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EXTENDED ABSTRACT

GENERALISED STRESSES YIELD SURFACES IN COMPUTERIZED STRUCTURAL ANALYSIS

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1 - INTRODUCTION

Yield surface such as Von Mises or Tresca conditions are well known. With addition of HILL'S principle and various hardening rules they are generally used in plastic analysis. This method seems conservative, but the analysis can be very expansive for many types of structures. This is due to the gread number of integrations to be performed. In some cases the resulting cost in term of computer run time is nearly prohibitive.

Conventional yield conditions are ponctual conditions expressed in conventional stresses, so integration is necessary over the volume of the whole structure. In many types of structures it is possible to use generalised stresses (e.g membrane stresses, bending-stresses, torsional stresses) to define a yield surface. Plastic analysis can be achieved by using the HILL'S principle and a hardening rule.

If such a method seems a rough approximation, it must be pointed out that the plastic response of materials is quite variable and that real materials are not homogeneous, as supposed in conventional methods. Moreover, with generalised stresses yield surface, it is possible to use test results on samples (of plates, or pipes) in order to have better approximation of the real behaviour of structure elements.

Two particuliar applications of the generalised stresses yield surface method are given : the first for plastic shell analysis and the other for plastic piping analysis.

2 - PLASTIC ANALYSIS OF SHELLS

For shell analysis the generalised stresses are membrane forces N_{ij} and bending (with torsional) moments M_{ij}. If the space of stress has six dimensions, there is only one condition for one point of the middle surface and integration accross the thickness is not requiered. The yield condition can be written

$$F\ (N_{ij},\ M_{ij},\ \alpha\ ,\ \beta\ \ldots) = 0$$

where α, β, .. are implicit variables taking hardening into account.

Vector notation can be used, with "reduced stresses" n_i et m_i

$$n_i = n_{kl} = \frac{N_{kl}}{t} \qquad m_i = m_{kl} = \frac{M_{kl}}{t^2/4}$$

A this point, some complementary assumptions can be made. First, to introduce the yield stress $\sigma^* = \sigma_y$, then to suppose that the yield condition is homogeneous of first order with respect to $\frac{n_i}{\sigma^*}$ and $\frac{m_i}{\sigma^*}$. Then the load surface may be written in the following form :

$$\phi\,(n_i,\, m_i,\, \alpha\, \ldots) = \sigma^*$$

and application of the HILL'S principle gives the increment of generalised strains, that is to say membrane elongation e_i and curvature χ_i. Obtained expressions are very like usual expressions for conventional strain increments

$$\delta_{ei} = \frac{\partial\phi}{\partial n_i}\,\delta\lambda \qquad \delta\,\chi_i = \frac{\partial\phi}{\partial m_i}\,\delta\lambda$$

where the parameter $\delta\lambda$ is related to the plastic strain increment $\delta\epsilon^*$ in the uniaxial tensile test.

If the "generalised Von Mises condition" is choosen as a yield condition, this condition can be written with only three "equivalant stresses", corresponding to three second order invariants. One invariant for the deviatoric tensor of membrane stresses, one for the deviatoric tensor of bending moments and one combining the two deviatoric tensors

$$\left\{\begin{aligned} n &= \sqrt{\left(n_{kl} - \delta_{kl}\, n_{ij}\right)\left(n_{kl} - \delta_{kl}\, n_{ij}\right)} \\ m &= \sqrt{\left(m_{kl} - \delta_{kl}\, m_{ij}\right)\left(m_{kl} - \delta_{kl}\, m_{ij}\right)} \\ p &= \sqrt{\left(n_{kl} - \delta_{kl}\, n_{ij}\right)\left(m_{kl} - \delta_{kl}\, m_{ij}\right)} \end{aligned}\right.$$

With such an assumption, the increments of generalised strains are given by relations very similar to Prandtl-Reuss formulas.

From a general point of view, the equations used for computations are very similar to conventional ones, but no integration accross the thickness is requiered. As a result, a substantial computation time reduction is obtained. Obviously some adjustements to the hardening rules are necessary. Experiments or local computation may be used for that purpose, but practical experience shows that results ingood agreement with experiments on structures, are obtained with simple rules. (Tests show that local hardening rules are often different accross the plate thickness, so integration validity with one hardening rule is also questionable).

The "generalised stresses yield surface model" had been used in the CEA-SEMT structural analysis system for more than two years and good experience had been gained. Some examples are given.

3 - PIPING ANALYSIS

When considering plasticity effects, piping systems analysis is practically impossible with conventional methods. Tridimensionnal integration is awfully expensive compared with the unidimensionnal integration (along the axis line) used in elastic analysis. So, it is desirable to find a plastic analysis method, with only "one dimension" integration. Generalised stresses yield condition is very suitable for such a purpose.

For piping, generalised stresses are :

- bending moment, or bending stress σ_b
- torsional moment, or torsional stress σ_t
- hoop stress (due to internal pressure) σ_p
- uniform tension stress σ_n

There is only a set of stresses (generalised stresses components) for a cross section and no integration over the cross section area is required.

Generalised strains corresponding to the generalised stresses are :

- axis curvature χ
- torsion (by unit of length) θ
- uniform strains

The generalised yield condition is given by :

$$F\ (\sigma_b,\ \sigma_t,\ \sigma_p,\ \sigma_n,\ \alpha,\ \beta\ \ldots) = \sigma^*$$

and the generalised strain increments can be determined with the help of HILL'S principle.

A very simple assumption about this relation is the "generalised Von Mises rule" :

$$\sqrt{\alpha_b^2 \sigma_b^2 + \alpha_t^2 \sigma_t^2 + \alpha_p^2 \sigma_p^2} = \sigma^*$$

where the α coefficients can be adjusted.

In fact, the most difficult problem is the choice of α_b values for curved segments of pipe (pipe bends or elbows). A rough, but useful approximation is :

$$\alpha_b = \frac{\pi}{3} \left(\frac{tR}{r^2} \right)^{-2/3} .$$

(t thickness, R bend radius, r pipe radius). Better approximations could be given by shell analysis of the curved segment.

Such yield surface models are used in the TEDEL modulus of the CEA-SEMT system for elastic-plastic piping analysis, but work is yet in progress on the determination of better rules for hardening effects and curved segments behaviour.

4 - CONCLUSION

Plastic behaviour of materials is very complex, the yield surface is not always well known, especially when yielding has occured. There is noticeable dispersion of mechanical properties for a given material, even in a given structure. As a result a perfect analysis is not possible and some assumptions must be made about physical laws (yield condition, hardening rule, etc..). and about conservative plastic response of the material.

For engineering needs, it is also necessary to limit computationnal costs. For these reason our opinion is that "Generalised stresses yield surface" models are valuable tools for plastic analysis of structures like shells and piping.

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