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HELICAL EQUILIBRIA AND CRITERIA FOR THE KINK INSTABILITY OF CYLINDRICAL TOKAMAK

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RESEARCH REPORT

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Abstract

Helical euilibria and criteria for the kink instability have been obtained numerically for various current distribution, including camel hump distribution. It is found that the unstable region expressed by q(a) is the largest in the case of uniform current.

Much work about MHD kink instability of the tokamak configulation has been done and Previous results indicate that the system subject to the kink instability has helical symmetry.[1),2)] And by means of MHD approximation non-linear equilibrium equation has been obtained and solved for a particular current distribution.[3)] In this paper we expand this perturbation method and a helical equilibrium and the displacement of the magnetic surface have been obtained numerically for an arbitrary current distri -bution. And also bifercating point of the kink instability has been obtained. This calculation was performed on IBM-360 and 370 computers in Plasma Physics Laboratory, Princeton University, Princeton and on the HITAC-8800 and 8700 computers in the Computer Center of the University of Tokyo.

Basic Equations

Tokamak system can be appreoximated as a cylindrical plasma(radius a) in a perfectly conducting cylinder (radius b) with periodicity $2\pi R$ (R is a major radius of the torus). Introducing helical cooreinates with $\varphi = k_z z + \ell\theta$ (r, θ, z are ordinary cylindrical coordinates), the φ component of the magnetic field B_φ and that of vector potential A_φ are given as

$$A_\varphi = k_z A_\theta - \ell A_z \equiv \psi \tag{1}$$

$$B_\varphi = k_z r B_\theta - \ell B_z \equiv B(\psi) \tag{2}$$

And the plasma equilibrium eqation $\vec{J} \times \vec{B} = \nabla p$ can be expressed as[3)]

$$\left((k_z r)^2+\frac{\ell^2}{r^2}\right)\frac{\partial^2\psi}{\partial\varphi^2}+r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{2}{r}\frac{\ell^2}{(k_z r)^2+\ell^2}\frac{\partial\psi}{\partial r}+\frac{2\ell k_z B_\varphi}{(k_z r)^2+\ell^2}=-\frac{\partial}{\partial\psi}\left(\frac{1}{2}B_\varphi^2\right)-\mu_0\frac{\partial p}{\partial\psi} \quad (3)$$

The other components of the magnetic field and the current density in the z-direction are

$$k_z r B_z + \ell B_\theta = \partial\psi/\partial r, \quad (4)$$

$$B_r = -\frac{1}{r}\frac{\partial\psi}{\partial\varphi}, \quad (5)$$

$$J_z = \frac{r^2}{\ell^2+(k_z r)^2}\frac{1}{\mu_0}\left(\frac{\partial\psi}{\partial r}-\frac{\ell}{r}\frac{B_\varphi}{r}\right)\frac{\partial B_\varphi}{\partial\psi}. \quad (6)$$

The boundary condition is given by $B_r=0$ at $r=b$ or

$$\frac{\partial\psi}{\partial\varphi}=0 \qquad \text{at } r=b. \quad (7)$$

Then in a toroidal plasma configuration with $(k_z r)^2 \ll 1$ and with the assumption of a strong magnetic field with $\beta=0$, we get following equations,

$$\frac{\ell^2}{r^2}\frac{\partial^2\psi}{\partial\varphi^2}+\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial\psi}{\partial r}+\frac{2k_z B_\varphi}{\ell}=-\frac{1}{2}\frac{\partial}{\partial\psi}(B_\varphi^2) \quad (8)$$

$$J_z(r) = -\frac{1}{\ell\mu_0}\frac{\partial}{\partial\psi}\left(\frac{1}{2}B_\varphi^2\right) \quad (9)$$

Equilibrium

To obtain the neibouring equilibrium solution, we expand up to the 1st order in the parameter α,

$$\psi = \psi_0(r) + \alpha\psi_1(r)\cos\varphi, \tag{10}$$

and by introducing strong constant magnetic field B_0 , B_φ is written as

$$B_\varphi = B_0 + B_1(r,\theta). \tag{11}$$

Then Eq.(8) is rewritten as

$$\frac{1}{r}\frac{d}{dr}r\frac{d\psi_0}{dr} - 2k_z B_0 = \ell\mu_0 J_z(r), \tag{12}$$

$$\frac{1}{r}\frac{d}{dr}r\frac{d\psi_1}{dr} - \frac{\ell^2}{r^2}\psi_1 = \frac{\ell\mu_0}{d\psi_0/dr}\left(\frac{d}{dr}J_z(r)\right)\psi_1 . \tag{13}$$

In vacuum $\psi_v(r,\varphi)$ is expanded as in plasma region and solved as

$$\psi_v = k_z B_0 r + \varepsilon\left(\left(\frac{b}{r}\right)^\ell - \left(\frac{r}{b}\right)^\ell\right)\cos\varphi \tag{14}$$

om the continuity condition imposed upon ψ on the plasma urface, we obtain

$$\frac{d\psi_1/dr}{\psi_1} = \frac{\ell}{a}\,\frac{(a/b)^{2\ell}+1}{(a/b)^{2\ell}-1} . \tag{15}$$

Stability Criterion

Equation (15) determines the $q(a)$ with which helical equilibrium is to exsist. Another marginal point is $q(a)=m$ (for any current distribution). As pointedout previously these critical q value is the bifercating point of the cylindrical equilibrium[2]. The critical q value has been calculated numerically for various current distribution and various (m,n) modes. Criteria of instability are shown in Fig.1 up to $q(a)\lesssim 5$ for the current profiles; $J_z(r)=J_0(1-(r/a)^2)^{\kappa-1}$ for $\kappa\gtrsim 2$ and $J_z(r)=J_0(1-(r/a)^{\frac{2}{\kappa-1}}$) for $1<\kappa\lesssim 2$.

Since $\kappa\equiv q(a)/q(0)$ is a measure of concentration, we call κ the concentration ratio. Note that κ is useful only when the current distribution is the monotonic function of r.

As shown in Fig.1, the unstable region becomes narrower as κincreases; more concentrated current distribution has larger stable region than the flat current distribution (even for $1<\kappa\lesssim 2$). In previous result the region $1<\kappa\lesssim 2$ is always unstable for $a/b=0$.[4] On the contrary, our result smoothly agree with the analysis by Shafranov in the case of the uniform current limit,$(\kappa\to 1)$. In addition to these result we have obtained a critical q value for the particular current distribution (we call it the camel hump distribution). This calculation shows that the unstable region for the camel hump current becomes narrower than that for the uniform current (Table 1). Displacements of the magnetic surface ($m=2,n=1$) of this distribution and the concentrated cases (κ=2.4, κ=3.0) are shown in Fig.2. In this particular case, the displacement is less localized than those of concentrated cases.

Discussion

By means of energy principle, the stability criterion was analyzed for non-uniform current. But from the view of a current distribution, in some calculation, the equation is overdetermined and assumption contradicts itself. On the other hand, in our method, the criterion itself is expressed explicitly by the current profile. And moreover for the analysis of nonlinear instability, we can obtain the criterion only by solving Eq.(3) without calculating the higher order magnetic energy change.

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TABLE CAPTION

Table 1. The lower criterion of instability of camel hump current distribution compared to that of flat current, in the case of a/b=0 and 0.8.

FIGURE CAPTIONS

Fig. 1 Unstable region of the kink instability is indicat-ed by the shaded portion. Solid line shows the case of a/b=0. As the value a/b is increased, the unstable region becomes narrower. The Criter-ion is shown by the dashed line for the case of a/b=0.8.

Fig. 2 Structure of helical equilibrium for m=2,n=1 mode are shown in terms of displacement of magnetic surface. Each displacement corresponds to the current profile as;

1 : $J_z = J_{01}(1+2(r/a)^2-3(r/a)^4)$,

2 : $J_z = J_{02}(1-(r/a)^2)^{K-1}$ $K=2.4$,

3 : $J_z = J_{03}(1-(r/a)^2)^{K-1}$ $K=3.0$.

Table 1

	a/b = 0			a/b = 0.8		
mode	(2,1)	(3,1)	(4,1)	(2,1)	(3,1)	(4,1)
"Camel Hump"	1.25	2.4	3.6	1.55	2.6	3.7
Flat	1	2	3	1.41	2.26	3.17

Fig. 1

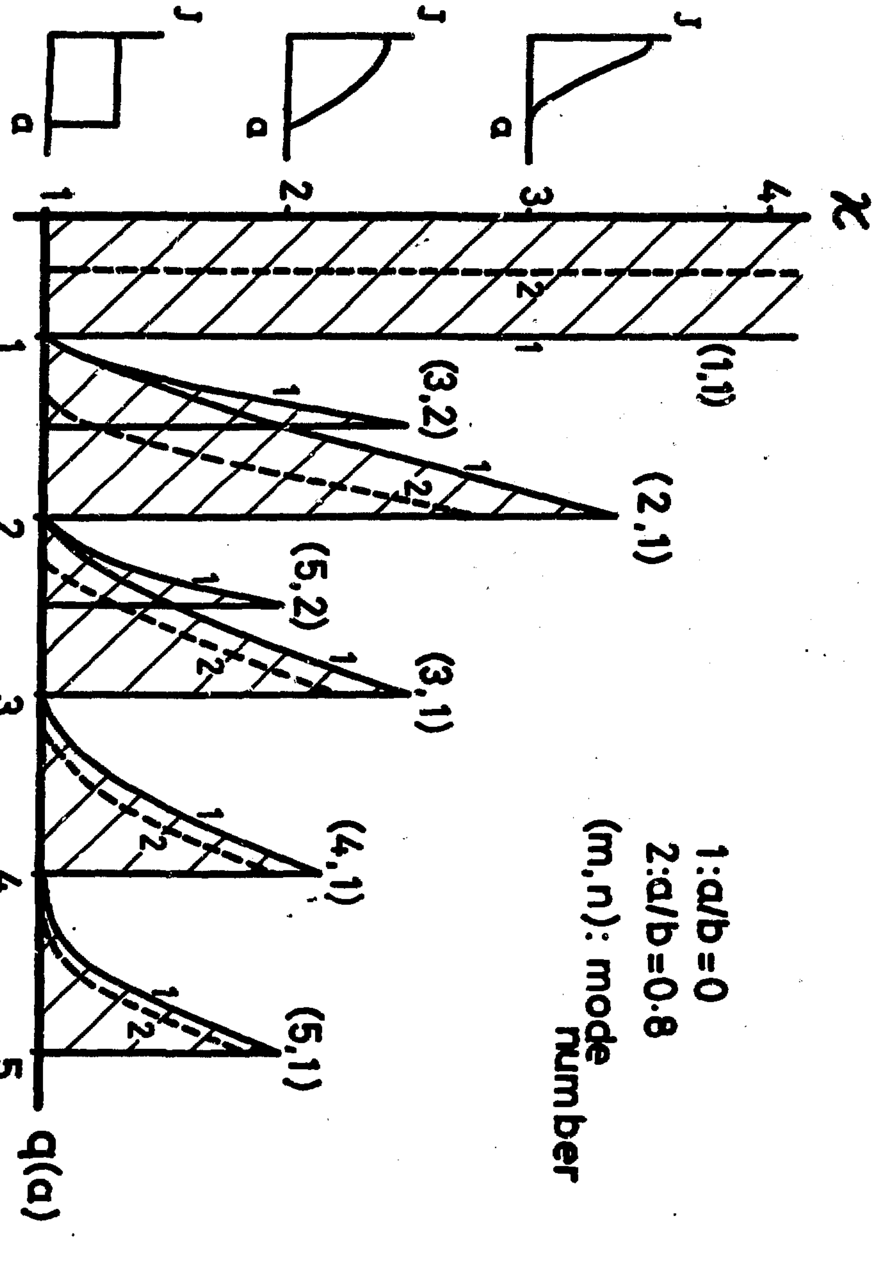

Fig. 2