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ELECTRON BOUNCE MODES IN MIRROR PLASMAS

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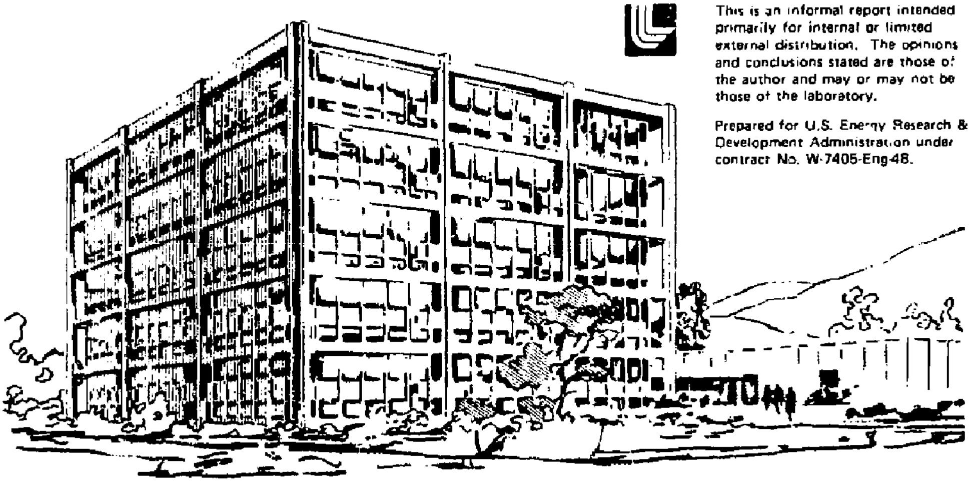

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ABSTRACT

Electron bounce modes can occur in mirror plasmas when the spread in the bounce frequency is small. This condition is satisfied in mirror devices when electrons are principally confined by an approximately quadratic electrostatic potential. These modes are examined by numerically solving an integral equation for the perturbed wave potential in a mirror plasma. A long wavelength mode is found that can be destabilized by ions because of the loss-cone nature of their distribution. Threshold densities and maximum growth rates are calculated using a perturbation method. The theoretical stability threshold predictions agree closely with Baseball II measurements.

Instabilities in low-density mirror machines such as Baseball I, Phoenix II, and Ogra II have been attributed to electron plasma waves driven by ion perpendicular energy.[1-5] According to infinite-medium theory these modes have a frequency $\omega = \omega_{pe}k_{\parallel}/k_{\perp}$, where $k_{\perp}$ and $k_{\parallel}$ are components of the propagation vector, and ω_{pe} is the electron plasma frequency. If ω is near the ion gyrofrequency ω_{ci}, and the ion gyroradius a_i is large enough that $k_{\perp}a_i > 1.85$, then the free energy of a peaked ion distribution can be effectively transferred to the wave. Instability occurs if this ion drive predominates over electron Landau damping. Since damping drops off when the wave phase velocity $\omega/k_{\parallel}$ exceeds the electron thermal velocity V_e, $\omega_{pe} \approx k_{\perp}V_e$ is taken as the condition for marginal stability. Threshold measurements in Baseball I appeared to support this description, but in Baseball II (BBII) instability occurred at densities well below threoretical predictions.[6]

In this letter we propose a mechanism involving electron bounce modes to explain BBII instability thresholds. Bounce modes occur when the electron bounce frequency ω_b is well enough defined for electrons to retain substantial phase coherence on successive transits. Collective electron behavior is then altogether unlike infinite medium response whenever the spread in bounce frequency $\Delta\omega_b$ satisfies $\Delta\omega_b \ll \omega_b^2/\omega$. This coherence condition is met in typical hot-ion mirror plasmas because the electrons are confined principally by a nearly quadratic electrostatic potential. The bounce modes can couple to the perpendicular ion motion just as electron plasma waves can, and the stability threshold is the density at which ion drive balances electron Landau damping.

The electron modes are treated here by solving an integral equation for the wave potential in a bounded mirror plasma, and a perturbation method is then used to determine the threshold density. The electron treatment is similar to that of Beasley et al.[7] and differs from earlier fluid[4] and WKB[5] models principally in using accurate electron trajectories.

We take magnetic field strength B to vary quadratically with distance s along a flux line up to a maximum value s_{max} and require the plasma potential ϕ to be a nondecreasing function for $s > 0$. For the electrons a thermalized distribution that vanishes continuously at the loss surface is chosen. In terms of the total energy E and magnetic moment μ of electrons, the distribution function is $F_e(E,\mu) = (\mu B_{max} + \psi_{max} - E)\exp(-E/T_e)$ for

$E \leq \mu B_{max} + \psi_{max}$ and zero otherwise. Here, B_{max} and ψ_{max} are maximum values of B and potential energy $\psi \equiv -e\Phi$, and $T = \frac{1}{2} m_e V_e^2$ is the mean electron thermal energy. The BBII ion distribution is represented by $F_i(s,v_\parallel,v_\perp^2) = N(s)F_\perp(v_\perp^2)F_\parallel(v_\parallel)$, where $v_\perp$ and $v_\parallel$ are the local velocity components. The functions used for $F_\perp$ model two typical situations. A low-density plasma decays principally by charge exchange with the background gas, so the initially monoenergetic ion distribution remains peaked about $<v_\perp^2>$. A delta function $F_\perp(v_\perp^2) = \delta(v_\perp^2 - <v_\perp^2>)$ is then appropriate. At higher densities ion-ion collisions can spread the distribution during the plasma lifetime, and this effect is accounted for by choosing $F_\perp(v_\perp^2) = v_\perp^2 \exp(-2v_\perp^2/<v_\perp^2>)$. Also, a thermalized parallel distribution $F_\parallel(v_\parallel) = \exp(-v_\parallel^2/2<v_\parallel^2>)$ and Gaussian number density $N(s) = N(0) \exp(-s^2/L_p^2)$ are used.

An integral equation for the wave potential is obtained in the usual way from Poisson's equation and the linearized Vlasov equation. Since the electrons largely determine the plasma response, the ion density is ignored in solving for the unperturbed eigenmodes and is then treated as a perturbation in the subsequent stability-threshold calculations. In Poisson's equation the parallel derivative of the Laplacian is neglected because the wavelength along flux lines is long compared with the perpendicular wavelength. Writing the wave potential as $\phi(s) \exp(ik_\perp x_\perp - i\omega t)$ yields the following integral equation:

$$k_\perp^2 \lambda_{De}^2 \phi(s) = 2\pi \frac{v_e^2 B(s)}{m_e} \int_0^\infty d\mu \int_{\mu B(s)+\psi(s)}^{\mu B_{max}+\psi_{max}} dE \frac{1}{|v_\parallel|} \frac{\partial F_e}{\partial E} \left(2\phi(s) + i\omega \int_{-\infty}^{t} dt' \left\{ \phi[s^+(t')] + \phi[s^-(t')] \right\} \exp[-i\omega(t'-t)] \right), \qquad (1)$$

where $|v_\parallel| = \{2[E - \mu B(s) - \psi(s)]/m_e\}^{1/2}$, and electron Debye length λ_{De} is defined as $(T_e/2\pi e^2 N)^{1/2} = V_e/\omega_{pe}$. The trajectories of positive and negative moving particles reaching s at time t are denoted here by $s^\pm(t')$. For the symmetrical unperturbed fields considered, solutions of Eq. (1) are either even or odd functions of s, and ϕ can be shown to vanish at s_{max}. The time integral in Eq. (1) is evaluated by representing ϕ along each electron trajectory by a Fourier series in harmonics of ω_b, given for quadratic fields by

$$\omega_b(E,\mu) \equiv \frac{2}{\pi}\left(\int_0^{s_t} \frac{ds'}{|v_\parallel'|}\right)^{-1} = \frac{1}{2m}\left(\mu \frac{\partial^2 B}{\partial s^2} + \frac{\partial^2 \phi}{\partial s^2}\right) . \tag{2}$$

Here, $s_t(E,\mu)$ is the electron turning point. The resulting equation for ϕ is

$$k_\perp^2 \lambda_{De}^2 \phi(s) = 4\pi \frac{v_e^2 B(s)}{m_e} \int_0^\infty d\mu \int_{\mu B(s)+\psi(s)}^{\mu B_{max}+\psi_{max}} dE \frac{1}{|v_\parallel|} \frac{\partial F_e}{\partial E} \Bigg\{ \phi(s)$$

$$- \sum_{\ell=0}^{\infty} \frac{\omega^2}{\omega^2 - (2\ell + \sigma)^2 \omega_b^2} \hat{\phi}_\ell \, sc[(2\ell + \sigma)\omega_b t] \Bigg\} , \tag{3}$$

where

$$\hat{\phi}_\ell = \frac{4}{\pi} \omega_b \int_0^{\pi/2\omega_b} dt \, \phi[s(t)] sc[(2\ell + \sigma)\omega_b t] .$$

The symbol sc in Eq. (3) denotes cosine for even solutions and sine for odd ones, and σ equals 0 and 1, respectively, in the two cases. Fourier transforming Eq. (3) over the length of the plasma then leads to a matrix equation that can be solved numerically:

$$k_{\perp 0}^2 \lambda_{De0}^2 \phi_m = 4\pi \frac{v_e^2 B_0}{m_e} \sum_{n=1}^{\infty} \phi_n \int_0^\infty d\mu \int_{\mu B_0}^{\mu B_{max}+\psi_{max}} dE \frac{1}{\omega_b} \frac{\partial F_e}{\partial E}$$

$$\times \left[S_{mn} - \frac{4}{\pi} \sum_{\ell=0}^{\infty} \frac{\omega^2}{\omega^2 - (2\ell + \sigma)\omega_b^2} T_{m\ell} T_{n\ell} \right] , \tag{4}$$

where 0 subscripts label quantities at $s = 0$. For quadratic fields, the matrices S and T may be written in terms of Bessel functions as

$$S_{mn} \equiv \omega_b \int_0^{s_t} \frac{ds}{|v_\parallel|} sc(k_m s) sc(k_n s)$$

$$= \frac{\pi}{4} \left\{ J_0[(k_m - k_n)s_t] + (-1)^\sigma J_0[(k_m + k_n)s_t] \right\} ,$$

$$T_{m\ell} \equiv \omega_b \int_0^{s_t} \frac{ds}{|v_\parallel|} \, sc(k_m s) sc[(2\ell + \sigma)\omega_b t]$$

$$= \frac{\pi}{2} J_{2\ell+\sigma}(k_m s_t) \ . \qquad (5)$$

Taking $k_m = [m + (\sigma - 1)/2] \, \pi/s_{max}$ in Eq. (5) is convenient because the basis functions $sc([m + (\sigma - 1)/2] \, \pi s/s_{max})$ satisfy the parity and endpoint constraints on ϕ. The imaginary part of Eq. (4) resulting from the singularity is neglected in lowest order when solving for ϕ_m, since for marginally stable modes it has the same magnitude as the ion term which has been assumed small.

For the idealized case in which the magnetic force on electrons is negligible, the eigenvalues of Eq. (4) for even ϕ form a family of curves shown in Fig. 1, with the longest wavelength solution for a particular eigenfrequency being associated with the largest value of $k_{\perp 0}^2 \lambda_{De0}^2$. Odd solutions have eigenvalue curves that resemble those in Fig. 1. The long wavelength even solutions for the shorter wavelength modes are of particular interest because for given ω and $k_\perp$ these can become unstable at the lowest plasma densities. An approximate analytic solution of Eq. (4) shows the important qualitative features of this mode. When ϕ is represented by the lowest-order Fourier component and the magnetic force on electrons is neglected, Eq. (4) reduces for even ϕ to

$$k_{\perp 0}^2 \lambda_{De0}^2 \approx \pi^2 \frac{V_e}{\omega_b s_{max}} \left\{ 4 \exp(-\nu) \left[I_0(\nu) + \frac{\omega^2}{\omega^2 - 4\omega_b^2} I_2(\nu) \right] - \exp(-2\nu) - 1 \right\} , \qquad (6)$$

where $\nu \equiv \pi^2 V_e^2 / 8\omega_b^2 s_{max}^2$. This analytic result is plotted as a dashed line in Fig. 1.

A mirror field affects the electron bounce modes principally by spreading the bounce frequency and weakening the resonances. The eigenvalue curves in Fig. 2 show the variation of the longest wavelength even mode with $B_{max}/B_0 > 1$.

Stability thresholds are calculated by the following perturbation procedure. The equation for ϕ is integrated over a flux surface to give a functional of the form $D(k,\omega) = \int ds \int ds' K(s,s',\omega)\phi(s)\phi(s') = 0$. The kernel K is split into an unperturbed part K_0 with real eigenfunctions ϕ_0 and eigenfrequencies ω_0, and a perturbing part K_1 that includes the imaginary response terms. A linear Taylor expansion then gives

$$s\omega = \frac{\int ds \int ds' K_1(s,s',\omega_0)\phi_0(s)\phi_0(s')}{\int ds \int ds' \frac{\partial K_0}{\partial \omega}(s,s',\omega_0)\phi_0(s)\phi_0(s')} = \frac{\Gamma(K_1,\omega_0)}{\frac{\partial \Gamma}{\partial \omega}(K_0,\omega_0)} . \tag{7}$$

If the perturbing ion and electron terms in the kernel are called respectively K_1^i and K_1^e, the condition for marginal stability is that the corresponding imaginary frequency shifts have the same magnitude:

$$\frac{\mathrm{Im}[\Gamma(K_1^i,\omega_0)]}{\mathrm{Im}[\Gamma(K_1^e,\omega_0)]} = 1 . \tag{8}$$

The procedure is valid so long as K_1^i and K_1^e are small compared with K_0.

For the chosen distribution functions, the appropriate ion and electron dissipative terms are

$$\mathrm{Im}(\Gamma^i) = \frac{-2\omega_{ci0}\omega_{pi0}^2}{B_0} \sum_{n=-\infty}^{\infty} n\Lambda_n \mathrm{Re} \int_0^{s_{max}} ds\phi(s) \frac{N(s)}{N_0} \int_0^{\infty} dv_\parallel \frac{F_\parallel}{v_\parallel}$$
$$\times \int_{-s_{max}}^{s_{max}} ds'\phi(s') \exp\left(-\frac{i|s-s'|}{v_\parallel}\left\{\omega - n\omega_{ci0}\left[1 + \frac{(s+s')^2}{4L_m^2} + \frac{(s-s')^2}{12L_m^2}\right]\right\}\right) , \tag{9}$$

and

$$\mathrm{Im}(\Gamma^e) = -2\pi^3 \frac{\omega^2\omega_{pe0}^2}{m} \sum_{\ell=0}^{\infty} \int_0^{\infty} dE \left(\frac{\partial F_e}{\partial E} \frac{\phi_e^2}{\left|\frac{\partial \omega_b^2}{\partial \mu}\right|}\right)_{\mu=\mu_\ell} , \tag{10}$$

where

$$\Lambda_n = \pi \int_0^{\infty} dv_\perp^2 \frac{\partial F_\perp}{\partial v_\perp^2} J_n^2\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right) .$$

The non-negative solutions of $\omega = (2\ell + \sigma)\omega_b(E,\mu_\ell)$ are denoted here by μ_ℓ, and L_m is a magnetic field scale length

$$\left(\frac{1}{2}\ \frac{d^2B}{ds^2}\right)^{-1/2} .$$

Since the ion term is strongly peaked around $\omega = \omega_{ci0}$, only the $n = 1$ term in the sum contributes significantly, and other terms may be neglected. To calculate the stability threshold for a particular eigenfrequency, the $k_{\perp 0}$ satisfaying Eq. (6) is first found numerically, and plasma density is then obtained using the value of $k_{\perp 0}^2 \lambda_{De0}^2$ associated with the eigenmode. The sharp peaking of the ion term ensures that the minimum threshold will occur when the eigenfrequency satisfies $\omega \approx \omega_{ci0}$ on some flux surface in the plasma. In BBII, ω varies about 15% radially, and the minimum threshold density found for ω within this range should correspond to the experimentally observed threshold for similar plasma parameters.

The stability thresholds predicted for the electron bounce modes are close to BBII values. The experimental values of the density parameter $\epsilon \equiv \omega_{pi0}^2/\omega_{ci0}^2$ in Table 1 were obtained by keeping perpendicular ion temperature constant while varying plasma length with an axial limiter.[8] With similar fields and plasma parameters, the theoretical thresholds calculated using the peaked and broad ion distributions bracket many of the experimental values. The lowest BBII thresholds are close to the values calculated using the peaked ion distribution, while the higher threshold cases match the broad distribution predictions better. This behavior is consistent with the expectation that at the higher threshold densities, collisions spread the initially peaked ion distribution.

Infinite medium wave theory is clearly unable to describe the stability of a mirror plasma in which the periodicity of particle orbits is important. The agreement of the calculated thresholds with values observed in BBII suggests that the proposed bounce-mode model is more adequate. Current work will extend the comparison to BBI and other low-density mirror experiments.

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Table 1. Experimental and theoretical values of $\varepsilon = (\omega_{pi0}/\omega_{ci0})^2$ in Baseball II for $\psi_{max}/T_e = 3.0$.

T_i (keV)	B_{max}/B_0	ψ_{max}/T_i	Experimental ε	Theoretical ε	
				Peaked distribution	Broad distribution
0.83	1.26	0.069	0.031	0.029	0.057
	1.64	0.090	0.074	0.047	0.087
	1.9	0.090	0.093	0.049	0.090
	2.2	0.095	0.104	0.053	0.098
1.34	1.26	0.049	0.025	0.024	0.045
	1.43	0.056	0.046	0.030	0.052
	1.64	0.060	0.070	0.034	0.064
	2.2	0.064	0.105	0.039	0.075

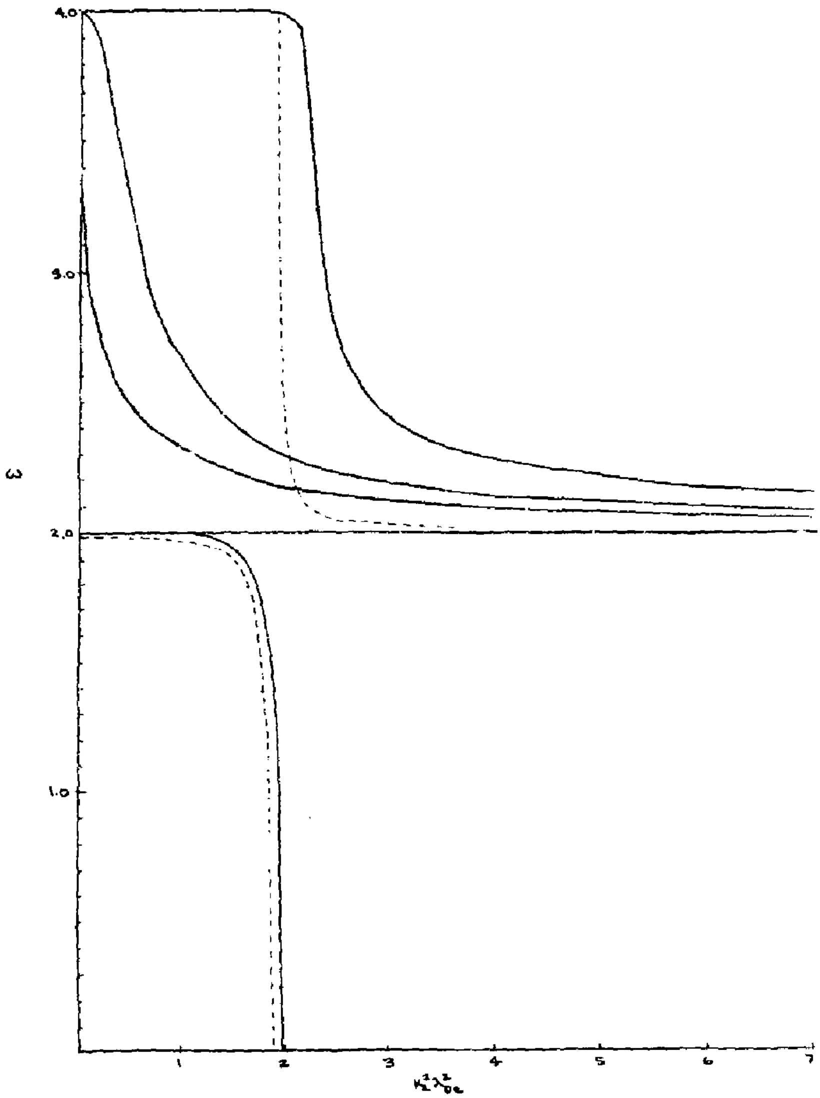

Fig. 1. Eigenvalues for even eigenfunctions in a quadratic potential $\psi = \psi_{max}(s/s_{max})^2$ with $\psi_{max}/T_e = 3.0$ and $B_{max}/B_0 = 1.0$. Dashed line is the analytic solution Eq. (6).

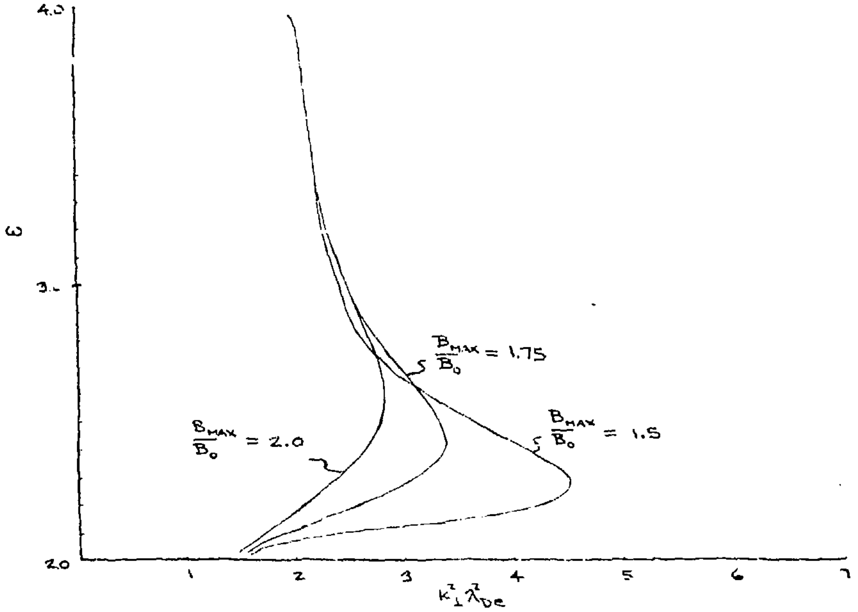

Fig. 2. Eigenvalues for even eigenfunctions with B_{max}/B_0 = 1.5, 1.75, and 2.0, and T_{max}/T_0 = 3.0.