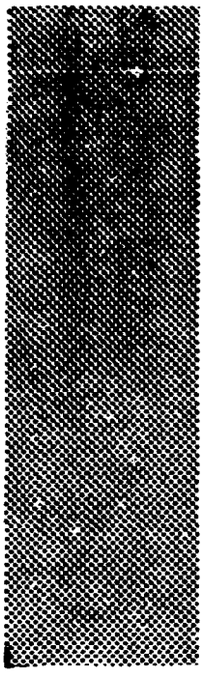
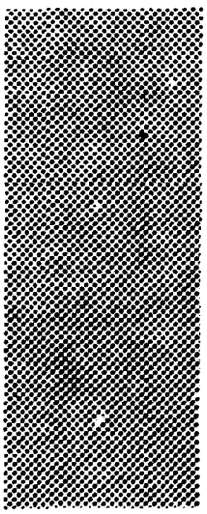
SSI: 1976-016



A COMPUTER PROGRAM FOR PROCESSING MICRODOSIMETRY SPECTRA

MATS BERGSTRÖM AND LENNART LINDBORG



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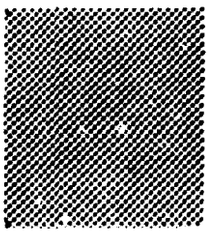

JUNE 1976

A COMPUTER PROGRAM FOR PROCESSING MICRODOSIMETRY SPECTRA

Mats Bergström*
Lennart Lindborg**

A small computer program for processing a microdosimetry single event energy deposition spectrum is presented. The program can perform smoothing of a spectrum and present a comparison of the smoothed and unsmoothed spectrum in order to detect distortions introduced by excessively rough smoothing. To increase the resolution of the spectrum and to reduce the influence of the uncertainty in the zero point setting of the multichannel analyzer, spectra are usually measured with different gain settings and are thereafter overlapped into one spectrum. The computer can perform such an overlapping and make a chi-square-analysis of the overlapping region. Such an analysis may reveal unsatisfactory experimental conditions, such as drifts in the gain between the two measurements, pile up effects or an unproper zero point setting of the multichannel analyzer. A method of dealing with the last mentioned problem is also presented.

The program was written for a Nuclear Data computer (ND 812) with a memory of 12 k but it should be easy to apply it to other computers.

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INTRODUCTION

There are several reasons for using a computer technique for the processing of microdosimetry single event energy deposition spectra. Such a spectrum for a low-LET radiation quality, may be described very roughly by $1/y^2$, where y is the deposited energy. The spectrum thus contains many events at small energies and very few at large energies (Fig. 1). In order to reduce the statistical fluctuations in the upper part of the spectrum, it usually has to be smoothed - a tedious task without a computer.

When the resolution of a spectrum is low, the number of pulses in the first few channels will be very large and mean values of the spectrum will become more sensitive to an erroneous zero setting of the multichannel analyzer. This problem is usually met by a new measurement with higher resolution but limited to the low-energy part of the spectrum. Overlapping of the two spectra gives a new spectrum which is less influenced by the uncertainty in the zero setting. The overlapping procedure is a time consuming procedure when made by hand, and is therefore suitable for computer processing.

A program suitable for a Nuclear Data computer (ND 812) with memory of 12 k is presented. The program has been used for the purposes mentioned above and for calculations of certain mean values of the measured spectrum.

ZERO SETTING OF A MULTICHANNEL ANALYZER

Determination of zero point error by overlapping microdosimetry spectra

When two spectra measured at different resolutions are overlapped, and when other errors than the zero point error are negligible, the result will be crossing curves. In the following it is assumed that at a correct zero-setting the position of the first few channels is in accordance with Fig. 2.

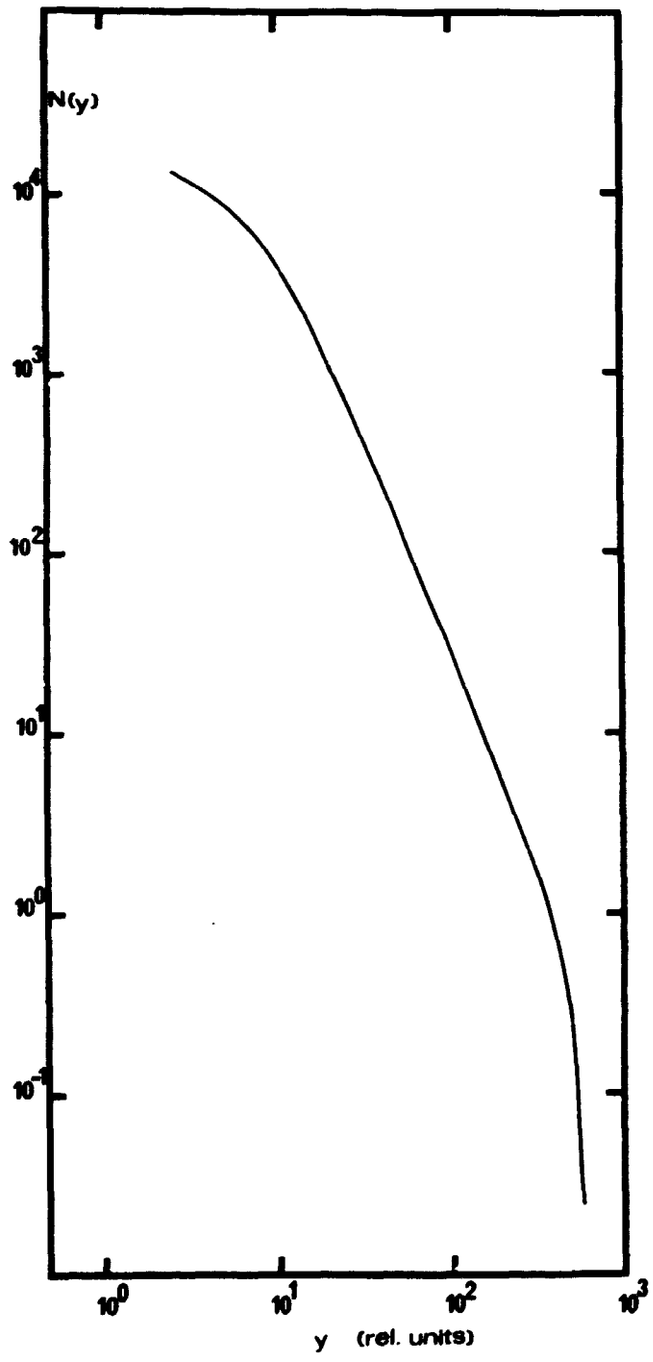


Fig. 1. A single event energy deposition spectrum measured with a spherical tissue-equivalent proportional chamber at 1 $\mu g/mm^2$. A beam of high energy electrons (15 MeV) was observed at a depth of 2 cm in a polystyren phantom.

The statistical analysis of the overlapping, performed by the program may reveal a crossing, but does not give the magnitude of the zero shift causing it. One method of determining a zero point shift is to plot a semilogarithmic graph for the lower part of each of the two spectra with channel number on the linear axis. One of these graphs is then laid on the other and adjusted vertically until the curves overlap in the extrapolation region (See p. 6). Then by moving one curve C channels sideways they can be made to overlap throughout the whole spectrum.

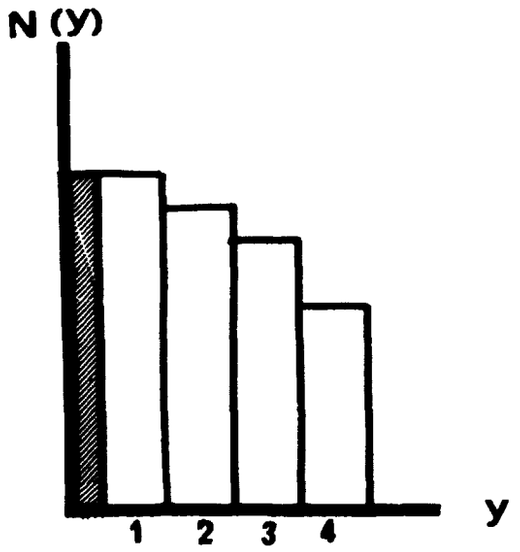


Fig. 2. The numbering of the first channels of the multichannel analyzer. (See also page 11 for further comments).

The error in the zero-setting (C′) is determined by:

$$C' = \frac{C \cdot MF}{MF-1}$$

For two spectra having the approximate form $N(y) = \frac{k}{y^2}$ and amplification factor (MF) 4 i.e. resolutions differing a factor 4, the formula becomes

$$C' = 16.7 \cdot \frac{D_{20} - D_{100}}{D_{100}}$$

if the overlapping region is taken to be the interval between channel number 20 and 100.

and

$$D_{20} = \frac{N_{20}}{N'_{20}}$$

$$D_{100} = \frac{N_{100}}{N'_{100}}$$

N_i = number of counts in channel i for the low resolution spectrum
N'_i = number of counts in channel i for the high resolution spectrum.

Effect on mean values of zero point error

The shape of a spectrum N(y) can be characterized by its mean values $\bar{y}_f$ and $\bar{y}_D$, which are defined on p. 11.

By manipulating the punched tape of measured spectra (spectra of high energy photons and electrons measured with resolutions of 18 eV/channel and 4.5 eV/channel), a shift in the zero setting equivalent to one channel was introduced.

For single spectra consisting of 10^5 counts, the percentage differences in mean values compared to the original spectra became:

$\Delta \bar{y}_F = 3.8 \pm 1.6$ ($\pm$ 2 SD)
$\Delta \bar{y}_D = 6.0 \pm 0.6$ ($\pm$ 2 SD)

If instead an overlapping technique with an amplification factor of 4 was used for the same spectra the differences were:

$\Delta \bar{y}_F = 1.6 \pm 1.2$ ($\pm$ 2 SD)
$\Delta \bar{y}_D = 2.9 \pm 1.3$ ($\pm$ 2 SD)

All mean values were increased when spectra were shifted downwards.

This shows that even a small error in setting the zero of the multichannel analyser introduce severe errors in the mean values of a low-LET microdosimeter single event spectrum and great care must be undertaken when calibrating the analyzer. The errors could be reduced by working with overlapped spectra.

WORKING PROCEDURE

Equipment

Microdosimetry spectra are stored in the multichannel analysing system Nuclear Data 4410 Single Parameter Data Acquisition and Display System with a resolution of 1024 channels. Spectra could advantageously be stored permanently on magnetic tape or punched paper tape.

For the calculations the computer in the system, ND 812, is used with the ORCAL-6 compiler. A minimum memory of 12 k is needed. The ORCAL compiler has been modified so as to store data in the same mode as the physics program (CARLÉN, 1974).

For in/output and extrapolation in the first channels ND 4410 Basic Physics Analyzer Program (41-1060) with overlays 41-1061 and 41-1062 are used.

Preliminaries

With the Basic Physics Analyzer Program the storage area of the multichannel analyzer is divided into groups of 1024 channels each. The spectrum with lower resolution is stored in group 5 and the spectrum with higher resolution is stored in group 4. When overlapping is not requested, the spectrum is stored in group 5.

In the overlapping technique the amplification factor between the two measurements is required by the program to be an even integer (e.g. 2, 4, 6 ....). The maximal number of pulses in a single channel is 83000 for the low resolution spectrum and 83000/(MF+1) for high resolution spectrum. (MF is the amplification factor between the resolutions).

The contents of the first channels are modified manually according to some mode of extrapolation (BRABY and ELLETT, 1971 and LINDBORG, 1975) using Basic Physics Analyzer Program. In case of overlapping only the high resolution spectrum needs to be modified.

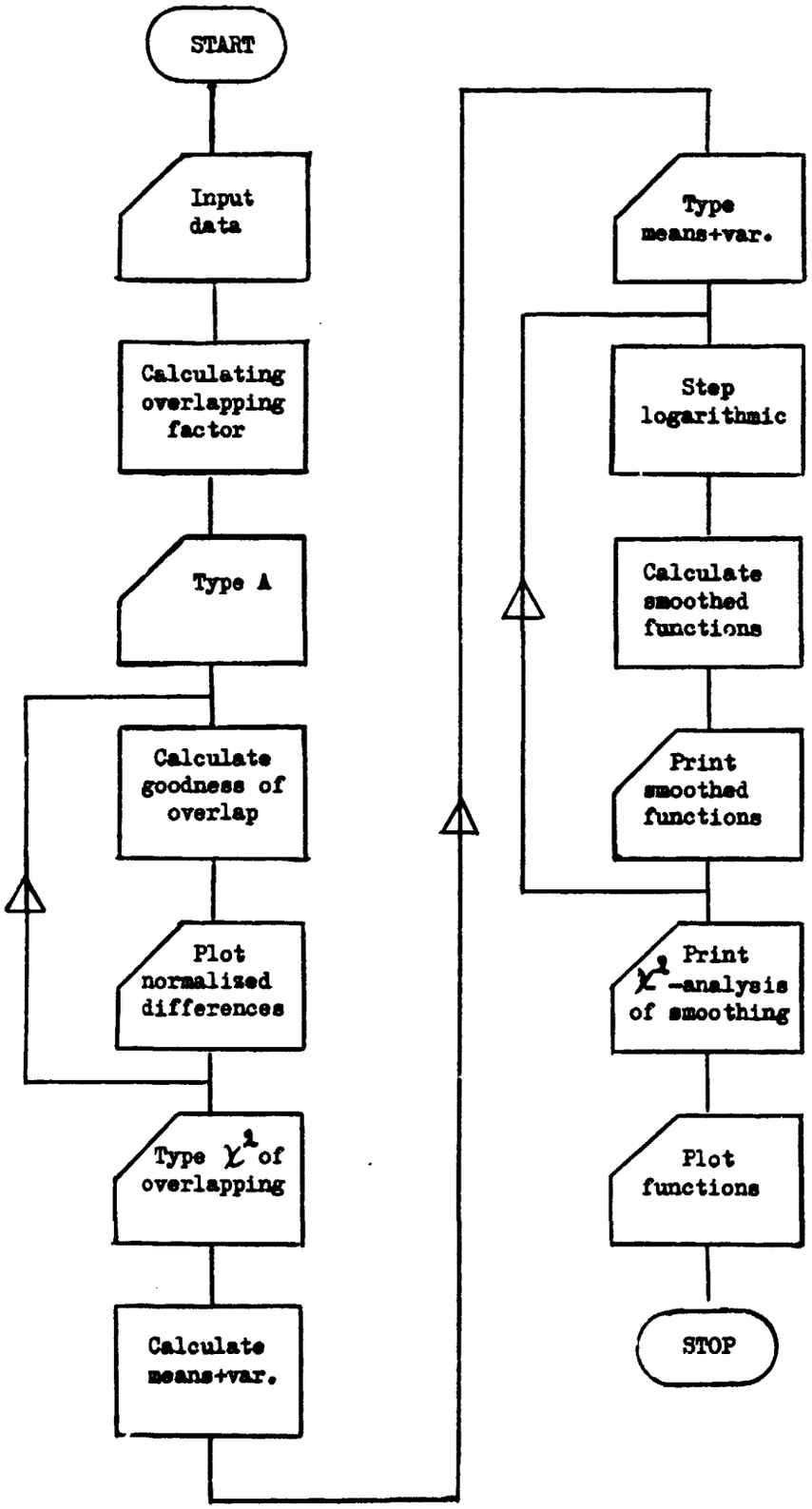


Fig. 3. Flow chart of the program HOPPLA.

The ORCAL-6 compiler and the calculation program (HOPPLA) are then fed into the computer and started. The flow chart for HOPPLA is shown in Fig. 3, and a print out of the complete program is given in appendix.

In the following text example instructions given by the user are underlined. Each instruction is terminated by pressing the RETURN button. When the program is started, questions are typed on the teletype (TTY).

Overlapping

When the TTY asks: AMPLIFICATION FACTOR: 4
respond with the amplification factor between the two spectra (an even integer).

In case of single spectrum respond with Ø
The TTY then asks: OVERLAPPING BETWEEN CHN: 10
and CHN: 50
Respond with the numbers of two channels in the spectrum of low resolution defining the region in which overlapping is to be performed (10 and 50 in the example above). The lower limit should not be set as to include the usually distorted lower energy deposition part of the spectrum. The upper limit should be less than 1024/MF, where MF is the amplification factor. If the overlapping region contains a channel with Ø content in the low resolution spectrum, the upper limit will automatically be set to the number of that channel. (The mathematics involved are described here and for continuation of the instructions see page 13).

A multiplication factor A is calculated representing the ratio between comparable channels with different resolution using the formula:

$$A = \frac{\sum_i \sqrt{x_i}}{\sum_i \frac{y_i}{\sqrt{x_i}}}$$

summed over all channels in the overlapping region.

x_i = content of channel i in low resolution spectrum;
y_i = sum of MF + 1 adjacent channels in high resolution spectrum.
The factor A is printed on TTY. Having calculated A the composite spectrum is put together from:

the factor A times the high resolution spectrum below the lower limit of the overlapping region
the unchanged low resolution spectrum above the upper limit
and a smooth change from high to low resolution spectrum in the overlapping region.

Due to factors like unlinearities in amplification, fault in setting the zero point of the multichannel analyzer, pile-up effect etc. the spectra may not overlap well.

In order to disclose poor overlapping, plotting of the differences between the two spectra in the overlapping region are performed together with a (modified) chi-square-analysis. Both moments are described below.

1. Plotting of differences.
The differences between comparable channels of the two spectra, normalized with expected statistical fluctuations are plotted on the TTY. A maximum of 50 channels spread evenly in the overlapping region are chosen and the corresponding differences are plotted together with indicators at each end showing deviations of $\pm$ 1 standard deviation. Good overlapping will be indicated by values evenly spread about the zero line with 2/3 of the points inside the limits $\pm$ 1 standard deviation (Fig. 4).

2. chi-square analysis.

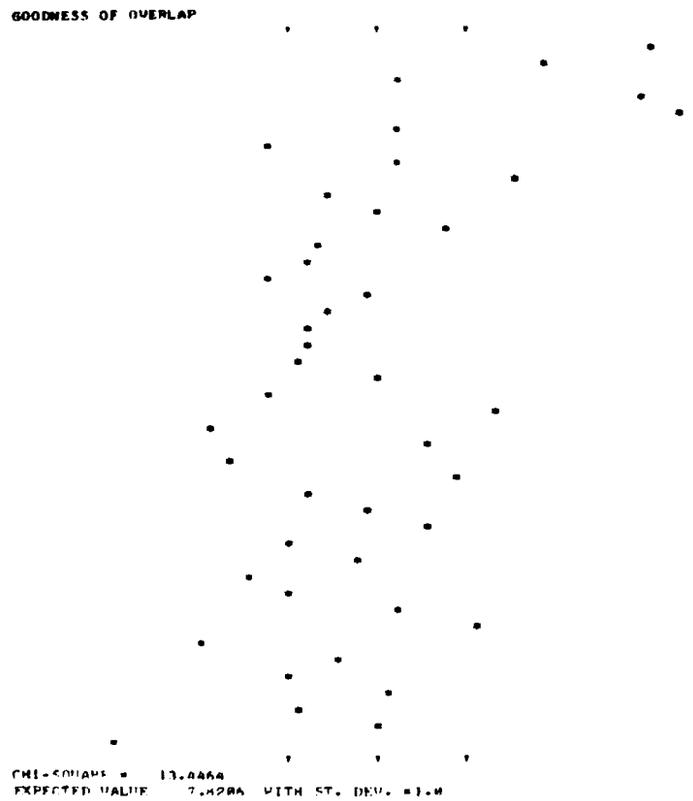
By forming the expressions

$$S = \sum_i \left[\frac{(x_i - A \cdot y_i)^2}{A^2 \cdot y_i} + \frac{(x_i - A \cdot y_i)^2}{x_i} \right]$$

summed over all channels in the overlapping region

and

$$S' = \frac{A \cdot S}{(1+A) \cdot \sqrt{A^2+1} \cdot \sqrt{2 \cdot F - 1}}$$



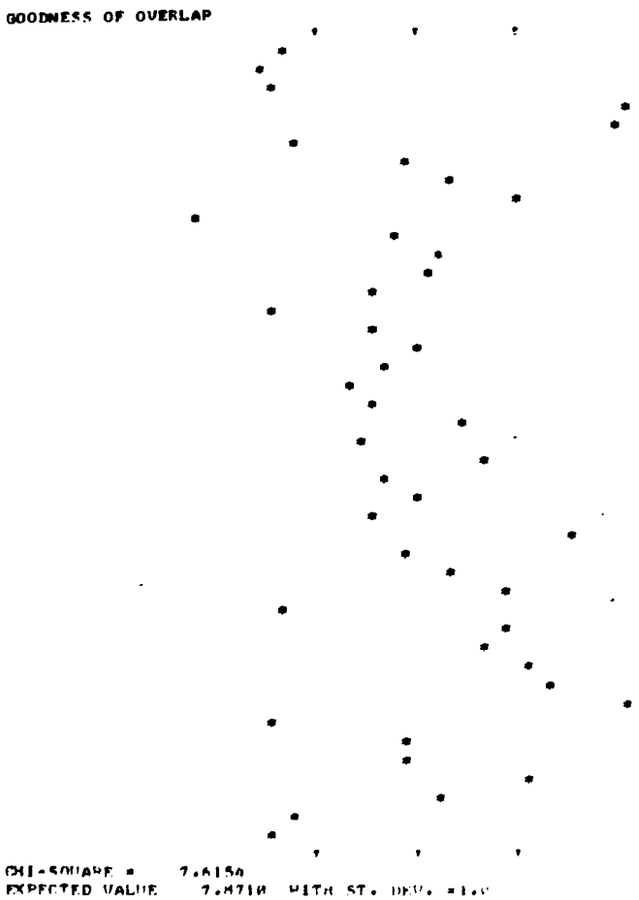


Fig. 4. Printout of goodness of overlapping. Lower figure shows good overlapping, upper figure shows bad overlapping due to a zero point error of 1 channel.

where F = the number of channels in the overlapping region,
it can be shown that S′ has the expectation value

$$\frac{(A+1)\cdot\sqrt{2F-1}}{2\cdot\sqrt{A^2+1}}$$

with standard deviation equal to one.

This is used as a modified chi-square analysis indicating whether the deviations between the two spectra in the overlapping region are within the limits of the expected statistical fluctuations or not. The TTY prints the value of S′ and its expectation value. (See Fig. 4).

Calculation of mean values and variance

Calculation of frequency mean value $\bar{y}_F$, dose mean value $\bar{y}_D$ and mean value due to $y^2 \cdot N(y)$ is calculated according to

$$\bar{y}_F = \frac{\sum_i i \cdot N(i)}{\sum_i N(i)}$$

$$\bar{y}_D = \frac{\sum_i i^2 \cdot N(i)}{\sum_i i \cdot N(i)}$$

$$\bar{y}_{y^2} = \frac{\sum_i i^3 \cdot N(i)}{\sum_i i^2 \cdot N(i)}$$

summed over all channels.
In order to simplify the calculations the channel number is taken to represent energy deposited. In order to do so the zero setting of the multichannel analyser must be in accordance with Fig. 2.

As can be seen from that figure account must be taken to the first half channel in the integrations. This is done in the program by assuming it to contain half of the events of channel one.

A relative variance ($V_{r,1}$) is calculated according to

$$V_{r,1} = \frac{\bar{y}_D}{\bar{y}_F} - 1$$

Mean values and variance are printed on TTY as are the partial sums (Fig. 5).

```
MEAN OF Y DUE TO N(Y)    12.6359
MEAN OF Y DUE TO Y*N(Y)   82.3721
MEAN OF Y DUE TO Y↑2*N(Y)  252.4210
REL. VARIANCE OF Y-DOSE    5.5189
 2741180.0 34637200 0.28531400E+10 0.72019100E+12
```

Fig. 5. Example of printout of mean values.

An estimation of the statistical uncertainty in the mean values as a function of the total pulse content has been given by BENGTSSON and LINDBORG, 1973.

Smoothing

The smoothing is performed on the unnormalized frequency function $i \cdot N(i)$. 66 channels are chosen evenly distributed on a logarithmic scale between channel 1/MF and an upper channel which depends on the choice of smoothing mode.

The method used is an extension of the method described by SAVITZKY and GOLAY (1969), in which smoothing is performed by convolution of the data points with a given set of integers. This was shown to be equivalent to a least squares fitting of data points to a cubic polynom. The convolution is performed using 5, 11, 17 or 25 consecutive channels.

In order to enable a more radical smoothing to be carried out, each data point in the convolution is represented by the mean of K adjacent channels (Fig. 6).

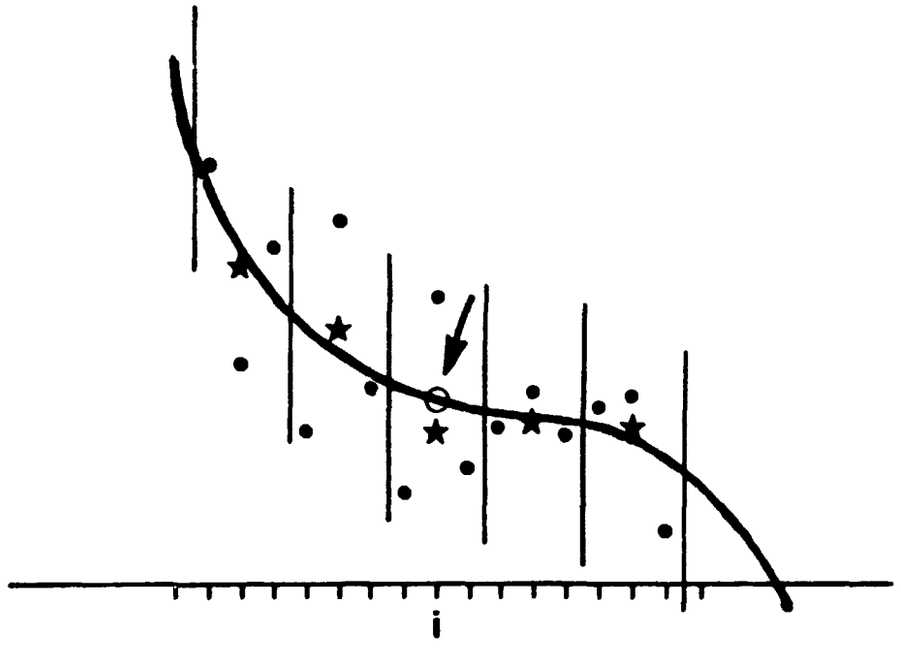


Fig. 6. Least squares fitting using 5 points, each the mean of 3 channels
• original values
* mean of 3 adjacent channels
o smoothed value

The mode of smoothing may be varied for intervals throughout the spectrum and is specified by giving a starting channel E in the low resolution spectrum for that interval, numbers of channels (K) and numbers of points (S) in the least squares fitting. Channel E must be in increasing order. The last interval is set by specifying channel Ø.

Given a set of smoothing modes, the computer makes equal steps in a logarithmic scale in the channel numbers, chooses the closest whole number channel (can be a fraction when smoothing in the part of the spectrum taken from high resolution spectrum) performs smoothing and prints out channel number, 100 times the normalized smoothed original

spectrum, the normalized smoothed frequency function, the normaliz.. smoothed dose function and the normalized smoothed $y^3 \cdot N(y)$ function (Fig. 7). When smoothing is performed below the lower limit of overlapping each channel in the low resolution spectrum is used.

Normalization is performed in such a way as to make the integrals of the functions plotted on a logarithmic channel number scale, equal to integrating a function of value 1 over 1 decade.

Y	100*F(Y)	Y*F(Y)	Y↑2*F(Y)	Y↑3*F(Y)
38.0000	0.2025	0.1770	0.5322	0.2455
43.0000	0.1550	0.1533	0.5215	0.2723
49.0000	0.1166	0.1314	0.5094	0.3030
56.0000	0.0882	0.1137	0.5037	0.3424
63.0000	0.0681	0.0986	0.4917	0.3760
71.0000	0.0518	0.0846	0.4753	0.4097
81.0000	0.0389	0.0724	0.4642	0.4565
92.0000	0.0288	0.0609	0.4430	0.4948
104.0000	0.0220	0.0526	0.4332	0.5469

Fig. 7. Printout of smoothed normalized functions.

Due to the wide range of values in the spectrum good judgement is needed when choosing modes of smoothing. Attempts to smooth too radically might destroy important characteristics of the spectrum and might also cause discontinuities in transition between smoothing intervals. This is due to the difficulty of fitting great parts of the spectrum to a cubic polynomial. A rule of thumb is to choose $E - \frac{1}{2} \cdot S \cdot K$ less than the channel containing the frequency function maximum, for channels above that maximum.

In order to provide a check on the effect of the smoothing, chi-square-analysis is performed in each smoothing interval and for the whole spectrum by comparing smoothed and unsmoothed spectra. The chi-square-values and the numbers of the channels used are printed, and with the help of a chi-square-table, it is possible to decide whether deviations between smoothed and unsmoothed spectra are in accordance with what is statistically expected. Inadequate smoothing will show up as an insignificant low chi-square-value and too drastic smoothing will show up as an insignificant high chi-square-value.(Fig. 8).

INTERVAL	5.0000	CHI-2=	6.7764	VALUES	4.0000
INTERVAL	6.0000	CHI-2=	18.5389	VALUES	5.0000

Fig. 8. Example of printout. Adequate smoothing in interval 5, but too strong smoothing in interval 6.

The appearance of frequency and dose functions gives important information about the measurement and therefore these normalized smoothed functions are plotted on the TTY in order to give immediate information, an example is shown in Fig. 9.

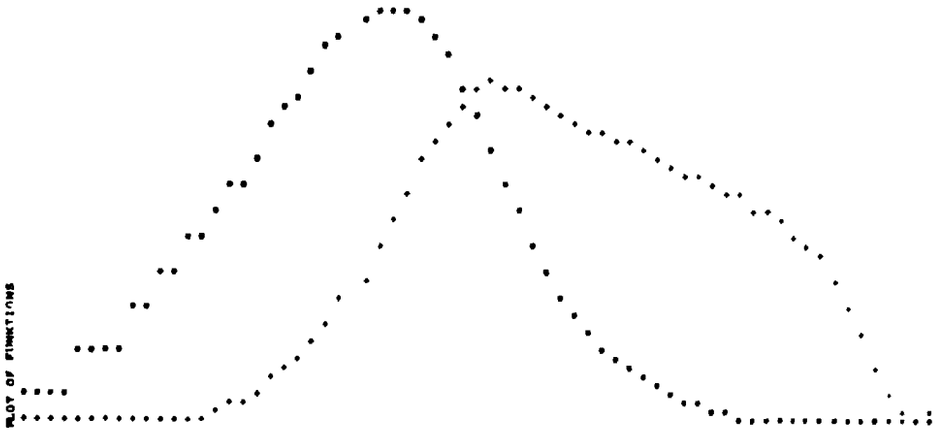


Fig. 9. Example of printout. Plotting of smoothed normalized frequency and dose functions.

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Braby L.A., Ellett W.H.: Ionization in microscopic volumes irradiated by energetic photons. Oregon state university, radiation center. Oct. 1971.

Carlén L.: Personal communication, 1971.

Lindborg L.: Microdosimetry measurements in beams of high energy photons and electrons: technique and results. Presented at fifth symp. on microdosimetry, Verbania-Pallanza (Italy), 1975.

Savitzky A., Golay M.: Smoothing and differentiation of data by simplified least squares procedures. Analytical chemistry, 36, 8, 1964.

APPENDIX

```
  .01 T "HOPPLA ";T !
01.02 A "AMPLIFICATION FACTOR"MF;T !
01.03 IF (MF)2.02,2.02,1.24
01.04 A "OVERLAPPING BETWEEN CHN"N1
01.05 A "  AND CHN "N2
01.06 GOTO 2.02
01.07 C OVERLAPPING FINISHED,CALCULATING MEAN
01.08 GOTO 2.58
01.09 C
01.10 FOR N=1,1,(N1-0.5)*MF-1;D 3
01.11 S T=A1*(MF+1)/MF;S S1=S1+T;S S2=S2+T;S S3=S3+T;S S4=S4+T
01.12 FOR N=N1,1,N2;D 4
01.13 FOR N=N2+1,1,1020;D 5
01.14 T !!"MEAN OF Y DUE TO N(Y)"S2/S1
01.15 T !"MEAN OF Y DUE TO Y*N(Y)"S3/S2
01.16 T !"MEAN OF Y DUE TO Y↑2*N(Y)"S4/S3
01.17 T !"REL. VARIANCE OF Y-DOSE"S3*S1/(S2↑2)-1
01.18 T !S1;T S2;T S3;T S4
01.19 C
01.20 C NUMBER OF EXTRA POINTS
01.21 S I=1;S U=0
01.22 IF (E(I)-N1)1.23,1.26,1.26
01.23 IF (S(I)*K(I)/2+1.5-U)1.25,1.25,1.24
01.24 S U=S(I)*K(I)/2+1.5;S V=U
01.25 S I=I+1;GOTO 1.22
01.26 IF (S(I)*K(I)/2+1.5-E(I)+N1-V)1.28,1.28,1.27
01.27 S V=S(I)*K(I)/2+1.5-E(I)+N1
01.28 IF (I-N3)1.29,1.30,1.30
01.29 S I=I+1;GOTO 1.26
01.30 S N4=1021.5-S(N3)*K(N3)/2+N1-V;S V1=U;S V2=V
01.31 IF ((V2-N1)*MF-V1)1.32,1.33,1.33
01.32 S V3=V1;GOTO 1.34
01.33 S V3=MF*(V2-N1);IF (MF)15.01,15.01,1.34
01.34 S A2=10238+2*N1;S A3=8190+2*(MF*N1+V3);S A4=FP(2,8192)*A1*(MF+1)+2.5
01.35 S I=0
01.36 S K=FP(2,A2+I);S K1=(FP(2,A2+I+2)-K)/MF
01.37 FOR N=0,1,MF-1;X FP(2,A3+2*N+MF*I,(K+N*K1)/100+0.5)
01.38 IF (I-2*V1/MF)1.39,1.40,1.40
01.39 S I=I+2;GOTO 1.36
01.40 FOR N=1,1,MF*N1-1;D 6.01
01.41 FOR N=0,1,V3-1;X FP(2,8190+2*V3-2*N,A4)
01.42 FOR N=0,1,N2-N1;D 6.02
01.43 S I=2
01.44 S Q1=0
01.45 FOR N=0,1,MF;S Q1=Q1+FP(2,A3-I*MF-MF+2*N)
01.46 X FP(2,A2-I,Q1/(MF+1)+0.5)
01.47 IF (I/2-V2)1.48,1.49,1.49
01.48 S I=I+2;GOTO 1.44
01.49 T !!"      Y      100*F(Y)   Y*F(Y)    Y↑2*F(Y)  Y↑3*F(Y)"
01.50 S L1=8190+2*V3;S I=1;S E(N3+1)=1024;S K3=MF;FOR N=1,1,N3;S C(N)=0
01.51 S K1=FLOG(1/MF);S K2=(FLOG(1000)-FLOG(1/MF))/66;S K2=1
01.52 S KF=A1*(MF+1);FOR N=1,1,N3;S D(N)=0
```

```
   53 S K=FITR(FEXP(K1)*K3+3.5)
01.54 IF (K-N1*K3)1.62,1.55,1.55
01.55 S L1=10238;S K3=1;S K=FITR(FEXP(K1)*K3+3.5)
01.56 IF (K-N2)1.57,1.58,1.58
01.57 S KF=((K-N1)↑2+A1*(N2-K)↑2)/(N2-N1)↑2;GOTO 1.60
01.58 S KF=1
01.59 IF (K-N4)1.60,1.70,1.70
01.60 IF (K-K3*E(I+1))1.61,1.80,1.80
01.61 GOTO 7.01
01.62 GOTO 8.01
01.63 S KR=K/K3;T !KR,S*100/(S1*KR),S*2.3/S1,S*2.3*KR/S2,S*2.3*KR↑2/S3
01.64 IF (S)1.65,1.65,1.66
01.65 S S=3.201
01.66 S C(I)=C(I)+(S/KR-FP(2,P+S(I)*K(I)-1))↑2/(S*KF/KR);S KP=KP+1
01.67 X FP(2,9202+4*KP,10↑5*S*2.3/S1)
01.68 S D(I)=D(I)+1;X FP(2,9202+4*KP,100*KR)
01.69 S K1=K1+K2;GOTO 1.53
01.70 C
01.71 C
01.72 S T1=0;S T2=0
01.73 FOR N=1,1,N3;D 14
01.74 T !!"OVERALL    CHI-2="T1;T "   VALUES"T2;T !!"PLOT OF FUNKTIONS"
01.75 GOTO 1.85
01.76 C
01.82 S I=I+1;GOTO 1.60
01.85 S KP=1
01.86 T !
01.87 FOR N=1,1,FP(2,9000+4*KP)/1502-2.5;T " "
01.88 T "*";T #
01.89 FOR N=1,1,FP(2,9202+4*KP)*S1*FP(2,9002+4*KP)/(152000*S2)-0.5;T " "
01.90 T "+"
01.91 C
01.92 IF (KP-66)1.93,1.94,1.94
01.93 S KP=KP+1;GOTO 1.86
01.94 QUIT

02.01 C INPUT SMOOTHING MODE
02.02 S I=2
02.03 T !;S I=I+1
02.04 A "FROM CHANNEL"E(I)
02.05 IF (E(I))2.10,2.10,2.06
02.06 A "MEAN OF"K(I)
02.07 A "SMOOTHING POINTS"S(I)
02.08 GOTO 2.03
02.09 C OVERLAPPING REGION
02.10 S J=2;S N3=I-1;IF (MF)13.02,13.22,2.11
02.11 S S1=0;S S2=2
02.12 S K=2;S S=0
02.13 S S=S+FP(2,8190+MF*(2*N1-1+J*2)+2*K)
02.14 IF (K-MF)2.15,2.16,2.16
02.15 S K=K+1;GOTO 2.13
02.16 IF (FP(2,10238+2*N1+2*J))2.17,2.22,2.17
02.17 S S1=S1+FSQT(FP(2,10238+2*N1+2*J))
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02.18 S S2=S2+S/FSQT(FP(2,10238+2*N1+2*J))
02.19 X FP(2,8193+MF*2+N1+2*J,100*S)
02.20 S J=J+1
02.21 IF (J-N2+N1)2.12,2.12,2.24
02.22 T !"OVERLAPPING FINISHED IN CHN"N1+J
02.23 S N2=N1+J
02.24 S A1=S1*(MF+1)/S2
02.25 T !"A1="A1
02.26 C
02.27 S A1=A1/(MF+1)
02.28 C GOODNESS OF OVERLAP
02.29 T !!"GOODNESS OF OVERLAP";T !
02.30 FOR N=1,.,28;T " "
02.31 T "!";FOR N=1,1,8;T " "
02.32 T "!";FOR N=1,1,8;T " "
02.33 T "!";S J=3;S S1=0;S N4=FITR((N2-N1)/50)+1
02.34 S X=FP(2,10238+2*N1+2*J)*100;S Y=FP(2,8192+MF*2*N1+2*J)
02.35 IF (J/N4-FITR(J/N4))2.39,2.36,2.39
02.36 S D=(X-A1*Y)*2.9/FSQT(X+A1*A1*Y)
02.37 T !;FOR N=1,1,D+37.5;T " "
02.38 T "*"
02.39 IF (Y)2.54,2.54,2.40
02.40 S S1=S1+((X-A1*Y)↑2)*(1/(A1*A1*Y)+1/X)
02.41 S E=(1-J/(N2-N1))*A1*Y+J*X/(N2-N1)
02.42 X FP(2,10238+2*N1+2*J,E)
02.43 IF (J-N2+N1)2.44,2.45,2.45
02.44 S J=J+1;GOTO 2.34
02.45 T !;FOR N=1,1,28;T " "
02.46 T "!";FOR N=1,1,8;T " "
02.47 T "!";FOR N=1,1,8;T " "
02.48 T "!";T !
02.49 S DS=S1*A1/(100*(A1+1)*FSQT((A1↑2+1)*(2*(N2-N1)-1)))
02.50 T "CHI-SQUARE ="DS
02.51 S S=0.5*(A1+1)*FSQT((2*(N2-N1)-1)/(A1↑2+1))
02.52 T !"EXPECTED VALUE"S
02.53 T "  WITH ST. DEV. =1.2";GOTO 1.07
02.54 S J=J+1
02.55 S X=X+FP(2,10238+2*N1+2*J)*100;S Y=Y+FP(2,8192+MF*2*N1+2*J)
02.56 GOTO 2.39
02.57 C
02.58 S K=8190+2*(N1-0.5)*MF; S L=N1-0.5-1/(4*MF)
02.59 S S1=FP(2,8192)/2+FP(2,K)/2
02.60 S S2=FP(2,8192)/(8*MF)+FP(2,K)*L/2
02.61 S S3=FP(2, 8192)/(2*(4*MF)↑2)+FP(2,K)*L↑2/2
02.62 S S4=FP(2,8192)/(2*(4*MF)↑3)+FP(2,K)*L↑3/2
02.63 GOTO 1.39

03.01 S X=FP(2,8190+2*N)
03.02 S S1=S1+X;S S2=S2+N*X/MF;S S3=S3+N*N*X/(MF*MF)
03.03 S S4=S4+(N/MF)↑3*X

04.01 S X=FP(2,10238+2*N)/100
04.02 S S1=S1+X;S S2=S2+N*X;S S3=S3+N*N*X;S S4=S4+N↑3*X
```

```
05.01 S X=FP(2,10238+2*N)
05.02 S S1=S1+X;S S2=S2+N*X;S S3=S3+N*N*X;S S4=S4+N↑3*X

06.01 S X=FP(2,A3-2*V3-2*N)*A1*(MF+1);X FP(2,A3-2*N,X+0.5)
06.02 S X=FP(2,A2+2*N)/100+0.5;X FP(2,A2+2*N,X)

07.01 S P=B1+2*K-(S(I)*K(I)-1);S J=0
07.02 S U1=0
07.03 FOR N=0,1,K(I)-1;D 7.08
07.04 S X(J)=U1
07.05 IF (J-S(I)+1)7.06,1.62,1.62
07.06 S J=J+1;GOTO 7.02
07.08 S U1=U1+FP(2,P+2*N+2*J*K(I))*(K-(K(I)*S(I)-1)/2+N+J*K(I))/K3

08.01 IF (S(I)-20)8.02,10.01,9.01
08.02 IF (S(I)-14)8.03,11.01,10.01
08.03 IF (S(I)-8)12.01,12.01,11.01

09.01 S S=-253*(X(0)+X(24))-138*(X(1)+X(23))
09.02 S S=S-33*(X(2)+X(22))+62*(X(3)+X(21))
09.03 S S=S+147*(X(4)+X(20))+222*(X(5)+X(19))
09.04 S S=S+287*(X(6)+X(18))+342*(X(7)+X(17))
09.05 S S=S+387*(X(8)+X(16))+422*(X(9)+X(15))
09.06 S S=S+447*(X(10)+X(14))+462*(X(11)+X(13))
09.07 S S=(S+467*X(12))/(5175*K(I))
09.08 GOTO 1.63

10.01 S S=-21*(X(0)+X(16))-6*(X(1)+X(15))
10.02 S S=S+7*(X(2)+X(14))+18*(X(3)+X(13))
10.03 S S=S+27*(X(4)+X(12))+34*(X(5)+X(11))
10.04 S S=S+39*(X(6)+X(10))+42*(X(7)+X(9))
10.05 S S=(S+43*X(8))/(323*K(I))
10.06 GOTO 1.63

11.01 S S=-36*(X(0)+X(10))+9*(X(1)+X(9))
11.02 S S=S+44*(X(2)+X(8))+69*(X(3)+X(7))
11.03 S S=S+84*(X(4)+X(6))+89*X(5)
11.04 S S=S/(429*K(I))
11.05 GOTO 1.63

12.01 S S=-3*(X(0)+X(4))+12*(X(1)+X(3))+17*X(2)
12.02 S S=S/(35*K(I))
12.03 GOTO 1.63

13.01 C MEAN OF SINGLE SPECTRA
13.02 T !"SINGLE SPECTRUM"
13.03 S A=FP(2,10240);S B=0.25
13.04 S S1=A/2;S S2=A*B/2;S S3=A*B↑2/2;S S4=A*B↑3/2
13.05 FOR N=1,1,1022;D 5
13.06 GOTO 1.14

14.01 T !!"INTERVAL "N;T "   CHI-2="C(N);T "   VALUES"D(N)
14.02 S T1=T1+C(N);S T2=T2+D(N)

15.01 FOR N=1,1,V2;S W(N)=FP(2,10238+2*N)
15.02 FOR N=1,1,V2; X FP(2,10240-2*N,W(N))
15.03 S MF=1;GOTO 1.49
```