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ERGODISATION OF  
MAGNETIC SURFACES IN TOKAMAKS

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## RESUME

Nous discutons un mécanisme de turbulence qui pourrait expliquer les disruptions internes et les impulsions de tension négatives observées dans les Tokamaks. Si les lignes de flux deviennent ergodiques entre deux surfaces magnétiques de rayon  $r_1, r_2$  ( $r_1 < r_2$ ), cette ergodicité tend à égaliser le profil de densité de courant dans l'intervalle  $(r_1, r_2)$ , ce qui crée par induction un champ électrique toroidal important, positif du côté  $r = r_1$ , et négatif du côté  $r = r_2$ . Du fait des connections magnétiques radiales, les valeurs positives et négatives du champ E tendent à s'opposer le long des lignes de flux dans le domaine  $(r_1, r_2)$ , et ce champ peut exister bien que la densité de courant varie faiblement. Cependant le champ E crée par effet de peau dans le plasma non perturbé, au voisinage des rayons  $r_1$  et  $r_2$ , des impulsions spatiales de courant électriques. De telles impulsions peuvent rendre instables des modes tearing ayant un nombre d'onde élevé, et étendre ainsi le domaine  $(r_1, r_2)$  où les lignes de flux sont ergodiques. Le point de départ du phénomène peut être une instabilité tearing et la séparation de îlots magnétiques qui sont normalement présents au voisinage des surfaces magnétiques  $q=1$  ou  $q=2$ .

ERGODISATION OF  
MAGNETIC SURFACES IN TOKAMAKS

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ABSTRACT

A turbulence mechanism which could explain the internal disruptions and the negative spikes which are observed in Tokamaks is discussed. If the flux lines become ergodic between two magnetic surfaces  $\Gamma = \Gamma_1$  and  $\Gamma = \Gamma_2$ ,  $\Gamma_1 < \Gamma_2$ , this ergodicity tends to flatten the current density profile between  $r_1$  and  $r_2$ , and this eventually induces a large electric field  $E$ , positive on the side  $\Gamma = \Gamma_1$  and negative on the side  $\Gamma = \Gamma_2$ . Due to the radial magnetic connections, the positive and the negative values of  $E$  tend to cancel each other along the flux lines, and the field  $E$  may exist although the current density experiences a small change in the interval  $(r_1, r_2)$ . However, the field  $E$  induces by skin effect spatial pulses of current density in the unperturbed domain near  $r = r_1$  and  $r = r_2$ . These pulses may drive unstable tearing modes with large transverse wave numbers, and so extend the domain  $(r_1, r_2)$  where the ergodicity takes place. The first step of the phenomena could be a tearing instability of the separatrix of the magnetic islands which are normally present near the magnetic surfaces  $q=1$  or  $q=2$ .

## I - INTRODUCTION

Two kinds of gross perturbations are currently observed in Tokamaks.

- 1) The "negative spikes" which appear as negative pulses of the voltage per turn, following the onset of a tearing mode on the magnetic surface  $q=2$  and a flattening of the temperature profile on each side of this surface<sup>1,2,3)</sup>.
- 2) The "internal disruptions" which consist of a sudden enlargement of the temperature profile on each side of the magnetic surface  $q=1$ <sup>3,4)</sup>. The two phenomena, as far as they are detected through soft X ray measurements, are similar. In both cases, the temperature profile - and probably the current density profile - are flattened on each side of the magnetic surface  $q=2$  or  $q=1$  over a relatively large distance ; on the other hand, the soft X ray signal originating near this surface often exhibits the same oscillatory structure, which is referred to the presence of a tearing mode in the case of the negative spikes. While the internal disruptions do not induce electromagnetic perturbations outside the plasma as the negative spikes do, these analogies suggest that the two phenomena have a similar mechanism.

The negative pulse of voltage associated with the negative spikes is not presently well understood<sup>5,6,7,8)</sup>. This pulse is not due to a sudden displacement of the plasma column. It has been proposed that, as a consequence of the onset of a tearing mode on the magnetic surface  $q=2$ , large size magnetic islands take place and eventually reach the limiter, leading to a complete reorganization of the discharge. However, the classical rising time of such large scale magnetic islands, which is of the order of the time of resistive penetration, is generally significantly longer than the experimental rising time. On the other hand, it is not clear why the fact that the islands reach the limiter would induce a short negative voltage pulse. Actually this pulse appears as a sudden increase of the poloidal magnetic flux (see Fig.1) embraced by the magnetic surface touching the limiter, without macroscopic change of this surface. Such a variation breaks the MHD invariant relation  $\Psi = G(\Phi)$  between the poloidal

flux  $\Psi$  and the toroidal flux  $\phi$  embraced by each magnetic surface. As the phenomenon originates as a tearing mode on the surface  $q=2$  this relation is probably broken inside the plasma, with  $\Psi$  increasing. Such an evolution corresponds, with the total discharge current remaining constant, to a flattening of the profile of current density  $I$ , i.e. to a decrease of the magnetic energy associated with the poloidal field  $B_{\theta}$ . It is then plausible that this liberation of energy is the reservoir which feeds the phenomenon.

In this note we present a scheme which could explain such a type of perturbation. We first note that the magnetic islands due to the development of a linearly unstable tearing mode on a resonant magnetic surface, e.g.  $q=2$ , may be themselves tearing unstable, for modes with relatively large azimuthal wave numbers taking place at the separatrix of the islands. This secondary instability is caused by the sudden change of the slope  $dI/dr$  at the separatrix, which has a nearly null value inside the islands and eventually a large value outside. The onset of the secondary unstable modes enlarges the domain where the flux lines are radially connected. This domain should extend further rapidly on each side of the original magnetic island. The physical mechanism of this explosive character is simple : in a region  $r_1 < r < r_2$  of the plasma where the flux lines are radially connected, the current density profile tends to equalize. The magnetic poloidal flux across this region (and the  $B_{\theta}$  magnetic energy) then tends to decrease, resulting in the presence of an inductive toroidal electric field  $E$ , which is positive on the magnetic axis side and negative on the outer side. The radial connection of the flux lines in the domain  $r_1 < r < r_2$  makes that the positive and negative values of the field  $E$  tend to balance as far as the current profile in this region is concerned. However, outside the interval  $(r_1, r_2)$ , where the flux lines are unperturbed, the field  $E$  produces by skin effect spatial pulses of current ; this configuration is likely to produce small scale tearing modes, which enlarge further the domain where radial connection

takes place. We are not able to consistently describe such an evolution in its early stage, i.e. when the connected domain is still near the original magnetic island. We content ourselves, in the second part of this note, to analyse the situation where, in a domain including the original magnetic islands, the magnetic surfaces have become ergodic, due to the presence of small scale magnetic eddies. On each side of this domain, the magnetic surfaces are unperturbed. The average density current in the ergodic region obeys a quasi-linear diffusion law. Together with this diffusion, an inductive electric field appears, which, as explained above, builds up spatial pulses of current at the frontiers of the ergodic domain, and makes further extension of this domain possible. If the external frontier reaches the limiter, the inductive electric field appears as an observable pulse of the voltage per turn. Of course the short-lived ergodicity of the magnetic surfaces induces a radial flux of electron thermal energy, symmetrical with respect to the original magnetic island, which is consistent with the flattening of the temperature profile observed during negative spikes and internal disruptions.

## II - TEARING INSTABILITY OF MAGNETIC ISLANDS.

We first consider the equilibrium of magnetic islands assumed to have developed from a linear tearing instability on the resonant magnetic surface  $\Gamma = \Gamma_0$  where, e.g.,  $q = 2$  <sup>9,8)</sup>. Initially, the magnetic components  $B_x, B_y, B_z$ , the current density  $I$  and the electric field  $E$  (in the toroidal direction  $\varphi$ ) have the values  $B_{x0}, B_{y0}, B_{z0}, I_0$  and  $E_0$ , respectively. We simulate the plasma near the surface  $\Gamma = \Gamma_0$  by a plasma sheet parallel the axis  $oy, oz$ , carrying the current density  $I$  along  $oz$ . The coordinates  $x, y, z$  are equal to  $r - r_0, r_0(\theta - \varphi/2), R\varphi$ . Initially the magnetic components in the sheet are  $B_x = B_x, B_y = 0, B_z = B_{z0} - x B_{\theta 0} (dq/qdr)$ . Generally the field  $B_x, B_y$  may be specified by a flux function  $\psi(x, y, t)$  such that  $B_x = \partial\psi/\partial y, B_y = -\partial\psi/\partial x, E = E_0 - \frac{1}{c} \partial\psi/\partial t$ . Initially  $\psi = \psi_0(x, y)$ , where

$$\psi_0(x, t) = \frac{1}{2} \alpha x^2 ; \alpha = B_{00} \frac{dq}{q dr} ; E_0 = -\frac{1}{c} \frac{\partial \psi_0}{\partial t}$$

The magnetic islands correspond to a perturbation of the flux function  $\psi_0$ , which becomes

$$\psi(x, y, t) = \frac{1}{2} \alpha x^2 - \Psi_1(x, t) \cos(k_0 y) ; k_0 = \frac{2}{\rho_0}$$

The magnetic surfaces near the islands are defined by the equation

$$\psi(x, y, t) = \frac{1}{2} \alpha x^2 - \psi_1(t) \cos(k_0 y) = \text{constant} ; \psi_1(t) = \Psi_1(0, t) > 0$$

The separatrix of the islands (see fig.(2)) is obtained by taking  $\psi = \psi_1$

Its half width at  $y = 0$  in the  $x$  direction is  $\delta = 2(\psi_1/\alpha)^{1/2}$ . For the perturbation

$\Psi_1$  to be consistent with MHD equations in the plasma bulk, we must have

$$\frac{\partial \Psi_1(x', t)}{\partial x} - \frac{\partial \Psi_1(-x', t)}{\partial x} = \Delta' \psi_1(t) \quad x' \gg \delta$$

where  $\Delta'^{-1}$  is the usual characteristic length in the linear theory of tearing modes<sup>10)</sup>. This means that if we write the current density (along  $oz$ ) in the form

$$\begin{aligned} I(x, y) &= I_0(x) + I_1(x) \cos(k_0 y) + \dots \\ I_1(x) &= 2 \int_{-\pi}^{+\pi} I(x, y) \cos(k_0 y) \frac{dk_0 y}{2\pi} \end{aligned} \quad (1)$$

we have, using the Ampère law

$$\frac{4\pi}{c} \int_{-\delta''}^{+\delta''} I_1(x) dx = \frac{\partial \Psi_1(\delta'', t)}{\partial x} - \frac{\partial \Psi_1(-\delta'', t)}{\partial x} = \Delta' \psi_1 \quad (2)$$

We assume that the width  $\delta$  of the magnetic islands is larger than the width of the resistive sheet in the linear theory of tearing modes. Then the plasma is at equilibrium in the magnetic islands and their neighbouring. The electron temperature  $T$ , the resistivity  $\eta$  and the current density  $I$  are functions of the flux  $\psi$ . Being given  $T(\psi)$  we have :

$$\begin{aligned} I(x, y) &= I(\psi) = \frac{E_0}{\eta(\psi)} + \frac{1}{c} \frac{\partial \psi(t)}{\partial t} \frac{I'(\psi)}{\eta(\psi)} \\ I(\psi) &= \iint_{\psi, \psi+d\psi} \cos(k_0 y) dx dy \left( \iint_{\psi, \psi+d\psi} dx dy \right)^{-1} = \frac{\int \cos u du}{\int (\psi + \psi_0 \cos u)^{1/2}} \\ &= \frac{\int \frac{\cos u du}{(\psi + \psi_0 \cos u)^{3/2}}}{\int \frac{du}{(\psi + \psi_0 \cos u)^{3/2}}} \end{aligned} \quad (3)$$

$\int_{\psi, \psi+d\psi}$  is taken between the surfaces  $\psi$  and  $\psi + d\psi$ ). The time evolution  $\partial\psi/\partial t$  of the magnetic islands is obtained from (1) (2) and (3) :

$$\frac{8\pi}{c} E_0 \int_{-\delta''}^{+\delta''} dx \int_{-n}^{+n} \frac{\cos k_0 y}{\eta(\psi)} \frac{dk_0 y}{2\pi} + \frac{8\pi}{c^2} \frac{\partial\psi(t)}{\partial t} \int_{-x'}^{+x'} \int_{-n}^{+n} \frac{\Gamma(\psi)}{\eta(\psi)} \cos k_0 y \frac{dk_0 y}{2\pi} = \Delta' \psi_1(t) \quad (4)$$

In the simplest case, the temperature  $T(\psi)$  has the form  $\bar{T}_0 + \delta T(\psi)$ , where  $\delta T$  is 0 inside the islands and has opposite values on symmetric magnetic surfaces outside. In that case the first integral in (4) vanishes and we have :

$$p \frac{8\pi}{c^2} \frac{\partial\psi}{\partial t} \frac{\delta}{\eta} = \Delta' \psi_1 ; \quad p = \frac{1}{\delta} \int_{-x'}^{+x'} dx \int_{-n}^{+n} \Gamma(\psi) \cos k_0 y \frac{dk_0 y}{2\pi} \sim \frac{1}{4}$$

The width  $\delta = 2 \left( \frac{\psi_1}{\alpha} \right)^{1/2}$  grows according to the law  $\frac{d\delta}{dt} = \frac{1}{4\pi} \Delta' \frac{1}{4p}$  if  $\Delta'$  is  $> 0$ , as it is the case if initially the plasma was linearly unstable. (Note that if the temperature  $T$  peaks inside the separatrix, due to a thermal instability mechanism, the first integral in (4) is positive and prevents the development of the islands if we have

$$\frac{8\pi}{c} \frac{E_0}{\eta} \frac{\delta T}{T} Y \delta = \Delta' \psi_1 \quad \text{or} \quad \frac{\delta T}{T} = \frac{1}{4Y} \delta \Delta' \frac{\beta_{00} (dq/q dr)}{(4\pi I_0/c)}$$

where  $\delta T$  is the variation of  $T$  inside the separatrix and  $Y$  is a numerical factor depending on the temperature profile).

The distribution of current  $I$  in the presence of the islands, when compared to the initial distribution experience a flattening in the  $x$  direction.<sup>(11)</sup> The difference :

$$I'(x) = \int_{-n}^{+n} I(x, y) \frac{dk_0 y}{2\pi} - I_0(x)$$

has the form shown on fig. (3) and provides a magnetic perturbation along  $oy$  corresponding to a flux function  $\psi(x, t)$  with odd parity in  $x$ . The value of  $\psi'$  is



of the order  $\frac{4\pi}{c} \delta^3 \frac{dI_0}{dt}$ . The electric field  $E(x) = -\frac{\partial \Psi'}{\partial t}$  along  $oz$  does not perturb the current inside the separatrix, due to its odd parity. Outside the separatrix, it induces a current  $I''$  which actually is the image in the plasma bulk of the current distribution  $I'(x)$ . For the normal growth of the islands we have :

$$I'' \sim \frac{1}{c} \frac{\partial \Psi'}{\partial t} \frac{1}{\eta} \sim \frac{4\pi}{\eta c^2} \frac{dI_0}{dt} \frac{d\delta^3}{dt} \sim \delta^2 \Delta' \frac{dI_0}{dt} \ll \delta \frac{dI_0}{dt}$$

The current  $I''$  plays a negligible role in that case. However this is due to the fact that the radial width  $\delta$  over which the flattening of the current profile  $I_0$  takes place increases slowly in time. The situation could be different if the island boundaries were unstable, so that the flattening region extends rapidly. Such a situation will be considered later.

The temperature  $T$  is determined by an equation of the type

$$\frac{3}{2} n \frac{\partial T(\psi, t)}{\partial t} \iint_{\psi, \psi+d\psi} dx dy = \frac{E_0^2}{\eta(\psi)} \iint_{\psi, \psi+d\psi} dx dy - \iint_{\psi, \psi+d\psi} W(x, y) dx dy - d\psi \frac{\partial}{\partial \psi} \left( \frac{\partial T}{\partial \psi} \mathcal{K}_0(\psi) \right) \quad (5)$$

$$\mathcal{K}(\psi) = \int_{\psi(x,y)=\psi} \chi(x, y) \frac{\partial \psi(x, y)}{\partial x} dy$$

where  $\chi$  is the transport coefficient for electron energy and  $W$  is the electron power loss density. A first approach is to assume that  $\chi$  is a constant in the neighbouring of the islands, and that the flux of energy  $\mathcal{K} \frac{\partial T}{\partial \psi}$  across a magnetic surface  $\psi$  on each side of the separatrix is large compared to the algebraic power which is generated in the islands. In that case Eq (5) is equivalent to express that  $\mathcal{K} \frac{\partial T}{\partial \psi}$  is a constant outside the separatrix and is null inside. This results in the usual plateau configuration, shown on fig. (4) at  $y = 0$

$$\begin{aligned} \frac{\partial T}{\partial \psi} &= 0 \quad (-\psi_1 < \psi < \psi_1) \\ \frac{\partial T}{\partial \psi} &= \delta \frac{dT_0}{dr} \frac{1}{2\sqrt{2}} \frac{1}{\psi_2} (G(\psi))^{-1} \frac{z}{|z|} \quad (\psi > \psi_2) \quad (6) \\ G(\psi) &= \int_{-n}^{+n} \left( \frac{\psi}{\psi_1} + \cos u \right)^{1/2} \frac{du}{2\pi} \end{aligned}$$

We may however make other assumptions. For instance we may suppose that the anomalous transport of electron energy across the plasma is essentially due to a repetitive relaxation process, each period of which involves the formation of magnetic islands and subsequent explosion of these islands across the plasma. The temperature profile flattens during explosions and recovers its original value in the time intervals between explosions. In these intervals the electron heat transfer is classical, and may be small compared to the Joule power in the islands. Accordingly the conduction term may be neglected in the balance equation (5). The solution  $T(\psi, t)$  of this equation consistent with the initial conditions is obtained by averaging between the surfaces  $\psi$  and  $\psi + \delta\psi$  the temperature  $T_0(r) = \bar{T}_0 + \frac{\partial T_0}{\partial r} x$  which would take place in the absence of the islands.

$$T = \bar{T}_0 \quad (-\psi < \psi < \psi_1)$$

$$T = \bar{T}_0 + \int_{\psi, \psi+\delta\psi}^{\psi_1} \frac{dT_0}{dr} x \, d\psi \left( \int_{\psi, \psi+\delta\psi}^{\psi_1} d\psi \right)^{-1} = \bar{T}_0 + \delta \frac{dT_0}{dr} \frac{1}{\sqrt{2} H(\psi)} \frac{2}{\sqrt{2}}$$

$$H(\psi) = \int_{-\pi}^{+\pi} \frac{d\psi}{2\pi \frac{\psi}{\psi_1} + \cos \alpha} \quad (\psi > \frac{\psi_1}{2})$$

This type of profile differs from that determined by (6) by the fact that for  $y = 0$  the derivative  $\partial T / \partial x$  has a large value near the separatrix (see Fig (4)) If we suppose that a radial flux of electron energy due to drift modes is present in the plasma during the formation of the islands, it is plausible from preliminary calculations that these drift modes are much more stable, for the same value  $(\partial T / \partial x)_{y=0}$ , at the edge of the separatrix than in the plasma bulk, because the strong shear effect which takes place near the separatrix. We may admit that even in that case a strong value  $(\partial T / \partial x)_{y=0}$  near the separatrix is present. In

what follows we will assume that the temperature  $T$  experiences along the line  $y = 0$  a variation of the order of  $\delta \frac{dT_0}{dr}$  over a distance from the separatrix  $\ll \delta$ . Such a variation, and the fact that  $T$  is constant inside the separatrix is likely to drive the plasma tearing unstable near the separatrix, as we will see now.

We specify a tearing mode near the separatrix by a magnetic perturbation  $\vec{\delta B} = \nabla \times \vec{A}$ , where  $\vec{A}$  is parallel to the magnetic lines and has the form

$$A_z = \exp i h z \exp i k \sigma \quad g(\sigma, \psi) \quad ; \quad k \gg k_0$$

where  $h$  and  $k$  are constants and  $\sigma = \sigma(x, y)$  is a coordinate along the lines  $\psi$  in the plane  $z = 0$  such that, along the flux lines

$$\frac{d\sigma}{dz} = \omega(\psi)$$

We choose  $\sigma$  so that  $\sigma = 0$  and  $\partial\sigma/\partial y = 1$  for  $y = 0$ . This means that  $\omega(\psi) = \frac{B_y(\psi)}{B_0}$ , if  $B_y(\psi)$  is the value of  $B_y$  on the surface  $\psi$  at  $y = 0$ .

It is easily verified that

$$\left| \frac{\partial\sigma}{\partial x} \right| < \frac{\partial\sigma}{\partial y} \sim 1 \quad \text{for} \quad \sigma < \sigma_0 \approx \left( \frac{24\delta}{k_0^2} \right)^{1/3} \leq \frac{1}{k_0} \quad (7)$$

To test the stability of the mode we consider the quantity (neglecting plasma pressure)

$$\delta\mathcal{L} = \frac{1}{4\pi} \iiint |\nabla \times \vec{A}|^2 dx dy dz - \frac{1}{c} \iiint \vec{\delta I} \cdot \vec{A}^* dx dy dz \quad (8)$$

where  $\vec{\delta I}$  is the perturbed current associated with the mode. The second integral in (7) must be taken as a principal part on each magnetic surface where the current  $\vec{\delta I}$ , as it results from MHD equations, is singular. The mode is unstable if  $\delta\mathcal{L}$  is  $< 0$ .<sup>12)</sup> We assume that  $k\sigma_0 \gg 1$  and take  $g(\sigma, \psi)$  localized inside the interval  $-\sigma_0 < \sigma < \sigma_0$ , in the form

$$g = \bar{g} \exp(-\alpha |x|) \quad (|\sigma| < \sigma_0)$$

$$g = 0 \quad (\sigma > \sigma_0)$$
(9)

where  $X(\psi)$  is the distance between the magnetic surface  $\psi$  and the separatrix at  $y=0$ . We first have, taking into account (7)

$$\frac{1}{4\pi} \iiint |\nabla \times \vec{A}|^2 dx dy dz = \frac{2\sigma_0}{4\pi} \int dz (k^2 + \alpha^2) |\bar{g}|^2 \frac{1}{\alpha} \quad (10)$$

Neglecting the plasma pressure, the current perturbation  $\vec{\delta I}$  is determined (except at singularities) by  $\vec{i} + \vec{\delta I}$  parallel to the perturbed flux lines and  $\text{div}(\vec{i} + \vec{\delta I}) = 0$ . We obtain

$$\vec{\delta I}_z = \exp i(hz + k\sigma) \cdot \exp(-i(h+k\omega)\frac{\sigma}{\omega}) \cdot v(\sigma, \psi) \quad (11)$$

where  $v$  satisfies the equation

$$\frac{\partial v}{\partial \sigma} = i k \frac{dI}{d\psi} \bar{g} \exp(-\alpha|x|) \exp\left(i(h+k\omega)\frac{\sigma}{\omega}\right)$$

The quantity  $\delta I_z$  is zero inside the separatrix where  $\frac{dI}{d\psi} = 0$ . We obtain outside

$$v(\sigma, \psi) = \left[ w(\psi) + \exp i(h+k\omega)\frac{\sigma}{\omega} \right] \frac{\omega}{h+k\omega} k \frac{dI}{d\psi} \bar{g} \exp(-\alpha|x|) \quad (|\sigma| < \sigma_0) \quad (12)$$

$$v(\sigma, \psi) = \left[ w(\psi) + \exp i(h+k\omega)\frac{\pm\sigma_0}{\omega} \right] \frac{\omega}{h+k\omega} k \frac{dI}{d\psi} \bar{g} \exp(-\alpha|x|)$$

$$(|\sigma| > \sigma_0)$$

On each surface  $\psi$ ,  $w(\psi)$  is determined by a periodicity condition for  $\delta I$  between the planes  $y = \pm \frac{\pi}{k_0}$ , corresponding to  $\sigma = \pm \sigma_1(\psi)$ . This condition may be written

$$w = w' + w''$$

$$w' = -\cos \frac{h+k\omega}{\omega} \sigma_0 ; \quad w'' = \frac{\cos(h\sigma_1/\omega)}{\sin(h\sigma_1/\omega)} \frac{\sin(h+k\omega)\sigma_0}{\omega} \quad (13)$$

When approaching the separatrix from outside, the quantity  $\sigma_0$  remains finite, while  $\sigma_1$  diverges logarithmically. The quantity  $\delta I$  is singular for the values of  $\psi$  for which  $h \sigma_1 / \omega = n\pi$ . The contribution to the integral  $\iiint \frac{\delta I_z A_z^*}{c}$  of that part of  $\delta I_z$  which is proportional to  $\omega^n$  consists of a sum of terms with alternate signs. This contribution is negligible. We finally obtain from (9), (11), (12), (13).

$$\iiint \vec{A}^* \cdot \frac{\delta \vec{I}}{c} dx dy dz = \iiint \left( A_z^* \frac{\delta I_z}{c} dx dy dz \approx \frac{1}{c} \int dz 2\sigma_0 \int \frac{|g|^2 k \frac{dI}{d\psi}}{h+k\omega} (\exp -2\alpha |X|) \left( 1 - \frac{\cos(h+k\omega)\sigma_0/\omega \sin(h+k\omega)\sigma_0/\omega}{(h+k\omega)\sigma_0/\omega} \right) dX \right) \quad (14)$$

We have assumed that the current density  $I(\psi)$  varies by a quantity  $\sim \delta \frac{\sigma_0}{dr}$  when  $X(\psi)$  varies by a distance  $\delta' \sim \delta$ . We choose  $h$  and  $k$  so that the quantity  $[h+k\omega(\psi)]\sigma_0/\omega(\psi)$  varies accordingly in the interval  $(\pi/2, \pi)$ . We have  $[\omega(\psi_1) - \omega(\psi_2)]/\omega(\psi_1) = [B_y(\psi_1) - B_y(\psi_2)]/B_y(\psi_1) = X(\psi)/\delta$ . Therefore

$$\frac{h+k\omega(\psi)}{\omega(\psi)} = \left( \frac{h}{\omega(\psi)} + k \right) - \frac{h}{\omega(\psi)} \frac{X(\psi)}{\delta} \quad (15)$$

and we have  $\frac{1}{2} \ll [h+k\omega(\psi)]\sigma_0/\omega(\psi) < \pi$  for  $|X(\psi)| < \delta'$  if

$$h \approx -k\omega(\psi); \quad |k\sigma_0| \approx \frac{\delta}{\delta'} \gg 1.$$

Assuming also that  $\delta' \ll 1/\alpha$  and that, for  $|x(\psi)| < \delta'$

$$\frac{dI}{d\psi} \approx - \left( \frac{dI_0}{d\tau} \delta \right) \frac{1}{B_y(\psi)} \delta' \approx \frac{dI_0}{d\tau} \frac{1}{B_{00} (dq/q d\tau)} \delta' \frac{z}{|z|}$$

We obtain from (14) and (15)

$$\mathcal{P} \int \int \int \frac{\vec{A}^* \cdot \vec{\delta I}}{c} dx dy dz \approx \frac{1}{c} \int dz 2\sigma_0 |\vec{q}|^2 k \frac{dI_0}{d\tau} \frac{1}{B_{00} (dq/q d\tau)} \sigma_0 \frac{z}{|z|} \quad (16)$$

Replacing  $\sigma_0$  by its value  $(24 \delta / k_0^2)^{1/3}$  and choosing  $\alpha \sim |k|$ ,

$k \frac{z}{|z|} < 0$ , we find that the quantity  $\omega \mathcal{G}$ , as it results from (8), (10) and (16), is negative, i.e. the mode is unstable, if we have

$$\frac{2\pi}{c} \frac{dI_0}{d\tau} \frac{1}{B_{00} (dq/q d\tau)} \left( \frac{24 \delta}{k_0^2} \right)^{1/3} > 1$$

a condition which may be satisfied in practice. Further calculations taking into consideration the resonant layers  $k_{\perp}(\psi)/\omega(\psi) = \dots$  results in the following estimation for the growth rate  $\gamma$  of the mode

$$\gamma \sim \frac{1}{\tau_R^{3/5}} \frac{1}{\tau_A^{2/5}}$$

where

$$\frac{\eta c^2 \tau_R}{4\pi} \sim \tau^2; \quad C_{AB} \tau_A \sim \tau$$

and  $C_{AB} = B_{00} / (\sqrt{\pi} \rho)^{1/2}$  is the Alfvén velocity in the field  $B_{00}$ .

If such an instability takes place, the current density  $I$  is equalized over a distance of the order  $\delta'$  in a time of the order  $\delta^{-1}$ . As explained above, such a reorganization induces an inductive electric field  $E'(\delta)$  ( $< 0$  for  $x > 0$  and  $> 0$  for  $x < 0$ ) consistent with the variation of the magnetic flux in the  $y$  direction inside the separatrix. In the present case, the variation  $I'(\delta)$  of the quantity  $\int_{-\pi}^{+\pi} I(x,y) \frac{dt_0 y}{2\pi}$  due to the instability is of the order of  $\frac{dI_0}{dr} \delta$  over a distance  $\delta'$ . It induces by itself the electric field

$$E' \sim \frac{\delta}{c} \delta \frac{dI_0}{dr} \delta \delta' \frac{4\pi}{c}$$

The field  $E'$  produces a current  $I''$  outside the separatrix of the order of

$$I'' \sim \frac{E'}{\eta} \sim \left( \frac{\tau_R}{\tau_A} \right)^{3/5} \left( \frac{dI_0}{dr} \delta \right) \frac{\delta \delta'}{r^2}$$

The current  $I''$  is the image in the plasma bulk of the current  $I'$  and is therefore localized over a distance  $\delta''$  such that  $I'' \delta'' \sim I' \delta'$ . The instability generates a new gradient

$$\frac{dI}{dr} \sim \frac{I''}{\delta''} \sim \frac{I''^2}{I' \delta'} \sim \left( \frac{\tau_R}{\tau_A} \right)^{6/5} \frac{\delta^3 \delta'}{r^4} \frac{dI_0}{dr}$$

which may be strong enough to produce a new generation of unstable tearing modes.

III - ERGODISATION OF THE MAGNETIC SURFACES.

We consider this situation as the first step of a progressive ergodisation of the magnetic surfaces. On each side of the resonant magnetic surface  $r = r_0$ . We assume that in the interval  $r_1 < r < r_2$ , the current density is equal to  $\bar{I}(r, t) + \delta I$  where  $\bar{I}$  is a local average value of  $I$  and  $\delta I$  is a strong fluctuation, corresponding to a fluctuation  $\delta B_r, \delta B_\theta$  of the magnetic field. Outside the interval  $(r_1, r_2)$  the current density  $I$ , the magnetic field  $B_\theta$  and the poloidal flux  $\Psi$  retain their initial value  $I_0(r), B_{\theta 0}(r)$  and  $\Psi_0(r)$ . The average current density  $\bar{I}$  in the interval  $(r_1, r_2)$  results from a flattening of the initial current  $I_0$  due to the fluctuation  $\delta B$ , and has therefore the structure shown on fig. (5), with  $\bar{I} - I_0 > 0$  for  $r > r_1$  and  $\bar{I} - I_0 < 0$  for  $r < r_1$ . In the interval  $(r_1, r_2)$  the field  $\bar{B}_\theta$  due to  $\bar{I}$  and  $I_0$ .

$$\bar{B}_\theta = \frac{1}{2\pi r} \frac{4\pi}{c} \int_0^r 2\pi r' I(r') dr' \quad (I(r) = I_0(r) \text{ for } r < r_1; I(r) = \bar{I}(r) \text{ for } r_1 < r < r_2)$$

$$\bar{B}_\theta - B_{\theta 0} \sim -\frac{4\pi}{c} \bar{I} - I_0 (r_2 - r_1) \quad (17)$$

is smaller than the initial field  $B_{\theta 0}(r)$ . The poloidal magnetic flux embraced by the interval  $(r_1, r_2)$  must be equal to the difference  $\Psi_0(r_2) - \Psi_0(r_1)$ . The simplest manner to achieve this condition is to introduce in the unperturbed region near the boundaries  $r_1, r_2$ , spatial pulses of current density  $J_1(r, t) > 0$  and  $J_2(r, t) < 0$  such that  $\int J_1 dr \simeq -\int J_2 dr$  (see fig. (5)). These currents produces an additional perturbation  $\delta B_\theta = \left( \int \frac{4\pi}{c} J_1 2\pi r dr \right) (2\pi r)^{-1}$  in the interval  $(r_1, r_2)$ , which must verify

$$\int_{r_1}^{r_2} (\bar{B}_\theta + \delta B_\theta) dr = \int_{r_1}^{r_2} B_{\theta 0} dr$$

This condition implies that

$$\int J_1 dr \sim -\int J_2 dr \sim |\bar{I} - I_0| (r_2 - r_1) \quad (18)$$



On the other hand, for the field  $B_\theta + \mathcal{B}_\theta$  to be equal to  $I_{cr}$  for  $r > r_2$  and  $r < r_1$ , we must have

$$\int (\bar{I}(r) + \mathcal{J}(r) - I_0(r)) 2\pi r dr = 0$$

At a given time  $t$ , the average poloidal flux  $\bar{\Psi}(r, t)$  has the structure shown on fig. (5). If the interval  $r_2 - r_1$  and the difference  $\bar{I} - I_0$  increase in time, an average electric field  $\bar{E}(r, t) = \frac{-1}{c 2\pi R} \frac{\partial \bar{\Psi}}{\partial t}$  appears in the interval  $(r_1, r_2)$  and near the boundaries  $r_1, r_2$ . The value of this field is approximatively given by

$$\bar{E} = \frac{-1}{c 2\pi R} \frac{\partial(\bar{\Psi} - \Psi_0)}{\partial t} \sim \pm v \frac{1}{2\pi R c} \frac{\partial(\bar{\Psi} - \Psi_0)}{\partial r} \quad (r \gtrsim r_0)$$

where  $v \approx \frac{dr_1}{dt} \approx \frac{dr_2}{dt}$ . Using the value of  $\bar{B}_\theta - B_{\theta 0} \sim \mathcal{B}_\theta$  given by (17) we obtain  $|\bar{\Psi} - \Psi_0| \sim \frac{4\pi}{c} (r_2 - r_1)^2 |\bar{I} - I_0|$  and

$$|\bar{E}| \sim v \frac{4\pi}{c^2} (r_2 - r_1) |\bar{I} - I_0| \quad (19)$$

The field  $\bar{E}$  is  $> 0$  for  $r < r_0$  and  $< 0$  for  $r > r_0$ . The variation of the energy associated to the average field  $\bar{B}_\theta + \mathcal{B}_\theta$  is given by

$$\frac{d\mathcal{E}}{dt} = - \int_{r_1}^{r_2} 2\pi r dr \bar{E} (\bar{I} + \mathcal{J}) \approx - \int_{r_1}^{r_2} 2\pi R 2\pi r dr \bar{E} I_0$$

We will find below that  $\int \bar{E} 2\pi r dr = 0$ . As  $\bar{E}$  is positive in the region where  $I_0$  is the largest, the quantity  $\frac{d\mathcal{E}}{dt}$  is negative and we have approximatively

$$\frac{d\mathcal{E}}{dt} \sim - 2\pi R 2\pi r \left| \frac{dI_0}{dr} \right| |\bar{E}| (r_2 - r_1)^2 \quad (20)$$

We try to analyse the self consistency of such a situation. We first note that the pulses  $\mathcal{J}_1, \mathcal{J}_2$  (which are the images in the unperturbed region of the current perturbation  $\bar{I} - I_0$  in the interval  $(r_1, r_2)$ ) are induced by the field  $\bar{E}$  in layers of width  $\delta r$  at the edges of the ergodic region. We have

$$|j_1| \sim |j_2| \sim \frac{E}{\eta \alpha}$$

where the coefficient  $\alpha > 1$  reflects an eventual increase of the plasma resistivity in these layers. The width  $\delta r$  is determined by (18) and (19)

$$\begin{aligned} \delta r &\sim \left| \frac{J - I_0}{j_1} \right| (r_2 - r_1) \\ \delta r &\sim \eta c^2 \frac{1}{4\pi v} \alpha \end{aligned} \quad (21)$$

The pulses  $J_1$  and  $J_2$ , because of the large value of  $\partial j_1 / \partial r$  and  $\partial j_2 / \partial r$ , are likely to drive unstable tearing modes in the unperturbed region near  $r_1, r_2$ , with a transverse wave number  $(\delta r)^{-1}$

We admit that these modes build up magnetic eddies with the transverse scale  $\delta r$ , which actually form the magnetic turbulence  $\delta B$ . We further admit that this turbulence is strong enough for a connection of the flux lines across the whole interval  $(r_1, r_2)$  to exist. This means that we have

$$\frac{\delta B}{\delta r} > B_{\theta 0} \frac{d\theta}{q dr} \quad (22)$$

However the eddies, while produced by instability near

$r = r_1$  and  $r = r_2$ , must be maintained afterwards by the inductive electric field  $\bar{E}$ . This is only possible if the turbulence  $\delta B$  is weak enough, so that each eddy has a magnetic axis consisting of a closed magnetic line. We admit that this condition is compatible with (22). An upper limit for the current fluctuation  $| \delta I | \sim \frac{c}{4\pi} \delta B / \delta r$  which causes the fluctuation  $\delta B$  is given by  $|\frac{\bar{E}}{\eta}|$ . Therefore we have

$$\frac{\delta B}{\delta r} < \frac{4\pi |\bar{E}'|}{c \eta} \quad (23)$$

Another limit for  $\delta B$  is obtained by expressing that the power  $\int \eta |\delta I|^2 2\pi r 2\pi R dr$  is smaller than the available power  $-\frac{d\mathcal{E}}{dt}$  given by (20). We must have

$$\left(\frac{\delta B}{\delta r}\right)^2 < \left(\frac{4\pi}{c}\right)^2 \frac{|\bar{E}'|}{\eta} \frac{dI_0}{dr} (r_2 - r_1) \quad (24)$$

The limit (24) is compatible with (22) and (2) if  $|\bar{E}'| \gg E_0 = \eta I_0$ .

The influence of the turbulence  $\delta B$  on the average current profile  $\bar{I}(r, t)$  may be calculated as follows. In the presence of  $\delta B$  the electron current, which is directed along the flux lines (neglecting pressure effects), induces a charge density  $\rho$  having the spatial structure of the magnetic eddies. We have

$$\frac{\partial \rho}{\partial t} = -\text{div} \left( \frac{I \vec{B}}{B_0} \right) = -(\nabla I)_r \frac{\vec{B}}{B_0} = -\frac{\partial \bar{I}(r)}{\partial r} \frac{\delta B_r}{B_0} - \frac{\vec{B}_0 \cdot (\nabla \delta I)}{B_0} \quad (25)$$

The variation  $\partial \rho / \partial t$  must be cancelled by the charge variation due to the time variation of an electric potential  $\phi_e$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t} \left( \frac{\epsilon}{4\pi} \Delta \phi_e \right) = \frac{\epsilon}{4\pi} (\delta r)^{-2} \frac{\partial \phi_e}{\partial t} \quad (26)$$

where  $\epsilon = c^2 / c_A^2$  is the plasma dielectric constant. ( $c_A = (B_0 / 4\pi \rho)^{1/2}$ )

The potential  $\phi_e$  induces an electric field  $E'$  along the flux lines such that

$$E' = -(\nabla \phi_e) \cdot \frac{\vec{B}}{B_0} = -\text{div} (\phi_e \vec{B} / B_0) \quad (27)$$

It results from (25) and (26) that

$$\frac{\partial \phi_e}{\partial t} = - \frac{4\pi (\delta r)^2}{\epsilon} \left( \frac{\partial \bar{I}}{\partial r} \frac{\delta B_r}{B_0} + \frac{\vec{B}_0 \cdot \nabla \delta I}{B_0} \right)$$

The value of  $\vec{B} \frac{\partial \phi_e}{\partial t}$  averaged over the scale of the magnetic eddies is given, neglecting the correlations between  $\delta B_r$ ,  $\delta B_\theta$  and  $\vec{B}_0 \cdot \nabla \delta I$ , by

$$\frac{1}{B_0} \overline{\vec{B} \frac{\partial \phi_e}{\partial t}} = - \frac{4\pi (\delta r)^2}{\epsilon} \frac{\partial \bar{I}}{\partial r} \frac{\overline{|\delta B_r|^2}}{B_0^2} \vec{e}_r$$

The average value  $\bar{E}'$  of the field  $E'$  given by (27) then satisfies, neglecting the average value of the quantity  $\left( \int \delta \vec{B} dt \right) \cdot \frac{\partial \delta \vec{B}}{\partial t}$

$$\frac{\partial \bar{E}'}{\partial t} = - \text{div} \left( \frac{\partial \phi_e}{\partial t} \frac{\vec{e}_r}{B_0} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{4\pi (\delta r)^2}{\epsilon} \frac{\partial \bar{I}}{\partial r} \frac{\overline{|\delta B_r|^2}}{B_0^2} \right] \quad (28)$$

The average equilibrium of electrons along the flux lines (at the scale of the magnetic eddies) implies that  $\bar{E} + \bar{E}' = \eta \bar{I}$ . We then obtain, using (28)

$$\eta \frac{\partial \bar{I}}{\partial t} - \frac{\partial \bar{E}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{4\pi (\delta r)^2}{c^2} c_{A0}^2 \frac{\overline{|\delta B_r|^2}}{B_{0\theta}^2} \frac{\partial \bar{I}}{\partial r} \right] \quad (29)$$

( $c_{A0} = (B_{0\theta} / 4\pi e)^{1/2}$  ;  $\overline{|\delta B_r|^2} \sim \delta B^2$ )

As the inductive field  $\vec{E}(r,t)$  is proportional to the rate of flattening of the profile  $\bar{I}(r,t)$ , the equation (29) gives this rate in terms of the profile  $\bar{I}(r)$  and the level  $\delta B$  of the magnetic turbulence. This equation is only applicable in the domain  $r_1' < r < r_2'$  where the turbulence  $\delta B$  is fully developed (see Fig. (5)) and  $\bar{I} \approx I_0$ . We need an additional

information on the field  $\bar{E}$  in the regions  $r_1'' < r < r_1'$  and  $r_1' < r < r_2''$  where the current pulses  $J_1$  and  $J_2$  take place : it results from (27) that  $\int_{r_1'}^{r_2''} E' r dr d\theta = 0$  and we have accordingly

$$\int_{r_1''}^{r_2''} [\bar{E} - \eta \bar{I} - \alpha \eta Y] r dr = 0 \quad (Y = Y_1 \text{ or } Y_2) \quad (30)$$

Combining (29) and (30) gives

$$\int_{r_1'}^{r_2''} \left[ \frac{\partial \bar{E}}{\partial t} - \frac{\partial(\eta I + \alpha \eta Y_{1,2})}{\partial t} \right] dr = \left[ \frac{4\pi(\delta r)^2}{c^2} \frac{CA_0}{B_{00}^2} \delta B_r^2 \frac{dI_0}{dr} \right]_{r=r_1', 2} \quad (31)$$

The field  $\bar{E}$  is larger than  $\eta \bar{I}$  and of the same order as  $\alpha \eta Y_{1,2}$ . The equations (29), (30) and (31) are approximately valid if one reduces the L.H.S. to the term proportional to  $\bar{E}$  or  $\partial \bar{E} / \partial t$ .

In the transition regions ( $r_1''$  -  $r_2'$ ) or ( $r_2'$  -  $r_2''$ ) we have

$$\frac{\partial \bar{E}}{\partial t} = \pm v \frac{\partial \bar{E}}{\partial r}. \text{ We obtain from (31) the value of } \bar{E} \text{ at } r = r_1' \text{ or } r_2':$$

$$(\bar{E})_{r=r_1', 2} \sim (\pm) \left[ \frac{4\pi \delta r^2}{c^2} \frac{CA_0}{B_{00}^2} \delta B^2 \frac{dI_0}{dr} \right]_{r=r_1', 2}$$

The same order of magnitude results from (29) in the whole interval ( $r_1'$  -  $r_2'$ )

if we assume that  $\left| \frac{\partial}{\partial t} \right| \sim v \left| \frac{\partial}{\partial r} \right|$ . Using the limiting value of  $\partial B$  given by (24) and the estimations (21) and (19), we obtain, taking  $\left| \frac{dI_0}{dr} \right| \sim \frac{I_0}{a^2} r_0$

$$I_0 \sim \frac{B_{00} c}{4\pi r_2} \quad \text{and} \quad r_2 - r_2' \sim r_0 \quad (a = \text{limiter radius})$$

$$\begin{aligned}
 \nu &\sim \alpha^{1/5} \frac{\Gamma_0}{\tau_A^{2/5} \tau_R^{3/5}} \left(\frac{r_0}{a}\right)^{2/5} \\
 \delta r &\sim \alpha^{1/5} \Gamma_0 \left(\frac{\tau_A}{\tau_R}\right)^{2/5} \left(\frac{a}{r_0}\right)^{2/5} \\
 |\dot{z}| &\sim \eta^{-1} \left|\frac{\bar{I} - I_0}{I_0}\right| \alpha^{1/5} \left(\frac{\tau_R}{\tau_A}\right)^{2/5} \left(\frac{r_0}{a}\right)^{2/5} \\
 \frac{\delta B}{B_0} &\sim \alpha^{3/5} \left|\frac{\bar{I} - I_0}{I_0}\right|^{1/2} \left(\frac{\tau_A}{\tau_R}\right)^{1/5} \left(\frac{r_0}{a}\right)^{3/10} \\
 \tau_R &\sim \frac{4\pi r_0^2}{\eta c^2} ; \tau_A \sim \frac{\Gamma_0}{c_{16}} ; \alpha \sim \frac{\bar{I}}{\eta^{5/3} I_0} \gg 1
 \end{aligned}
 \tag{32}$$

In practical cases the ratio  $\frac{\tau_A}{\tau_R}$  is  $\sim 10^{-5}$  and the time scale  $\tau_R^{3/5} \tau_A^{2/5}$  is  $\sim 10^{-4}$  sec.

Let us consider the situation when the boundary at  $r = r_2$  of the ergodic region reaches the limiter at  $r = a$ . We assume to simplify that the coupling between the primary current  $I_{pr}$  and the plasma current  $I_{pl}$  is perfect and that the impedance of the primary circuit is infinite. Then  $I_{pr}$  is constant but the toroidal electric field at the plasma edge is free from any outer constraint. At the time  $t_0$  when  $r_2 = a$ , the average poloidal flux  $\bar{\psi}(r, t)$  has the structure A indicated on the fig. (6). After  $t_0$ , the evolution of  $\bar{\psi}$  inside the plasma continue to be determined by Eq. (29), (taking into account that  $\bar{E} = -\frac{1}{c} \frac{\partial \bar{\psi}}{\partial t}$ ). This equation imposes a relatively small variation of  $\bar{E}$  and therefore  $\bar{\psi}$  experiences the evolution shown on fig. (6). To preserve the condition (30), which expresses that the integral  $\int \bar{\psi} r dr$  keeps a small value, the ergodic domain must continue to extend towards the magnetic axis at  $r = r_1$ , while the value of  $\bar{\psi}$  at  $r = a$  increases. This latter circumstance corresponds to the presence of an observable negative voltage per turn  $V = -\frac{1}{c} \left(\frac{\partial \bar{\psi}}{\partial t}\right)_{r=a}$  at the plasma edge. Note also that a strong variation of the slope  $\frac{\partial \bar{\psi}}{\partial r}$  must exist at the plasma edge between a radius  $r = a' < a$  and  $r = a$ . This discontinuity corresponds to a stationary current pulse  $j_1$ .

which replaces the moving pulse  $y_2$ . The total current in the plasma is equal to

$$I_{P2} = y_2' - \left( \frac{\partial \Psi}{\partial r} \frac{c}{4\pi} \frac{r}{R} \right)_{r=a'}$$

and the pulse  $y_2'$  in the interval  $(a'a)$  must exist as long as the quantity  $\left( \frac{\partial \Psi}{\partial r} \right)_{r=a'}$  inside the plasma has not recovered its initial value  $\left( \frac{\partial \Psi_0}{\partial r} \right)_{r=a'}$ , i.e. up to the time  $t_1$ , when the flux  $\Psi$  takes the structure B indicated on the Fig. (6). During the period  $(t_0, t_1)$ , the electric field  $\frac{V}{2\pi R}$  must be large enough to induce the current  $y_2'$  and is eventually larger than the field  $\bar{E}$  estimated above during the phase of ergodisation. Because of the condition (30) such an enhancement near  $r=a$  must be balanced by an enhancement of the positive value of  $\bar{E}$  near  $r=r_1$ . After the time  $t_1$  the electric field at  $r=a$  and therefore at  $r=r_1$  vanishes. The progression of the ergodic domain stops and the whole ergodisation disappears.

The negative voltage  $V$  is therefore present during the period  $(t_0, t_1)$  only. The quantity  $-\frac{1}{c} \int_{t_0}^{t_1} V dt$  is equal to the variation of  $\Psi$  at  $r=a$  between the states A and B; and we have

$$\left| \int_{t_0}^{t_1} V dt \right| \sim 2\pi R (r_2 - r_1)^2 \frac{4\pi \bar{E} - I_0}{c}$$

Similar considerations show that the ergodisation vanishes if the internal boundary  $r=r_1$  of the ergodic domain reaches the magnetic axis before the external boundary  $r=r_2$  reaches the limiter.

Then no change of  $\Psi$  and therefore no voltage pulse at the plasma edge can be observed. This could be the case of the internal disruptions. The radius  $r=r_0$  would then be the radius of the magnetic surface where  $q=1$ , which is a small fraction of the limiter radius.

IV - CONCLUDING REMARKS

The most important practical consequence of the scheme reported above is that the strong thermal conduction of electrons along the flux lines should result in a strong radial conduction when the flux lines are ergodic. The corresponding law of evolution of the average electron temperature  $\bar{T}(r,t)$  may be shown to be (neglecting collisions)

$$\frac{\partial \bar{T}(r,t)}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} (r K \frac{\partial T}{\partial r})$$

$$K \sim \delta r \frac{\delta \beta}{\beta_0} v_{the}$$

where  $v_{the}$  is the thermal velocity of electrons. Integrating these equations during the time  $\sim \frac{r_0}{v}$  when ergodisation is present and using the estimations (32) of the quantities  $\delta r$ ,  $\delta \beta$  and  $\delta r$ , we obtain the temperature variation  $|\bar{T} - T_0|$  during the ergodisation period, if  $\frac{\partial T}{\partial r} \sim T_0 \frac{r_0}{a^2}$ ,  $\frac{\partial^2 T}{\partial r^2} \sim \frac{T_0}{a^2}$

$$\left| \frac{\bar{T} - T_0}{T_0} \right| \sim \frac{v_{the}}{c_A} \left| \frac{\bar{I} - I_0}{I_0} \right|^{1/2} \left( \frac{r_0}{a} \right)^{1/2} \quad (33)$$

$$(c_A = \beta_0 / (4\pi e))^{1/2}$$

If the ergodisation mechanism is to explain the internal disruptions, the variations  $\bar{T} - T_0$  and  $\bar{I} - I_0$  must be cancelled during the regeneration periods of duration  $\tau_d$  between disruptions. Let us assume that the plasma behaviour is classical during these periods, so that the temperature  $T(r,t)$ , the current density  $I(r,t)$  and the electric field  $E(r,t) = -\frac{1}{c} \frac{\partial \psi(r,t)}{\partial t}$  are governed by the equations, on the magnetic axis



$$n \frac{\partial T}{\partial t} = \eta I^2$$

$$E = \eta(T) I$$

$$(\delta E = \delta \eta I + \eta \delta I = -\frac{3}{2} \frac{\delta T}{T} \eta I + \eta \delta I \approx -\frac{3}{2} \frac{\delta T}{T} \eta I; \text{ if } \frac{\delta T}{T} \gg \frac{\delta I}{I})$$

where  $n$  is the particle density. It is readily shown that the temperature and current density variations  $(T-T_0)'$  and  $(I-I_0)'$  during the time  $\tau_d$  then satisfy

$$n |(T-T_0)'| \sim \eta I_0^2 \tau_d$$

$$\frac{(T-T_0)'}{T_0} \eta I_0 \tau_d \sim \frac{1}{c} r_0^2 \frac{4\pi}{c} |(I-I_0)'|$$

Therefore we have :

$$\left| \frac{(T-T_0)'}{T_0} \right| \sim \frac{CA\theta}{v_{the}} \left( \frac{m_i}{m_e} \right)^{1/2} \left[ \frac{(I-I_0)'}{I_0} \right]^{1/2} \quad (34)$$

where  $m_e$  and  $m_i$  are the mass of ions and electrons. The compatibility of (33) and (34) imposes that

$$\beta_\theta = \frac{n T_0}{\beta_{\theta 0}^2 / 4\pi} \sim \left( \frac{m_e}{m_i} \right)^{1/2} \frac{\beta_0}{\beta_{\theta 0}} \left( \frac{a}{r_0} \right)^{3/2} \quad (35)$$

As pointed out by Dei-Cas<sup>13)</sup> imposing a limit  $\sim 1$  to  $\beta_\theta$  on the magnetic surface  $r = r_0$  where  $q = 1$  provides a method to fit the experimental values of the life time  $\tau_e$  of electron energy on the magnetic axis, for a given applied toroidal electric field  $E_0$  and magnetic field  $B_0$ . We find from (35) taking

$$\beta_{\theta 0} / B_0 = r_0 / R$$

$$\tau_e = \frac{n T_0}{\eta I_0^2} \sim \left( \frac{m_e}{m_i} \right)^{1/2} \frac{B_0}{c E_0} \frac{a^{3/2}}{r_0^{1/2}}$$

The reality of the proposed mechanism could be tested by comparing the electron temperature variation during internal disruptions or negative spikes with the variations of the ion temperature or the particle density, which should be less sensible to the ergodicity of flux lines. Also the radial distribution of energetic ions which are trapped in the ripples of the main field should be poorly influenced by the ergodicity. As suggested by Launois and Smeulders the comparison of the soft X-ray signals emitted by electrons of different energy could also give a useful information.

REFERENCES

- (1) - S.V. Mirnov; I.B. Semenov, Atomnaya Energiya 30, (1971) p.4.
- (2) - J.C. Hosea, C. Bobeldijk, D.J. Grove, Madison Conference (1971)  
Vol. II, p. 425.
- (3) - S. Von Coeler, W. Stodiek, N. Santhoff, Phys. Rev. Lett. 33, (1974)  
p. 1201.
- (4) - Groupe T.F.R. Lausanne Conference (1975).
- (5) - P.H. Rutherford, H.P. Furth, M.N. Rosenbluth, Madison Conference (1971)  
Vol. II, p. 553.
- (6) - B.B. Kadomtsev, O.P. Pogutze, Sov. Phys. JETP 38, (1974) p. 283.
- (7) - M.N. Rosenbluth, R.Y. Dagazian, P.H. Rutherford, Phys. Fluids 16, (1973)  
p. 1894.
- (8) - P.H. Rutherford, Garching Conference (1973) B21.
- (9) - P.H. Rebut, A. Samain, Grenoble Conference (1972) Vol. I, p. 29 and  
Vol. II p. 232
- (10) - H.P. Furth, J. Killeen, M.N. Rosenbluth, Phys. Fluids, 6, (1963) 459.
- (11) - E. Minardi, Tokyo Conference (1974) IAEACN 33/A13-2.
- (12) - See eg. H.P. Furth, P.H. Rutherford and H. Selberg, Matt. 897 - May 1972  
p. 5. The quantity  $\mathcal{L}$  is equal to the time integrated flux of electromagnetic energy into the resistive layers from the M.H.D. domains. The growth rate of the mode may be obtained by expressing  $\mathcal{L} \sim \mathcal{E}_J + \mathcal{E}_K$ , where  $\mathcal{E}_J + \mathcal{E}_K$  is the energy which appears in these layers in the form of Joule energy and kinetic energy.
- (13) - R. Dei-Cas To be presented at the A. P. S. Meeting (nov. 75)  
St Petersburg.

## FIGURE CAPTIONS

- Fig. (1) Tokamak Geometry
- Fig. (2) Magnetic island near the magnetic surface  
 $r = r_0$ ,  $q = 2$ .  $x = r - r_0$ ,  $y = r(\theta - \frac{\varphi}{2})$ ,  $z = R\varphi$ ,  $\kappa_0 = 2 \frac{z}{r_0}$
- Fig. (3) Structure of the averaged perturbation of the current density  $\bar{j}'(x) = \int_{-\pi}^{\pi} j(x, y) d\kappa_0 y / 2\pi = \bar{j}_0(x)$ , associated flux function  $\bar{\psi}'(x)$  and image current "I" in the bulk of plasma.
- Fig. (4) Temperature profile near the separatrix - Thermal conduction of the electrons dominant A), neglected B).
- Fig. (5) Structure of the average current density  $\bar{j}$  image currents  $j_{1,2}$  and associated flux function  $\bar{\psi}$  in the ergodic domain.
- Fig. (6) Evolution of the flux function  $\bar{\psi}$  when the ergodic domain reaches the limiter. A) at time  $t_0$ , B) at time  $t_1$

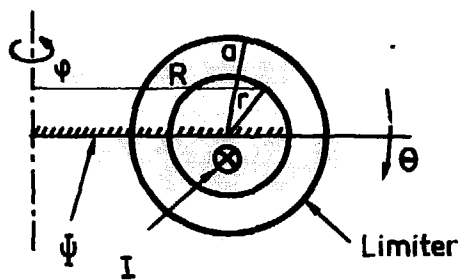


Fig. 1

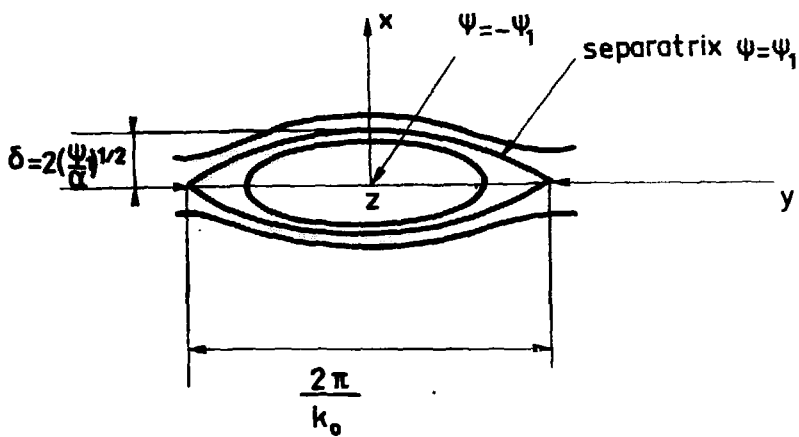


Fig. 2

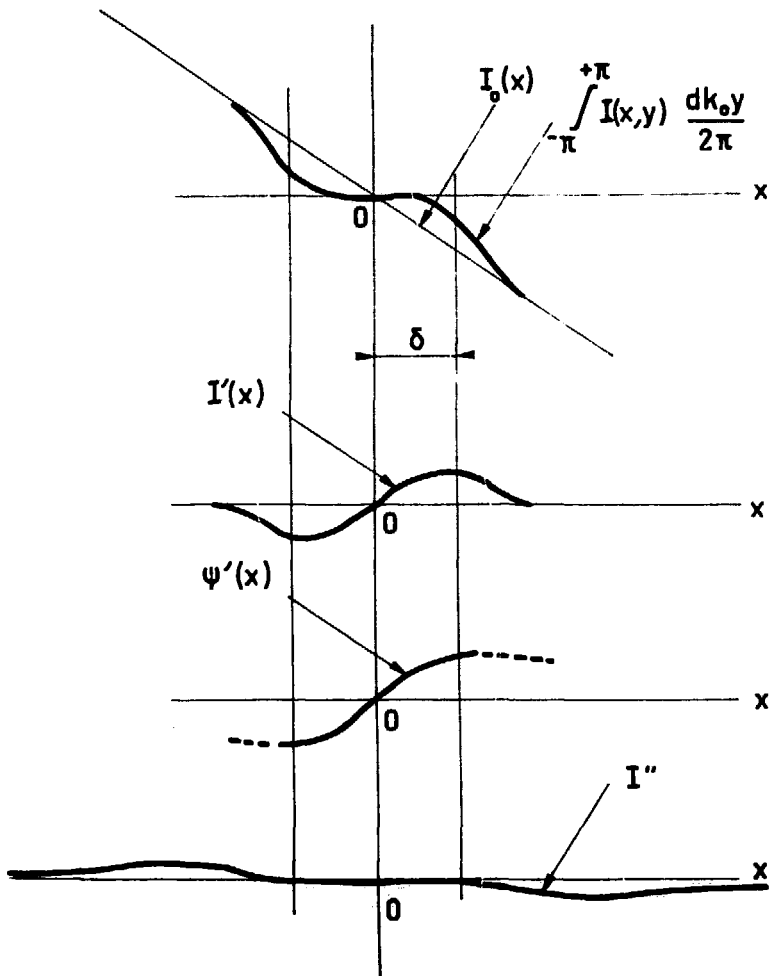


Fig. 3

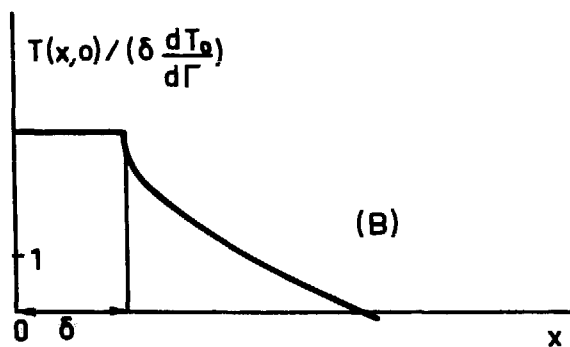
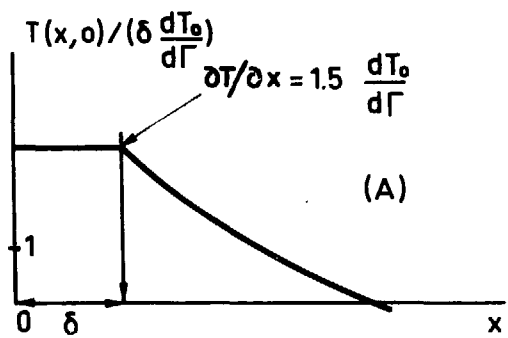


Fig. 4



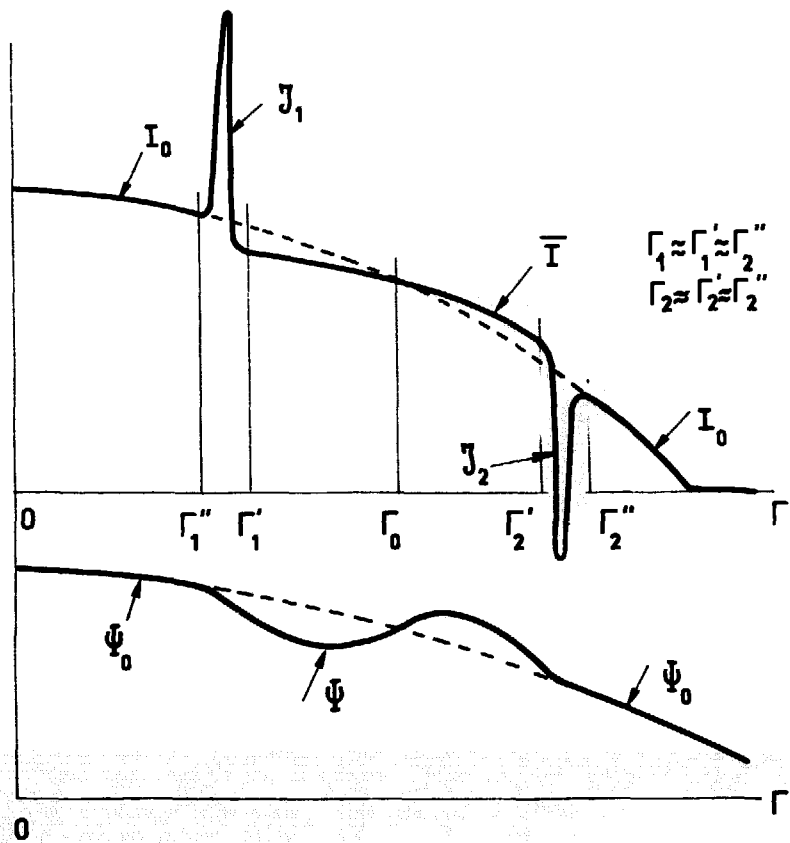


Fig. 5

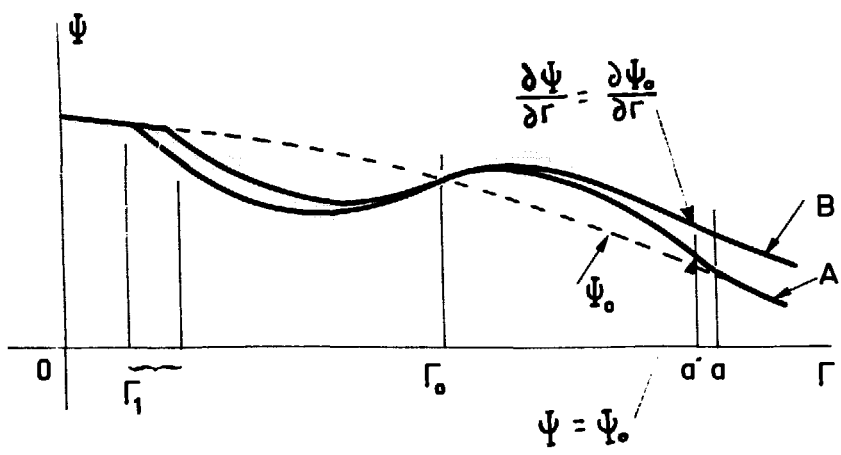


Fig. 6