approach the same case to be studied in both papers. Nevertheless, we have detected significant differences in the wave amplitude evolution. The somewhat higher value of the absolute maximum of $F^2(\eta) \approx 2,9$ and the almost same values of amplitude at the second and the third maximum are typical for their results. Only the distances between the corresponding maxima of $F^2(\eta)$ have remained practically unchanged. Searching for reasons of the mentioned discrepancies we turned our attention to the choice of the initial conditions different in every case. In our approach we have taken the purely growing selfconsistent solution of the linearized problem as initial conditions, the amplitude of the velocity perturbation being 0,03 v_b for $\Lambda = 0,13$ at $t = 0$. The exponential growth of $F^2(\eta)$ at least for $\eta \leq 2$ is obvious Fig. 1a. By choosing a higher value of the initial beam velocity perturbation $v = v_b(1 + \alpha \cos \omega t)$, $\alpha = 0{,}1$ not accompanied by the appropriate density perturbation, the transition from the linear to the nonlinear region has not been achieved in the case studied by Rogashkova et al. (1973). (The region of an initial exponential growth is missing in their results). Since in the off-resonance case beam and phase velocities are very close one another, also the further nonlinear stage of interaction may be strongly influenced by the deeper modulation of the beam velocity. They are, therefore, worth comparing also the beam particles velocity distributions Fig. 2a, as well as the phase (density) distributions Fig. 2a. Whereas, following Rogashkova et al. (1973), the beam splits into two clusters rotating in the phase space the beam distribution being as a rule a double humped function with an expressive minimum at $v = v_b$ (excepting $\eta \approx 2{,}8;\ 5{,}6;\ 8{,}3$ where these two peaks fuse temporarily), our distribution functions have three maxima (one at $v = v_b$) if the velocity spread of the beam is not too small. Also the density distributions indicate that in our case the portion of beam electrons near the bottom of the wave potential well remains steadily at a relatively high level. We may conclude that for the instability starting at lower initial levels (within the region of validity of the linear approximation) the beam does not split into two separate clusters, making thus understandable the relative suppression of the spatial oscillations of the excited wave amplitude.

As far as the maximum value of the actual wave electric field intensity is concerned, we may get only qualitative information if $\nu_{eff} \neq 0$. (The actual value of ν_{eff} i.e. of $\varkappa_L$ remains undetermined in our approach and has to be taken from experiment). Nevertheless, the numerical results and scaling eqs. (2.10), (3.1) imply that for

$$(3.2) \qquad \varkappa_L v_b > 3^{3/4} \mu_L$$

the amplitude of the stationary wave reaches higher values $|E_{max}(z)| > |E_{max}(t)|$. Since it is reasonable to assume that $\nu_{eff} \ll \mu_L$, the inequality (3.2) should be satisfied if plasma eigen-modes are excited at not too high pressures. If, however, in the off-resonance case $|\omega_M - \omega_p| > (\mu_L, \nu_{eff})$, the inequality (3.2) is no more valid. Therefore, lower field intensities are then attained than in cases adequately described by the time model.

In Figs. 1 also the normalized difference between the unperturbed beam velocity v_b and the instantaneous phase velocity v_{ph} is plotted. In the off-resonance case the phase velocity remains very close to v_b (they coincide exactly in the linear region and $\mu(\eta) \ll 1$ in latter stages of interaction). The absorption of the wave energy results in expressive abrupt changes of $\mu(\eta)$ (Figs. 1b,c) - which are apparently greater than those in the time problem (Fig. 1d) - superposed on the systematic $\mu(\eta)$ increase. The abrupt local shortening of the wave-length near each minimum of the stationary wave amplitude is the very cause of the excited wave to grow again downstream in the dissipative model.

Evolution of the wave calculated in the time and in the spatial dissipative and off-resonance models has, thus, certain common features (oscillating character of the wave amplitude, decrease of the phase velocity). There are, however, also significant differences which it should be possible to distinguish experimentally. (Note e.g. the apparently larger changes in the phase velocity established in spatial dissipative models or the differences in the distance between the first and the second maximum of the wave amplitude. Following Figs. 1b,c,d these distances are related roughly as 1 : 2/3 : 2 for a common value of the linear spatial growth rate). Detailed comparison between numerical and experimental results has been performed by Šunka and Jungwirth (1974). Very good agreement with the spatial dissipative model has been found in the investigated case of excitation of the upper branch of electron oscillations of a magnetised plasma by a modulated beam.

To get a more complete insight into the physical mechanism of the interaction, we turn now the attention to the phase-space diagrams describing the beam evolution. In Figs. 3a,b the dimensionless velocity $-\dfrac{d\varphi_j}{d\eta}$ and phase related to the wave ($\beta - \varphi_j$) of each third beam-sheet are plotted. It is a common feature of all single-wave models that beam particles become effectively trapped by the excited wave in the nonlinear region. Also the formation of a heavily populated bunch of beam particles preceeding the first maximum of the wave amplitude always takes place ($\eta = 2,8$ in the off-resonance model Fig. 3a, $\eta = 2,6$ in the dissipative model Fig. 3b). Further, however, significant differences in the beam evolution occur.

In the time model almost all beam particles are trapped by the excited wave and a dominant part of them performs quasiperiodic oscillations near the bottom of the potential well (Shapiro, Shevchenko, 1972). This results in quasiperiodical oscillations of the fundamental wave amplitude. During each period the well trapped particles perform a complete cycle consisting of the decelerating and the accelerating phase, the beam energy varying thus complementary to that of the wave.

In the off-resonance case almost all beam particles are trapped by the wave, too. Since, however, beam particles have to conserve their energy in averaged, the momentum transferred by the beam particles decreases with the increasing spread in beam particles velocities and vice versa, leading to oscillations of the wave amplitude. As it can be seen from Fig. 3a trapped beam particles oscillate within the whole potential trap (they are not well trapped in general), the oscillations of the amplitude being, therefore, far less regular than in the time model. Rotation of trapped particles in the phase space is characterized by the acceleration of one half of the trapped particles accompanied by the simultaneous deceleration of the second half in each moment.

In the dissipative model ($\mu = 1/\sqrt{3}$) Fig. 3b only the decelerating phase of the trapped particles oscillations takes place at the distance of one spatial period of the wave amplitude. The abrupt shortening of the wave-length near each minimum of the wave amplitude returns namely these particles into the region of deceleration which results in a permanent decrease of energy of the trapped particles, as well as in an increased portion of the untrapped particles (1/3 approx.).

4. CONCLUSION

It was the aim of this paper to demonstrate that theoretical results concerning the time evolution of the beam-plasma interaction are sometimes irrelevant to the spatial problem. Such situation occurs if either the group velocity of the wave determined in the absence of the beam is far less than its phase velocity or if the beam is modulated at an off-resonance frequency. In the former case energy absorption of the excited wave caused by weakly nonlinear parametric--like processes may become dominant for the beam-plasma interaction, whereas in the latter the energy transfer is negligible and the whole interaction is limited to the exchange of momentum between beam particles and the single excited strong wave.

We have further proved by comparing the numerical results of all three models (the time-like model, the dissipative model and the off-resonance one) that even at a common value of the linear spatial growth rate the nonlinear evolution of the excited wave amplitude and phase, as well as the beam dynamics differ significantly. It is, of course, possible to distinguish these differences also experimentally. Moreover, under appropriate conditions each model may become the adequate one for description of the single strong wave excitation in real experimental facilities.

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FIGURE CAPTIONS

Fig. 1.a. Squared wave amplitude $F^2(\eta)$ (solid line) and phase velocity $\mu(\eta)$ (dashed line) for $\mu_L = 0$

Fig. 1.b. Squared wave amplitude $F^2(\eta)$ (solid line), total beam energy losses $\Delta W_b = \frac{2\Delta v_b}{A v_b}$ (dotted line), and phase velocity $\mu(\eta)$ (dashed line) for $\mu_L = \frac{1}{\sqrt{3}}$

Fig. 1.c. Squared wave amplitude $F^2(\eta)$ (solid line), total beam energy losses $\Delta W_b = \frac{2\Delta v_b}{A v_b}$ (dotted line), and phase velocity $\mu(\eta)$ (dashed line) for $\mu_L = 1$

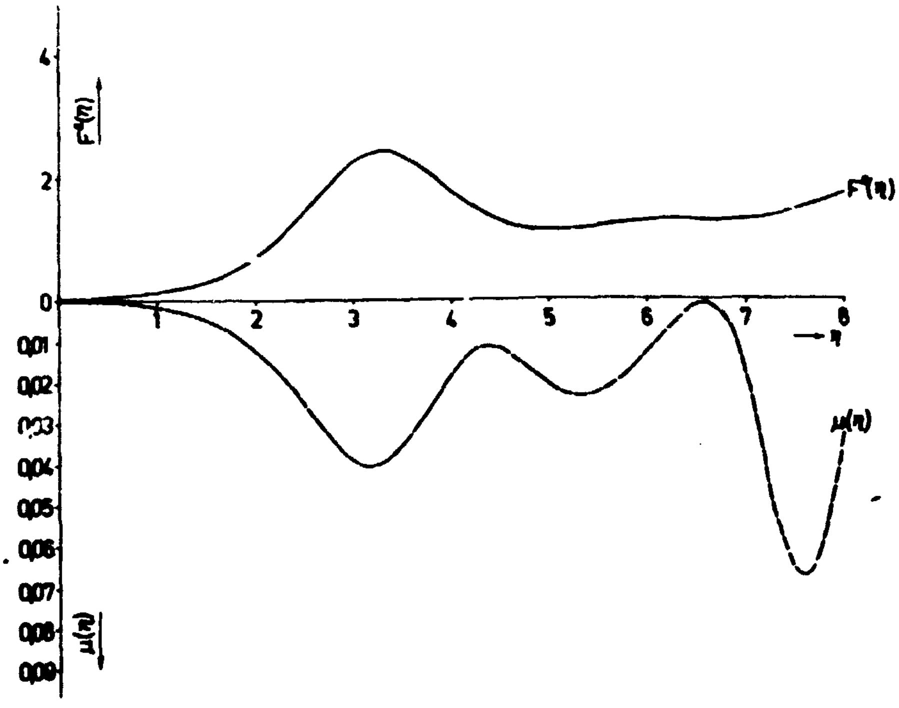
Fig. 1.d. Squared wave amplitude $F^2(\tau)$ (upper curve) and normalized phase velocity (lower curve) in time problem $\left(\Delta W_b = \frac{\omega}{k_L} \frac{W_b - W_b(\tau)}{W_b} = \frac{9}{16\sqrt{3}} F^2(\tau)\right)$

Fig. 2.a. Averaged beam-particles velocity distributions calculated for $\mu_L = 0$

Fig. 2.a'. Beam-particles phase (density) distributions calculated for $\mu_L = 0$

Fig. 3.a. Phase-space diagrams of beam particles calculated for $\mu_L = 0$

Fig. 3.b. Phase-space diagrams of beam particles calculated for $\mu_L = \frac{1}{\sqrt{3}}$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7
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η
$F^q(\eta)$
$\mu(\eta)$

Fig. 1.a.

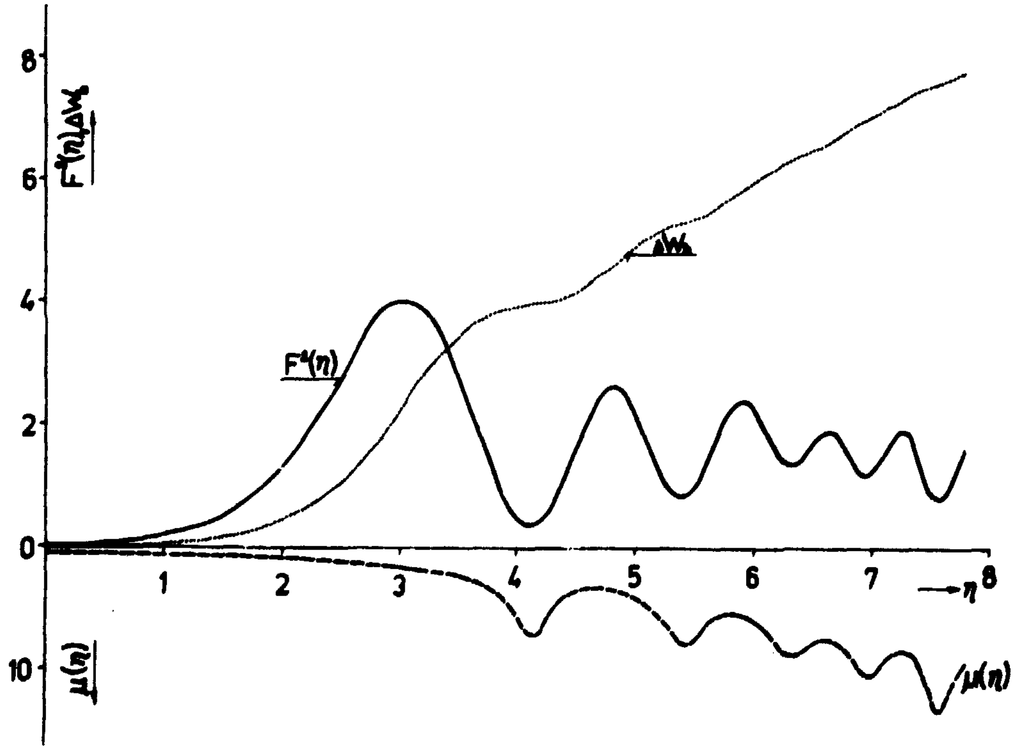
8
6
4
2
0
10
1
2
3
4
5
6
7
8
η
$F^q(\eta),\Delta W_0$
$\mu(\eta)$
ΔW_0
$F^q(\eta)$

Fig. 1,b.

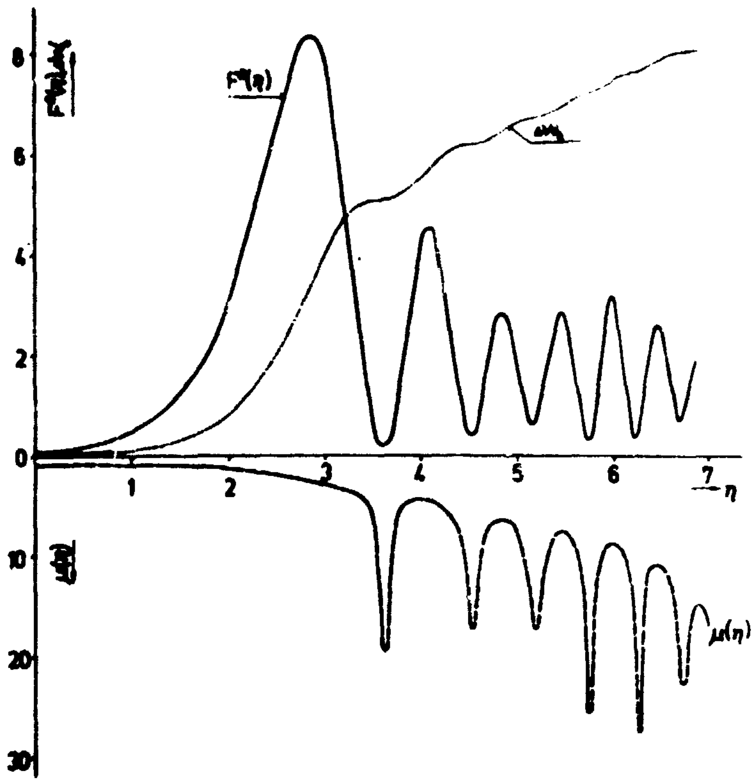

Fig. 1.c.

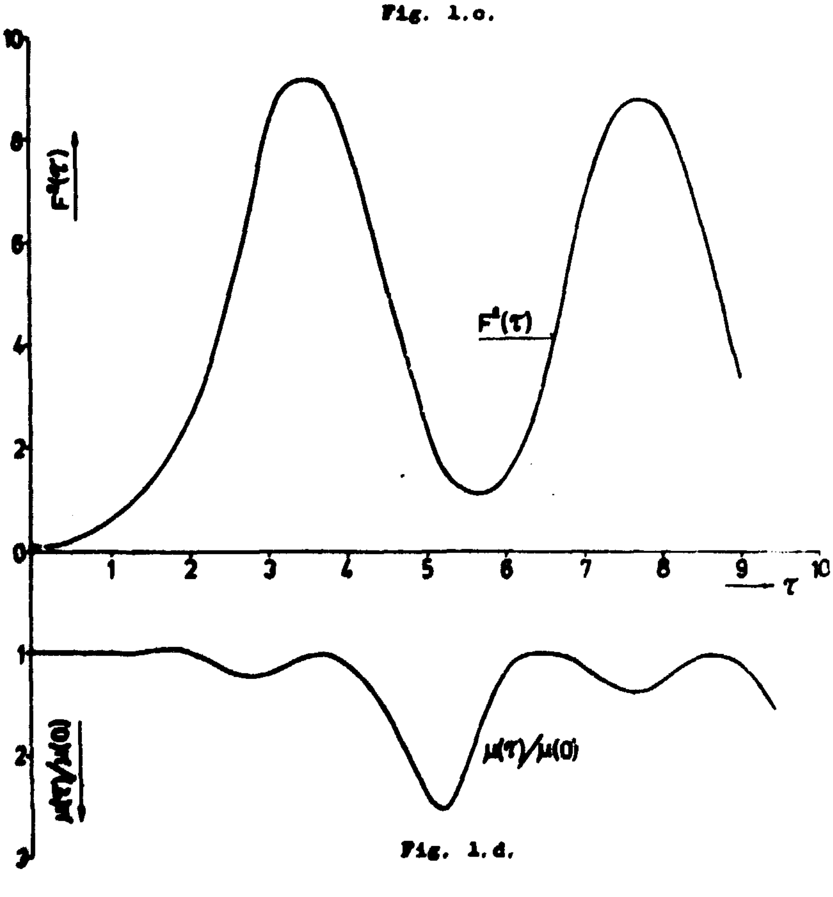

Fig. 1.d.

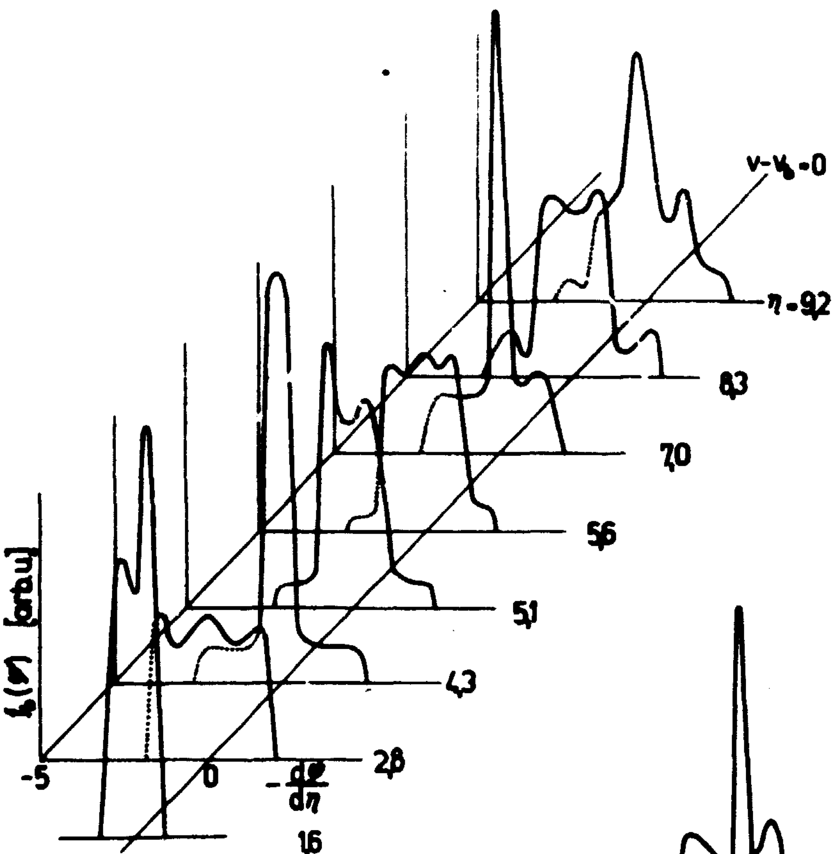
V-V₀=0
η=9,2
8,3
7,0
5,6
5,1
4,3
2,8
1,6
-5
0
-dφ/dη

Fig. 2.a.

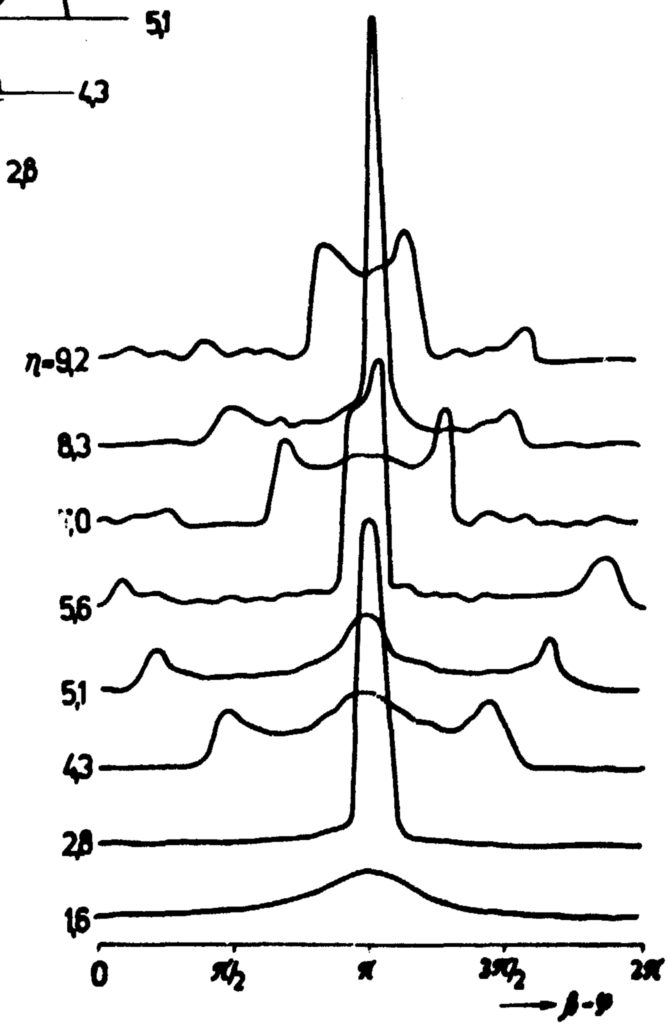
η=9,2
8,3
7,0
5,6
5,1
4,3
2,8
1,6
0
π/2
π
3π/2
2π
→ β-φ

Fig. 2.a.'

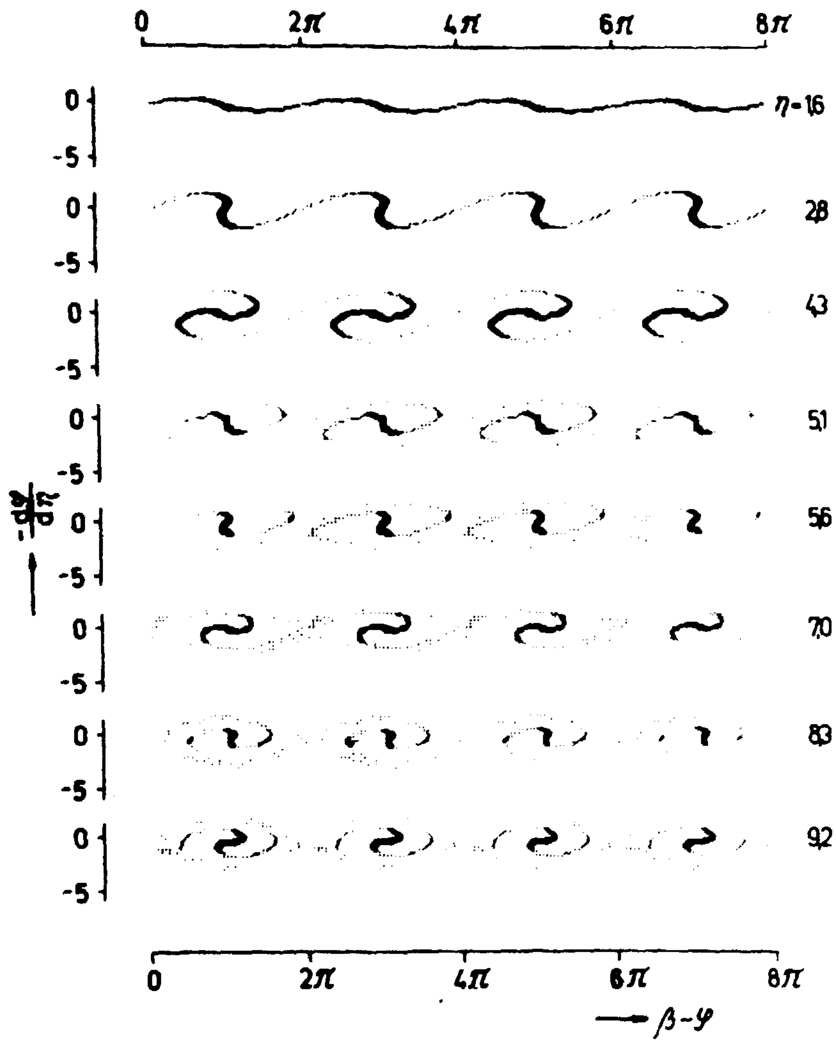

Fig. 3.a.

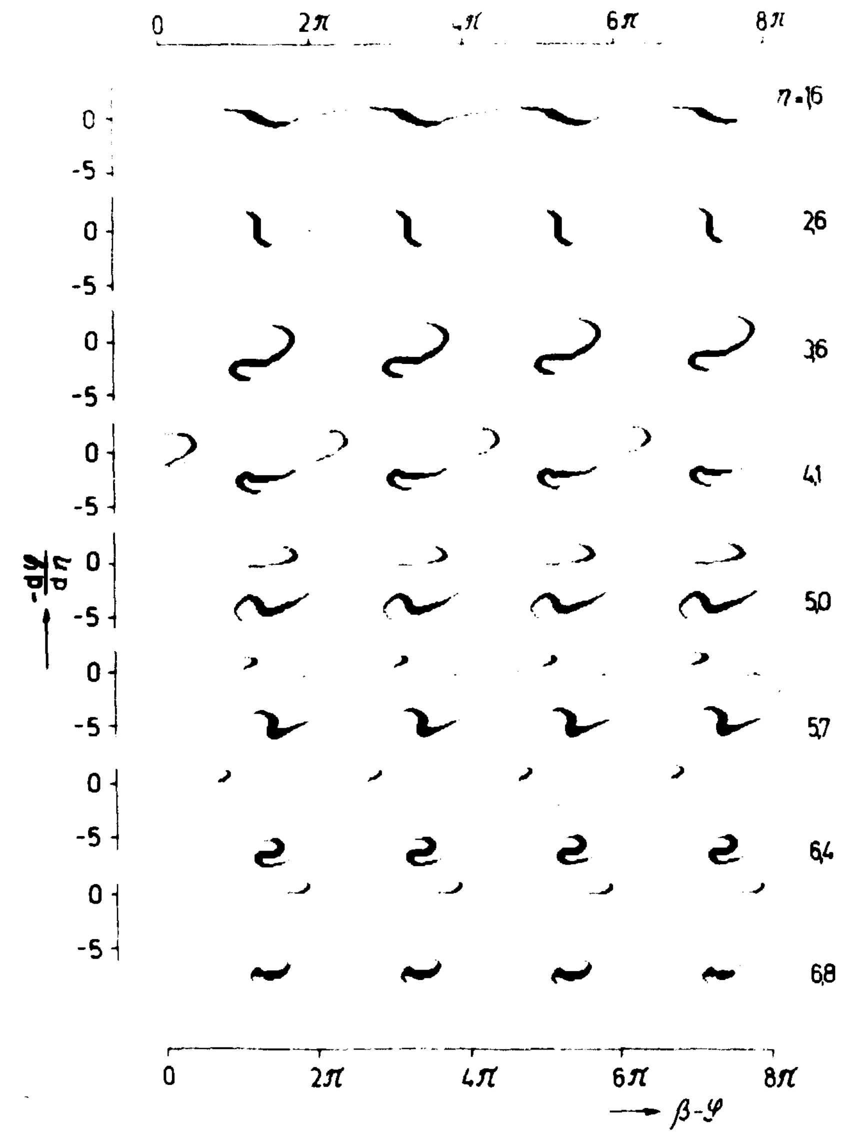

Fig. 3.b.

Vytiskla Státní tiskárna, n. p., závod 0, Praha 1