INSTITUTE OF PLASMA PHYSICS

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Note on the Motion of Charged Particles in an Axi-Symmetric Crossed Electro-Magnetic Field

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RESEARCH REPORT

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Abstract

Rotational frequency of a plasma cylinder immersed in a uniform magnetic field and an axisymmetric radial electric field which is proportional to radial distance from the axis is determined by a single particle model without collisions. The effects of the centrifugal and the Corioli's forces are taken into account which lead to final expressions of the rotational frequency and the effective magnetic field as seen by the particles in the rotating plasma cylinder with correction terms which are proportional to the radial electric field.

... plasma cylinder is immersed in a uniform magnetic field, a radial electric field is produced very often. Under such a condition, the plasma cylinder rotates azimuthally. Particularly, in a PIG discharge, the radial electric field is strong enough to excite the "drag instability".[1] In the theory by Hoh,[2] the cross field rotation plays an important role in the excitation of the drag instability. In the investigation of azimuthal wave propagation in plasma cylinder, too, the information regarding the plasma rotation, due to cross field, is very important for the Doppler shift correction.

It will be assumed that the rotational velocity is determined only by the velocity $\left|\frac{\vec{E} \times \vec{B}}{B^2}\right| = \frac{E}{B}$. But, with a low magnetic field and/or a high electric field, the velocity is different from E/B. In such a case, Corioli's force and centrifugal force must be taken into account. In this note, we have tried, by a simple analysis, to determine the rotational velocity of plasma cylinder in a uniform magnetic field B and a radial electric field E which is proportional to the radial distance from the axis.

§2. Analysis

When a plasma is immersed in uniform magentic field $\vec{B}$ and uniform electric field $\vec{E}$ which is perpendicular to $\vec{B}$, plasma particles drift with a velocity E/B in a collisionless regime. This is true as long as E/B is much smaller than the light velocity c. But it is not the case in an axisymmetric geometry.

Suppose crossed fields as

$$\vec{B} \equiv (B_r, B_\theta, B_z) = (0, 0, B)$$
$$\vec{E} \equiv (E_r, E_\theta, E_z) = (\varepsilon r, 0, 0) \tag{1}$$

in cylindrical coordinates and a plasma cylinder is produced within a region of $r < R$. If $\vec{B}$ is sufficiently strong and $\vec{E}$ is weak enough, all plasma particles rotate around the z-axis with an angular frequency $\frac{\varepsilon}{B}$. Thus the plasma cylinder rotates as a solid without any velocity shear. But this is only an approximation, and we must, in some cases, take into account finite Larmor radius, which plays an important role at low magnetic fields.

The equation of motion of a charged particle in the crossed fields of axial symmetry, as given in Eq.(1), is

$$m\ddot{\vec{r}} = e\vec{E} + e\dot{\vec{r}} \times \vec{B} \tag{2}$$

where we neglect collisions. Expecting that the particle will drift azimuthally, we transform our coordinates into a system rotating around the z-axis with an angular frequency ω. In this frame, $\dot{\vec{r}}$ and $\ddot{\vec{r}}$ are transformed as

$$\dot{\vec{r}} = \dot{\vec{r}}_R + \vec{\Omega} \times \vec{r}_R$$
$$\ddot{\vec{r}} = \ddot{\vec{r}}_R + 2\vec{\Omega} \times \dot{\vec{r}}_R + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_R) \tag{3}$$

where O is a vector representing rotation with the angular velocity ω, which is

$$\vec{O} = (O_r, O_\theta, O_z) = (0, 0, \omega)$$

and the suffix "R" denotes the rotating system. Thus the new equation of motion is

$$m\ddot{\vec{r}}_r = e\vec{E} - m\vec{O} \times (\vec{O} \times \vec{r}_R) - 2m\vec{O} \times \dot{\vec{r}}_R + e(\dot{\vec{r}}_R + \vec{O} \times \vec{r}_R) \times \vec{B} \tag{4}$$

where the second and the third terms on the right hand side are the centrifugal force and Corioli's force, respectively. After some rearrangements of Eq.(4), we get the following as

$$m\ddot{\vec{r}}_R = (eB\omega - e\varepsilon + m\omega^2)\vec{r}_R + (eB + 2m\omega)(\dot{\vec{r}}_R \times \vec{k}) \tag{5}$$

where $\vec{k}$ is the unit vector parallel to $\vec{B}$, and the second term is perpendicular to $\dot{\vec{r}}$. If we choose ω so as to fulfil the equation

$$eB\omega - e\varepsilon + m\omega^2 = 0 \tag{6}$$

we can completely eliminate the first term in Eq.(5). Thus the rotational frame is rotating with an angular frequency ω given by one of the roots of Eq.(6), which is also given by F. F. Chen. The only force which remains is a type of Lorentz force, perpendicular and proportional to $\dot{\vec{r}}$. The final equation in the rotational frame, rotating with an angular frequency ω, is given by

$$m\ddot{\vec{r}}_R = e\dot{\vec{r}}_R \times \vec{B}' \tag{7}$$

$\vec{B}'$ is

$$\vec{B}' = (0,\ 0,\ B(1 + \frac{2\omega}{\Omega_c})) \tag{8}$$

where ω is given by Eq.(6) and Ω_c is the cyclotron frequency. We find that now in the new system, due to the effect of the Corioli's force, the magnetic field experienced by a charged particle is not the applied field $\vec{B}$ but a different magnetic field $\vec{B}'$.

§3. Considerations

To get the angular frequency of the new rotating coordinate system, we solve Eq.(5), which gives

$$\omega = \frac{1}{2m}(-eB \pm \sqrt{e^2B^2 + 4me\varepsilon}) \tag{9}$$

The first root

$$\omega_1 = \frac{1}{2m}(-eB + \sqrt{e^2B^2 + 4me\varepsilon}) \tag{10}$$

tends to zero in the limiting case of $\varepsilon \to 0$, and corresponds to drift velocity when E_r is very weak. Expanding Eq.(10) with ε, we get

$$\omega = \omega^*(1 - \frac{\omega^*}{\Omega_c}) \tag{11}$$

where

$$\omega^* = \frac{\varepsilon}{B}$$

is the angular frequency directly corresponding to the cross fields drift velocity E/B. The second root of Eq.(9) for

$\varepsilon = 0$, is simply Ω_c. This rotating system with the angular frequency ω_2, which will be discussed later, also eliminates the radial force completely. These situations are shown in Fig.1.

§4. Conclusion

By the transformation of coordinates from the laboratory system to the rotating system, we can separate the motion of a charged particle in crossed electric and magentic fields into a pure gyration and a drift motion around the axis of symmetry. We obtain, as the angular frequency or drift motion, two solutions. One corresponds to the $\vec{E} \times \vec{B}/B^2$ drift and the other to the cyclotron gyration in the limiting case of $\varepsilon \to 0$. This means that the motion of a charged particle in crossed fields can be separated into a pure gyration and drifting rotation in two ways. This can be explained as follows: Suppose that the particle motion is represented as

$$\vec{r}(t) = \vec{r}_1 \exp(i\omega_1 t) + \vec{r}_2 \exp(i\omega_2 t) \tag{12}$$

it is clear that there are two ways of interpreting Eq.(12), i.e., the first term as the pure gyration component and the second as the drift rotation around the axis, or the vice versa (cf. Fig.1). It will be very natural that, when $\omega_1 >> \omega_2$, the first term is the pure gyration component and the second is the drift motion around the axis.

At the same time, it turns out that the particle will experience a different magnetic field from the one in the laboratory system. This effect results from the Corioli's force.

References

1) T. Kuroda, G. Horikoshi and S. Nagao: J. Phys. Soc. Japan 26 (1969) 181

2) F. C. Hoh: Ark. Fys. 24 (1963) 285

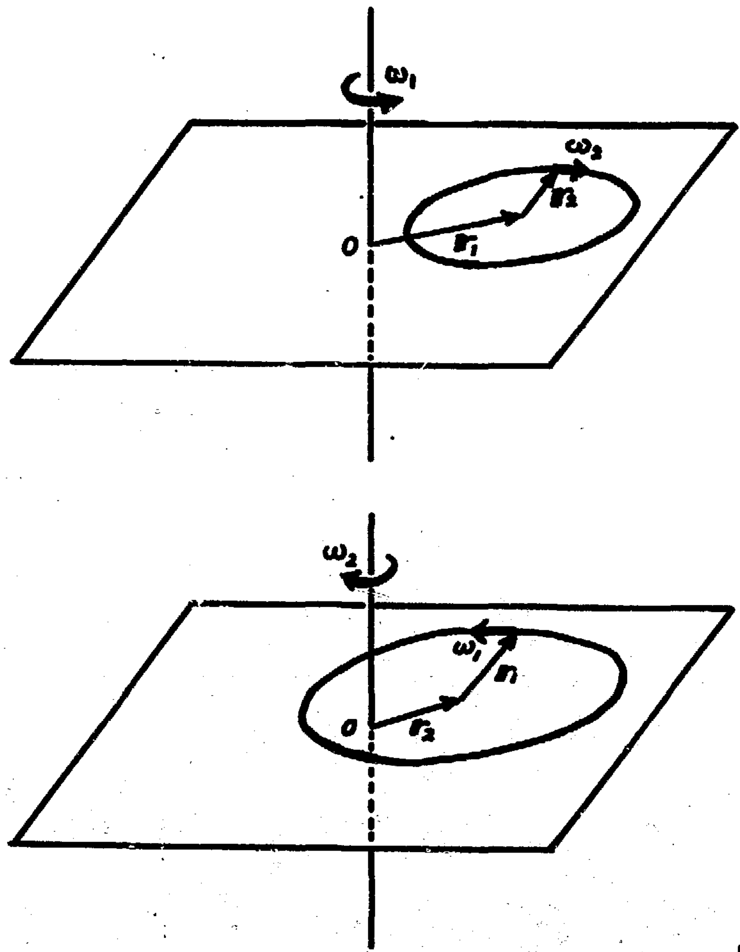
3) F. F. Chen, Phys. Fluids 9 965 (1966)

<u>Figure Caption</u>

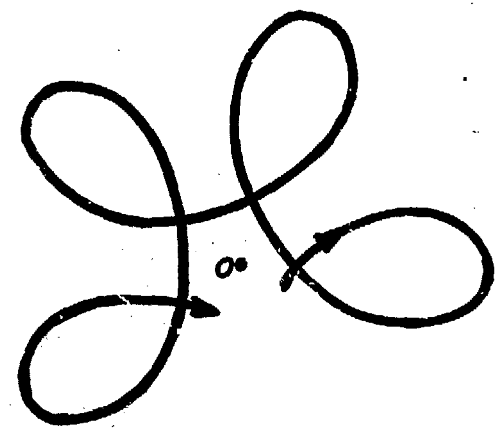
Fig.1. Motion of a charged particle in a crossed electromagnetic field.

(a) Orbit as seen from a laboratory system.

(b) Orbit of the particle motion, interpretable in two ways (upper and lower Figs.).



(b)



(a)

Fig. 1