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DIFFUSION IN TOKAMAKS WITH IMPURITIES IN THE PFIRSCH SCHLUTER REGIME

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ABSTRACT.

We consider the transport in Tokamaks due to collisions between impurity ions in the Pfirsch-Schlüter regime and light ions. The diffusion coefficients for the particles have a Pfirsch-Schluter like structure for all the regimes of light ions. They experience however a transition from a set of values (A) to a different set of values (B), when the quantity $v_{thl}^2 \tau_{Dl} \tau_{Ml} (qR)^{-2}$ varies through a value of the order unity. (v_{thl} and τ_{Dl}, τ_{Ml} are the thermal velocity and the relaxation times for deflection and Maxwell'sation of light ions). The values (A) (applicable in particular when the light ions are in the Banana and Plateau regimes) allow in principle plasma purification by the ion temperature gradient.

INTRODUCTION

The interest given to the behaviour of impurities in Tokamaks has led to study again [1, 2, 3] the Pfirsch-Schlüter (P.S.) regime of diffusion. [4]. This is due to the fact that in practical conditions the impurities are in the P.S. regime rather than in the Plateau or Banana regime. The papers [1] and [2, 3] have given different results concerning the particle transport coefficients when both impurities and light ions are in the P.S. regime. In particular the sign of the temperature gradient in the expression of the particle flux is different, leading to different conclusions concerning the possibility of preventing the accumulation of impurities by this gradient. In this paper, we first show that the two results correspond to two different regimes inside the P.S. regime for both light species (ions l) and impurity species (ions I). In the two regimes we have $v_{th\alpha}\tau_{D\alpha}(qR)^{-1} < 1$, where $v_{th\alpha}$ and $\tau_{D\alpha}$ are respectively the thermal velocity and the relaxation time for orthogonal deflection of ions $\alpha = l$, I. The time τ_{Dl} for light ions is due to (l - l) collisions as well as to (l - I) collisions. We will assume that

$$\frac{m_I}{m_l} \gg 1 \quad , \quad z = \frac{n_I Z_I^2}{n_l Z_l^2} \ll \frac{m_I}{m_l}$$

where n_α , m_α and Z_α are the density, the mass and the charge of ions $\alpha = l$, I. In these conditions the distribution function of light ions approaches a Maxwellian in a time τ_{Ml} for which $(l$ - I$)$ collisions are much less effective than $(l$ - $l)$ collisions. We have : $\tau_{Dl} \sim \tau_{Ml}(1 + O(z))$. For $z \gg 1$, it is possible that, while having $v_{thI}\tau_{DI}(qR)^{-1} > 1$, we have $v_{thI}^2 \tau_{DI} \tau_{Ml}(qR)^{-2} > 1$. We will show that the particle transport coefficients reported in [1] and [2], [3] correspond to the assumptions $v_{thl}^{2}\tau_{Dl}\tau_{Ml}(qR)^{-2} < 1$ and > 1, respectively, and are in fact the asymptotic forms of a general expression which will be calculated inside the P.S. regime for light ions by an appropriate kinetic treatment. We will show also that the values of these coefficients obtained for $v_{thl}^{2}\tau_{Dl}\tau_{Ml}(qR)^{-2} > 1$ apply when the light ions are in the Plateau and the Banana regimes.

II. BASIS OF THE KINETIC CALCULATION IN THE P.S. REGIME FOR BOTH SPECIES.

The starting point is, as usual, the Fokker Planck equation for species α written in the drift approximation [5]

$$\frac{d\, f(\vec{x},\vec{v},\alpha)}{dt} = \frac{-\sin\theta}{2\,\omega_{c\alpha} R_0}\, v^2(1+p^2)\,\frac{\partial f_0(r,v,\alpha)}{\partial r}$$

$$+ \frac{p\,v}{R_0\, q}\,\frac{\partial f_1(r,\theta,v,p,\alpha)}{\partial\theta}$$

$$- \frac{r\,v\,(1-p^2)}{2\,q\,R_0\,R}\,\sin\theta\,\frac{\partial f_1(r,\theta,v,p,\alpha)}{\partial p} \qquad (1)$$

$$= \sum_{\alpha'} C\left[f(\vec{x},\vec{v},\alpha),\; f(\vec{x},\vec{v},\alpha')\right]$$

where θ, φ, r, R ($r \ll R$) and R_o are defined in fig. 1, $v = |\vec{V}|$, $p = \frac{V_{\parallel}}{v}$ ($V_{\parallel}$ is the component of the velocity $\vec{V}$ along the field $\vec{B}$), $q = r B_\varphi R_o^{-1} B_\theta^{-1}$ ($q \gg 1$), and $\omega_{c\alpha} = Z_\alpha e B_o (m_\alpha c)^{-1}$ is the cyclotron frequency on the magnetic axis. We assume that the cross sections of the magnetic surfaces are circles centered on the magnetic axis. We use the frame of reference rotating around the major axis where the electrostatic field is zero. The distribution function $f(\vec{x}, \vec{V}, \alpha)$ for species α has been written

$$f(\vec{x}, \vec{V}, \alpha) = f_o(r, v, \alpha) + f_1(r, \theta, v, p, \alpha) \qquad (2)$$

where $f_o(r, v, \alpha)$ is Maxwellian on the magnetic surface r at a temperature $T(r, \alpha)$ (we assume $T(r, 1) \approx T(r, I)$)

$$f_o(r, v, \alpha) = \left(\frac{m_\alpha}{2\pi T(r, \alpha)}\right)^{3/2} n(r, \alpha) \exp \frac{-m_\alpha v^2}{2 T(r, \alpha)} \qquad (3)$$

In the P.S. regime for both species, the collisions reduces quickly strong anisotropies in velocity space and we can assume that the perturbation f_1 has the form

$$f_1(r, \theta, p, v, \alpha) = \left[X_o(r, \theta, v, \alpha) + p\, X_1(r, \theta, v, \alpha)\right] f_o(r, v, \alpha)$$

It is easy to show that X_o (which represents the isotropic part of f_1) varies along θ as $\sin\theta$, and that X_1, apart from a term which represents a rotation of the whole plasma along the flux lines at an angular frequency Ω around the major axis, varies along θ as $\cos\theta$. We write accordingly, on the magnetic surface r

$$f_1(r,\theta,v,p,\alpha) = f_0(r,v,\alpha)\left[Y_0(v,\alpha)\sin\theta + \frac{pv}{v_{th\alpha}} Y_1(v,\alpha)\cos\theta + 2vp\frac{\Omega R_0^2}{v_{th\alpha}^2 R}\right] \quad (4)$$

$$v_{th\alpha} = \left(\frac{2T(r,\alpha)}{m_\alpha}\right)^{1/2}$$

The transport coefficients may be obtained by two equivalent expressions [5] which involve either the even or the odd part in p of the perturbation f_1 . If F_α and $F_{E\alpha}$ are the averaged radial particle flux and the energy flux for species α , we have, when f_1 has the form (4)

$$F_{\{E\}\alpha} = -\frac{4n}{3R_0\omega_{c\alpha}}\int_0^\infty v^4 dv\, Y_{0\alpha}\, f_{0\alpha}\left\{\frac{m_\alpha v^2}{2}\right\} \quad (5)$$

or

$$F_{\{E\}\alpha} = \frac{-2n q\, m_\alpha c}{Z_\alpha e\, B_0}\int_0^\infty v^3 dv\int_{-1}^{+1} p\, dp\, \left\{\frac{m_\alpha v^2}{2}\right\}$$

$$\sum_{\alpha'}\left(C\left[f_{0\alpha}\frac{pv}{v_{th\alpha}}Y_{1\alpha}, f_{0\alpha'}\right] + C\left[f_{0\alpha}, \frac{pv}{v_{th\alpha'}}Y_{1\alpha'} f_{0\alpha'}\right]\right) \quad (6)$$

$$\left(f_{0\alpha} = f_0(r,v,\alpha), \dots\right)$$

As stated above, the regimes we have in view correspond to various values of the relaxation times $\tau_{D l}$ and $\tau_{M l}$. If $\tau_{M l}$ is small enough, the isotropic part of the distribution function $f(\vec{x}, \vec{v}, \alpha)$ must be Maxwellian at each point of any magnetic surface, and $Y_0(v, l)$ is then of the form

$$Y_0(v, l) = \alpha + \beta \frac{v^2}{v_{th l}^2} \tag{7}$$

corresponding to local values of the density and the temperature for the species l

$$n(r, \theta) = n(r, l)\left(1 + \left(\alpha + \frac{3}{2}\beta\right) \sin\theta\right)$$

$$T(r, \theta) = T(r, l)\left(1 + \beta \sin\theta\right)$$

In that case one may use the BRAGINSKII coefficients /6/ to calculate the fluxes of particle and energy for ions l in terms of the parallel gradients of $n(r, \theta)$ and $T(r, \theta)$ i.e. in terms of thé constants α and β. By expressing that the divergence of these fluxes cancells out the divergence of the corresponding transverse fluxes associated with the field curvature, one may calculate α and β. The radial fluxes F_l and $F_{E l}$ are then obtained from (5) and are those reported in /1/. On the other hand if the time $\tau_{M l}$ is long enough, $Y_0(v, l)$ may depart from representing a Maxwellian perturbation. In that case the terms of $\sum_{\alpha'} C[f_l, f_{\alpha'}]$ which are even in p are of the order of $f_{0l}\, Y_0(v, l)\, \tau_{Ml}^{-1}$. By considering the terms of (1) for species l which are odd in p we obtain $v_{th1}\, Y_0(v,1)(qR)^{-1} \sim Y_1(v,1)\tau_{D1}^{-1}$ and therefore

Even terms of $\sum_{\alpha'} C[f_1, f_{\alpha'}] \sim f_{01}\,\frac{Y_0(v,1)}{\tau_{M1}} \sim f_{\bullet 1}\,\frac{Y_1(v,1)}{\tau_{D1}\,\tau_{M1}}\,\frac{qR}{v_{th1}}$

It is then readily verified that if $v_{th1}^2\,\tau_{D1}\,\tau_{M1}\,(qR)^{-2} \gg 1$ cancelling the even terms of (1) provides the equation

$$Y_1(v,1) = a + b\,\frac{v^2}{v_{th1}^2}$$

obviously incompatible with (7). Again the constants a and b may be determined by expressing the continuity of the local fluxes of particle and energy for species 1 (these constants will be given by (23)). The fluxes F_1 and F_{E1} may then be calculated using (6) and are those reported in [2] and [3].

We will study in the next section the P.S. regime for species 1, I, for arbitrary values of τ_{M1}, by a variationnal method, taking for $Y_0(v,1)$ and $Y_1(v,1)$ the following trial functions, (which are hoped to be general enough)

$$\left.\begin{aligned} Y_0(v,1) &= \alpha + \beta\,\frac{v^2}{v_{th1}^2} + \gamma\left(\frac{v^2}{v_{th1}^2}\right)^{\nu} \\ Y_1(v,1) &= a + b\,\frac{v^2}{v_{th1}^2} + c\left(\frac{v^2}{v_{th1}^2}\right)^{2} \end{aligned}\right\} \quad (8a)$$

where α, ..., a, ... and ν are adjustable constants. On the other hand we will admit that the impurity population remains Maxwellian at the temperature $T(r, I)$. We take accordingly

$$\left.\begin{aligned} Y_0(v, I) &= \alpha_I \\ Y_1(v, I) &= a_I \end{aligned}\right\} \quad (8b)$$

It may be shown that, with $m_I \gg m_l$, this is equivalent to neglect $\partial T(r,I)/\partial r$. (In fact the gradient $\partial T(r,I)/\partial r$ is involved in the expression of the fluxes F_l and F_{El} through quantities of the form $\partial T(r,l)/T(r,l)\partial r - 0(Z_l/Z_I)\partial T(r,I)/T(r,I)\partial r$ and, if $Z_I \gg Z_l$, plays a minor role when calculating these fluxes. The energy flux F_{EI} associated with ions I is small compared to F_{El} except if $n_I/n_l > (m_l/m_I)^{1/2}$. In that case F_{EI} is mainly due to (I-I) collisions and may be calculated by standard P.S. formula involving ions I only.) Replacing the functions $Y_0(v, \alpha)$ by their expressions (8) in (5), we obtain

$$\left.\begin{aligned} F_l &= - \frac{n_l \, m_l \, c \, v_{thl}^2}{Z_l \, e \, B_0 \, R_0} \, \frac{2}{3 \pi^{1/2}} \left[\alpha \, \Gamma\left(\frac{5}{2}\right) + \beta \, \Gamma\left(\frac{7}{2}\right) + \gamma \, \Gamma\left(\nu + \frac{5}{2}\right)\right] \\ F_I &= - \frac{n_l \, m_l \, c \, v_{thl}^2}{Z_I \, e \, B_0 \, R_0} \, \frac{2}{3 \pi^{1/2}} \, \alpha_I \, \Gamma\left(\frac{5}{2}\right) \\ F_{El} &= - \frac{n_l \, m_l^2 \, c \, v_{thl}^4}{Z_l \, e \, B_0 \, R_0} \, \frac{1}{3 \pi^{1/2}} \left[\alpha \, \Gamma\left(\frac{7}{2}\right) + \beta \, \Gamma\left(\frac{9}{2}\right) + \gamma \, \Gamma\left(\nu + \frac{7}{2}\right)\right] \end{aligned}\right\} \quad (9)$$

$(n_l = n(r, l))$

III. VARIATIONAL CALCULATION OF THE TRANSPORT COEFFICIENTS IN THE P.S. REGIME FOR BOTH SPECIES.

An extremum principle equivalent to the Fokker Planck equation is necessarily based on the well known symmetry properties of the collision operator and therefore is necessarily closely related to the principle of minimum entropy production which has been used, e.g. by ROSENBLUTH et al. [5] However this principle does not involve the operator $\frac{d}{dt} f(\vec{x}, \vec{v}, \alpha)$ and does not allow the determination of the function $f(\vec{x}, \vec{v}, \alpha)$ without imposing other constraints to this function . To obtain a variational principle equivalent to the Fokker-Planck equation, we put first

$$f(\vec{x}, \vec{v}, \alpha) = A_\alpha \exp - \frac{m_\alpha v^2/2 - \mathcal{U}(\vec{x}, \vec{v}, \alpha)}{T} \qquad (10)$$

where A_α and T are constants.

The set of functions $\mathcal{U}(\vec{x}, \vec{v}, \alpha)$ represents the departure of the plasma from thermodynamical equilibrium. We may write the collision operator (in the Landau form [6]) as

$$\sum_{\alpha'} C\left[f(\vec{x}, \vec{v}, \alpha), f(\vec{x}, \vec{v}, \alpha')\right] = \mathcal{D}_f\left[\mathcal{U}(\vec{x}, \vec{v}, \alpha)\right]$$

where $\mathcal{D}_f[\,\cdot\,]$ is a linear operator acting on the functions $\mathcal{V}(\vec{x}, \vec{v}, \alpha)$ of the variables $\vec{x}$, $\vec{v}$ and indice α , which is specified for each function $\mathcal{V}(\vec{x}, \vec{v}, \alpha)$ by

$$\mathcal{D}_f \left[\mathcal{V}(\vec{x}, \vec{v}, \alpha) \right] = \frac{1}{T} \sum_{\alpha'} \iiint \frac{\partial}{\partial m_\alpha v_r} \Big\{ A_{rs} (\vec{x}, \vec{v}, \vec{v}', \alpha, \alpha')$$

$$f(\vec{x}, \vec{v}, \alpha) \; f(\vec{x}, \vec{v}', \alpha') \left[\frac{\partial \mathcal{V}(\vec{x}, \vec{v}, \alpha)}{\partial m_\alpha v_s} - \frac{\partial \mathcal{V}(\vec{x}, \vec{v}', \alpha')}{\partial m_{\alpha'} v'_s} \right] \Big\} \, d_3 \vec{v}' \tag{11}$$

$$A_{rs}(\vec{x}, \vec{v}, \vec{v}', \alpha, \alpha') = 2\pi e^4 \, z_\alpha^2 \, z_{\alpha'}^2 \, \log \Lambda \; \frac{|w|^2 \delta_{rs} - w_r w_s}{|w|^3}$$

$$\vec{W} = \vec{v} - \vec{v}' \quad ; \quad r, s = 1, 2, 3.$$

The operator $\mathcal{D}_f$ is symmetric in the sense that

$$\sum_\alpha \iiint\iiint d_3 \vec{x} \, d_3 \vec{v} \quad \mathcal{D}_f \left[\mathcal{V}(\vec{x}, \vec{v}, \alpha) \right] \mathcal{W} (\vec{x}, \vec{v}, \alpha) =$$

$$\sum_\alpha \iiint\iiint d_3 \vec{x} \, d_3 \vec{v} \quad \mathcal{V}(\vec{x}, \vec{v}, \alpha) \quad \mathcal{D}_f \left[\mathcal{W}(\vec{x}, \vec{v}, \alpha) \right]$$

Because of this symmetry, if we define the two functionals of the three functions $\mathcal{U}(\vec{x}, \vec{v}, \alpha)$, $\frac{d f(\vec{x}, \vec{v}, \alpha)}{dt}$ and $f(\vec{x}, \vec{v}, \alpha)$, considered as independent

$$\dot{\Sigma} \left(\frac{df}{dt} , \mathcal{U} \right) = \frac{2}{T} \sum_\alpha \int \cdots \frac{d f(\vec{x}, \vec{v}, \alpha)}{dt} \, \mathcal{U}(\vec{x}, \vec{v}, \alpha) \; d_3 x \, d_3 v \tag{12}$$

$$\dot{S} (f, \mathcal{U}) = - \frac{1}{T} \sum_\alpha \int \cdots \mathcal{D}_f \left[\mathcal{U} (\vec{x}, \vec{v}, \alpha) \right] \mathcal{U}(\vec{x}, \vec{v}, \alpha) \, d_3 x \, d_3 v.$$

the Fokker Planck equation is obviously equivalent to the principle that the functional of $\frac{df}{dt}$, f, $\mathcal{U}$

$$\dot{\Sigma}\left(\frac{df}{dt}, \mathcal{U}\right) + \dot{S}(f, \mathcal{U})$$

is extremum for all variations of the function $\mathcal{U}$. Actually the value of $\dot{S}(f, \mathcal{U})$, when f and $\mathcal{U}$ describe effectively populations of particles (and in particular verify (10)), is the entropy production in the plasma. It is easily deduced from (11) that we have (for any f and $\mathcal{U}$)

$$\dot{S}(f, \mathcal{U}) = \frac{1}{2T^2} \sum_{\alpha,\alpha'} \int \dots A_{rs}(\vec{x}, \vec{v}, \vec{v}', \alpha, \alpha')$$
$$f(\vec{x}, \vec{v}, \alpha)\, f(\vec{x}, \vec{v}', \alpha') \left(\frac{\partial \mathcal{U}(\vec{x}, \vec{v}, \alpha)}{m_\alpha \partial v_r} - \frac{\partial \mathcal{U}(\vec{x}, \vec{v}', \alpha')}{m_{\alpha'} \partial v'_r}\right)$$
$$\left(\frac{\partial \mathcal{U}(\vec{x}, \vec{v}, \alpha)}{m_\alpha \partial v_s} - \frac{\partial \mathcal{U}(\vec{x}, \vec{v}', \alpha')}{m_{\alpha'} \partial v'_s}\right) d_3\vec{x}\, d_3\vec{v}\, d_3\vec{v}' \qquad (13)$$

If the plasma is not far from thermodynamical equilibrium, the function $\mathcal{U}(\vec{x}, \vec{v}, \alpha)$ is small and may be calculated at first order with respect to the external constraints replacing f by $A_\alpha \exp(-m_\alpha v^2/2T)$ in $\dot{S}(f, \mathcal{U})$, which is a quadratic form in $\mathcal{U}$. In the functional $\dot{\Sigma}\left(\frac{df}{dt}, \mathcal{U}\right)$, which is a linear form in $\mathcal{U}$, the function $\frac{df}{dt}$ must be replaced by its expression at first order in $\mathcal{U}$ (i.e. by $A_\alpha \exp(-m_\alpha v^2/2T) \frac{d\mathcal{U}(\vec{x}, \vec{v}, \alpha)}{T\, dt}$). If one substitutes for $\mathcal{U}$ a trial function $\mathcal{U}(\vec{x}, \vec{v}, \alpha, P)$ depending on a set of adjustable parameters P, these parameters must be varied, when calculating the variation of

$\dot{\Sigma}(\frac{df}{dt}, \mathcal{U}) + \dot{S}(f,\mathcal{U})$, only if they appear through the function $\mathcal{U}$ in the explicit form of the functional $\dot{\Sigma}(\frac{df}{dt}, \mathcal{U}) + \dot{S}(f, \mathcal{U})$. It is conveni nt to under_line the parameters P when they appear in this way, i.e. to write

$$\dot{\Sigma}\left(\frac{df}{dt}, \underline{\mathcal{U}}\right) + \dot{S}(f, \underline{\mathcal{U}}) = \dot{\Sigma}\left(\frac{df(\vec{x},\vec{v},\alpha,P)}{dt}, \mathcal{U}(\vec{x},\vec{v},\alpha,\underline{P})\right) + \dot{S}\left(f(x,v,\alpha,P), \mathcal{U}(x,v,\alpha,\underline{P})\right)$$

The parameters $P = \underline{P}$ may then be determined by expressing that $\dot{\Sigma} + \dot{S}$ is extremum for all the variations of $\underline{P}$.

For the present problem, we may restrict the integration in space which appears in the expressions (12) and (13) of the functionals $\dot{\Sigma}(\frac{df}{dt}, \mathcal{U})$ and $\dot{S}(f, \mathcal{U})$ to the domain between the magnetic surfaces r and $r + \delta r$. We may take

$$T = T(r, l) = T(r, I) = \frac{m_\alpha V_{th\alpha}^2}{2}$$

$$A_\alpha = \frac{n(r,\alpha)}{\pi^{3/2} \, V_{th\alpha}^3} = \frac{n_\alpha}{\pi^{3/2} \, v_{th\alpha}^3}$$

By comparing (2) (3) (4) with (10) we obtain

$$\frac{\mathcal{U}(\vec{x},\vec{v},\alpha)}{T} = \log\left(\frac{n(r,\alpha)}{n_\alpha}\right) + \frac{1}{2} m v^2 \left(\frac{1}{T} - \frac{1}{T(r,\alpha)}\right) - \frac{3}{2}\log\left(\frac{T(r,\alpha)}{T}\right) + Y_0(v,\alpha)\sin\theta + Y_1(v,\alpha)\frac{v_p}{v_{th\alpha}}\cos\theta + 2v\Omega p\frac{R_0^2}{R}$$

The function $\frac{df(\vec{x},\vec{v},\alpha)}{dt}$ may be calculated from (1), (3) and (4)

$$\frac{df(\vec{x},\vec{v},\alpha)}{dt\;\; F_0(r,v,\alpha)} = \left[-\frac{\partial n(r,\alpha)}{n_\alpha\,\partial r} + \frac{\partial T(r,\alpha)}{T\,\partial r}\left(\frac{v^2}{v_{th\alpha}^2}-\frac{3}{2}\right)\right]$$

$$\frac{v^2(1+p^2)}{2\,\omega_{c\alpha}\,R_0}\;\sin\theta \;+\left[Y_0(v,\alpha)\cos\theta - Y_1(v,\alpha)\frac{v\,p\sin\theta}{v_{th\alpha}}\right]\frac{v\,p}{R_0\,q}$$

$$-\;\frac{\Omega}{v_{th\alpha}^2}\;\frac{r}{R_0\;q}\;\;v^2(1-3p^2)\sin\theta$$

Substituting the expressions of $\frac{df}{dt}$ and $\mathcal{U}$ in (12) gives the functional $\dot{\Sigma}\left(\frac{df}{dt},\mathcal{U}\right)$ as a functional of $Y_0(v,\alpha)$, $Y_1(v,\alpha)$ $\underline{Y_0}(v,\alpha)$, $\underline{Y_1}(v,\alpha)$ and a function of Ω and $\underline{\Omega}$. It appears in fact that $\dot{\Sigma}\left(\frac{df}{dt},\mathcal{U}\right)$ is independent of Ω and $\underline{\Omega}$. Replacing the functions $Y_0(v,\alpha)$, $Y_1(v,\alpha)$ by their trial expressions specified by (8), we obtain

$$\dot{\Sigma}\left(\frac{df}{dt},\underline{\mathcal{U}}\right) = \dot{\Sigma}_{ps}\left(Y_0(v,\alpha),\, Y_1(v,\alpha),\, \underline{Y_0}(v,\alpha),\, \underline{Y_1}(v,\alpha)\right)$$

$$= \frac{(2\pi)^3}{3\,\pi^{3/2}}\;n_i\,v_{thi}\,q^{-1}\,r\,\delta r$$

$$\Big\{\left(\rho_i-\frac{3}{2}\rho_{\varepsilon i}\right)\left(\underline{\alpha}\,\Gamma\!\left(\tfrac{5}{2}\right)+\underline{\beta}\,\Gamma\!\left(\tfrac{7}{2}\right)+\underline{\gamma}\,\Gamma\!\left(\underline{\nu}+\tfrac{5}{2}\right)\right)+\rho_{\varepsilon i}\left(\underline{\alpha}\,\Gamma\!\left(\tfrac{7}{2}\right)+\underline{\beta}\,\Gamma\!\left(\tfrac{9}{2}\right)+\underline{\gamma}\,\Gamma\!\left(\underline{\nu}+\tfrac{7}{2}\right)\right)$$

$$-\underline{\alpha}\left(a\,\Gamma\!\left(\tfrac{5}{2}\right)+b\,\Gamma\!\left(\tfrac{7}{2}\right)+c\,\Gamma\!\left(\tfrac{9}{2}\right)\right)-\underline{\beta}\left(a\,\Gamma\!\left(\tfrac{7}{2}\right)+b\,\Gamma\!\left(\tfrac{9}{2}\right)+c\,\Gamma\!\left(\tfrac{11}{2}\right)\right)$$

$$-\underline{\gamma}\left(a\,\Gamma\!\left(\underline{\nu}+\tfrac{5}{2}\right)+b\,\Gamma\!\left(\underline{\nu}+\tfrac{7}{2}\right)+c\,\Gamma\!\left(\underline{\nu}+\tfrac{9}{2}\right)\right)$$

$$+\alpha\left(\underline{a}\,\Gamma\!\left(\tfrac{5}{2}\right)+\underline{b}\,\Gamma\!\left(\tfrac{7}{2}\right)+\underline{c}\,\Gamma\!\left(\tfrac{9}{2}\right)\right)+\beta\left(\underline{a}\,\Gamma\!\left(\tfrac{7}{2}\right)+\underline{b}\,\Gamma\!\left(\tfrac{9}{2}\right)+\underline{c}\,\Gamma\!\left(\tfrac{11}{2}\right)\right)$$

$$+\gamma\left(\underline{a}\,\Gamma\!\left(\nu+\tfrac{5}{2}\right)+\underline{b}\,\Gamma\!\left(\nu+\tfrac{7}{2}\right)+\underline{c}\,\Gamma\!\left(\nu+\tfrac{9}{2}\right)\right)$$

$$+\left(\rho_I\,\underline{\alpha_I}-\underline{\alpha_I}\,a_I+\alpha_I\,\underline{a_I}\right)\Gamma\!\left(\tfrac{5}{2}\right)\frac{n_I\,v_{thI}}{n_i\,v_{thi}}\Big\} \qquad (14)$$

where we have underlined the adjustable parameters $\nu, \alpha, \beta, \gamma$, a, b, c, α_I, a_I according to the convention stated above, and we have put

$$\rho_\alpha = -\frac{2q\,v_{th\alpha}}{\omega_{c\alpha}}\frac{\partial n(r,\alpha)}{n_\alpha \partial r}\ , \ \rho_{t l} = -\frac{2q\,v_{thl}}{\omega_{cl}}\frac{\partial T(r,l)}{T\partial r} \tag{15}$$

The entropy production $\dot{S}(f, U)$ is independent of the angular velocity Ω and of the parameters α, α_I which simply reflects a change of density of species l, I. It does not depend also on β which reflects a change of temperature of the species l, because we neglect the energy exchange between particles l and I. Also it depends on the parameters a and a_I, which reflects a shift of the Maxwellian for light ions and impurity ions proportional to $a\,v_{thl}/2$ and $a_I v_{thI}/2$ through the difference $a' = a - \frac{v_{thI}}{v_{thl}} a_I$

Actually we have

$$\dot{S}(f, U) = \dot{S}(f_0, U) = \dot{S}(f_0, U_0') + \dot{S}(f_0, U_1')$$

with

$$\frac{U_0'(\vec{x}, \vec{v}\,l)}{T} = \gamma\left(\frac{v^2}{v_{thl}^2}\right)^{\nu}\cos\theta$$

$$\frac{U_1'(\vec{x}, \vec{v}, l)}{T} = \left(a' + b\frac{v^2}{v_{thl}^2} + c\frac{v^2}{v_{thl}^2}\right)\frac{v\,p\cos\theta}{v_{thl}}$$

$$U_0'(\vec{x}, \vec{v}, I) = U_1'(x, v, I) = 0\,.$$

We then obtain from the explicit expression (13) of $\overset{\circ}{S}$, after some algebra

$$\dot{S}(f, \underline{u}) = \dot{S}_{ps}\left(\underline{Y_0(v, \alpha)}, \ \underline{Y_1(v, \alpha)}\right)$$

$$= \frac{(2\pi)^3}{3\pi^{3/2}} \, n_i \, v_{thi} \, q^{-1} \, r \, \delta r$$

$$\Big(\mathcal{L}(\underline{\nu})\,\underline{\gamma}^2 + \mathcal{A}\,\underline{a}'^2 + \mathcal{B}\,\underline{b}^2 + \mathcal{C}\,\underline{c}^2 + 2\mathcal{A}'\,\underline{a}'\,\underline{b}$$

$$+ 2\mathcal{B}'\,\underline{b}\,\underline{c} + 2\mathcal{C}'\,\underline{a}'\,\underline{c}\Big) \qquad (16)$$

where

$$\left.\begin{aligned}
&\mathcal{L}(\nu) = g \, I(\nu) \\
&I(\nu) = \frac{g}{2^{3/2}} \, \frac{\nu^2}{2^{2\nu}} \, \Gamma\!\left(2\nu + \tfrac{3}{2}\right) \int_{-1}^{+1} dp \int_0^{\frac{\pi}{2}} d\theta \\
&\qquad (1 - p^2) \cos\theta \sin^4\theta \left[(1 - p \sin 2\theta)^{\nu-1} - (1 + p \sin 2\theta)^{\nu-1}\right]^2 \\
&\mathcal{A} = \mathcal{A}' = \frac{\mathcal{C}'}{2} = g \cdot \pi^{1/2} \, \frac{3}{8} \, z \\
&\mathcal{B} = g \, \pi^{1/2} \, 3 \left(\frac{1}{4 \times 2^{1/2}} + \frac{1}{4} z\right) \\
&\mathcal{C} = g \, \pi^{1/2} \, 3 \left(\frac{157}{16 \times 2^{1/2}} + 3 z\right) \\
&\mathcal{B}' = g \, \pi^{1/2} \, 3 \left(\frac{22}{16 \times 2^{1/2}} + \frac{3}{4} z\right)
\end{aligned}\right\} (17)$$

and

$$g = \frac{q R}{2 \tau_{lI} v_{thl}} \; ; \; \frac{1}{\tau_{lI}} = \frac{4 (2\pi)^{1/2} 2^{3/2} n_I Z_l^2 Z_I^2 \, e^4 \, \mathrm{Log}\, \Lambda}{3 \, m_l^2 \, v_{thl}^3}$$

Expressing that $\delta(\dot{\Sigma}_{PS} + \dot{S}_{PS}) = 0$ for a variation $\delta \underline{\alpha_I}$ and for variations $\delta \underline{a}$ and $\delta \underline{a_I}$ such that $\delta \underline{a}' = \delta \underline{a} - \delta \underline{a}_I v_{thI}/v_{thl} = 0$, we obtain

$$\rho_I - \alpha_I = 0 \tag{18}$$

$$\alpha\, \Gamma\left(\tfrac{5}{2}\right) + \beta\, \Gamma\left(\tfrac{7}{2}\right) + \gamma\, \Gamma\left(\nu + \tfrac{5}{2}\right) + \alpha_I\, \Gamma\left(\tfrac{5}{2}\right) \frac{n_I}{n_l} = 0 \tag{19}$$

In view of (9), eq.(19) implies the ambipolarity of the particles fluxes : $Z_l \Gamma_l + Z_I \Gamma_I = 0$. Taking also into account (18), $\dot{\Sigma}_{PS}$ may be rewritten so that

$$\dot{\Sigma}_{PS} + \dot{S}_{PS} = \frac{(2\pi)^3}{3\, \pi^{3/2}} \, n_l v_{thl} \, q^{-1} \, r \, \delta r$$

$$\Big\{ \left(\rho_l - \tfrac{3}{2} \rho_{El} - \rho_I \frac{v_{thI}}{v_{thl}} - a' \right) \left(\underline{\alpha}\, \Gamma\left(\tfrac{5}{2}\right) + \underline{\beta}\, \Gamma\left(\tfrac{7}{2}\right) + \underline{\gamma}\, \Gamma\left(\underline{\nu} + \tfrac{5}{2}\right) \right)$$

$$+ \left(\rho_{El} - b\right) \left(\underline{\alpha}\, \Gamma\left(\tfrac{7}{2}\right) + \underline{\beta}\, \Gamma\left(\tfrac{9}{2}\right) + \underline{\gamma}\, \Gamma\left(\underline{\nu} + \tfrac{7}{2}\right) \right)$$

$$- c \left(\underline{\alpha}\, \Gamma\left(\tfrac{9}{2}\right) + \underline{\beta}\, \Gamma\left(\tfrac{11}{2}\right) + \underline{\gamma}\, \Gamma\left(\underline{\nu} + \tfrac{9}{2}\right) \right)$$

$$+ \alpha \left(\underline{a}'\, \Gamma\left(\tfrac{5}{2}\right) + \underline{b}\, \Gamma\left(\tfrac{7}{2}\right) + \underline{c}\, \Gamma\left(\tfrac{9}{2}\right) \right) + \beta \left(\underline{a}'\, \Gamma\left(\tfrac{7}{2}\right) + \underline{b}\, \Gamma\left(\tfrac{9}{2}\right) + \underline{c}\, \Gamma\left(\tfrac{11}{2}\right) \right)$$

$$+ \gamma \left(\underline{a}'\, \Gamma\left(\nu + \tfrac{5}{2}\right) + \underline{b}\, \Gamma\left(\nu + \tfrac{7}{2}\right) + \underline{c}\, \Gamma\left(\nu + \tfrac{9}{2}\right) \right)$$

$$+ \mathcal{D}(\underline{\nu})\, \underline{\gamma}^2 + \mathcal{A} \underline{a}'^2 + \mathcal{B} \underline{b}^2 + \mathcal{C} \underline{c}^2 + 2\mathcal{A}' \underline{a}' \underline{b} + 2 \mathcal{B}' \underline{b}\, \underline{c} + 2 \mathcal{C}' \underline{a}\, \underline{c} \Big\}$$

Minimisation of $\dot{\Sigma}_{PS} + \dot{S}_{PS}$ (for a given ν), with respect to $\underline{a}'$, $\underline{b}$, $\underline{c}$ and α, β, γ results in a system of linear equations from which the value of these parameters is readily obtained. In particular the combinations $\alpha\,\Gamma(\frac{5}{2}) + \beta\,\Gamma(\frac{7}{2}) + \gamma\,\Gamma(\nu + \frac{5}{2})$ and $\alpha\,\Gamma(\frac{7}{2}) + \beta\,\Gamma(\frac{9}{2}) + \gamma\,\Gamma(\nu + \frac{7}{2})$ which appear in the expression (9) of the fluxes F_l and F_{El} are given by

$$\alpha\,\Gamma\left(\frac{5}{2}\right) + \beta\,\Gamma\left(\frac{7}{2}\right) + \gamma\,\Gamma\left(\nu + \frac{5}{2}\right) =$$
$$\frac{1}{\pi^{3/2}}\left(\rho_l - \frac{3}{2}\rho_{El} - \rho_I\frac{v_{thI}}{v_{thl}}\right)\left[\mathcal{A}\left(1 - \frac{\mathcal{P}}{\mathcal{R}}\right) + \frac{4}{5}\mathcal{A}'\frac{\mathcal{P}}{\mathcal{R}} - \frac{4}{35}\mathcal{C}'\frac{\mathcal{P}}{\mathcal{R}}\right]$$
$$+ \frac{1}{\pi^{3/2}}\rho_{El}\left[\mathcal{A}'\left(1 - \frac{\mathcal{P}}{\mathcal{R}}\right) + \frac{4}{5}\mathcal{B}\frac{\mathcal{P}}{\mathcal{R}} - \frac{4}{35}\mathcal{B}'\frac{\mathcal{P}}{\mathcal{R}}\right]$$
$$\alpha\,\Gamma\left(\frac{7}{2}\right) + \beta\,\Gamma\left(\frac{9}{2}\right) + \gamma\,\Gamma\left(\nu + \frac{7}{2}\right) =$$
$$\frac{1}{\pi^{3/2}}\left(\rho_l - \frac{3}{2}\rho_{El} - \rho_I\frac{v_{thI}}{v_{thl}}\right)\left[\mathcal{A}'\left(1 + \frac{4}{5}\frac{\mathcal{Q}}{\mathcal{R}}\right) - \mathcal{A}\frac{\mathcal{Q}}{\mathcal{R}} - \frac{4}{35}\mathcal{C}'\frac{\mathcal{Q}}{\mathcal{R}}\right]$$
$$+ \frac{1}{\pi^{3/2}}\rho_{El}\left[\mathcal{B}\left(1 + \frac{4}{5}\frac{\mathcal{Q}}{\mathcal{R}}\right) - \mathcal{A}'\frac{\mathcal{Q}}{\mathcal{R}} - \frac{4}{35}\frac{\mathcal{Q}}{\mathcal{R}}\mathcal{B}'\right] \qquad (20)$$

where

$$\mathcal{P} = 4\left(\frac{35}{4}\mathcal{A} - 7\mathcal{A}' + \mathcal{C}'\right)\frac{\mathcal{L}(\nu)}{\Gamma(\nu + \frac{5}{2})(\nu^2 - \nu)}$$

$$\mathcal{Q} = 4\left(\frac{35}{4}\mathcal{A}' - 7\mathcal{B} + \mathcal{B}'\right)\frac{\mathcal{L}(\nu)}{\Gamma(\nu + \frac{5}{2})(\nu^2 - \nu)}$$

$$\mathcal{R} = \Gamma\left(\nu + \frac{5}{2}\right) - \frac{4}{5}\Gamma\left(\nu + \frac{7}{2}\right) + \frac{4}{35}\Gamma\left(\nu + \frac{9}{2}\right)$$
$$+ 4\left(\frac{35}{4}\mathcal{A} + \frac{28}{5}\mathcal{B} + \frac{4}{35}\mathcal{C} - 14\mathcal{A}' - \frac{8}{5}\mathcal{B}' + 2\mathcal{C}'\right)\frac{\mathcal{L}(\nu)}{\Gamma(\nu + \frac{5}{2})(\nu^2 - \nu)}$$

Choosing the value of ν , we obtain from (9) and (20) (17) (15) the flux F_l of light particles l and the flux of energy F_{El} in terms of the relative density and temperature gradients $\partial n(r,\alpha)/n_\alpha \partial r$ and $\partial T(r,l)/T\partial r \approx \partial T(r,I)/T\partial r$ in the form

$$F_l = -F_I \frac{Z_I}{Z_l} = -\frac{n_l \rho_{thl}^2}{\tau_{lI}} q^2 \left[K \left(\frac{\partial n(r,l)}{n_l \partial r} - \frac{Z_l}{Z_I} \frac{\partial n(r,I)}{n_I \partial r} \right) + H \frac{\partial T(r,l)}{T \partial r} \left(1 + o\left(\frac{Z_l}{Z_I}\right) \right) \right] \qquad (21a)$$

$$F_{El} = -\frac{n_l \rho_{thl}^2}{\tau_{lI}} q^2 T \left[\left(K_E \left(\frac{\partial n(r,l)}{n_l \partial r} - \frac{Z_l}{Z_I} \frac{\partial n(r,I)}{n_I \partial r} \right) + H_E \frac{\partial T(r,l)}{T \partial r} \left(1 + o\left(\frac{Z_l}{Z_I}\right) \right) \right) \right] \qquad (21b)$$

$$\rho_{thl} = \left(\frac{2T}{m_l}\right)^{1/2} \frac{1}{\omega_{cl}} \quad , \quad \frac{1}{\tau_{lI}} = \frac{4 (2\pi)^{1/2} n_I Z_I^2 Z_l^2 e^4 \mathcal{L}og \Lambda}{3 m_l^{1/2} T^{3/2}}$$

where the coefficients K, H, K_E, H_E are functions of $g = q R / z V_{thl} \tau_{lI}$ and $z = n_I Z_I^2 / n_l Z_l^2$

The table I gives the computed values of these coefficients for $\nu =$. In principle we may fit the value of the adjustable parameter ν by expressing that $\dot{\Sigma}_{PS} + \dot{S}_{PS}$ is an extremum with respect to ν . It is easily shown that this condition is equivalent to the condition that the function

$$f'(\nu) = \frac{[I(\nu)]^{1/2}}{(\nu^2-\nu)\cdot\Gamma(\nu+5/2)\quad 2^{-1/2}\,\pi^{3/4}}$$

is extremum with respect to ν. Numerical calculation (cf. fig.2) of $J(\nu)$ gives $\nu = 5.1$, but the minimum of $f'(\nu)$ is poorly marked. Actually choosing $\nu = 5$ or $\nu = 3$ gives approximatively the same values of the transport coefficients K, H, K_E, H_E

The values of the coefficients k, H, k_E, H_E given by the table (I) are valid only if the light ions are in the P.S. regime, i.e. if $v_{thl}\tau_{Dl}(qR)^{-1} < 1$. As the deflection time τ_{Dl} is given by $\tau_{Dl}^{-1} \sim \tau_{lI}^{-1}(1 + O(z^{-1}))$, this condition implies that $g\,(z + O(1)) > 1$. The coefficients k, h, k_E h_E experience a transition from the values quoted (A) (cf. [2, 3]) in the table (I) to the values quoted (B) (cf. approximatively [1]), when the quantity $g^2(z + O(1))$ increases through unity. As the Maxwellisation time τ_{Ml} is given by $\tau_{Ml} \sim \tau_{lI}\;O(z)$, the transition takes place for $v_{thl}^2\;\tau_{Ml}\;\tau_{Dl}\;(qR)^{-2} \sim 1$.

IV. PARTICLE TRANSPORT IN THE CASE WHEN LIGHT IONS ARE IN THE BANANA OR PLATEAU REGIME, AND IMPURITY IONS IN THE P.S. REGIME.

We will see now that if the light ions are in the banana regime $(\tau_{Dl}\,v_{thl}(qR)^{-1} > (R/r)^{3/2})$ or in the Plateau regime $(1 < \tau_{Dl}\,v_{thl}\,(qR)^{-1} < (R/r)^{3/2})$

the particle flux $\Gamma_1 = -(Z_I/Z_1)\,\Gamma_I$ is given by the P.S. like formula (21a), taking for the coefficients K, H the values (A) of the table (I). This may be proved without detailed calculations. We note that, in the Banana or the Plateau regime for ions 1, the expression (4) for the perturbation $f_1(r, \theta, v, p, 1)$ which appears in (2) must be replaced by an expansion in the complete set of the Legendre Polynomials $P_n(p)$

$$
\begin{aligned}
f_1(r, \theta, v, p, 1) = {} & f_0(r, v, 1) \\
& \Big[Y_0(v, 1) \sin\theta + \frac{p v}{v_{th\alpha}} Y_1(v, 1) \cos\theta \\
& + 2 v p \frac{\Omega R_0^2}{v_{th\alpha}^2 R} + \sum_{n \geq 2} P_n(p)\, x_n(v, \theta) \Big]
\end{aligned}
\tag{22}
$$

However, as the impurity ions are kept in the P.S. regime, we retain the expression (4) for $f_I(r, \theta, v, p, I)$. In view of a variational calculation to determine f_1 we will use the trial expressions (8) for $Y_0(v, \alpha)$ and $Y_1(v, \alpha)$. (In fact the conclusions reached below are independent of this choice). Using the expression (22) of $f_1(r, \theta, v, p, 1)$ we obtain from (1) and (10) the expressions of $\underline{\frac{d f(\vec{x}, \vec{v}, 1)}{dt}}$ and $\mathcal{U}(\vec{x}, \vec{v}, 1)$. Then the expression of $\dot{\Sigma}\left(\underline{\frac{d f(\vec{x}, \vec{v}, \alpha)}{dt}}, \underline{\mathcal{U}(\vec{x}, \vec{v}, \alpha)}\right) + \dot{S}\left(f(\vec{x}, \vec{v}, \alpha), \mathcal{U}(\vec{x}, \vec{v}, \alpha)\right)$ is obtained from (12) and (13) as a functional of $Y_0(v, \alpha)$, $\underline{Y_0(v, \alpha)}$, $Y_1(v, \alpha)$, $\underline{Y_1(v, \alpha)}$, $x_n(v, \theta)$, $\underline{x_n(v, \theta)}$ and as a function of Ω, $\underline{\Omega}$. It is readily verified, using the orthogonality of the Legendre Polynomials, that the functional $\dot{\Sigma} + \mathring{S}$ has the form

$$\dot{\Sigma}\left(\frac{df}{dt}, u\right) + \dot{S}(f, u) =$$

$$\dot{\Sigma}_{PS}\left(Y_0(v,\alpha), \underline{Y_0(v\alpha)}, Y_1(v,\alpha) \quad \underline{Y_1(v,\alpha)}\right)$$

$$+ \dot{S}_{PS}\left(Y_0(v,\alpha) \; \underline{Y_1(v,\alpha)}\right)$$

$$+ \sigma\left(Y_1(v,l), \underline{Y_1(v,l)}, \Omega, \underline{\Omega}, X_n(v,\theta), \underline{X_n(v,\theta)}\right)$$

Where $\dot{\Sigma}_{PS}$ and $\dot{S}_{PS}$ are given by (14) and (16).
By minimizing $\dot{\Sigma}_{PS} + \dot{S}_{PS}$ with respect to the coefficients $\underline{\alpha}, \underline{\beta}, \underline{\gamma}$, which do not appear in σ, we obtain the set of linear equations

$$\left(\rho_l - \tfrac{3}{2}\rho_{El} - a\right)\Gamma\left(\tfrac{5}{2}\right) + \left(\rho_{El} - b\right)\Gamma\left(\tfrac{7}{2}\right) - c\,\Gamma\left(\tfrac{9}{2}\right) = 0$$

$$\left(\rho_l - \tfrac{3}{2}\rho_{El} - a\right)\Gamma\left(\tfrac{7}{2}\right) + \left(\rho_{El} - b\right)\Gamma\left(\tfrac{9}{2}\right) - c\,\Gamma\left(\tfrac{11}{2}\right) = 0$$

$$\left(\rho_l - \tfrac{3}{2}\rho_{El} - a\right)\Gamma\left(\nu + \tfrac{5}{2}\right) + \left(\rho_{El} - b\right)\Gamma\left(\nu + \tfrac{7}{2}\right) - c\,\Gamma\left(\nu + \tfrac{9}{2}\right) = - 2\gamma\mathcal{L}(\nu) = -2\gamma g \; I(\nu)$$

which results in

$$\left.\begin{aligned} a &= \rho_l - \tfrac{3}{2}\rho_{El} = -\frac{2q\,v_{thl}}{\omega_{cl}}\left(\frac{\partial n(r,l)}{n_l\,\partial r} - \frac{3}{2}\,\frac{\partial T(r,l)}{T\,\partial r}\right) \\ b &= \rho_{El} = -\frac{2q\,v_{thl}}{\omega_{cl}}\;\frac{\partial T(r,l)}{T\,\partial r} \\ c &= 0 \end{aligned}\right\} \qquad (23)$$

if

$$\gamma g \sim \gamma \frac{qR}{v_{th l}\, \tau_{M l}} \ll \frac{q\, v_{th l}}{\omega_{c l}} \left[O\left(\frac{\partial T(r,l)}{T \partial r}\right) + O\left(\frac{\partial n(r,l)}{n_l \partial r}\right) \right] \tag{24}$$

To obtain an upper limit of the quantity γ , we note that the averaged flux ψ of a quantity of the form $\left(\frac{v}{v_{th l}}\right)^{2\mu} \frac{1}{1+p^2}$ transported by the light particles across the magnetic surface r is given by

$$\psi = -\iiint \frac{\sin\theta\, v^2 (1+p^2)}{2\, \omega_{cl}\, R_0}\; f_1(r, \theta, v, p, l) \left(\frac{v}{v_{th l}}\right)^{2\mu} \frac{1}{1+p^2}\, dp\, \frac{R}{R_0}\, v^2 dv\, d\theta$$

In view of the expression (22),(8) of f_1 and of the orthogonality of the Legendre Polynomials we obtain

$$\psi = - n_l \frac{1}{\omega_{cl} R_0} \frac{v_{th l}^2}{2\, \pi^{1/2}} \left(\alpha\, \Gamma\left(\mu + \tfrac{5}{2}\right) + \beta\, \Gamma\left(\mu + \tfrac{7}{2}\right) + \gamma\, \Gamma\left(\mu + \nu + \tfrac{5}{2}\right) \right) \tag{25}$$

On the other hand an upper limit of ψ is obtained from the diffusion process experienced by individual particles l in the Banana or Plateau regime. This means that ψ has at most the neoclassical order of magnitude, namely

$$|\psi| = n_l \frac{\rho_{th l}^2}{\tau_{D l}} q^2 \left(\frac{R}{r}\right)^{3/2} \left[O\left(\frac{\partial T(r,l)}{T \partial r}\right) + O\left(\frac{\partial n(r,l)}{n_l \partial r}\right) \right] \tag{26a}$$

and

$$|\psi| = n_l\, \rho_{th l}^2 \frac{q\, v_{th l}}{R} \left[O\left(\frac{\partial T(r,l)}{T \partial r}\right) + O\left(\frac{\partial n(r,l)}{n_l \partial r}\right) \right] \tag{26b}$$

in the banana and in the Plateau regime, respectively. Comparing (25) and (26) for any value of μ gives

$$\gamma \lesssim \frac{q\, v_{thl}}{\omega_{cl}}\left[O\left(\frac{\partial T(r,l)}{T\,\partial r}\right) + O\left(\frac{\partial n(r,l)}{n_l\,\partial r}\right)\right]\left(\frac{R}{r}\right)^{3/2}\frac{qR}{\tau_{Dl} v_{thl}}$$

If $\quad \tau_{Dl}\, v_{thl}\,(qR)^{-1} > (R/r)^{3/2}$

$$\gamma \lesssim \frac{q\, v_{thl}}{\omega_{cl}}\left[O\left(\frac{\partial T(r,l)}{T\,\partial r}\right) + O\left(\frac{\partial n(r,l)}{n_l\,\partial r}\right)\right]$$

If $\quad 1 < \tau_{Dl}\, v_{thl}\,(qR)^{-1} < (R/r)^{3/2}$

From these estimations and from the fact that $\tau_{Dl} \leq \tau_{Ml}$, it results that the condition (24) is fulfilled in both the Banana and the Plateau regime. The equation (23) therefore applies in these regimes. Expressing that $\dot{\Sigma}_{PS} + \dot{S}_{PS}$ is extremum with respect to $\underline{\alpha_I}$ and $\underline{a_I}$, which again do not appear in σ, and using (23), we obtain the value of α_I. The value of the flux F_I of ions I then results from (9). The flux F_I happens to verify (21a) taking for K and H the values (A) of the table (1). It must be noticed that the condition (24) is verified in the whole Plateau regime, including the frontier $v_{thl}\,\tau_{Dl}\,(qR)^{-1} \sim 1$ with the P.S. regime, if we have $\tau_{Dl} \gg \tau_{Ml}$. In that case the particle transport coefficients K and H experience a transition from the values (A) to the values (B) inside the P.S. regime, for $v_{thl}^2\,\tau_{Dl}\,\tau_{Ml}\,(qR)^{-2} \sim 1$. If $\tau_{Dl} \sim \tau_{Ml}$, this transition appears for $v_{thl}\,\tau_{Dl}(qR)^{-1} \sim 1$, i.e. at the same time as the transition Plateau- P.S. The coefficients K and H at the transition could be calculated in that case by the variational method which has been used above, by adding the term proportional to $P_2(p)$ to the truncated expansion (4) of

$f_1(r, \theta, v, p, l)$ in Legendre Polynomials $P_n(p)$.

V. CONCLUSION.

Our conclusion is that, in a configuration where impurities are in the P.S. regime and the light ions are in the Banana, Plateau or P.S. regime, the neoclassical particle flux $\Gamma_l = -\frac{Z_I}{Z_l}\Gamma_I$ is given by the P.S. like formula (21), with the coefficients K and H experiencing a transition from the values (A) to the values (B) of the table (1) when the quantity $v_{thl}^2 \tau_{Dl} \tau_{Ml} (qR)^{-2}$ decreases through a value $\sim$ 1. It should be noticed that the values (A) of K and H allow in principle the purification of the plasma by the temperature gradient of the light ions. If the Maxwellian time τ_{Ml} is of the same order as the deflection time τ_{Dl}, the transition coincides with the transition Plateau-P.S. for the light ions. If the time τ_{Ml} is significantly larger than the time τ_{Dl}, the transition takes place inside the P.S. regime for the light ions. The values of the coefficients K, H (and the coefficients K_E, H_E giving the energy flux Γ_{lE}) at the transition have been computed in that case and are given by the table (1).

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REFERENCES

[1] P.H. RUTHERFORD, Phys. of Fluids 17, 1732 (1974)

[2] A. SAMAIN, Rapport EUR.CEA.FC.745 (1974).

[3] P. ROLLAND, F. WERKOFF, in 7th European Conference on Controlled Fusion and Plasma Physics, Paper 120 (see also supplementary pages), Lausanne 1975.

[4] D. PFIRSCH, A. SCHLÜTER, Max Planck Institute, ~~Report MPI~~/PA.7/62, 1962.

[5] M.N. ROSENBLUTH, R.D. HAZELTINE, F.L. HINTON, Phys. of Fluids. 15, 116 (1972).

[6] S.I. BRAGINSKII in Review of Plasma Physics (Edited by M.A. Leontovitch) vol. 1, p.205 - Consultant Bureau, New-York (1965)).

FIGURE CAPTIONS

(1) Tokamak Geometry.

(2) Values of the function $F'(\nu)$ versus ν .

TABLE I

$g = 10^{-3}$

K	H	K_E	H_E
1	- 0.5	1	0.5 + 1.41/Z

← A

$g = 0.1$

z	K	H	K_E	H_E
10	0.99	- 0.49	1	0.64

$g = 0.3$

z	K	H	K_E	H_E
3	0.98	-0.46	1.00	0.97
5	0.96	-0.44	1.00	0.79
10	0.93	-0.4	0.99	0.65

$g = 0.5$

z	K	H	K_E	H_E
2	0.96	-0.43	1.01	1.20
5	0.90	-0.36	0.99	0.9
10	0.84	-0.28	0.98	0.67

$g = 1$

z	K	H	K_E	H_E
1	0.93	-0.36	1.03	1.85
3	0.83	-0.24	1.00	0.97
5	0.77	-0.16	0.98	0.81
10	0.47	-0.06	0.96	0.70

$g = 2$

z	K	H	K_E	H_E
1	0.83	-0.17	1.08	1.76
3	0.69	-0.04	1.01	0.96
5	0.63	0.024	0.98	0.82
10	0.57	0.09	0.94	0.72

$g = 5$

z	K	H	K_E	H_E
1	0.74	0.01	1.12	1.68
3	0.61	0.09	1.01	0.96
5	0.57	0.12	0.97	0.83
10	0.53	0.15	0.94	0.73

$g = 10$

z	K	H	K_E	H_E
1	0.71	0.06	1.13	1.66
3	0.59	1.2	1.01	0.96
5	0.55	0.14	0.97	0.83
10	0.52	0.17	0.94	0.73

← B

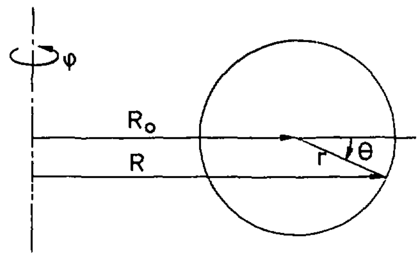

Fig.1

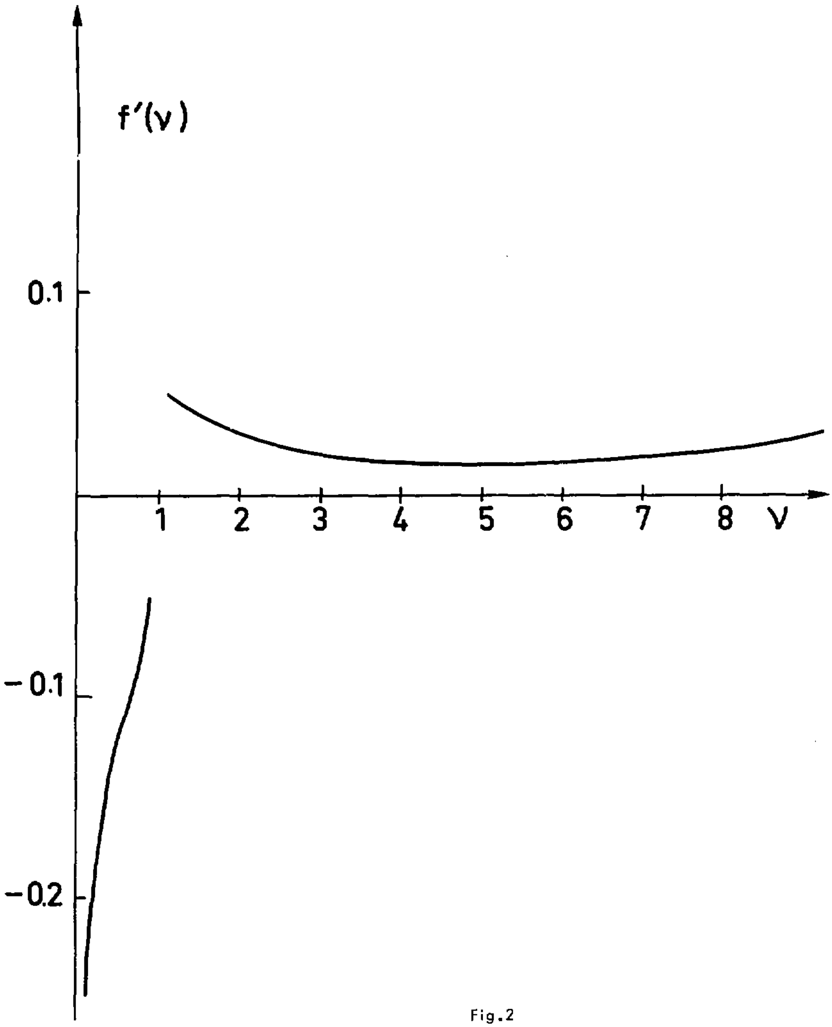

Fig.2