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Dissipative Trapped Electron Modes in the Presence of Impurities

K. T. Tsang

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by

K. T. Tsang

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OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
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DISSIPATIVE TRAPPED ELECTRON MODES IN THE PRESENCE OF IMPURITIES *

K. T. Tsang
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37830

ABSTRACT

The effect of impurities on low frequency drift modes of a toroidally-confined plasma is investigated by the gyro-kinetic equation. It is assumed that electrons are in the banana regime and ions in the plateau regime. Impurity collision damping is found to be significant in the usual trapped electron mode. A new instability due to the impurities can occur for normal profiles and impurities peaked at the center. Quasi-linear considerations show that impurities will be driven outward if such an instability occurs.

I. INTRODUCTION

Extensive studies have recently appeared on the various aspects of dissipative trapped electron modes[1-6] due to the possibility that they may be responsible for the anomalous electron transport in present or next generation tokamak experiments. Analysis of ORMAK data[7] tends to indicate that shear is not always strong enough to stabilize these modes in their pure form.[3] However, this analysis is inconclusive since there are cases when half of the plasma column does not satisfy the stability criterion and the other half does. For this reason, we want to treat the mode in a more realistic way, i.e., to include the effects of impurities.

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The obvious effect of impurities is to increase the electron and ion collision frequencies. The main ion collisions can contribute to the damping directly, while electron collisions cannot, since the growth rate is a complicated function of $\varepsilon\omega/\nu_e$. However, the increase of electron collisionality can decrease the trapped electron population and increase the number of electrons that can resonate with the wave. This effect has already been treated in Reference 6.

The more direct way in which impurities enter the dispersion relation is through their contribution, as additional ion species, to the charge neutrality condition. This leads to two important effects. First, the reduction of growth rate due to impurities collisions. This is the same effect found in the trapped ion mode.[10] Second, a new interval in phase velocities of the modes is introduced.

For convenience, we consider the case with only one species of impurity present. Usually we have the relation: $v_I < v_i < v_e$, where v_I, v_i and v_e are the thermal velocities of impurity, main ion and electron respectively. Hence, we can distinguish two modes. The first one is the usual trapped electron mode, which occurs when the parallel phase velocity $\omega/k_{||}$ (where ω is the mode frequency and $k_{||}$ is the parallel wave number) is between v_i and v_e. It is modified only slightly by the presence of impurities. When $\omega/k_{||}$ is between v_I and v_i we obtain the impurity trapped electron mode similar to the impurity drift mode of Reference 8. The impurity trapped electron mode usually has a mode frequency less than the electron diamagnetic drift frequency. Because of this difference, the trapped electron term is more destabilizing because i) the universal unstable term is never small even when $k_\perp \rho_i \ll 1$, where $k_\perp$ and ρ_i are respectively the perpendicular and wave number and ion gyro-radius, and ii) the electron toroidal magnetic drift frequency, ω_{De}, can no longer be neglected compared with ω, so that the Landau resonance associated with ω_{De} greatly amplifies the trapped electron terms.

Effects of impurities on drift waves, collisionless trapped particle modes and trapped ion instabilities have been previously treated before by various authors.[8-10] Dissipative trapped electron modes in the presence of impurities have also been discussed briefly by Bhadra.[11] However, his treatment is local, and finite Larmor radii effects are neglected. Because of the recent progress in the linear theory of trapped electron modes,[5-6] a more complete and improved calculation is needed in order to analyze the experimental data in a more realistic way.

In the following sections, we first discuss the usual trapped electron mode in an impure plasma. Then we will treat the impurity trapped electron mode. The treatment is based on the gyro-kinetic equation recently derived[12] so that we can easily retain the gyro-radius effects of main ion and impurity. This is a handier and more logical way than starting from the Vlasov equation and integrating along the characteristic. Impurities and main ion are assumed to be in higher collisional regimes (plateau or even Pfirsch-Schluter) than the electrons so that no significant part of those with $v_\parallel < \sqrt{\varepsilon} v_\perp$ is trapped in the magnetic well. Electrons are assumed to be deep in the banana regime so that all of those with $v_\parallel < \sqrt{\varepsilon} v_\perp$ are trapped. In the following, we assume one species of impurity for convenience. The results can readily be generalized to multispecies impurities.

II. USUAL TRAPPED ELECTRON MODE

We assume a low β toroidal plasma with toroidal coordinates r, θ and ζ, where θ and ζ are the angles measured poloidally and toroidally. The electrostatic perturbation is assumed to be $\phi = \hat{\phi}\,(r, \theta)\exp(-i\omega t - i\ell\zeta + im\theta)$, where $\hat{\phi}$ has only slow θ dependence compared with $\exp(im\theta)$. The toroidal and poloidal mode numbers satisfy $lq(r_0) = m$, where $q(r)$ is the safety factor and r_0 is the radial location of the rational surface of interest.

The equilibrium distribution functions are assumed to be Maxwellians: F_e, F_i and F_I. The subscripts e, i and I denote electron, main ion and impurity respectively.

The perturbed ion and impurity distribution functions f_i and f_I can be shown to satisfy[12]

$$(\omega - \omega_{D\beta} + iv_{\parallel}\ \underline{n}\cdot\underline{\nabla})g_{\beta} - (\omega - \omega_{*\beta}^{T})\frac{e}{T_{\beta}}F_{\beta}\langle\exp(iL_{\beta})\rangle = i\,\nu_{\beta}\,g_{\beta} \qquad (1)$$

$$\beta = i, I$$

where

$$f_{\beta} \equiv -\frac{Z_{\beta}e\phi}{T_{\beta}}F_{\beta} + g_{\beta}\exp(-iL_{\beta}),$$

$$\left\langle \phi e^{iL_{\beta}} \right\rangle \equiv J_0\left(\frac{kv_{\perp}}{\Omega_{\beta}}\right)\phi + J_1\left(\frac{kv_{\perp}}{\Omega_{\beta}}\right)\frac{v_{\perp}}{2k\Omega_{\beta}}\frac{\partial^2\phi}{\partial r^2},$$

$L_{\beta} = kv_{\perp}/\Omega_{\beta}\cos\phi$, ϕ being the gyrophrase angle,

$$\omega_{*\beta}^{T} = \omega_{*\beta}\left[1 + \eta_{\beta}\left(\frac{M_{\beta}v^2}{2T_{\beta}} - \frac{3}{2}\right)\right],$$

$$\omega_{*\beta} = k \frac{cT_\beta}{Z_\beta eB} \frac{\partial \ell n\, N_\beta}{\partial r}, \quad \omega_{D\beta} = -\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right) \frac{\cos\theta}{\Omega_\beta R},$$

$$\eta_\beta = \frac{\partial\ \ell n\ T_\beta}{\partial\ \ell n\ N_\beta},$$

$$k = \frac{m}{r}, \quad \Omega_\beta = \frac{Z_\beta eB}{M_\beta C},$$

J_0 and J_1 are the Bessel functions. Note that the form we wrote down for $\langle\phi \exp(iL_\beta)\rangle$ is valid only in the so-called prime coordinates.[12] The magnetic drift frequencies $\omega_{D\beta}$ in Eq. (1) will be neglected later. However, ω_{De} for the trapped electrons will be retained. There are two reasons to justify this. First, in present experiments T_β is much less than T_e. Secondly, since ions and impurities are in plateau regime, the cos θ-dependence of $\omega_{D\beta}$ tends to average to zero, due to free-streaming or collisional randomization in the poloidal direction. Only those electrons trapped in the unfavorable curvature of the tokamak field will have nonzero average ω_{De} and have significant effect on the mode. A Bhatnagar-Gross-Krook (BGK) collision model is used in Eq. (1). This model is valid in the limit of small $k_\theta \rho_i$, but not accurate enough when $k_\theta \rho_i$ is of order one and larger.[12] However, to include any realistic collision model in the treatment will lead to immense complication, which would render the attempt worthless.

Equ. (1) contains the finite gyro-radius effects of the ions and impurities. It is straightforward to obtain the perturbed densities $\tilde{N}_\beta$ from Eq. (1):

$$\frac{\tilde{N}_\beta}{N_\beta} = -\frac{Z_\beta e\phi}{T_\beta} + \int d\underline{V}\, \frac{(\omega - \omega_{*\beta}^T) Z_\beta e F_\beta}{\omega + i\nu_\beta - k_\parallel v_\parallel}\, \frac{1}{T_\beta}\left[J_0^2 + J_0 J_1 \frac{v}{k\Omega_\beta}\frac{2}{r^2}\right]\cdot\phi$$

where J_0 and J_1 have the argument $kv_\perp/\Omega_\beta$ and $k_\parallel = (m - lq)/Rq$. In the limit $b_\beta = (k\,\rho_\beta)^2 \ll 1$, with $\rho_\beta^2 = T_\beta/M_\beta\Omega_\beta^2$, we have

$$\frac{\tilde{N}_\beta}{N_\beta} = -\frac{Z_\beta e_\phi}{T_\beta} - \frac{Z_\beta e}{T_\beta}\left\{\left[\left[\omega - \omega_{*\beta}\,(1 + \eta_\beta/2)\right]\, Z(\xi_\beta)\right.\right. \tag{2}$$

$$\left. - \omega_{*\beta}\,\eta_\beta\,(\xi_\beta + \xi_\beta^2\, Z(\xi_\beta)\right]\left(1 - b_\beta + \rho_\beta^2 \frac{\partial^2}{\partial r^2}\right)$$

$$\left. + \omega_{*\beta}\,\eta_\beta\, Z(\xi_\beta)\right\}\phi/k\, v_\beta,$$

where $\xi_\beta = (\omega + i\nu_\beta)/k\,v_\beta$, $Z(\xi_\beta) = 1/\sqrt{\pi}\int_{-\infty}^{+\infty} \exp(-x^2)dx/x-\xi$ is the plasma dispersion function, and $v_\beta = (2T_\beta/M_\beta)^{1/2}$ is the thermal velocity.

Since $\omega \gg \nu_\beta$, we obtain the following when $v_I < v_i < \frac{\omega}{k} < v_e$:

$$\frac{\tilde{N}_\beta}{N_\beta} = \frac{Z_\beta e}{T_\beta}\left\{-\frac{\omega_{*\beta}}{\omega} - \frac{i\nu_\beta}{\omega}\left(1 - \frac{\omega_{*\beta}}{\omega}\right)\right. \tag{3}$$

$$\left. + \left[1 - \frac{\omega_{*\beta}}{\omega}(1 + \eta_\beta)\right]\left(\rho_\beta^2 \frac{\partial^2}{\partial r^2} + \frac{1}{2\xi_\rho^2} - b_\beta\right)\right\}\phi$$

For the electron part, it is a well-known result[3,4] that if we assume a flute-like perturbed potential, then

$$\frac{\tilde{N}_e}{N_e} = \frac{e\phi}{T_e} + \sqrt{2\varepsilon}\,\frac{e\phi}{T_e}\left[\left(1 - \frac{\omega_{*e}}{\omega}\right) A_2 - \eta_e \frac{\omega_{*e}}{\omega} A_1\right] \tag{4}$$

$$+ i\,\frac{e\phi}{T_e}\left[\left(1 - \frac{\omega_{*e}}{\omega}\right) R_2 + \eta_e \frac{\omega_{*e}}{\omega} R_1\right],$$

with

$$\begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \sum_{n=1}^{\infty} \frac{8\pi^{-1/2}}{\alpha_n^2} \int_0^{\infty} \frac{v^2 dv\, e^{-v^2}}{1 - \omega_{De}/\omega + i\nu_e \alpha_n^2/8\varepsilon\omega} \begin{Bmatrix} v^2 - \frac{3}{2} \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \frac{\pi^{1/2}\omega}{|k_{\|}| v_e} \begin{Bmatrix} \frac{1}{2} \\ 1 \end{Bmatrix} \left[\exp\left(- \frac{\omega^2}{k^2 v_e^2} \right) - \exp\left(- \frac{\omega^2}{\varepsilon k^2 v_e^2} \right) \right],$$

and α_n being the n^{th} zero of $J_0(x)$. Equations (3) and (4), and the charge neutrality condition leads to the dispersion relation:

$$\phi + \sqrt{2\varepsilon}\, \phi \left[\left(1 - \frac{\omega_{*e}}{\omega} \right) A_2 - \eta_e \frac{\omega_{*e}}{\omega} A_1 \right] + i \left[\left(1 - \frac{\omega_{*e}}{\omega} \right) R_2 + \eta_e \frac{\omega_{*e}}{\omega} R_1 \right] \quad (5)$$

$$= \sum_{\beta = i, I} \left\{ \frac{\omega_{*e}}{\omega} \frac{r_{ne}}{Z_\beta r_{n\beta}} - i \left(\frac{T_e}{T_\beta} + \frac{\omega_{*e}}{\omega} \frac{r_{ne}}{Z_\beta r_{n\beta}} \frac{\nu_\beta}{\omega} \right) \right.$$

$$\left. + \left[\frac{T_e}{T_\beta} + \frac{\omega_{*e}}{\omega} \frac{r_{ne}}{Z_\beta} \left(\frac{1}{r_{n\beta}} + \frac{1}{r_{T\beta}} \right) \right] \left(\rho_\beta^2 \frac{\partial^2}{\partial r^2} + \frac{1}{2\xi_\beta^2} - b_\beta \right) \right\} \phi,$$

where $\alpha_\beta = Z_\beta^2 N_\beta/N_e$. $r_{n\beta} = (\partial \ln N_\beta/\partial r)^{-1}$, and $r_{T\beta} = (\partial \ln T_\beta/\partial r)^{-1}$.

Assuming the terms involving trapped electrons, collisions, and finite gyro-radius are all small, and using the equilibrium neutrality condition, we obtain

$$\omega = \omega_{*e} + \delta\omega, \text{ where } |\delta\omega| \ll |\omega_{*e}| .$$

In the next order equation, after we expand $k_{\|}$ around $r = r_0$, Eq. (5) becomes a Weber equation and can be solved by imposing the outgoing wave boundary condition. Then the eigenvalue of the solution is $\delta\omega$, and we obtain

$$\frac{\gamma}{|\omega_{*e}|} \equiv \frac{\mathrm{Im}\,\delta\omega}{|\omega_{*e}|} \tag{6}$$

$$= \left(\frac{kc}{eB}\right)^2 T_e\, M' \left[(2\varepsilon)^{1/2}\ |\mathrm{Im}\, A_2| + \frac{1}{2}\left(\frac{m_e|Ls|}{M|r_{ne}|}\right)^{1/2} \ln\frac{1}{\varepsilon}\right]$$

$$+ \eta_e \left[(2\varepsilon)^{1/2}\ \mathrm{Im}\, A_1 \left(|\frac{\nu_e}{\varepsilon\omega}|\right) - \frac{1}{4}\left(\frac{m_e|L_s|}{M|r_{ne}|}\right)^{1/2} \ln\frac{1}{\varepsilon}\right]$$

$$- \sum_\beta \alpha_\beta \frac{\nu_\beta}{|\omega_{*e}|} \left(\frac{T_e}{T_\beta} + \frac{r_{ne}}{Z_\beta r_{n\beta}}\right)$$

$$- \left(\frac{M'}{M''}\right)^{1/2} \left|\frac{r_{ne}}{L_s}\right| ,$$

where

$$M' = \sum_\beta \alpha_\beta Z_\beta^{-2} M_\beta \left[1 + \frac{T_\beta}{T_e} \frac{r_{ne}}{Z_\beta}\left(\frac{1}{r_{n\beta}} + \frac{1}{r_{T\beta}}\right)\right],$$

$$M'' = \left\{\sum_\beta \alpha_\beta M_\beta^{-1} \left[1 + \frac{T_\beta}{T_e} \frac{r_{ne}}{Z_\beta}\left(\frac{1}{r_{n\beta}} + \frac{1}{r_{T\beta}}\right)\right]\right\}^{-1},$$

$$M = (M'\ M'')^{1/2},\ L_s^{-1} = rq'/Rq^2.$$

Eq. (6) looks almost the same as the result given in Ref. 5, except for the ion and impurity collision stabilization terms. The temperature gradient term due to the trapped electrons is exactly the same as in Ref. 5, whereas all the other terms are slightly changed due to the presence of the impurities.

In general, M' and M" are positive even though $r_{ne}\,(1/r_{n\beta} + 1/r_{T\beta})$ can have either sign, because $T_\beta/T_e Z_\beta$ is much less than unity. There may exist some remote possibilities that M'/M" is negative. In that case, magnetic shear will contribute to the real part of ω instead of the imaginary part and there will be no more shear stabilization.

Eq. (6) in the limit $k\rho_i << 1$ has been used to determine the stability of typical ORMAK discharge parameters. It is found[7] that the impurity collision term overwhelms all other terms, so that the plasma is stable everywhere.

III. IMPURITY TRAPPED ELECTRON MODE

When the parallel phase velocity of the mode is between the ion and impurity thermal velocities, we have the impurity trapped electron mode. Eq. (3) is still valid for the impurities. For the ions, the BGK model in Eq. (1) is no longer as satisfying. Following Ref. 6, we use a pitch angle collision model for the ion, which is justified by the presence of the impurities. The ion equation can be written as

$$(\omega + i v_\parallel \vec{n} \cdot \vec{\nabla})\, g_i - (\omega - \omega_{*i}^T) \frac{eF_i}{T_i} \left[J_0 + J_1 \frac{v_\perp}{2k\Omega_i} \frac{\partial^2}{\partial r^2}\right] \phi$$

$$= i \frac{\nu_i}{2} \frac{\partial}{\partial v_\parallel} \left[(v^2 - v_\parallel^2) \frac{\partial \hat{g}}{\partial v_\parallel} \right] .$$

Treating only the case of small $k\rho_i$, we get

$$\widetilde{N}_i = - \frac{e\phi}{T_i} N_i + \widetilde{N}_i' ,$$

where

$$\widetilde{N}_i' = - 4\pi \frac{\omega - \omega_{*i}^T}{|k_\parallel|}\, i \frac{e\phi}{T_i} \int_0^\infty v dv\, F_i \int_0^\infty dp \frac{\sin(pv)}{p} \exp\left(i \frac{\omega}{|k_\parallel|} p - \frac{\nu v^2 p^3}{6|k_\parallel|}\right), \quad (7)$$

by the same method used in Ref. 6.

The charge neutrality condition then becomes

$$\phi + (2\varepsilon)^{1/2}\,\phi\left[\left(1 - \frac{\omega_{*e}}{\omega}\right) A_2 - \eta_e \frac{\omega_{*e}}{\omega} A_1\right] \tag{8}$$

$$+ i\,\phi\left[\left(1 - \frac{\omega_{*e}}{\omega}\right) R_2 + \eta_e \frac{\omega_{*e}}{\omega} R_1\right]$$

$$= -\alpha_1\left[\frac{T_e}{T_I}\phi - \frac{T_e}{e}\frac{\widetilde{N}_i'}{N_i}\right]$$

$$+ \alpha_I \frac{T_e}{T_I}\left[-\frac{\omega_{*I}}{\omega} - i\,\frac{\nu_I}{\omega}\left(1 - \frac{\omega_{*I}}{\omega}\right)\right]\phi$$

$$+ \alpha_I \frac{T_e}{T_I}\left[1 - \frac{\omega_{*I}}{\omega}(1 + \eta_I)\right]\left(\rho_I^2\,\frac{2}{r^2} + \frac{1}{2\xi_I^2} - b_I\right)\phi\ .$$

Only the adiabatic responses in Eq. (8) are lowest-order quantities; hence, it implies

$$\omega = \frac{1}{Z_I}\quad\frac{\alpha_I\omega_{*e}}{1 + \alpha_1\,T_e/T_i}\;\frac{r_{ne}}{r_{nI}} + \delta\omega \tag{9}$$

with $\delta\omega$ much smaller than the first right-hand term.

The terms R_1, R_2 and $\widetilde{N}_i'$ are treated as perturbations in the next order equation. In calculating the contribution from $\widetilde{N}_i'$, a technique similar to that in Ref. 6 is used. The part of $\delta\omega$ due to $\widetilde{N}_i'$ is

$$(1 + \alpha_i\tau)\,\frac{T_e}{e}\,\frac{\alpha_i}{N_e}\,\omega\int_{-\infty}^{+\infty} dx\,\widetilde{N}_i'\,\phi^{(0)}\Big/\int_{-\infty}^{+\infty} dx\;\;\phi^{(0)\,2},$$

where $x = r - r_0$, $\phi^{(0)}$ is the lowest-order radial eigen solution.

The integral $\int_{-\infty}^{+\infty} dx\,\widetilde{N}_i'\,\phi^{(0)}$ can be reduced to the form:

$$I = \int_0^\infty \frac{dx}{x} \cdot \exp\left[ia - b/x - \sigma x^2\right]$$

$$\simeq \int_0^{x_0} \frac{dx}{x} \exp\left[(ia - b)/x\right]$$

$$\simeq \ell n \;\; x_0/(a^2 + b^2)^{1/2} \quad ,$$

where $a = \omega L_s p/k$, $b = \nu v^2 p^3 L_s/6k$, $\sigma = i|k| (T_I/M_I)^{1/2}/|L_s|\rho_I\omega$, and $x_0 = (\sigma/i)^{-1/2}$. Neglecting the slow μ-dependence in the logarithm, and inserting the most probable value of p, i.e., v_i^{-1}, we obtain the following growth rate due to the ions:

$$\frac{\gamma_i}{|\omega|} \simeq \sqrt{2}\, \alpha_i\, \mu^{1/2}\, (\ell n\, \mu^{-1/2}) \left[\frac{\tau}{1 + \alpha_i\tau} + \frac{Z_I}{\alpha_I}\frac{r_{nI}}{r_{ni}}\left(1 - \frac{\eta_i}{2}\right)\right] \qquad (10)$$

where $\mu = M_i|L_s|\tau\alpha_I/2M_I|r_{nI}|(1 + \alpha_i\tau)$, and $\tau = T_e/T_i$. Equation (10) is similar to the well-known result of electron-wave Landau resonance in slab geometry. However, we should recognize that it is an asymptotic result for $\mu << 1$. For $\mu \gtrsim 1$, we should replace the right-hand side of Eq. (10) by zero. The exact value of γ_i when $\mu \sim 1$ involves very complicated calculation, and it is not clear whether the perturbative method makes sense in this case. Taking into account all the other terms, the total growth rate of this mode is:

$$\frac{\gamma}{|\omega|} \simeq \left(\frac{Z_I r_{nI}}{\alpha_I r_{ne}} - \frac{1}{1+\alpha_i\tau+\alpha_I\, b_I\, T_e/T_I}\right)\left[(2\varepsilon)^{\frac{1}{2}}\left|\mathrm{Im}\ A_2\right| + \frac{1}{2}\left(\frac{m_e}{M_I}\left|\frac{L_s}{r_{nI}}\right|\frac{\alpha_I}{1+\alpha_i}\right)^{\frac{1}{2}}\ell n\frac{1}{\varepsilon}\right] \qquad (11)$$

$$+\ \eta_e\frac{Z_I r_{nI}}{\alpha_I r_{ne}}\left[(2\varepsilon)^{1/2}\ \mathrm{Im}\ A_1\ \left(\left|\frac{\nu_e}{\varepsilon\omega}\right|\right) - \frac{1}{4}\left(\frac{m_e}{M_I}\left|\frac{L_s}{r_{nI}}\right|\frac{\alpha_I}{1+\alpha_i\tau}\right)^{1/2}\ell n\ \frac{1}{\varepsilon}\right]$$

$$- \alpha_I \frac{T_e}{T_I} \frac{\nu_I}{|\omega|} \left(\frac{1}{1 + \alpha_i \tau} + \frac{T_I}{\alpha_I T_e} \right) + \frac{\gamma_i}{|\omega|}$$

$$- \left[1 + \frac{T_I}{\alpha_I T_e} (1 + \alpha_i \tau) (1 + \eta_I) \right] |\frac{r_{nI}}{L_s}|$$

Eq. (11) tells us that shear is only provided by the impurities. Although apparently the size of the shear term is not reduced, it is smaller compared with the destabilizing trapped electron terms, since $Z_I \, r_{nI}/\alpha_I \, r_{ne}$ is usually greater than one. Ion collision stabilization is missing from our assumed collision model, but it will appear if we use a hybrid pitch angle and BGK collision model. In any case, judging from our experience in last section, it is not as important as the impurity collision. The total effect of the ion pitch angle collisions is to induce an ion-wave collisional resonance which gives rise to γ_i. From Eq. (10), we observe that when $\mu \ll 1$, the ion term is destabilizing if

$$\frac{\tau}{1 + \alpha_i \tau} + \frac{Z_I r_{nI}}{\alpha_I r_{ni}} \left(1 - \frac{\eta_i}{2} \right) < 0.$$

This condition is also given in Ref. 8. The most interesting terms are those due to the trapped electrons. When $Z_I r_{nI}/\alpha_I r_{ne} > 0$ and $\eta_e > 0$, a violent instability will occur. Unlike the usual trapped electron mode, Im A_2 and the circulating electron resonance term is not multiplied by a small number even when $k\rho_i$ is very small. Hence, these two sources of instability are always present. If we assume r_{nI} is of the same order as r_{ne}, $Z_I r_{nI}/\alpha_I r_{ne}$ is usually much larger than one for the impurity level in present-day tokamaks. Thus, the effect of A_1, A_2 and the untrapped electron resonance will be magnified.

When r_{nI} is negative, i.e., when impurities are peaked at the center, the resonance associated with the magnetic curvature drift sets in. For the deeply-trapped electron, we have

$$\omega_{De}/\omega \simeq - \frac{m_e v^2}{2T_e} \frac{r_{nI}}{R} \frac{Z_I}{\alpha_I} (1 + \alpha_i \tau).$$

Since the ratio $r_{nI} Z_I \ (1 + \alpha_i \tau)/R \ \alpha_I$ is of order unity, this resonance is more pronounced than that in the usual trapped electron mode.

The discussion above implies that the condition $r_{nI}/r_{ne} > 0$, $\eta_e > 0$ is a very unstable situation with respect to the impurity trapped electron mode. Hence, the quasi-linear consequence is that impurities start to diffuse outward until the driving term of the instability vanishes. This is confirmed by the calculation of the quasi-linear diffusion flux for impurities. We have, by integrating the quasi-linear Vlasov equation over velocity space,

$$\frac{\partial N_I}{\partial t} = - \frac{1}{r} \frac{\partial}{\partial r} r \, \Gamma_I,$$

and

$$\Gamma_I = 2 \sum_k \frac{c}{B} k \ |\tilde{\phi}_k|^2 \ \mathrm{Im} \ \frac{\tilde{N}_{Ik}}{\tilde{\phi}_k} \quad .$$

Substituting $\tilde{N}_{Ik}$ for this mode in, we get

$$\Gamma_I = - \sum_k \left(\frac{Z_I e \phi_k}{T_I} \right)^2 \frac{(v_I k)^2}{|\omega|} \rho_I^2 \frac{N_I}{r} \qquad (12)$$

$$\cdot \left\{ \left(\frac{Z_I r_{nI}}{\alpha_I r_{ne}} - \frac{1}{1 + \alpha_i \tau + \alpha_I \ T_e/T_I \ b_I} \right) \left[\sqrt{2\varepsilon} \ |\mathrm{Im} \ A_2| \right. \right.$$

$$+ \frac{1}{2}\left(\frac{m_e}{M_I} \left|\frac{L_s}{r_{nI}}\right| \frac{\alpha_I}{1 + \alpha_i\tau}\right)^{1/2} \ln \frac{1}{\varepsilon}\Bigg]$$

$$+ \eta_e \frac{Z_I r_{nI}}{\alpha_I r_{ne}}\left[\sqrt{2\varepsilon}\ \mathrm{Im}\ A_1 - \frac{1}{4}\left(\frac{m_e}{M_I} \left|\frac{L_s}{r_{nI}}\right| \frac{\alpha_I}{1 + \alpha_i\tau}\right)^{1/2} \ln \frac{1}{\varepsilon}\right] + \frac{\gamma_i}{\omega}\Bigg\}$$

Whenever the instability occurs, the sum of the terms inside the curly bracket in Eq. (12) is positive. For the case $\partial N_I/\partial r < 0$ (peaked at the center), the impurity particle flux is positive (outward). This may provide us with an explanation for the experimental fact that so far no impurities peaked at the center as predicted by neoclassical theory are observed.

IV. CONCLUSION

The introduction of impurities in the consideration of trapped electron modes leads us to distinguish between two modes: the usual trapped electron mode and the impurity trapped electron mode, which are basically the usual drift mode and the impurity drift mode, respectively, in toroidal geometry. Both modes are studied in great detail. The growth rate of the usual trapped electron mode is given in Eq. (6). For ORMAK discharges data, the collisional damping term due to impurity overwhelms all other terms and the mode is stabilized. The growth rate of the impurity trapped electron mode is given in Eq. (11). If is found that a strong instability will occur if $r_{nI}/r_{ne} > 0$ and $\eta_e > 0$. Whenever such an instability occurs, the quasi-linear impurity particle flux is found to be positive (outward). This may explain the lack of impurity peaking at the center as predicted by neoclassical theory in most tokamak discharges.

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