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Abstract.

A study of the trapped ion instability in the real geometry of the large Tokamaks leads to the consideration of a new branch of that instability, driven by a resonance with the magnetic drift of the particles, both in collisional and non collisional regimes.

The existence of trapped particles in a geometry of the Tokamak type is known to give rise to instabilies in either the coilisionless or collisional regime /1/. The Tokamaks of the next generation (PLT, T10, JET) will confine plasmas with parameters in the theoretical range for the dissipative trapped ions instability /1,2/. Simultaneously, those installations will have small aspect ratios ($A \equiv \frac{R}{a} \sim 3\text{-}4$) and may have flat density profiles and peaked temperature profiles : such a plasma can result from the limited length of penetration of cold neutrals.

In the present work we analyse the consequences of a large magnetic drift expected in a fat torus for different values of the ratio $\mu = \frac{d \operatorname{Ln} n}{d \operatorname{Ln} T}$ between the temperature and density scale heights. A negative value of μ (inverted density profile) has already been proposed as a stabilizing scheme /3/.

The main result is the following : for $\mu < \frac{3}{2}$ there is a bifurcation of the purely growing collisionless trapped ion mode /1/ into two modes : the ion mode which resonates with the ion magnetic drift frequency ; and the electron one, with the electron magnetic drift frequency. The electron mode disappears in the collisional regime but the ion one extends sufficiently to predict instability of the future large installations even for flat or inverted density profiles.

Our conclusion is consequently quite different from the previous ones /4/ : for a fat torus, the expansion in inverse aspect ratio breaks and the magnetic drift frequency modifies completely the dispersion relation ; in the collisional regime, the sign of the real part of the frequency is opposite for the main range of parameters (the phase speed is in the direction of ion drift). Future work should be devoted to a deeper study of the non-linear stage of such instabilities including real geometrical factors.

In the following, we give the main analytical and numerical results successively in the collisionless and collisional regimes.

Using a simple collision model, and assuming $T_i = T_e$, we start from the usual dispersion relation /1/:

$$\sqrt{A\pi} = \int_0^\infty d\varepsilon \; \varepsilon^{1/2} e^{-\varepsilon} \left[\frac{\omega - \omega_n^* - \omega_T^*(\varepsilon - 3/2)}{\omega - \omega_D \varepsilon + i \nu_{ief} \varepsilon^{-3/2}} + \frac{\omega + \omega_n^* + \omega_T^*(\varepsilon - 3/2)}{\omega + \omega_D \varepsilon + i \nu_{eef} \varepsilon^{-3/2}} \right] \qquad (1)$$

where

$$A = R/r \,, \quad \varepsilon = E/T, \quad \nu_{jef} = \nu_j A$$

$$\omega_T^* = -\frac{lq}{r} \frac{cT}{eB} \frac{d\ln T}{dr} = \omega_i^* \,, \quad \omega_n^* = \frac{d\ln n}{d\ln T} \omega_T^* \,,$$

$$\omega_D = \frac{2lq}{r} \frac{cT}{eB} \frac{1}{R_o} \langle G \rangle$$

and G, which contains the dependence of ω_d to the reflection angle for a trapped particle, is approximately constant and equal to $\frac{1}{2}$ for a realistic shear.

We first study Kadomtsev's non-collisional mode (interchange mode). Assuming $\nu_{eef} \ll \omega_D$, $\omega \ll \omega_T^*$, we find :

$$\tau = \frac{2}{\sqrt{\pi}} \int_0^\infty d\varepsilon \; \varepsilon^{3/2} e^{-\varepsilon} \; \frac{(\varepsilon - 3/2 + \mu)}{\varepsilon^2 + y^2} \qquad (2)$$

Where we have introduced the dimensionless parameters :

$$\tau \equiv \frac{\omega_D \sqrt{A}}{\omega_T^*} = 2 \frac{\rho_T}{r} \sqrt{\frac{r}{R}} \;\; \langle G \rangle \quad , \quad y = \gamma / \omega_D$$

$$\rho_T = \left(\frac{d \, Ln \, T}{d \, r} \right)^{-1} \quad , \quad \mu = \frac{d \, Ln \, n}{d \, Ln \, T} = \frac{\omega_n^*}{\omega_T^*}$$

and assumed a purely growing mode ($\omega = i\gamma$).

For a given μ, the integral on the R H S of eq. (2) has a maximum for a certain value of y. It is the maximum value of τ (i.e. the minimum temperature gradient) for the instability to exist.

Fig. 1 shows the value of τ as a function of y, for the case $\mu = \frac{1}{2}$. Fig. 2 shows the maximum possible τ as a function of μ. We see that, for $\mu < 1$, the interchange mode is easily stabilized (as a reference, we note that with typical P L T parameters : $\frac{R}{a} = 3$, $\frac{r}{a} = \frac{2}{3}$, $\frac{\rho_T}{a} = \frac{1}{2}$, we obtain : $\tau \simeq .35$).

For $\mu \geq \frac{3}{2}$, the maximun τ is obtained for $y = 0, \tau = 2\mu - 2$. For $\mu < \frac{3}{2}$, it is obtained for a value $y > 0$. In that case, for τ less than this maximum, one has two purely growing modes. One is Kadomtsev's, while the second one has a smaller growth rate and is stabilized by a strong temperature gradient. However, no marginal mode ($\gamma = 0$) appears first when the gradient is increased, as would be expected.

We are thus led to suspect the existence of a new branch, starting from a marginal mode.

We then look for a marginal mode (ω real) in eq. (1), which now becomes :

$$\tau = \frac{1}{\sqrt{\pi}} \int_0^\infty d\varepsilon \; \varepsilon^{1/2} e^{-\varepsilon} \left(\varepsilon - \frac{3}{2} + \mu \right) \left[\frac{1}{\varepsilon - \frac{\omega}{\omega_D}} + \frac{1}{\varepsilon + \frac{\omega}{\omega_D}} \right] \quad (3)$$

Where we have assumed that $\omega \ll \omega_T^*$.

We see that a real frequency mode is possible only if the $i\pi\delta$ contribution, due to the resonance $\varepsilon = \left|\frac{\omega}{\omega_D}\right|$, is killed by $\left|\frac{\omega}{\omega_D}\right| = \frac{3}{2} - \mu$.

This gives

$$\tau = 1 - 2\lambda + 2\sqrt{\pi}\ \lambda^{3/2} e^{\lambda}\, \mathrm{erfc}(\sqrt{\lambda})$$

where $\lambda = \frac{3}{2} - \mu$.

We have thus found two modes, corresponding to ion ($\omega = \lambda\omega_{di}$) and electron ($\omega = \lambda\omega_{de}$) magnetic drift resonances. Retaining ω in the numerator of eq. (3) gives an equation that must be solved numericaly, yielding a somewhat higher τ. Fig. 2 shows the curve of the maximum τ for the resonant mode, above the one corresponding to the interchange mode. The curves merge for $\mu \geq \frac{3}{2}$, where the resonance is no more possible, giving $\tau = 1 - 2\lambda$.

If we retain the possibility of a positive growth rate in eq. (3), and solve it for ω and τ as functions of γ (with fixed μ) we can see that, as the growth rate increases from zero, ω and τ decrease. Eventually, as ω goes to zero, the new branch merges with the interchange branch at its maximum.

Fig. 3 shows the graph of both branches in (γ, τ) coordinates, and fig. 4 the value of ω as a function of γ, for various values of μ. It is then visible that, for $\mu < \frac{3}{2}$, the domain of instability is considerably extended by the new branch.

We now investigate how these modes extend into the collisional regime. Eq. (1) now becomes, for the marginal mode :

$$\alpha^2 \int_0^\infty d\varepsilon\ \varepsilon^2 e^{-\varepsilon} \left(\varepsilon - \tfrac{3}{2} + \mu\right) \left[\frac{1}{(\varepsilon^{3/2} - \alpha\zeta\varepsilon^{5/2})^2 + \alpha^2} - \frac{60}{(\varepsilon^{3/2} + \alpha\zeta\varepsilon^{5/2})^2 + 3600.\,\alpha^2} \right] = 0 \qquad (4)$$

$$\tau = -\frac{\alpha\zeta}{\sqrt{\pi}} \int_0^\infty d\varepsilon\ \varepsilon^2 e^{-\varepsilon} \left(\varepsilon - \tfrac{3}{2} + \mu\right) \left[\frac{\varepsilon^{3/2} - \alpha\zeta\varepsilon^{5/2}}{(\varepsilon^{3/2} - \alpha\zeta\varepsilon^{5/2})^2 + \alpha^2} - \frac{\varepsilon^{3/2} + \alpha\zeta\varepsilon^{5/2}}{(\varepsilon^{3/2} + \alpha\zeta\varepsilon^{5/2})^2 + 3600.\,\alpha^2} \right] \qquad (5)$$

for its imaginary and real part respectively, where $\alpha = \nu_{ieff}/\omega$, $\zeta = \omega_D/\nu_{ieff}$ and we have assumed that

$$\frac{\nu_e}{\nu_i} = \sqrt{\frac{m_i}{m_e}} = 60 \qquad \text{(Deuterium).}$$

We note that, given the physical conditions, ζ is proportional to l, the toroidal mode number.

We proceed numericaly, first solving eq. (4) for α, with given ζ and μ, and then eq. (5) for τ. We also have provided for the presence of ω in the numerator, though it only modifies slightly the results.

Decreasing ζ (increasing the collision frequency), we see that :

- the electron mode is very quickly stabilized by electron collisions but the ion mode extends further, and the frequency remains positive ($\frac{\omega}{\omega_{Di}} > 0$) until a certain value of ζ where it becomes negative and we recover the usual collisional mode.

-the τ for marginal stability also decreases, and becomes weakly dependent on μ, for $\mu < 1$, in the collisional region.

Eqs (4-5) have also been solved including a positive growth rate, when $\mu = \frac{1}{2}$. Fig. 5 shows the value of τ for marginal stability, as a function of ζ, for various values of μ, and fig. 6 the growth rate as a function of τ and ζ, with $\mu = \frac{1}{2}$; fig. 7 shows the value of ω as a function of ζ for the marginal mode.

With $n_e = n_i = 10^{14}\ cm^{-3}$, $T_e = T_i = 3$ keV, B = 50 kG and q = 2, and the geometrical values already used, we find : $\zeta \simeq 2\,l$. We then see that, with $\tau = 0.35$, all modes with $l \geqslant 2$ should be unstable. Typically, γ is a fraction of ω_d and smaller than ω :

$$\gamma \lesssim \omega_D \quad , \quad \omega \sim \omega_D$$

It is interesting to compare these results with those obtained for the "classical" (non resonant) mode.

In a recent paper /4/, Tang included the stabilizing effect of the small number of ions trapped in regions of favourable mean magnetic gradient (barely trapped ions), but neglected the destabilizing effect due to the bulk of the trapped ions, which is included here.

In a typical case ($n = 10^{14}$, $T_e = T_i = 3$ keV, $\omega_n^* = \omega_T^*$) he finds marginal stability for the $l = 4$ mode, and maximum growth rate for $l = 6$ and $\gamma \simeq 4.10^3\ s^{-1}$. Higher - l modes are stabilized by ion-Landau resonances which we have neglected. In our case, we find similar growth rates but instability beginning with the $l = 2$ mode.

We wish to point out the interest of the dimensionless parameters used here, especially τ. These may enable one to take into account different effects which we have neglected herein.

Namely, to derive eq. (l) it is necessary to make the assumption that $\Phi = \bar{\Phi}$, where $\bar{\Phi}$ is the mean perturbed potential seen by a particle along its orbit. An approximate calculation, assuming either $\Phi \sim \cos\frac{\theta}{2}$ or $\Phi \sim \cos^2\frac{\theta}{2}$, and taking into account the exact number of trapped particles, then leads to a slight correction of τ, namely $\tau \simeq 0.8\ \frac{\rho_T}{r}\sqrt{\frac{r}{R}}$.

Finally, we wish to thank Dr. J.C. Adam and W.M. Tang for fruitful discussions about this work, and N.AUBY for helpful assistance in the numerical calculations.

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FIGURE CAPTIONS

Fig. 1 : Growth rate as a function of the temperature gradient for the purely growing mode with $\mu = \frac{1}{2}$

Fig. 2 : Minimum temperature gradient for the purely growing (G) and resonant (R) modes as a function of μ.

Fig. 3 : Growth rate as a function of τ, for both branches, for various values of μ.

Fig. 4 : Real port of the frequency os a function of the growth rate, for the non-collisionol mode.

Fig. 5 : Marginal stobility as a function of ζ, for various volues of μ, in the collisional regime.

Fig. 6 : Growth rate as a function of τ and ζ, with $\mu = \frac{1}{2}$.

Fig. 7 : Real part of the frequency as a function of τ and ζ.

Fig.1

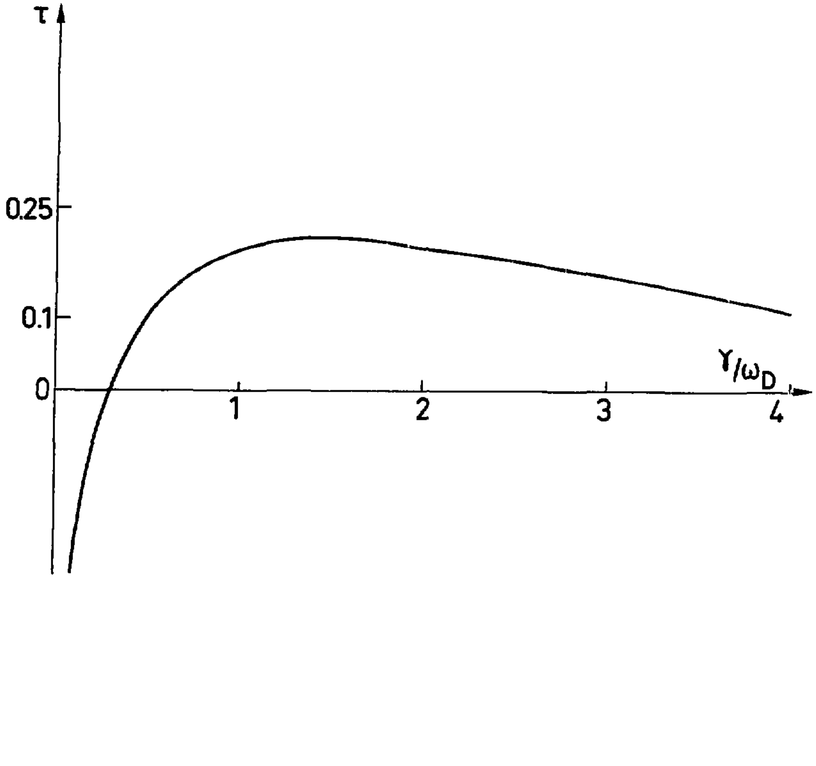


Fig.2

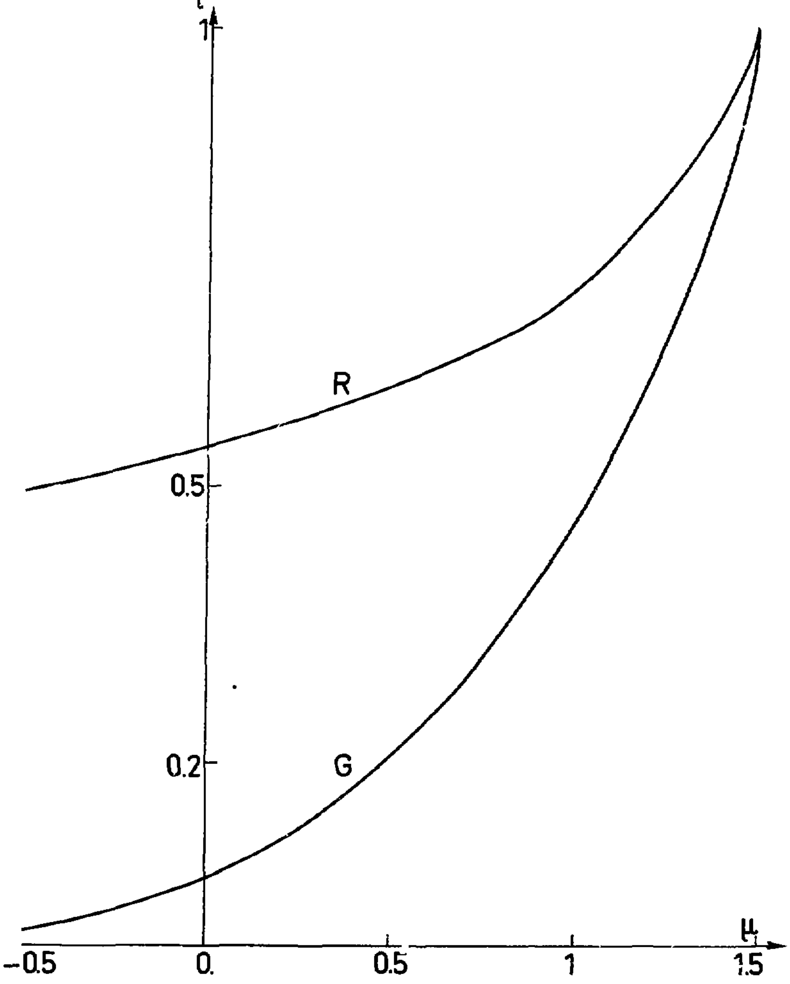


Fig.3

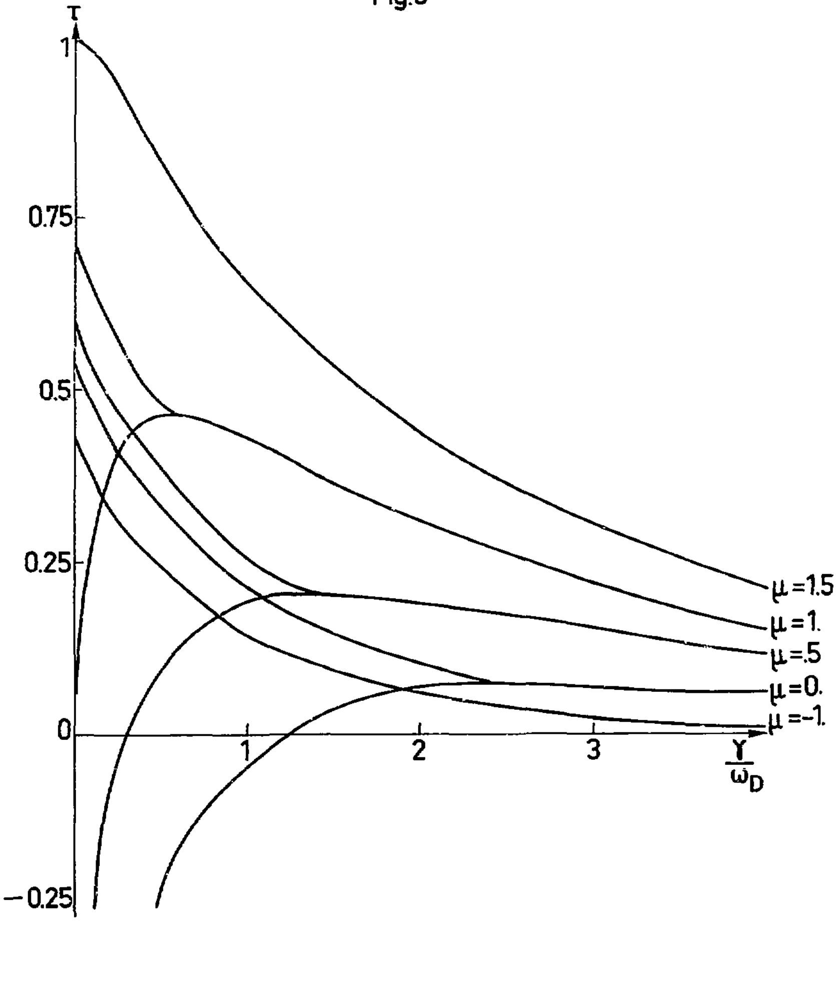


Fig.4

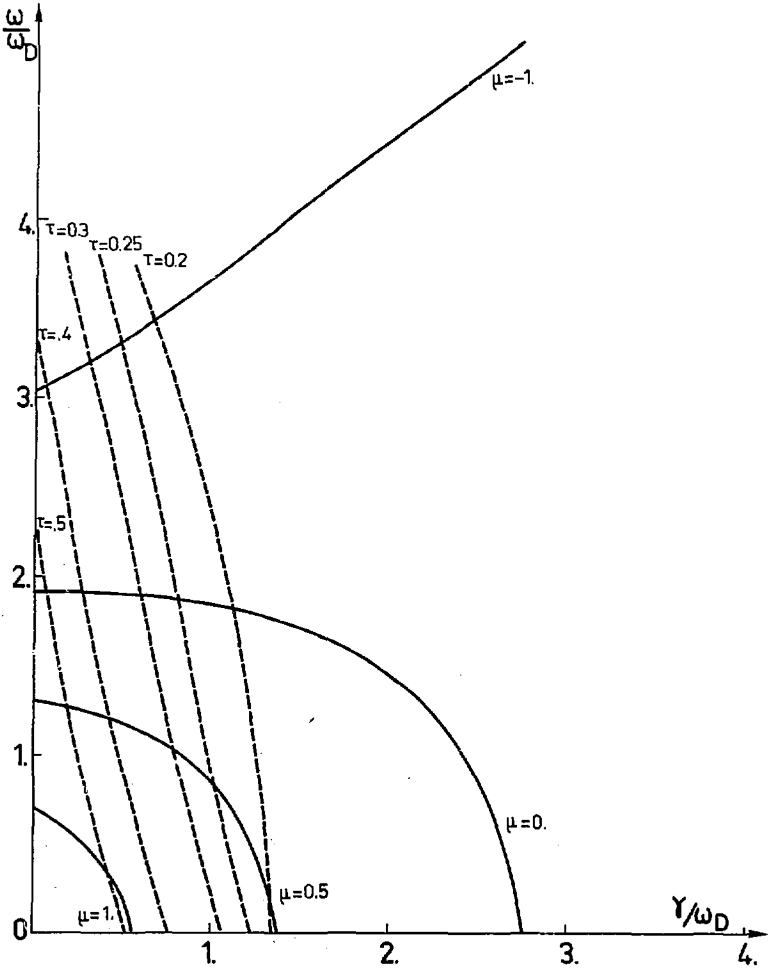


Fig.5

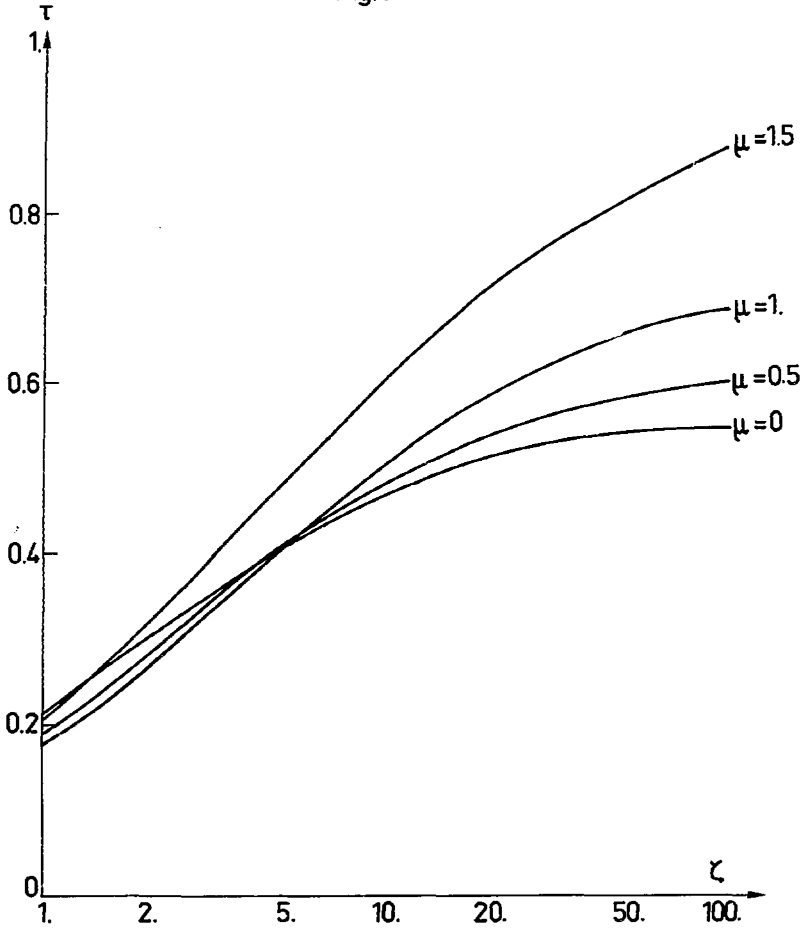
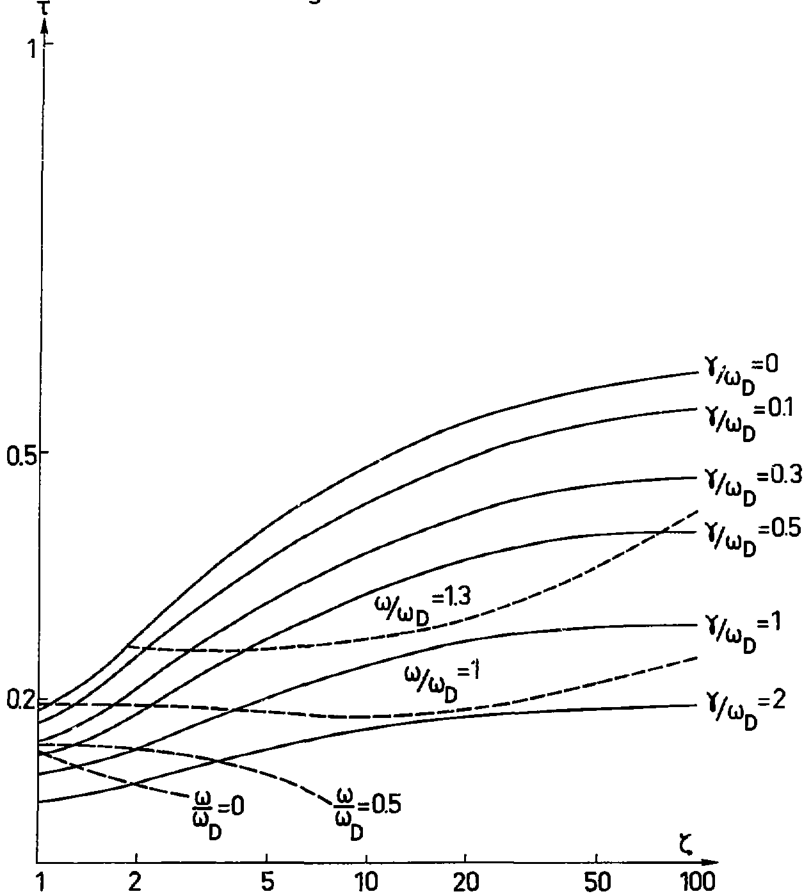


Fig.6



τ
1
0.5
0.2
γ/ω_D=0
γ/ω_D=0.1
γ/ω_D=0.3
γ/ω_D=0.5
γ/ω_D=1
γ/ω_D=2
ω/ω_D=1.3
ω/ω_D=1
ω/ω_D=0
ω/ω_D=0.5
ζ
1
2
5
10
20
50
100

Fig.7