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EFFECT OF BEAM DENSITY AND OF HIGHER HARMONICS ON BEAM-PLASMA INTERACTION

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ABSTRACT

The interaction of a cold electron beam-plasma system is investigated numerically in the region of densities ratio $n_B / n_P = 2x10^{-3} - 2x10^{-2}$. The one-dimensional model of a collisionless plasma is used. The time development of the wave with maximal growing rate and its spatial harmonics is studied. The plasma effect is simulated by the direct computation of plasma particle trajectories (this being different from the usual plasma simulation by means of a dielectric). These calculations show the following effects of the finite parameter $(n_B / n_P)^{1/3}$: the ratio of the plasma energy to the electric field energy is increased, the damping character of field and macroscopic amplitudes appears and the influence of the second harmonic is not negligible for $n_B / n_P \geq 10^{-2}$.

1. INTRODUCTION

There have been considerable series of papers devoted to the numerical simulation of cold electron beam-plasma interaction, e.g. (MATSIBORKO et al., 1972; O'NEIL et al., 1971; THOMPSON, 1971; JUNGWIRTH et al., 1973 α ; JUNGWIRTH et al., 1973 β). The authors MATSIBORKO et al., (1972), O'NEIL et al. (1971), THOMPSON (1971) have investigated the time development of the beam-plasma instability, the authors JUNGWIRTH et al. (1973 α), JUNGWIRTH et al. (1973 β) have dealt with a stationary space development of this instability. The general feature of these papers is the model of a single wave and the assumption of the smallness of the parameter $\eta^{1/3} \equiv (n_B/n_P)^{1/3} \ll 1$. In the paper of MATSIBORKO et al. (1972) the weaker condition $\eta \ll 1$ has been assumed, nevertheless the results of their paper are valid only in the limit $\eta^{1/3} \ll 1$. In the actual experiments it is $\eta \sim 1 \times 10^{-2}$; hence the parameter $\eta^{1/3}$ possesses a finite value. The results of our paper show the effects of the finite parameter $\eta^{1/3}$, which are neglected in the papers considering only the limit $\eta^{1/3} \ll 1$.

The approximation of the single wave (with neglecting of higher harmonics) a priori does not fulfil the energy-momentum conservation laws. Thus the system of the plasma, the beam and of the single wave (eventually satelites including) is not a conservative one. The system is conservative only if higher harmonics are included. As far as the approximation of $\eta^{1/3} \ll 1$ is concerned, it consists in the papers of MATSIBORKO et al. (1972), O'NEIL et al. (1971), THOMPSON (1971), JUNGWIRTH et al. (1973 α), JUNGWIRTH et al. (1973 β) in the two following simplifications. The first from them replaces the system of the plasma and the wave by the wave propagating in a dielectric, the second one uses only the linear expansion of the dielectric function near the plasma frequency.

The aim of our paper is the investigation of the interaction of the beam with the plasma for the values of η , which corresponds to real experiments, the effects of higher harmonics including. Similarly as in papers of MATSIBORKO et al. (1972), O'NEIL et al. (1971), THOMPSON (1971), JUNGWIRTH et al. (1973 α), JUNGWIRTH et al. (1973 β) the satelites are neglected. The influence of finiteness of the parameter $\eta^{1/3}$ is investigated in the region $1.26 \times 10^{-1} - 2.72 \times 10^{-1}$ (which corresponds to $\eta = 2 \times 10^{-3} - 2 \times 10^{-2}$).

We shall therefore deal with the time development of the collisionless interaction of the following system: the cold electron beam, the cold plasma with the immobile ion background, the mode with maximum growing rate (further only basic mode) and its second and third space harmonics. The mathematical formulation of this problem is following:

$$\frac{\partial f_\alpha}{\partial t} + \{f_\alpha, \mathcal{H}_\alpha\} = 0, \qquad \mathcal{H}_\alpha = \frac{P_\alpha^2}{2m} + eU, \qquad \{\ \} \equiv \begin{matrix}\text{Poisson}\\ \text{bracket}\end{matrix}$$

(1) $$\Delta U = -4\pi e \sum_\alpha \left[\int f_\alpha dP - n_\alpha\right], \qquad U = \sum_{n=-3}^{+3} U_n(t)\, e^{ink_0 X}$$

$$\int_{-\infty}^{+\infty}\int_0^L f_\alpha dP\, dX = n_\alpha L, \qquad L = 2\pi/k_0, \qquad P_0 = mV_0$$

$$f_B(P, X; t = -\infty) = n_B\, \delta(P - P_0), \qquad f_P(P, X; t = -\infty) = n_P\, \delta(P)$$

where e is the charge of an electron, m is the mass of an electron, $\mathcal{H}_\alpha$ is the Hamiltonian of a particle, P_α is the particle momentum, f_α are the distribution functions, n_α are the unperturbed particle densities, V_0 is the unperturbed beam velocity, U is the potential of the electrostatic waves, k_0 is the wavenumber of the basic mode; the index $\alpha = P$ and $\alpha = B$ denotes plasma particles and beam particles, respectively.

2. BASIC EQUATIONS

Mathematical formulation

The following method presented by one of the authors (Lacina, 1973) was used for the numerical calculations. The principle of this method consists in the solution of the motion of beam and plasma particles as functions of the time and initial conditions. Using this method for our starting equations (1) we get the following set of equations.

The time evolution of the complex wave amplitudes $U'_n(t')$ is given by the particle trajectories in the following way:

$$U'_n(t') = -\frac{4\pi}{k_n'^2}\left[N'_{Pn}(t') + N'_{Bn}(t')\right], \qquad n = \pm 1, \pm 2, \pm 3$$

(2) $$N'_{Pn} = \frac{1}{L'}\int_0^{L'} \exp\{-ik'_n [X'_P(X'_0, t') - t']\}\, dX'_0,$$

$$N'_{Bn} = \frac{\eta}{L'}\int_0^{L'} \exp\{-ik'_n [X'_B(X'_0, t')]\}\, dX'_0, \qquad k'_n = n.k'_0,$$

where $N'_{\alpha n}$ are the complex density amplitudes and where the particle motion is given by canonical euqations (3)

(3) $$\dot{X}'_P = \frac{\partial \mathcal{H}'_P}{\partial P'}, \qquad \dot{P}'_P = -\frac{\partial \mathcal{H}'_P}{\partial X'_P}$$

$$\mathcal{H}'_P = \frac{P_P'^2}{2} - \sum_{n=-3}^{+3} U'_n \cdot e^{ik'_n(X'_P - t')}$$

(3) $$\dot{X}'_B = \frac{\partial \mathcal{H}'_B}{\partial P'}, \qquad \dot{P}_B = -\frac{\partial \mathcal{H}'_B}{\partial X'_B}$$

$$\mathcal{H}'_P = \frac{P_B'^2}{2} - \sum_{n=-3}^{+3} U'_n(t') e^{ik'_n X_B}$$

The particle initial conditions are determined as follows:

(4) $$X'_P(0) = X'_0 + 2k'_1 \cdot \mathrm{Im}\left\{\frac{U'_1(0) e^{ik'_1 X'_0}}{[\omega'_1(0) + k'_1]^2}\right\}$$

$$P'_P(0) = -2k'_1 \cdot \mathrm{Re}\left\{\frac{U'_1(0) e^{ik'_1 X'_0}}{\omega'_1(0) + k'_1}\right\}$$

$$X'_B(0) = X'_0 + 2k'_1 \cdot \mathrm{Im}\left\{\frac{U'_1(0) e^{ik'_1 X'_0}}{[\omega'_1(0)]^2}\right\}$$

$$P'_B(0) = -2k'_1 \cdot \mathrm{Re}\left\{\frac{U'_1(0) e^{ik'_1 X'_0}}{\omega_1(0)}\right\}$$

$$\omega'_1(0) = \Omega'_1(0) + i\mu'_1(0), \qquad \Omega'_1(0) = \mathrm{Re}\,[\omega'_1(0)] .$$

The initial conditions (4) were determined in such a way as to correspond to the unstable wave with a small amplitude $U'_1(0)$ developing in the linear region with the maximal growing rate $\mu'_1(0)$. The values $\Omega'_1(0)$, $\mu'_1(0)$, k'_1 were obtained from the expressions, given in the paper of THOMPSON, (1971).

The momentum of the plasma ϕ'_P and that of the beam ϕ'_B are given as follows:

$$\phi'_P = V'_P = \frac{1}{L'}\int_0^{L'} P'_P(X'_0, t')\, dX'_0 , \qquad \phi'_B = \eta V'_B = \frac{\eta}{L'}\int_0^{L'} P'_B(X'_0, t')\, dX'_0 .$$

Similarly the plasma kinetic energy W'_P and the beam kinetic energy W'_B both in the initial plasma frame (i.e. in the laboratory frame) are given in the following way:

$$W'_P = \frac{1}{2L'}\int_0^{L'} [P'_P(X'_0, t')]^2 dX'_0$$

$$W'_B = \frac{\eta}{2L'}\int_0^{L'} [P'_B(X'_0, t')]^2 dX'_0 + \phi'_B(t'), \qquad W'_{BS} = W'_B - \frac{1}{2}\eta V_B'^2 .$$

The expression W'_{BS} determines the smearing energy of the beam. The electric field energy is given as follows

$$W'_{E(TOT)} = \sum_{n=1}^{3} W'_{En}, \qquad W'_{En} = \frac{k_n'^2}{4\pi} \cdot U'_n U_n'^* .$$

The set of equations (2)-(4) is expressed in a useful dimensionless form by means of similarity laws (LACINA, 1971). The relation between the dimensionless (dashed) quantities and the dimensional quantities is given by the following transformation

$$m = m' m , \qquad e = e' |e| , \qquad n_\alpha = n_p \, n'_\alpha$$

$$(5) \qquad P = m V_0 P' , \qquad X = \frac{m V_0}{|e| (n_p m)^{1/2}} X' , \qquad t = \frac{m^{1/2}}{(e^2 n_p)^{1/2}} t'$$

$$U = \frac{m V_0^2}{|e|} U' , \qquad \omega = \left(\frac{e^2 n_p}{m} \right)^{1/2} \omega' , \qquad k = \frac{|e| (n_p m)^{1/2}}{m V_0} k'$$

From this transformatior it follows:

$$(6) \qquad e' = -1 , \quad m' = 1 , \quad n'_p = 1 , \quad n'_B = \eta , \quad V'_0 = 1$$

which was used in equations (2)-(4).

The transformation (5) appears very useful due to the fact that an arbitrary physical quantity and also an arbitrary equation for physical quantities are invariant with respect to the transformation (5). This means that for the transformation of an arbitrary physical quantity or an equation we need not use the transformation (5), but it is sufficient to carry out the normalisation with respect to the equation (6). Further, due to the above mentioned transformation our results are invariant with respect to the choice of the beam velocity and the plasma density.

The set of equations (2), (3), (4) has been solved numerically using the computer IBM 370/135. Using the method of LACINA (1973), the starting Eqs. (2)-(4) obtain a useful form for the computer calculations. The advantage consists in the possibility of the use of the more precise numerical methods for space integrals in Eq. (2) and thus this method substantially reduces the number of numerical particles. (By the numerical particles we define here particles, the trajectory of which are necessary to compute in order to obtain the desired accuracy.) This differs from the numerical experiments (e.g. MORSE et al., 1969) and from the method of O'NEIL et al. (1971), where the summation over physical particles is carried out; thus it is not possible to reduce the number of particles by means of numerical procedure.

In our equations (2)-(4) the plasma quantities are related to the laboratory frame whereas the beam quantities are related to the initial beam frame.

In what follows we shall omit the dashes above the dimensionless quantities.

The numerical procedure

The time integration of the canonical Eqs. (3) has been carried out by means of Runge-Kutte method with accuracy h^5 . The Simpson method has been used for the space integration in Eq. (2), differing from the simple sum of particles used by O'NEIL et al. (1971). The energy-momentum conservation laws have been checked in order to be sure about the accuracy of the calculation. Using the Simpson method it was sufficient to calculate the trajectories of only 100 numerical particles in the case of simple mode approximation and 300 numerical particles in the case of three modes approximation (the basic mode, the second, the third harmonic).

3. NUMERICAL RESULTS

The computer output

The following quantities have been computed in order to appreciate the given physical model sufficiently:

W_{En} - the electric field energy of the n-th mode
$W_{E(TOT)}$ - the total electric field energy
W_P - the plasma kinetic energy
W_B - the beam kinetic energy
W_{BS} - the smearing beam kinetic energy ("thermal" energy)
V_{Ph} - the phase velocity of the basic mode
V_B - the average beam velocity
Φ_P, Φ_B - the total plasma momentum, the total beam momentum, respectively
ω_n - the complex frequency of the n-th mode
U_n - the complex amplitude of the potential of the n-th mode
N_{Pn} - the complex amplitude of the n-th mode of the plasma density
N_{Bn} - the complex amplitude of the n-th mode of the beam density
W_{TOT} - the total energy
Φ_{TOT} - the total momentum.

To be able to compare the influence of higher harmonics, we have computed both the case of basic mode only and the case of basic mode and the second and the third harmonics including for various values of η . As it is mentioned above, the initial conditions were determined in agreement with linear theory. In order to start the computation in the linear region we have put $U_1(t=0) = 10^{-4}$ for the initial conditions. This value is at least two orders of magnitude lower than that of maximum value $U_{1\,max}$.

Single mode approximation

This model has been computed for the values η = 2x10^{-3}; 6x10^{-3}; 1x10^{-2} and 2x10^{-2}. The review of the linear wave parameters is presented in the Table I.

η	2x10^{-3}	6x10^{-3}	1x10^{-2}	2x10^{-2}
k_1	3.5981	3.6555	3.7004	3.7917
$Re(\omega_1)$	-0.2038	-0.3110	-0.3808	-0.5053
$Im(\omega_1)$	0.2917	0.4108	0.4801	0.5902

Table I. Linear wave parameters

The review of obtained results for η = 2x10^{-3}, 6x10^{-3} is given in the Fig. 1 and Fig. 2.

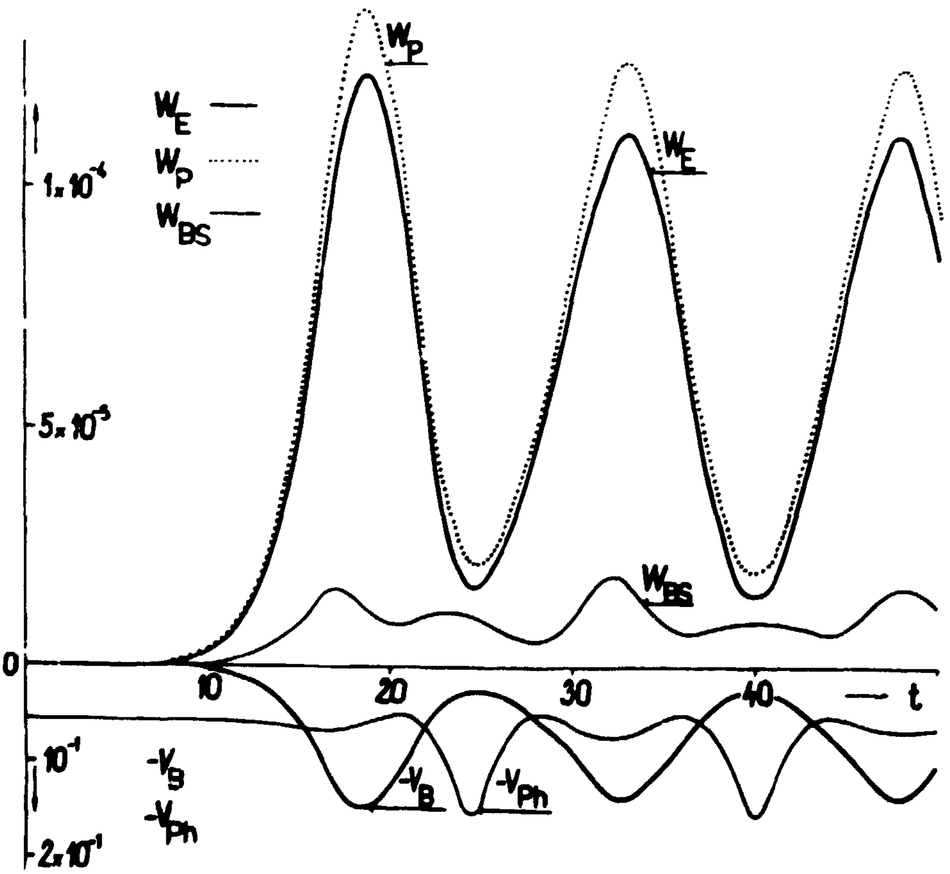

Fig. 1 (η = 2x10^{-3}). Time variation of W_{E1}, W_p, W_{BS}, V_{Ph}, V_B.

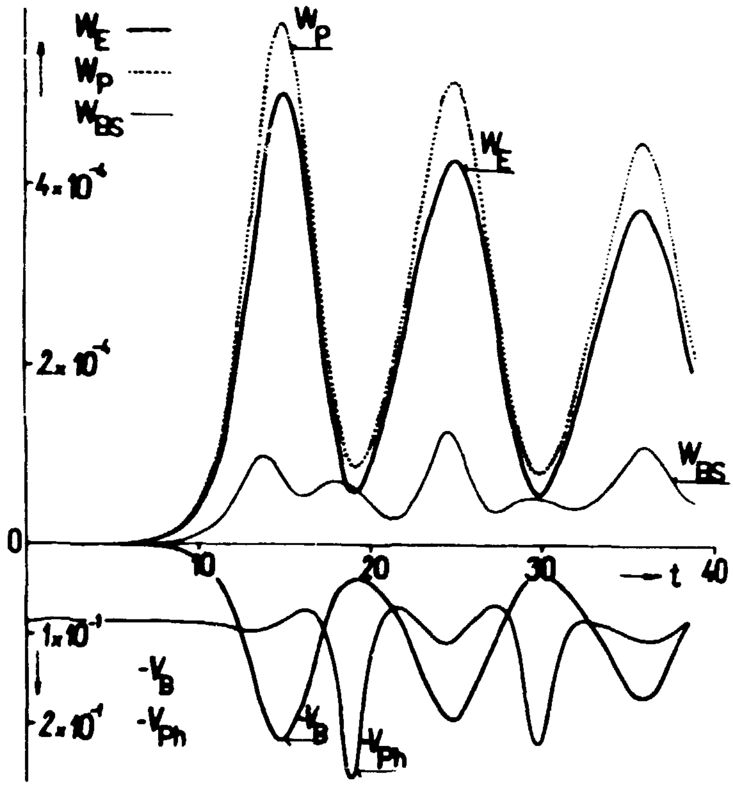

Fig. 2 ($\eta = 6\times10^{-3}$). Time variation of W_{E1}, W_P, W_{BS}, V_{Ph}, V_B.

The influence of higher harmonics for densities $\eta = 10^{-2}$ and $\eta = 2\times10^{-2}$ cannot be neglected in the strong nonlinear region and hence the single wave approximation is for the above mentioned densities insufficient. Therefore we did not put the respective results of single wave approximation separately. In order to be able to compare the single wave approximation with that of higher harmonics, the course of W_{E1} and V_B from the single wave model is plotted in the corresponding figure for higher harmonics (Fig. 3).

Higher harmonics approximation

This model has been computed for $\eta = 1\times10^{-2}$ and η 2×10^{-2}; for $\eta =$ $= 2\times10^{-3}$ and $\eta = 6\times10^{-3}$ the influence of higher harmonics appears to be negligible. The results are plotted in the Figs. 3,4 for $\eta = 2\times10^{-2}$.

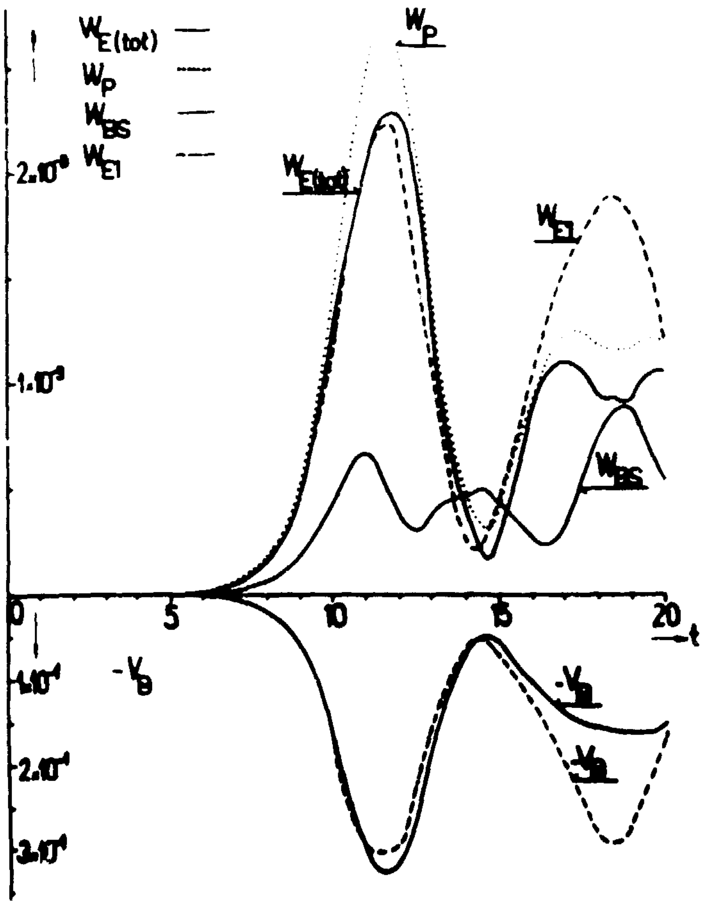

Fig. 3 ($\eta = 2\times10^{-2}$). Time variation of $W_{E(TOT)}$, W_P , W_{BS} , V_B ; the quantities W_{E1} , V_B from the single wave approximation are plotted with a dashed line.

4. THE DISCUSSION OF THE RESULTS

The effects of finite values of $\eta^{1/3}$ and of higher harmonics

Comparing papers of MATSIBORKO et al. (1972), O'NEIL et al. (1971) and THOMPSON (1971) with our results presented in the previous chapter, the following differences appear. First of all we have found the "nonequipartition of energy distribution", by which we understand the unequal transfer of beam energy to the electric field and to the plasma. The finite $\eta^{1/3}$ effect appears also in the damping of maximum value of quantities, oscillating with the characteristic bounce frequency. Further, for densities $\eta \geq 1\times10^{-2}$ the energy of the second harmonic W_{E2} is not negligible in comparison with the energy of the first

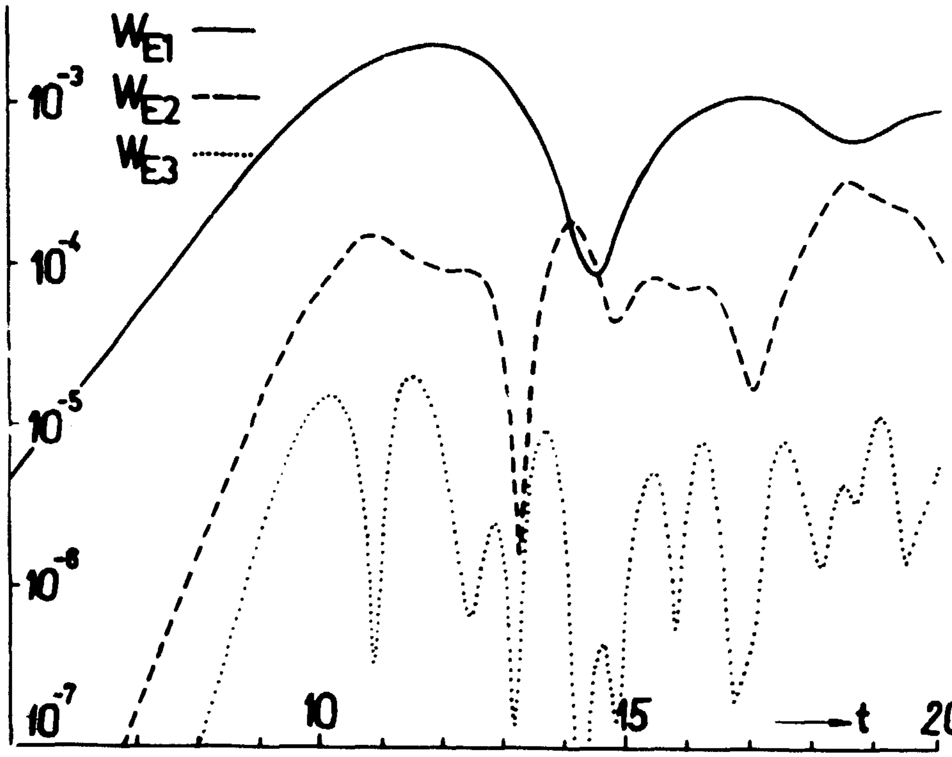

Fig. 4 ($\eta = 2x10^{-2}$). Time variation of harmonic field energies W_{En} (n = 1, 2, 3) plotted in the logarithmic scale.

harmonic W_{E1} (W_{E2} and W_{E1} are of the same order in the region of the first minimum for $\eta = 2x10^{-2}$). The second harmonic effects a decay of field energy in the range of second maximum (this effect being rather strong for $\eta = 2x10^{-2}$).

Nonequiparticity of energy transfer

From the approximation $\eta^{1/3} \ll 1$ (used by MATSIBORKO et al. (1972), O'NEIL et al. (1971) and DRUMMOND et al. (1970)) the equiparticity of the beam energy transfer to the electric field and to the plasma follows. That means: one half of beam energy is transfered to the plasma and the second half to the electric field. The actual energy distribution between the plasma and the field following from our results is plotted in Figs. 1 and 2 for the single wave approximation and in Fig. 3 for the higher harmonics approximation. As we can see from these figures, the ratio $W_P / W_{E(TOT)}$ strongly changes its value in time (especially in the regions of minimum values of $W_{E(TOT)}$); generally this ratio is growing with growing density ratio η. In the time, when the energy W_E possesses the minimum values the ratio W_p / W_E possesses maximum values from 1.3 for $\eta = 2x10^{-3}$ up to 1.95 for $\eta = 2x10^{-2}$.

Comparing the time-dependence of W_p / W_E with the time-dependence of the phase velocity V_{ph} (see Figs. 1, 2) we can find that the time development of W_p / W_E is strongly influenced by the change in the instantaneous phase velocity in the single wave case. The equipartition energy distribution is usually derived in the theory for $Re(\omega) \sim \omega_p$ (which is valid in the limit $\eta^{1/3} \rightarrow 0$). Since for finite values of $\eta^{1/3}$ the difference between $Re(\omega)$ and ω_p is not negligible, the equiparticity of energy distribution is violated, as follows from our results. ASTRELIN et al. (1973) have dealt also with the beam-plasma interaction for $\eta = 2x10^{-2}$. Differently from us they started from the single-wave model, however they considered a finite temperature of the beam and of the plasma. They also have found the nonequiparticity of energy distribution W_p / W_E. The higher value of W_p / W_E, in their case, appears to be due to trapped plasma particles.

The damping of oscillations

As it follows from Fig. 2, the oscillations of quantities with the characteristic frequency of trapped beam particles exhibit damped character. This effect appears both in the single wave and higher harmonics approximations; in the latter case there is an additional decay due to the influence of higher harmonics (as it follows from Fig. 3). In this Chapt. we shall only deal with the damping appearing in the single wave approximation (with the supposed negligible effect of higher harmonics).

The maximum values of the following quantities exhibit the damped character: the field energy W_{E1}, the plasma energy W_p, the phase velocity V_{ph} and the average beam velocity V_B. The damping rate reaches its largest value for $\eta = 6x10^{-3}$ (for the discussed set of parameters η). As it follows from this case, the damping does not appear to be entirely irreversible; the values of quantities in the fourth maximum exhibit again growing character. This growing may be caused by some slower oscillation process, not yet known. However, the single wave approximation ceases its accuracy behind the third maximum due to the effect of higher harmonics and satelites. Usually it is supposed (DRUMMOND et al., 1970; MATSIBORKO et al., 1972) that such a damping is caused by means of a smearing process in trapped particles. Nevertheless, as it follows from the time behaviour of the beam smearing energy W_{BS}, this effect is insufficient for the explanation of such a damping rate.

The influence of higher harmonics

O'NEIL et al. (1971) have estimated in their paper the energy of the higher harmonics on the assumption $\eta^{1/3} \ll 1$. Since they have supposed a priori that the energy of the higher harmonics is negligible in comparison with the energy of the basic mode, they have not calculated the amplitudes of higher harmonics self-consistently. The amplitudes of the higher harmonics have been determined in their case by means of Fourier analysis of the beam density and the influence of plasma has been simulated by a dielectric with $\epsilon = 1$; the backward effect of the higher harmonics on the particle motion has been neglected.

We have solved the higher harmonics approximation self-consistently; it means that we have calculated their influence on the motion of particles and at the same time the influence of the beam and plasma particle motion on the higher harmonics generation (differently from the simulation of the influence of plasma by means of a dielectric). The effect of higher harmonics is negligible for densities $\eta \ll 10^{-2}$; and is not negligible for densities $\eta \geq 10^{-2}$ as it can be deduced from our results. The influence of the higher harmonics is obvious from Figs. 3, 4.

The energy of the -n-th harmonic is growing in time with the initial increment $\gamma_n = n.2\gamma_1$, where γ_1 = the linear increment (see Tab. I). In the paper of O'NEIL et al. (1971) the initial growth rates for the higher harmonics are lower, which is probably due to the model of plasma $(\epsilon = 1)$ used there.

Whereas the difference for the first maximum between the single mode and higher harmonics approximations is not essential, considerable differences appear for the second maximum. The energy of electric field in the higher harmonics approximation decreases according to the single wave approximation for $\eta = 1x10^{-2}$ up to 88% and for $\eta = 2x10^{-2}$ already up to 49%. The same situation appears for the plasma energy and for the average beam velocity. The strong influence of higher harmonics in the case $\eta = 2x10^{-2}$ is given by relatively high value of energy of the second and third harmonics, as can be seen from Fig. 4. The energy of the second harmonic approaches the first harmonic energy and the ratio of the energy of the third to the first harmonic reduces up to one order.

The comparison of our results with references

The comparison of our results with corresponding results, presented in cited papers is given in the summary Tab. II. The comparison is carried out both with theoretical papers (MATSIBORKO et al., 1972; O'NEIL et al., 1971) and with experimentel paper (APEL, 1967). The compared papers are referred in the first column of the Table II. Since in different papers different normalisation of physical quantities has been used, we have carried out the following common renormalisation

$$(7)\quad \overline{W}_E = 2\,W_E\cdot\eta^{2/3},\qquad \overline{\Omega} = \frac{\Omega}{\sqrt{4\pi}}\cdot\eta^{-1/3},\qquad \overline{\gamma} = \frac{\gamma}{\sqrt{4\pi}}\cdot\eta^{-1/3}$$

where W_E , Ω and γ are physical quantities recalculated in the CGSE system of units.

We have chosen for the comparison both the first maximal values of W_E , Ω which the system reaches during its time development and the linear values of $\Omega(0)$, $\gamma(0)$. The corresponding renormalized values given by the relation (7) are given in Tab. II in the columns 2-5; our data are computed for $\eta = 2\times10^{-2}$. The 6-th and the 7-th column contain relative increments (i.e. the ratio of the second and of the third harmonic increment to the first harmonic increment) for the beginning of the interaction. The 8-th and the 9-th column present the ratio f the 2-nd and of the 3-rd harmonic energy to the energy of the 1-st harmonic; all of them are taken for the first maximal values of W_{E1} , W_{E2} , W_{E3} in the course of time. Since there were mainly graphical results in cited papers to our disposal, the values presented in the Table II possess some degree of inaccuracy.

References	$\overline{W}_{E1MAX}$	$\overline{\Omega}_{1MAX}$	$\overline{\Omega}_1(0)$	$\overline{\gamma}_1(0)$	γ_2/γ_1	γ_3/γ_1	W_{E2}/W_{E1}	W_{E3}/W_{E1}
MATSIBORKO	1,06	1,05	0,40					
O'NEIL	1,12	1,01	0,40	0,70	1,69	2,52	$6,3\times10^{-3}$	$3,1\times10^{-3}$
APEL							$3,3\times10^{-2}$	$1,3\times10^{-3}$
OURS	0,80	2,9	0,53	0,62	2,00	3,00	$6,8\times10^{-2}$	$6,8\times10^{-3}$

Table II. The comparison of our results with references

The differences of $\overline{W}_{E1MAX}$, $\overline{\Omega}_{1MAX}$, $\overline{\Omega}_1(0)$ and $\overline{\kappa}_1(0)$ between our values and those in the papers of MATSIBORKO et al. (1972), O'NEIL et al. (1971) are obviously due to the approximation $\eta^{1/3} \ll 1$, used in these papers. Our relative energies W_{E2}/W_{E1}, W_{E3}/W_{E1} are in good agreement with the experimental results of APEL (1967).

In the paper of O'NEIL et al. (1971) the comparison of the harmonic energies with the experimental values of APEL (1967) is carried out, too. Let us note, however, that in the paper of O'NEIL et al. (1971) the approximation $\eta^{1/3} \ll 1$ has been used, whereas in the paper of APEL (1967) the quantity $\eta^{1/3}$ equals approximately $\eta^{1/3} \approx 0.26$ (for $\eta = 0.018$).

5. CONCLUSION

This paper presents theoretical results of the nonlinear beam-plasma interaction in the region of the parameter $\eta = n_B/n_p$, closely related to experimental values.

From the given results it follows that

in the region $0 < \eta < 10^{-3}$ the approximation based on the model of the linear dielectric (MATSOBORKO et al., 1972; O'NEIL et al., 1971) is valid,

in the region $10^{-3} < \eta$ new effects appear (the nonequipartiticity of energy distribution and the damped character of the oscillations) and

in the region $10^{-2} < \eta$ there are further effects caused by higher harmonics.

These new effects, caused by the finiteness of $\eta^{1/3}$, will obviously appear in the case of dense beams (e.g. in relativistic beams which are usually working in the higher density region).

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REFERENCES

APEL J.R. (1967) Phys. Rev. Letters 19, 744.

ASTRELIN V.T. and BUCHELNIKOVA N.S. (1973) Proc. XIth International Conf. on Phenomena in Ionized Gases, Prague, 392.

DRUMMOND W.E. et al. (1970) Phys. Fluids 13, 2422.

JUNGWIRTH K. and KRLÍN L. (1973) Preprint IPPCZ-186α.

JUNGWIRTH K. and ŠUNKA P. (1973) Preprint IPPCZ-181β.

LACINA J. (1971) Plasma Phys. 13, 303.

LACINA J. (1973) VIth European Conf. on Plasma Physics and Controlled Fusion, Moscow.

MATSIBORKO N.G. et al. (1972) Plasma Physics, 14, 591.

MORSE R.L. and NIELSON G.W. (1969) Phys. Fluids. 12, 2418.

O'NEIL T.M., WINFREY J.H. and MALMBERG J.H. (1971) Phys. Fluids 14, 1204.

THOMPSON J.R. (1971) Phys. Fluids 14, 1532.

Vytiskla Státní tiskárna, n. p. závod 6, Praha 1