

HARTREE-FOCK CALCULATION OF NUCLEAR BINDING ENERGY  
OF SODIUM ISOTOPES

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Recent measurements [1] of the nuclear binding energy ( $B$ ) of exotic sodium isotopes have shown that the two neutron separation energy  $B_{2n}$ , defined as  $B_{2n}(A, Z) = B(A, Z) - B(A-2, Z)$  displays a sudden discontinuity for the mass number  $A=31$  (see fig.1) which is very similar to those observed in the rare earth region. In the latter case this phenomenon has been explained in terms of a shape transition of the intrinsic ground state.

Previous Hartree-Fock (H.F.) calculations using a Skyrme type effective interaction [2] (called SIII) have accurately reproduced the binding energies  $B$  and the quadrupole moments  $Q$  of a large number of nuclei along the stability line. Using the same Skyrme interaction S-III we have performed H.F. calculations for the sodium isotopes  $A=19$  to  $A=35$ . For mass number  $A$  between 28 and 34 we find that there are two prolate H.F. solutions. This can be seen in the figure 2 where we show the deformation energy curves of the same sodium isotopes plotted as a function of the proton quadrupole moment. These curves have been obtained by a constrained H.F. calculation using the quadrupole moment as a constraint. For  $A$  smaller (larger) than 31 the less (more) deformed solution is more stable. The curve SIII in fig. 1 has been calculated using the lowest H.F. solution. We have corrected this curve by adding the rotational energy

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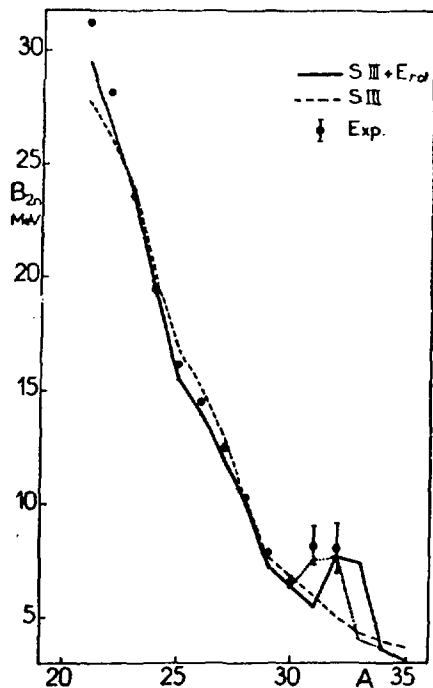


Fig.1 - Experimental and calculated values of the separation energies of the last pair of neutrons

calculated in an approximate way :  $E_{rot} = \langle J^2 \rangle / 2I$  where the rigid body value has been taken for the moment of inertia  $I$ . The corresponding curve ( $S_{III} + E_{rot}$ ) as shown in fig.1, agrees well with experiment. In particular its slope is very similar to the experimental one. This agreement shows that the (surface and volume) symmetry energy corresponding to our interaction is correct. Moreover the discontinuity in the experimental  $B_{2n}$  curves is also reproduced. We also show in fig.1 (dotted line) how the previous results change when one assumes the shape transition to occur between  $^{30}\text{Na}$  and  $^{31}\text{Na}$ . In fact this assumption is reasonable since the calculated rotational

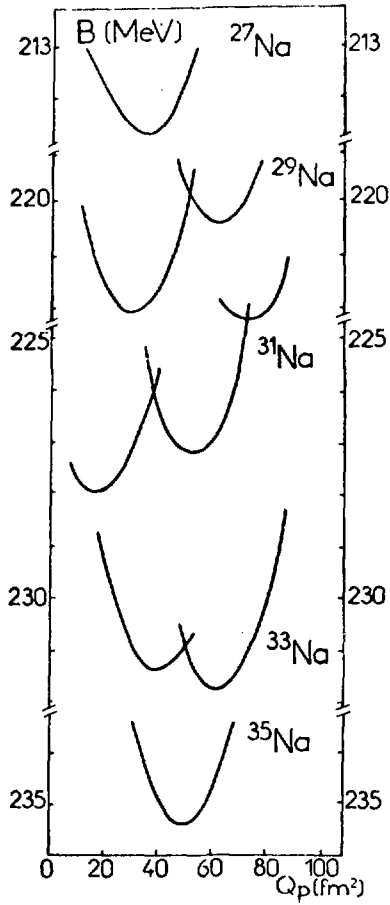


Fig. 2 - Deformation energy curves of sodium isotopes as a function of the proton quadrupole moment

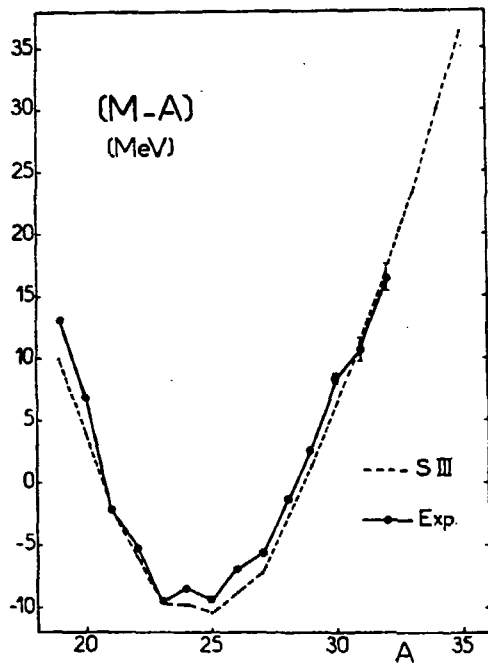


Fig.3 - Experimental and calculated mass excesses

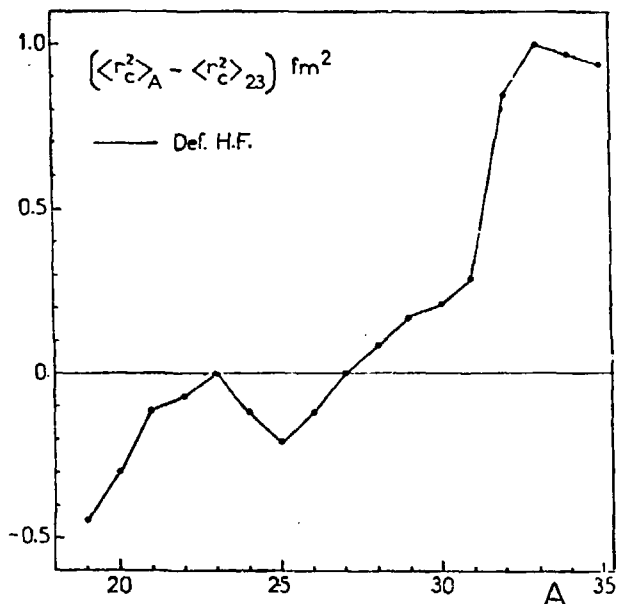


Fig.4 - Calculated isotope shift  $\delta \langle r_C^2 \rangle$  relative to the isotope  $^{23}\text{Na}_C$

energy (2.1 MeV) of the very prolate state of  $^{31}\text{Na}$  compensates the small excitation energy of this state relative to the lowest minimum of the constrained H.F. energy curve (see figure 2).

In Fig.3 we compare the experimental values of mass defects of Na isotopes to the H.F. results. The mean curvature of the experimental curve is correctly reproduced and our calculation accounts for a fraction of the odd-even staggering effect.

In table (1) we give the calculated values of the quadrupole moments of the mass and protons distributions. We have also indicated the values of the deformation parameter  $\beta_2$  of the liquid drop with the same r.m.s. radius and quadrupole moment. One notices the very large values of  $\beta_2$  for the isotopes  $A=32$  to  $A=34$ , larger than those previously found in H.F. calculations of the ground state properties of rare earth and actinide nuclei [3]. It is likable that this region of large deformation extends to the neutron rich isotopes of neighbouring elements. Calculations in this direction are in progress.

In table (1) one also sees that the values of proton  $\beta_{2p}$  and mass  $\beta_{2M}$  deformation parameters of a same isotope can be very different. This is particularly striking for the quasi-spherical isotopes preceding the transition region ( $A=28$  to  $31$ ) where the ratio  $\beta_{2p}/\beta_{2M}$  is close to 1.9. This phenomenon is remarkable since Skyrme H.F. calculations in other regions of the chart of elements, including neutron rich nuclei like  $^{150}\text{Ce}$  have always led to values of  $\beta_{2p}/\beta_{2M}$  close to one. It seems then that this strange behaviour is characteristic of light exotic nuclei. It should be noted that in many phenomenological calculations of ground state properties one assumes the equality  $\beta_{2M}=\beta_{2p}$ . Our calculations show that this assumption must be released for light nuclei.

In Fig.4 we have plotted the variation of the charge radius  $\langle r_C^2 \rangle$  (relative to the isotope  $^{23}\text{Na}$ ) as a function of the mass number  $A$ . The structure of this curve is essentially due to the deformation and the comparison of table (1) and figure (4) shows a clear correlation between the changes in  $\beta_{2p}$  and the changes in radii.

From the previous discussion and results the H.F. method appears as a powerful tool to calculate the properties of exotic nuclei. In particular the experimental

A	$Q_p$	$\beta_{2p}$	$Q_M$	$\beta_{2M}$
19	25.80	.212	33.72	.172
20	36.89	.296	59.68	.276
21	46.55	.365	86.55	.364
22	47.28	.369	93.42	.368
23	48.33	.374	101.14	.373
24	40.67	.319	83.82	.296
25	32.63	.259	64.90	.221
26	32.04	.252	66.86	.213
27	33.51	.260	74.88	.223
28	31.42	.242	69.59	.196
29	29.45	.225	64.30	.170
30	22.91	.175	45.61	.115
31	16.60	.127	28.11	.068
32	55.89	.396	174.12	.367.
33	60.37	.421	198.37	.394
34	53.90	.376	177.25	.339
35	47.23	.330	154.77	.286

Table 5 - Calculated values of the quadrupole moments of the mass ( $Q_M$ ) and protons ( $Q_p$ ) distributions

total binding energies are well reproduced. Contrary to many models based on extrapolations the H.F. method is able to predict sudden transitions like the one which seems to occur in the vicinity of the drip line ( $N=20$ ). However further experimental work measuring either the mass of  $^{33}\text{Na}$  (see fig.1) or the radius isotope shift (see fig.4) is still necessary to definitively prove the existence of the calculated shape transition.

#### REFERENCES

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