CHAPTER 6

INFLUENCE OF THE EFFECTIVE INTERACTION ON SPECTRA OF SUPERFLUID NUCLEI

Abstract: It is attempted to improve the description of the collective 2^+ and 3^- states in even single-closed-shell nuclei by using a "realistic" effective interaction based on the Reid soft core potential, instead of the Gaussian forces which are commonly used. As an example the Sn isotopes are considered.

The results of the Reid force are not better than those of the Gaussian are. The too large excitation-energies of the collective 2^+ and 3^- states found earlier, apparently are not due to particular features of the Gaussian effective forces used. Explicit admixture of a quadrupole and an octupole part in the effective interaction does improve the description of the collective 2^+ and 3^- states, but it makes the description of the other excited states worse.

1. Introduction

The properties of single-closed-shell(SCS) nuclei can be accounted for reasonably well by the number-projected quasiparticle model. In the even SCS nuclei the low-lying levels can be described by two-quasiparticle excitations of the nucleons in the open shell. However, it appears that the excitation-energies of the collective 2_1^+ and 3_1^- states in these nuclei are calculated systematically too large, especially for the Sn isotopes. There the calculated energies of the 2_1^+ states lie 0.2-0.5 MeV too high, and those of the 3_1^- states lie 1 MeV too high[1,2]. For the N=50 and N=82 nuclei the energy difference is smaller[2-5].

The problem is how to improve the excitation-energies of the collective states without allowing unreasonable results for the non-collective states. The solution of this problem is important not only for itself, but also for an adequate description of the spectra of neighbouring odd nuclei. For in the odd Sb, In or Sn isotopes states occur, which consist of a particle, a hole or a quasi-particle coupled to the even Sn 2_1^+ and 3_1^- core states. If the 2_1^+ and 3_1^- states in the even Sn isotopes are calculated too high, one will find the corresponding multiplets in the adjacent nuclei also too high.

In order to lower the energies of the collective states one may follow different methods.

Firstly, one might expect that an extension of the shell model space lowers the collective states. It has been found however that the collective states do not come down by extending the model space from one to three major shells, for the one type of nucleons considered [6].

Secondly, particle-hole excitations in the closed shell may contribute to the collective states. These p-h contributions to the 2_1^+ and 3_1^- states have been calculated by different authors, using the quasiparticle model without performing number projection [2,7]. Their conclusion is that the p-h excitations contribute considerably to the 3_1^- states, bringing down their excitation-energies to the correct value, but for the 2_1^+ states their contribution is not so large. The excitation-energies of the 2_1^+ states are reduced by only 0.1-0.2 MeV. However this energy shift of the 2_1^+ states seems not large enough to account completely for the observed discrepancy of 0.2-0.5 MeV.

A third possibility to lower the energy of the excited states is taking into account the change in the BCS pair distribution due to the excitation of the nucleus. The usual way to determine the pair distribution in quasi-particle calculations is to solve the gap equations, which means that the pair distribution is optimized for the ground state. It appears [8] however that the pair distribution of the excited states should be less diffuse than that of the ground state, due to the excitation of the quasiparticles. In a two-quasiparticle calculation the effect of varying the pair distribution in the excited states is that the excitation-energies are lowered 0.0-0.2 MeV.

Our aim is now to investigate whether the remaining small but systematic discrepancy is due to the use of a Gaussian force in the quasiparticle calculations [1-5]. Therefore we compare the results of a calculation using a "realistic" force with those calculated with a Gaussian force. Here "realistic" means that the effective interaction has been derived from a bare interaction, viz. the Reid soft core potential[9],which fits the two-nucleon data. Besides, we consider also the Schiffer interaction,which has been designed to reproduce p-p, p-h and h-h spectra of odd-odd nuclei with two almost closed shells throughout the periodic table [10]. The Schiffer force is also able to reproduce rather well the spectra of the odd-odd Sb and In nuclei [11], which have one open shell. We want to investigate whether the T=1 part of this interaction may be used as an effective interaction in the open shell. Finally it is examined whether the collective 2_1^+ and 3_1^- states can be brought down, without disturbing the positions of the other excited states, by adding to the Gaussian force a quadrupole and an octupole force.

As examples of SCS nuclei we consider $^{115-119}$Sn.

In sect.2 the various effective interactions are listed. The results of the calculations using these different interactions are compared in sect.3. Sect.4 contains a summary and conclusions.

CHAPTER 6

2. The effective interactions

Four different effective interactions will be compared.

Firstly, a Gaussian force is considered. It has been used by a number of authors [1-6,12]. It is defined by

$$V(r) = -V_0(P_S+tP_T)e^{-r^2/\mu^2} \qquad (V_0 \text{ in MeV}) \qquad (2.1)$$

where $r=|\vec{r}_1-\vec{r}_2|$ and P_S and P_T are the singlet-even and triplet-odd projection operators. The range parameter μ is chosen such that $\mu\nu=0.90$, where ν is the harmonic-oscillator strength constant,which has the value $\nu=0.44\text{fm}^{-1}$. The triplet-to-singlet ratio t has the value t=-0.4. The choice of these force parameters is the same as in refs.[1,2,6].

As an example of a "realistic" force, we use an effective interaction G which has been derived from the Reid soft core potential V_R [9]. To take effectively into account the short-range correlations which cannot explicitly be considered in the model space, we solve the Bethe-Goldstone equation

$$G=V_R+V_R \frac{Q}{W-H_0(1)-H_0(2)} G \qquad (2.2)$$

The Pauli operator Q is approximated by an Eden-Emery operator [13], which forbids scattering into two particle oscillator states with $2n_1+l_1+2n_2+l_2 \leqslant N$ (n=1,2,......). The choice N=20 turned out to give a good approximation for a more realistic Pauli operator for the neutrons.

With this choice one also takes care that the two particle states of the model space ($N\leqslant 14$) are not considered in solving the Bethe-Goldstone equation and thereby avoids double counting of the corresponding two particle correlations. The starting energy W should roughly be equal to the sum of two single particle energies for the active nucleons and has been chosen to be W=-10 MeV. The spectrum for the intermediate particle states (H_0) is approximated by that of a harmonic oscillator shifted down by a constant C=25 MeV. Since the inclusion of more complicated core polarisation effects is not yet understood, we allow for a global renormalization of this G interaction by multiplying it with a factor V_0 consistent with the determination of the overall force strength of the phenomenological interactions used. This parameter V_0 is extracted from the odd-even Sn mass differences(cf. sect.3).

The Schiffer interaction [10] has been designed to reproduce p-p, p-h and h-h spectra of odd-odd nuclei with two nearly closed shells throughout the periodic table:

$$\left.\begin{array}{l} V_{SE}=-160e^{-r^2}+6.4e^{-0.1r^2} \\ V_{TO}=-225e^{-r^2}+6.4e^{-0.1r^2} \end{array}\right\} T=1$$
$$\left.\begin{array}{l} V_{SO}=-195e^{-r^2} \\ V_{TE}=-195e^{-r^2} \end{array}\right\} T=0 \qquad (2.3)$$

(V in MeV, r in fm)

The remarkable feature in this interaction is the repulsive long range term $e^{-0.1r^2}$ which has to be included in order to reproduce the p-p, p-h and h-h spectra [10]. Also for the rare earth nuclei the experimental data suggest the use of a repulsive long range term in the effective interaction [14].

Finally we consider a Gaussian force (t=0) to which a quadrupole and an octupole force have been added. The extra terms added to eq. (2.1) are

$$Q(\lambda)=-V_0 c_\lambda (\nu r_1)^\lambda (\nu r_2)^\lambda \sum_{\mu=-\lambda}^{+\lambda} Y_{\lambda\mu}(\hat{r}_1) Y^*_{\lambda\mu}(\hat{r}_2) \quad \lambda=2,3 \qquad (2.4)$$

The strength parameters c_λ have been chosen such that the 2_1^+ and 3_1^- states are lowered in energy without disturbing the positions of the other levels too much. They are set equal to c_2=0.005 and c_3=0.00075.

For all interactions the overall force strength is treated as a parameter,which will be determined by the odd-even Sn mass differences (cf. sect.3).

3. Comparison of results of different interactions

3.1. The odd nuclei

The shell model space for the neutrons in ^{115,117,119}Sn consists of one major shell, viz. the 1g7/2, 2d5/2, 3s1/2, 2d3/2 and 1h11/2 subshells. The parameters of the calculation, viz. the overall force strength V_0 and the single-particle energies ε_a are to be chosen in a consistent way for the different effective interactions; otherwise a comparison of the results for the different forces is not meaningful.

A consistent manner to determine the parameters is extracting them from spectroscopic data on the odd nuclei in the same way for the various interactions. The method of extraction which is used, has been extensively described in ref. [3] and has been applied to the Sn isotopes in refs.[1,6].

The starting point lies in the adopted experimental quasiparticle energies given in ref.[6]. Following refs.[1,3] the force strength V_0 is determined from the experimental quasiparticle energies by solving the inverse modified gap equations (IMGE). With the obtained force strength V_0 the relative single-particle energies are fitted to the relative quasiparticle energies in a number-conserving way, viz. by performing a one-quasiparticle generator coordinate method (1qpGCM) fit [6].

We find that solving the IMGE the correct average particle number is much better reproduced by the Schiffer force and the Reid force than by the Gaussian forces. When the single-particle energies ε_a and the overall force strength V_0 are plotted as functions of particle number, the curves for the Schiffer force and the Reid force are somewhat more smooth and constant than those for the Gaussians. The smoothness and constancy of these curves give an indication of the quality of the interaction, as is discussed in ref.[3]. So one can conclude that the Schiffer force and the Reid force produce more satisfactory results for the odd nuclei than the Gaussian forces do. Furthermore, the use of the "realistic" Reid interaction is supported by the fact that the renormalizing constant V_0 is very close to 1 ($0.93 \leqslant V_0 \leqslant 1$).

3.2. The even nuclei

The number projected two-quasiparticle formalism which we use for the calculation of $^{116,118}Sn$ is that of ref.[15]. The parameters of the calculation are obtained by interpolation from those for the odd nuclei. The results of the calculations are given in tables 1 and 2. The effective charge for the neutrons is arbitrarily taken equal to $e_n = 1.0$.

Firstly, we compare the results calculated with the Reid force with those calculated with the Gaussian (t=-0.4) force. One may conclude that some excitation-energies are improved, others become worse by applying the Reid force.

Table 1

Excitation-energies of two-quasiparticle neutron states in $^{116,118}Sn$ for various effective interactions

	J^{π}	Gauss	Reid	Schiffer	Gauss+Q+O	exp.[16]
^{116}Sn	0^{+}	1.95	1.67	1.99	2.40	1.76
		2.80	2.47	2.71	2.74	2.03
	2^{+}	1.70	1.95	2.50	1.00	1.29
		2.79	2.61	2.95	2.88	2.11
		2.94	3.17	3.08	2.98	2.23
	4^{+}	2.67	2.53	2.94	2.96	2.39
		3.23	3.34	3.23	3.29	2.80
		3.47	3.44	3.36	3.52	3.05
	8^{+}	3.77	3.75	3.47	3.96	(3.23)
	10^{+}	3.81	3.96	3.56	3.81	(3.30)
	3^{-}	3.48	3.75	3.49	3.01	2.27
	5^{-}	2.87	2.45	2.96	3.08	2.37
	6^{-}	3.36	3.16	3.18	3.32	2.77
	7^{-}	2.84	3.25	3.16	3.05	2.91
^{118}Sn	0^{+}	1.96	2.05	1.88	2.30	1.76
		2.38	2.78	2.54	2.42	2.06
	2^{+}	1.56	2.18	2.39	0.90	1.23
		2.64	2.60	2.88	2.66	(2.04)
		3.01	3.15	2.88	2.88	(2.40)
	4^{+}	2.81	2.75	2.86	2.91	2.28
		3.36	3.37	3.39	3.30	(2.73)
		3.52	3.62	3.54	3.67	(2.96)
	8^{+}	3.50	3.22	3.15	3.68	(3.06)
	10^{+}	3.54	3.41	3.22	3.54	(3.11)
	3^{-}	3.56	3.85	3.64	3.11	(2.31)
	5^{-}	2.74	2.50	2.73	2.90	2.32
	7^{-}	2.52	2.86	2.92	2.82	2.57

All energies are given with respect to the corresponding ground states in MeV. The experimental data are taken from ref.[16]. The heading *Gauss+Q+O denotes the Gaussian (t=0) force to which a quadrupole and an* octupole term are added (cf. sect.2).

Table 2

BEλ-values of the collective 2_1^+ and 3_1^- states in $^{116,118}Sn$ for various effective interactions

		Gauss	Reid	Schiffer	Gauss+Q+O	exp.[16,17]
^{116}Sn	BE2↑(g.s.→2_1^+)	1340	805	1700	1570	2160±50
	BE3↑(g.s.→3_1^-)	40.8×10^3	48.6×10^3	10.4×10^3	49.6×10^3	$(220\pm90)\times10^3$
^{118}Sn	BE2↑(g.s.→2_1^+)	1130	880	1540	1300	2160±50
	BE3↑(g.S.→3_1^-)	40.9×10^3	47.6×10^3	12.6×10^3	49.6×10^3	$(170\pm70)\times10^3$

The BEλ-values are given in $e^2fm^{2\lambda}$. The experimental values are taken from refs.[16,17].
The different effective interactions are listed in sect.2.
The effective charge equals e_n=1.0.

The description of the properties of the collective 2_1^+ and 3_1^- is worsened. Our conclusion is that the Reid force yields no real improvement over the Gaussian force for the even nuclei.

Also the Schiffer force does not give an improvement of the description of the states of the even nuclei. Although its BE2-value is enlarged, the excitation-energy of the 2_1^+ state is found much too large. The small BE3-value of the 3_1^- states is due to the fact that using the Schiffer force this state has mainly the (1g7/2, 1h11/2) configuration, instead of the(2d5/2, 1h11/2) configuration, as for the other effective interactions.

Adding explicitly a quadrupole and an octupole force to the (Gaussian) effective interaction improves the description of the collective 2_1^+ and 3_1^- states. However it worsens the positions of the other excited states. Especially the first excited 0^+ state is shifted too much, indicating that the pairing part of the interaction is not adequate. So by admixing a quadrupole interaction one improves the energy of the 2_1^+ state at the cost of a less good description of the other excited states.

As for other effective interactions, in refs.[1,3] it has been shown that also the surface-delta-interaction(SDI) does not yield better results than the Gaussian forces do. We find also that a change in the range parameter μ of the Gaussian forces does not improve the results.

4. Summary and conclusions

It was attempted to improve the description of the collective 2^+ and 3^- states in even SCS nuclei. This was done by using different effective interactions, viz. a "realistic" interaction derived from the Reid soft core potential [9] and the Schiffer interaction[10], instead of the Gaussian forces which were used earlier[1-6]. As an example the Sn isotopes were considered. The model parameters were determined consistently for each effective interaction.

For the odd nuclei the results of the Reid force and the Schiffer force are better than those of the Gaussian forces. However the spectra of the even nuclei are not improved by using these interactions. One may conclude that the "realistic" interaction does not yield better results than the Gaussian one. Explicit admixture of a quadrupole and an octupole part in the effective interaction does improve the description of the collective 2^+ and 3^- states, but it makes the description of the other excited states worse.

So we conclude that the too large excitation-energies of the collective 2^+ and 3^- states found in earlier work [1-6], occur also for other reasonable forces considered here. This conclusion is also supported by recent results of other authors[18].

Part of the discrepancy between the calculated and experimental excitation-energies can be removed by using a BCS pair distribution which has been optimized for the excited states, as in ref.[8]. Also inclusion of particle-hole excitations in the closed shell will bring down the collective 2^+ and 3^- states, especially the latter[2,7]. We think that for the N=50, 82 nuclei inclusion of these two effects can account for the observed discrepancy. However for the Z=50 nuclei a small discrepancy is left, which might be removed by taking into account four-quasi-particle excitations in the open shell. An indication is that for the Z=50 nuclei there is experimental evidence of the occurrence of low-lying four-quasiparticle states[1], whereas in the N=82 nuclei the lowest states appear to be almost pure two-quasiparticle states[4].

CHAPTER 6

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