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SHORT DISTANCE BEHAVIOUR OF THE WAVE FUNCTION AND QUARK CONFINEMENT

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Recently some models have been proposed in which the forces between quarks are increasing with the relative distance that results in the confinement of quarks inside a particle and their unobservability in the free state.

We shall consider the problem of quark confinement in the framework of the quasipotential approach[1], namely by using the Kadyshevsky equation[2]. In quasipotential equations, in contrast to the Bethe-Salpeter equation, the momenta of all the particles belong to the mass shell. Therefore it is convenient to pass here to the relativistic configurational representation (RCR), introduced earlier[3] in the framework of the Kadyshevsky approach. The difference of the RCR from the nonrelativistic coordinate representation consists in application here of the Shapiro transformation[4] instead of the conventional Fourier transformation. The Shapiro transformation has the meaning of the expansion over the principal series (PS) of the unitary irreducible representations of the Lorentz group SO(3.1) - the group of motions of the mass shell hyperboloid $p_0^2 - \vec{p}^2 = M^2$.

With notations[3] this expansion for the wave function of relative motion reads

$$\Psi(\vec{p}) = \int \xi(\vec{p},\vec{r})\,\Psi(\vec{r})\,d\vec{r} \quad ; \xi(\vec{p},\vec{r}) = \left(\frac{p_0 - \vec{p}\vec{n}}{M}\right)^{-1-irM} \qquad (1)$$
$$\vec{r} = r\vec{n} \quad ; \quad \vec{n}^2 = 1 .$$

Here $\vec{p}$ is the quark momentum in the c.m.s. $(\vec{p}_1 = -\vec{p}_2 = \vec{p})$. The parameter r defines eigenvalues χ^{-2} of the Casimir operator of the SO(3.1). $\hat{C} = \frac{1}{4} M_{\mu\nu} M^{\mu\nu}$ ($M_{\mu\nu}$ - are the generators of the SO(3.1))

$$\hat{C}\,\xi(p,r) = \chi^{-2}\xi(p,r)\ ,\quad \chi^{-2} = \frac{1}{M^2} + r^2 \quad (0 < r \leq \infty) \qquad (2)$$

and, as was shown in[3], it has the meaning of a relativistic generalization at the relative coordinate. In the quasipotential equation written in the RCR the transforms of the Feynman propagators in the new r -space play the role of potentials. Thus, to the propagator $\frac{1}{(p-\kappa)^2}$, describing the massless gluon exchange there corresponds the attractive relativistic Coulomb potential[3]

$$V(r) = -\frac{1}{4\pi r}\ cth\ \pi r M \qquad (3)$$

Due to the proven in[5] equality $\langle r_c^2 \rangle \equiv 6 \frac{\partial F(t)}{\partial t}\Big|_{t=0} = \left\{ \hat{C}\, F(t) \right\}\Big|_{t=0}$ the invariant mean square radius of a particle has the meaning of the average of the eigenvalue of the SO(3.1) Casimir operator $\hat{C} = \chi^2$ over the transforms F(2) of the form factor $F(t)$ in the RCR. In the case when these distributions $F(r)$ in the new r -space are the functions of constant sign, the relativistic coordinate r describes the distances larger than the Compton wave length.

The transition to the distances, smaller than the Compton wave length, may be achieved, following[5], by including into the wave function expansion the supplementary series (SS). characterized by the subsequent values of the Casimir operator $\hat{C} \to \chi^2 = \frac{1}{M^2} - \rho^2$, where $0 \leq \rho \leq \frac{1}{M^2}$. The coordinate ρ is reckoned beginning from the boundary of the sphere to its center, and the value $\rho = \frac{1}{M}$ corresponds to the origin $\chi^{-2} = 0$.

For the SS the analogs of the plane waves of PS $\xi(\vec{p},\vec{r})$ are the functions $\zeta(\vec{p},\vec{\varrho}) = \left(\frac{p_0 - \vec{p}\vec{n}}{M}\right)^{-1-\varrho M}$ $\left(0 < \varrho \le \frac{1}{M}\right)$ which formally can be found from $\xi(\vec{p},\vec{r})$ by the change $r \to i\varrho$. The expansion of $\Psi(\vec{p})$ with account of SS for the states with $l=0$ has the form:

$$\Psi_{l=0}(p) = 4\pi \int_0^\infty \frac{\sin rM\chi}{rM\, sh\chi} \Psi(r) r^2 dr + 4\pi \int_0^{1/M} \frac{sh\,\varrho M\chi}{\varrho M\, sh\chi} \Psi(\varrho)\varrho^2 d\varrho \quad (4)$$

Consider now the analog of the relativistic Coulomb potential for distances smaller than $1/M$. Passing in (3) to the SS through the change $r \to i\varrho$ we arrive at the potential (see Fig.1)

$$V(\varrho) = \frac{1}{4\pi\varrho} ctg\, \pi\varrho M; \quad 0 < \varrho \le \frac{1}{M} , \quad (5)$$

confining quarks inside the sphere with $R^2 = X^{-2} = \frac{1}{M^2}$. The operator of the free Hamiltonian $\hat{H}_0$ for the plane waves of the SS $\hat{H}_0 \zeta(\vec{p},\vec{\varrho}) = 2E_p \zeta(\vec{p},\vec{\varrho})$; $E_p = M ch\chi = \sqrt{M^2 + \vec{p}^2}$

$$\hat{H}_0 = 2M ch \frac{i}{M}\frac{\partial}{\partial\varrho} + \frac{2}{\varrho} sh \frac{i}{M}\frac{\partial}{\partial\varrho} - \frac{\Delta_{\theta,\varphi}}{\varrho^2} e^{\frac{i}{M}\frac{\partial}{\partial\varrho}} \quad (6)$$

as in the case of /3/ is the finite-difference operator. The solution of the quasipotential equation with the potential (5)

$$\left(\hat{H}_0 + V(\varrho)\right)\Psi_q(\vec{\varrho}) = 2E_q \Psi_q(\vec{\varrho}) \quad (7)$$

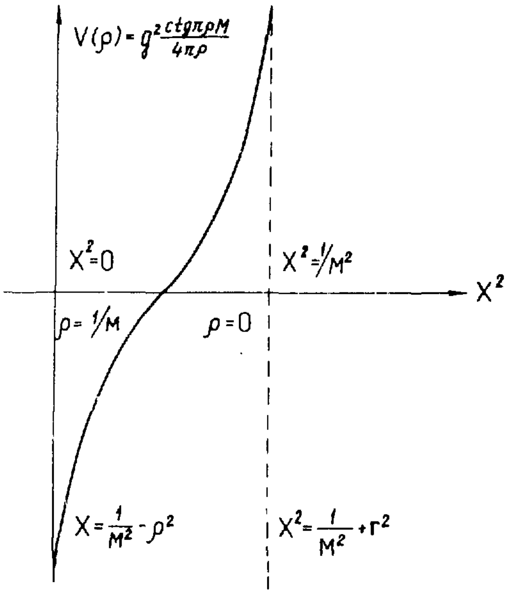

Fig.1

in the domain $0 \leq \chi^2 < \frac{1}{(2M)^2}$, where $ctg\, \pi \rho M < 0$ and the $M_{bound} \equiv 2E_q = 2M \cos x$, for the states with $\ell = 0$ has the form

$$\Psi_{q,\ell=0}(\rho) = (e^{-ix} \sin x) \cdot e^{-ix\rho} \cdot \exp\left[x \cdot \frac{ctg\,\pi\rho M}{2 \sin x}\right] \cdot F\left(1+\rho M,\ 1+i\frac{ctg\,\pi\rho M}{2\sin x};\ 2;\ 2ie^{-ix}\sin x\right). \tag{8}$$

The function $ctg\,\pi\rho M$ in (5), constant with respect to the operation of the finite-difference differentiation (cf. /3/), plays a role of the effective interaction constant in equation (6). The requirement of the regularity of the solution at $\chi^2 = 0$ $(\rho = 1/M)$ leads to the condition $\sin 2x = x$, which determines two energy levels. One with $M_{bound} \equiv 2E_q = 1.38\, M$, another with $M_{bound} \equiv 2E_q = 2M$. In the region $\frac{1}{(2M)^2} \leq \chi^2 < \frac{1}{M^2}$, where $ctg\,\pi\rho M > 0$ and $2E_q = 2M\, ch\chi \geq 2M$, the wave function can be obtained from (8) by the change $x \to -i\chi$. The requirement of the regularity at $\chi^2 = \frac{1}{M^2}$ $(\rho = 0)$ leads to another condition $2 sh\chi\, e^{-\chi} = \chi$, that determines the third level with $M_{bound} \equiv 2E_q = 2.93\, M$. Therefore in the quark-antiquark system, moving in the field of potential (5) in the state with $\ell = 0$ there are possible three energy levels, or three excited states of one particle (for example ρ, ρ' and ρ'').

The functions of SS $\xi(\vec{p}, \vec{\rho})$ do not belong to the class of square- integrable functions /7/. This leads to necessity to include into the definition of the scalar product of the wave functions (8) in the momentum space the regularing kernel $K\left[(p-k)^2\right]$, i.e.,

$$(\Psi_1, \Psi_2) = \int \Psi_1(\vec{p})\, K[(p-k)^2]\, \Psi_2(\vec{k}) \frac{d^3\vec{p}}{p_0} \frac{d^3\vec{k}}{k_0} .$$

The questions of the normalization of the wave functions (8) and the description of the meson spectrum and Ψ -particles in our model with the quark confining potential (5) will be the subject of the next publications.

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