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CONDUCTING GRIDS TO STABILIZE MHD GENERATOR PLASMAS AGAINST IONIZATION INSTABILITIES

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ABSTRACT

Ionization instabilities in MHD generators may be suppressed by the use of grids that short circuit the AC electric field component corresponding to the direction of maximum growth. An analysis of the influence of the corresponding boundary conditions has been performed in order to obtain more quantitive information about the stabilizing effect of this system.

INTRODUCTION

Two types of stabilization of MHD-generator plasmas are possible:

i) First order stabilization, which means that a suitable change in plasma conditions or boundary conditions must prevent the growth of the instabilities in the initial stage of their development. This kind of stabilization can be analyzed using linearized perturbed equations.

ii) Second order stabilization, which means that the stabilizing system must keep the fluctuation amplitude at a sufficiently low level by the extraction of wave energy. The description of this kind of stabilization must be given in terms of quasi linear or nonlinear theories.

The knowledge about second order stabilization mechanisms in the case of ionization instabilities is very limited upto now. The suggestions about first order stabilization, known in the litterature, concern different methods. It has been proposed to influence the dynamics of the instabilities, either by the use of fully ionized seed [1], or by the addition of Nitrogen [2, 3, 4]. A possibility of using active circuits has been described [5]. Passive circuits have been suggested by Ricateau [6]. This stabilization method has been checked experimentally [7]. However, the result (reduction of the fluctuation amplitude with 25 %) shows that the stabilization has been of the second order type, so that no good description of the phenomena is available.

This paper describes first order stabilization by a particular passive circuit system, namely grids that short circuit the AC component of the electric field without affecting its DC component. By taking the orientation of the grids parallel to the direction of maximum growth of the instabilities (see for instance [8]), the anticipated stabilizing effect is expected to have a maximum.

ASSUMPTIONS AND BASIC EQUATIONS

Two basic assumptions have been made about the effect of the grids:

i) The coupling between the plasma and the grids is ideal, so that the grids provide in an equipotential surface in the plasma.
ii) The grids do not affect the DC quantities. This may be realized by composing each grid from segments, shorter that the wavelength and connected with each other by suitable LCR circuits.

The MHD plasma considered is assumed to be homogeneous in the direction of the magnetic field, so that the analysis considers only the XY-plane perpendicular to $\vec{B}$. The X-axis coincides with the direction of maximum growth of ionization instabilities, determined in the absence of the grids [8]. The orientation of the grids has been chosen parallel to this direction in order to affect the electric field related to the most unstable waves as much as possible. The location of the grids is given by y = 0 and y = d (Fig. 1).

The usual basic equations are employed, i.e. the Saha equation, the Ohm's law, the electron energy equation and the field equations. The The influence of radiation heat conduction, convection and compression on the electron energy has been neglected (see [9]). In the gas frame the set of equations becomes

$$\frac{N_e^2}{N_s - N_e} = \left(\frac{2\, m_e k T_e}{h^2}\right)^{3/2} \exp\left(-\frac{E_i}{kT_e}\right) \qquad (1)$$

$$\vec{J} = \sigma \frac{\vec{F} + \overrightarrow{\omega\tau} \times \vec{F}}{1 + \omega\tau^2} \quad , \quad \vec{F} = \vec{E} + \frac{\Delta p_e}{N_e e} \qquad (2)$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2} N_e k T_e + E_i N_e\right) = \frac{J^2}{\sigma} - \frac{3}{2} N_e k (T_e - T) \cdot \left(\nu_{es} \frac{m_e}{m_s} + \nu_{ei} \frac{m_e}{m_s} + \nu_{en} \frac{m_e}{m_n}\right) \qquad (3)$$

$$\nabla \cdot \vec{J} = 0 \tag{4}$$

$$\nabla \times \vec{E} = 0 \tag{5}$$

CALCULATIONS

The method of calculation as given by Nelson [9] is used, and therefore described here only very shortly and incompetentely. The equations (1 - 5) are linearized and reduced to three equations in the unknowns n_e^*, ψ and ϕ representing the AC parts of the electron density, current potential and electrical potential. Taking into account that the plasma is infinte in the X-direction, the resulting set of equations can be given in tensor notation using the Einstein summation convention as follows [9]

$$A_{\nu\rho}\left(\frac{\partial}{\partial t}\right)\xi_\rho + B_{\nu\rho}\frac{\partial}{\partial y}\xi_\rho = 0 \tag{6}$$

where ξ_ρ is the column vector with components n_e^*, ψ, ϕ.

The Laplace transform of (6) is

$$A_{\nu\rho}(-Z)\overline{\xi}_\rho + B_{\nu\rho}\frac{\partial}{\partial y}\overline{\xi}_\rho = \eta_\nu e^{i\vec{\kappa}\cdot\vec{r}}$$

with $\eta_\nu = (\alpha, 0, 0)$, where we assumed that a plane wave perturbation of n_e^* with an amplitude α initiates the instability at $t = 0$. $\overline{\xi}_\rho$ is the Laplace transform of the dependent variable ξ_ρ

$$\overline{\xi}_\rho = \int_0^\infty \xi_\rho e^{Zt}dt \tag{7}$$

The solution of the homogeneous part of eq. (7) is

$$\overline{\xi}_\rho = e^{i\kappa_x x}(\mu_\rho e^{\lambda_1 y} + \mu_{\rho+3}e^{\lambda_2 y}) \tag{8}$$

where λ_1 and λ_2 are given as the roots of the following quadratic equation

$$\det\left[A_{\nu\rho}(-Z) + \lambda B_{\nu\rho}\right] = (P_z Z + P)\lambda^2 +$$

$$Q_z Z + Q)\lambda + (R_z Z + R) = 0 \qquad (9)$$

The coefficients μ_ρ are determined by the boundary conditions and by requiring that the functions $\mu_\rho e^{\lambda_1 y}$ and $\mu_\rho e^{\lambda_2 y}$ are solutions of the homogeneous part of eq. (7). The grids impose the following boundary conditions on the plasma

$$\overline{\xi}_3 = 0 \qquad \text{at } y = 0 \text{ and } y = d$$

The coefficients μ_ρ are then given by the following equation

$$P_{\nu\rho}\mu_\rho = \zeta_\nu \qquad (10)$$

where $P_{\nu\rho}$ can be given in the matrix notation as

$$\begin{matrix}
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & e^{\lambda_1 d} & 0 & 0 & e^{\lambda_2 d} \\
A_{11}(-Z) & A_{12} + \lambda_1 B_{12} & 0 & 0 & 0 & 0 \\
A_{31} + \omega\tau_0 A_{21} & \omega\tau_0 A_{22} - \lambda_1 & (\omega\tau_0 + \frac{1}{\omega\tau_0})A_{23} & 0 & 0 & 0 \\
0 & 0 & 0 & A_{11}(-Z) & A_{12} + \lambda_2 B_{12} & 0 \\
0 & 0 & 0 & A_{31} + \omega\tau_0 A_{21} & \omega\tau_0 A_{22} - \lambda_2 & (\omega\tau_0 + \frac{1}{\omega\tau_0})A_{23}
\end{matrix}$$

The stability analysis is reduced to the solution of Z from the equation

$$\det(P_{\nu\rho}) = 0$$

Expansion of $\det(P_{\nu\rho})$ yields

$$\det(P_{\nu\rho}) = (e^{\lambda_1 d} - e^{\lambda_2 d})D_1 D_2 = 0 \qquad (11)$$

where

$$D_1 = \begin{vmatrix} A_{11}(-Z) & A_{12} + \lambda_1 B_{12} \\ A_{31} + \omega\tau_0 A_{21} & \omega\tau_0 A_{22} - \lambda_1 \end{vmatrix}$$

and D_2 is the same determinant with λ_1 replaced by λ_2. D_1 and D_2 depend on Z only through $A_{11}(-Z)$, which is linear in Z and through λ_1 and λ_2 via the coefficients of eq. (9). The product D_1D_2 thus yields a cubic equation in Z.

Eq. (11) determines several classes of modes

i) Modes making $e^{\lambda_1 d} - e^{\lambda_2 d} = 0$. They are given by $\lambda_1 - \lambda_2 = 2\pi in/d$, n = 1, 2, 3, ... Substitution of this relationship into eq. (9) yields two values of Z for each value of n, denoted by Zn(n, 1) and Zn(n, 2).

ii) Modes making $D_1D_2 = 0$. This results in three values of Z denoted by ZD_1, ZD_2 and ZD_3.

RESULTS

Values of Z have been computed ') for an A-Cs mixture in the regime where neutral collisions are dominant as well as in the regime where Coulomb collisions are also important. Therefore the calculations have been carried out for two different electron temperatures, i.e. $T_e = 2500$ K and $T_e = 3500$ K. The other plasma conditions considered are: $T = 1500$ K, $N_A = 2.5 \times 10^{25}$ m^{-3} (the gas pressure is about 5 atm), $sf = 10^{-3}$. The Hall parameter has been varied between 0 and 5. The wavenumber K_x has been taken equal to 628 m^{-1} corresponding to a wavelength $\lambda_x = 1$ cm. The grid distance d has been varied between 0.5 and 2 cm.

Under the conditions considered some of the modes mentioned in the previous section are damped. One of the two Zn-modes is always damped, say Zn(n, 1). The modes denoted by ZD_1, ZD_2 and ZD_3 are also always damped. The modes Zn(n, 2) can grow if i) the Hall parameter is large enough, and ii) the ratio d/λ_x is large enough.

The influence of the Hall parameter on the real part Rn(n, 2) is shown in Fig. 2 for $d/\lambda_x = 2$ and n = 1, 2, 3. The damping constant of non bounded plane waves, R_{pw}, is shown in the figure as well. The fact that the modes of higher n become unstable at higher Hall parameter values can be explained by noticing that n denotes the harmonic. When n = 2 the fluctuating electric field pattern is such that another equipotential surface is formed in the plasma at y = d/2. When n = 3 two equipotential surfaces are formed between the grids, and so on. With increasing number of equipotential surfaces the pattern departs ever more from a plane wave in the X-direction, and thus the growth rate decreases. (Rn > 0 and $R_{pw} < 0$

') The computations have been carried out on the EL X8 computer of the computer center of the Eindhoven University of Technology.

means an unstable wave in Fig. 2.) Furthermore it follows from n representing the harmonic that some combinations of n and d yield the same instability pattern. In terms of values of Z this means for instance that Zn(1, 2) at d = 1 cm is equal to Zn(2,2) at d = 2 cm, if the same plasma conditions and λ_x value are involved.

From a practical point of view it is important to know what the grid distance must be in order to stabilize waves of a certain wavelength λ_x. Therefore the critical Hall parameter has been determined as the value of $\omega\tau$ at the stability boundary of the most unstable mode (n = 1). The critical Hall parameter as a function of d/λ_x has been given in Fig. 3. This figure shows that in order to have an effect on $\omega\tau$ of more that 100%, d must be $\leq \lambda_x$.

Fig. 4 shows the amplitude profile of the electrical potential fluc-fluctuations at t = 0. According to the assumption made earlier this profile is consistent with a pure plane wave structure of the electron density perturbation, with the wave vector parallel to the grids. If $d/\lambda_x = 2$ the profile still approximates the plane wave structure in a considerable part of the plasma, indicating that the ultimate mode structure will not differ too much from a plane wave in the X-direction and will be highly unstable. Ir $d/\lambda_x = 1$ or 0.5 the departure from plane waves is much more significant, which shows that approximate plane waves in the X-direction will not match the boundary condition. This situation corresponds to higher values of $\omega\tau_{cr}$.

CONCLUSIONS

As could have been expected beforehand, the stabilization method described is a wavelength selective one. Waves with wavelengths much smaller than the grid distance will not be stabilized. This work shows more quantitively that the grid distance must be equal to or smaller than the wavelength considered, keeping in mind that the critical Hall parameter must be enlarged drastically in order to have a valuable improvement of the MHD generator performance [10]. Alternatively, having a fixed grid distance d the result of this work is that only waves with wavelengths equal to or larger than that distance will be stabilized appreciably. For the wavelengths smaller than d an additional stabilizing scheme has to be found.

It should be noted that the assumptions of ideal coupling between the plasma and the grids leads to an optimistic prediction about the stabilizing effect of the method described.

Furthermore it should be stressed that the analysis performed gives no information about any effect of a grid system in the second order stabilization regime.

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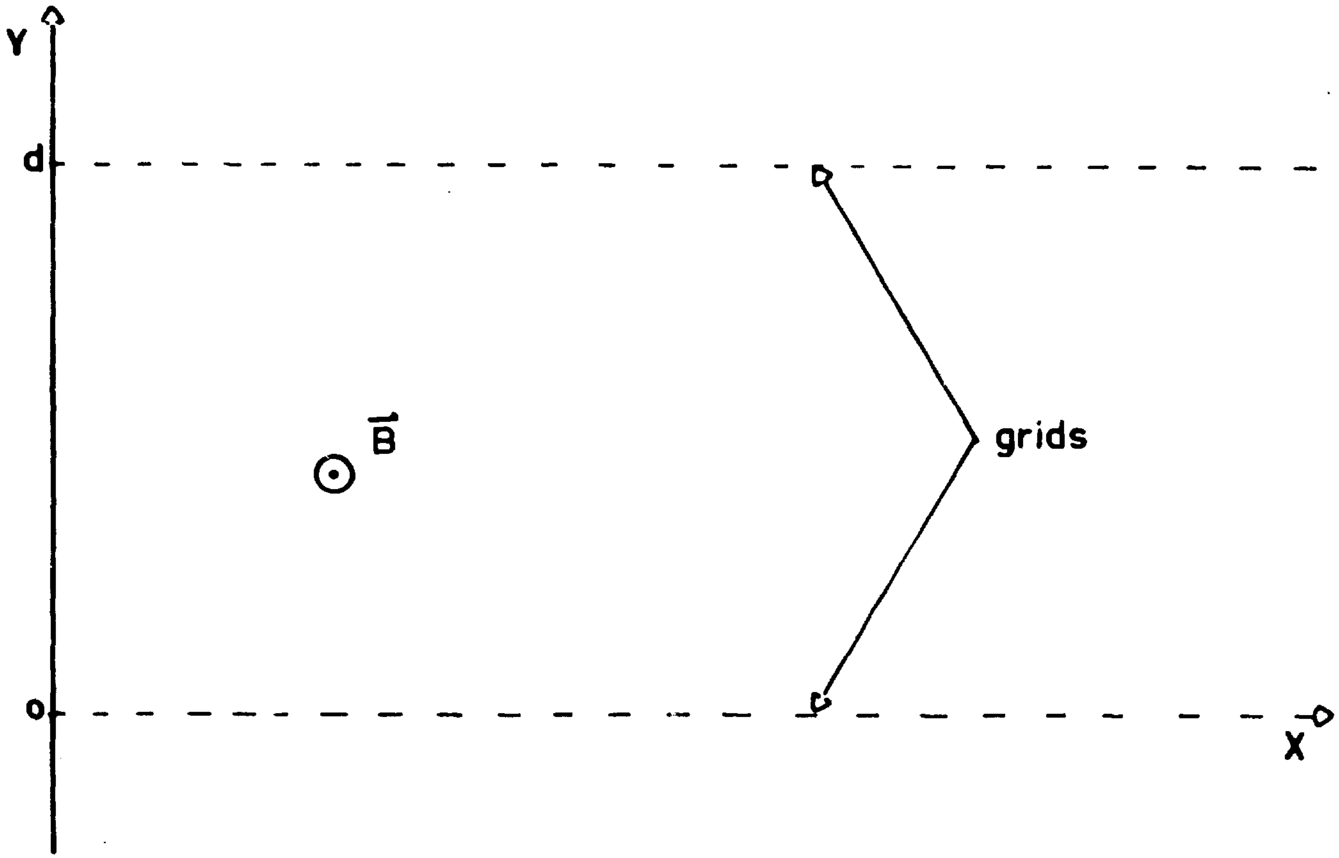

Fig. 1. Co-ordinate frame and position of the grids.

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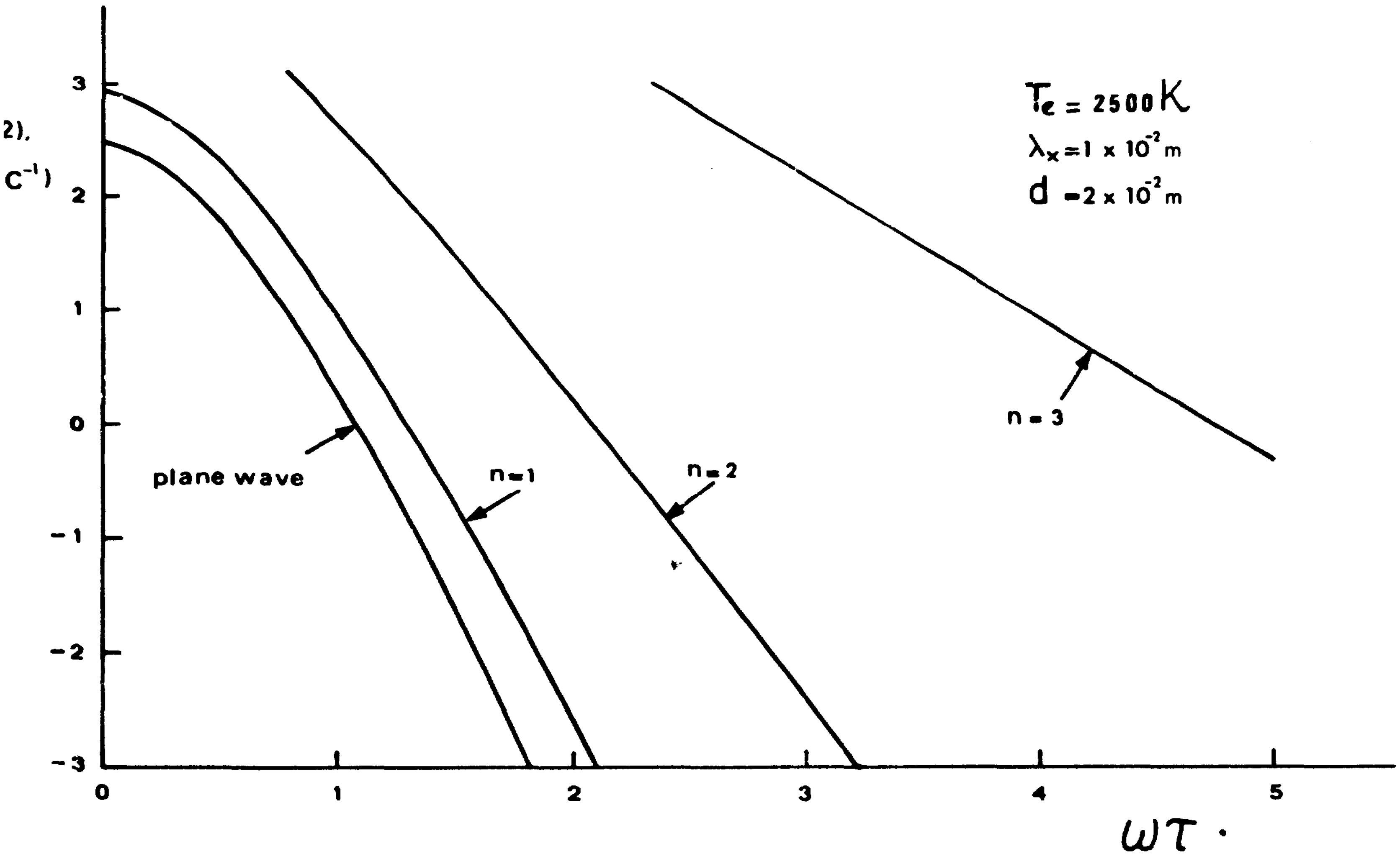

Fig. 2. Damping constant of plane waves R_{pw} and of the Zn(n, 2) mode as a function of the Hall parameter.

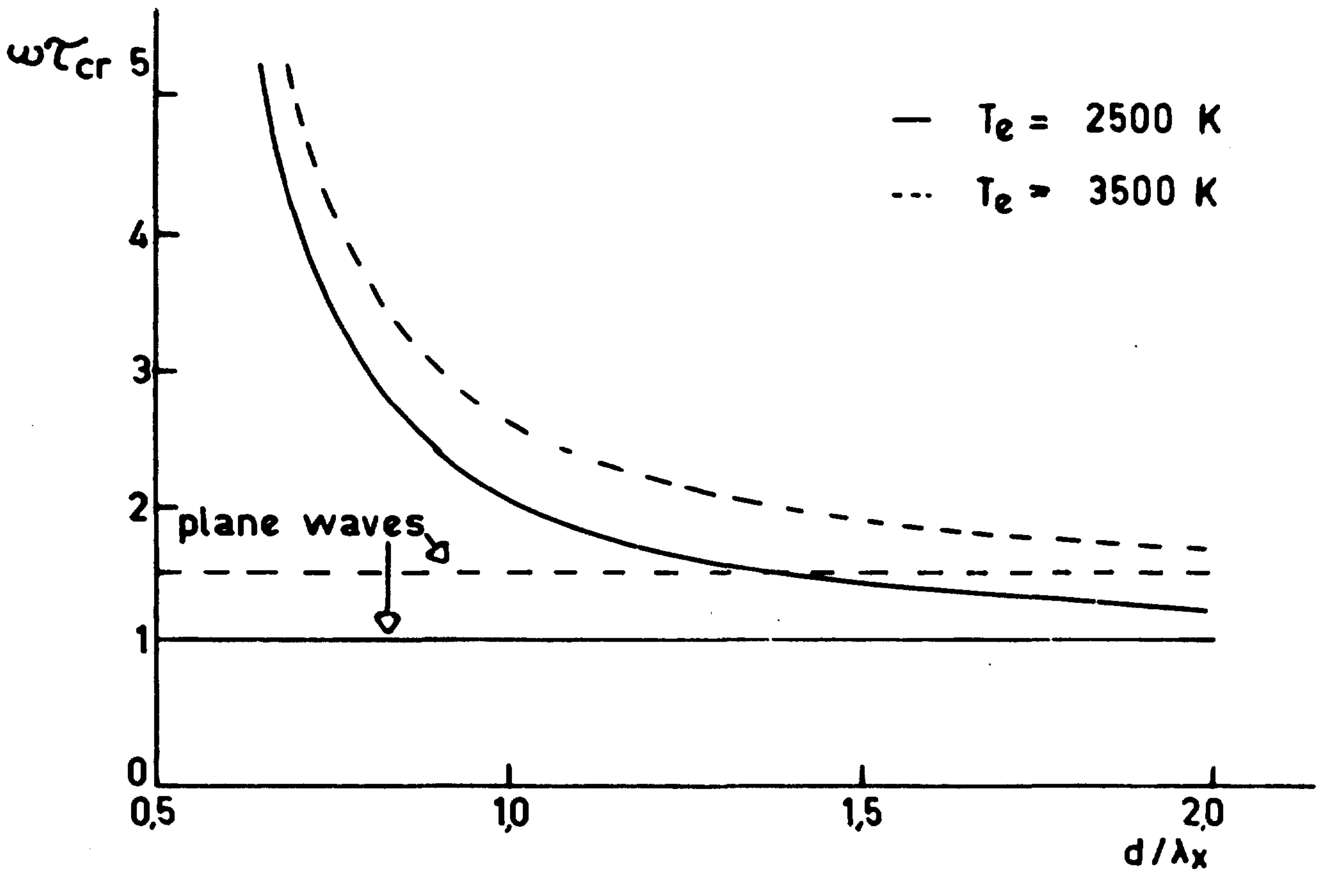

Fig. 3. Critical Hall parameter vs. The ratio of grid distance d and wavelength in the X-direction λ_x. Values of $\omega\tau_{cr}$ for plane waves are also indicated.

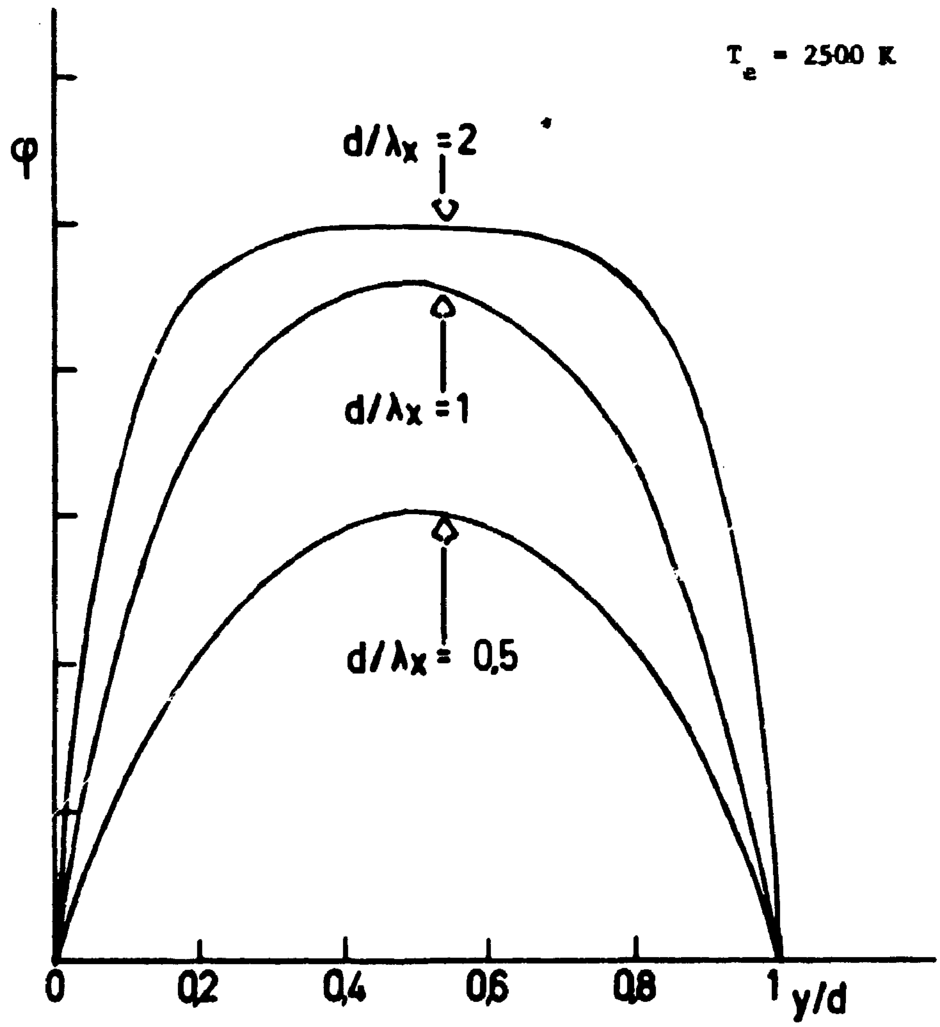

Fig. 4. Profile of the electrical potential amplitude at $t = 0$ for some values of the ratio of grid distance d and wavelength in the X-direction λ_x.

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