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INTERFERENCE EFFECTS IN INELASTIC PROTON SCATTERING

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ABSTRACT : Interference effects in ^{28}Si (p,p') inelastic scattering are studied in the Coulomb barrier region. The interference occurs between compound contributions and direct inelastic transitions. The asymmetric resonance shape is analysed to extract the intrinsic shape of the excited target nucleus.

KEYWORDS

E | NUCLEAR REACTION ^{28}Si(p,p'), E = 3.1-3.8 MeV ; measured $\sigma(E,\theta)$.Deduced ^{28}Si sign of β_2,optical parameters. Natural target.

1. INTRODUCTION

Inelastic processes due to direct Coulomb or/and nuclear effects have been extensively employed in measurements of B (E_λ) values or deformation parameters of collective nuclei. Unfortunately, the cross-sections are not sensitive to the nature of the intrinsic shape since, in first order, they involve β_λ^2 or B (E_λ), whereas the intrinsic shape (oblate, prolate) is defined by the square-root of these quantities.

In sect. 2. we describe a search for interference effects in proton inelastic scattering which could give this information. The proton beam was provided by the 4 MV Van de Graaff accelerator of Bordeaux. As target nucleus we have chosen ^{28}Si for which the static deformation is known. The experimental evidence of the interference effect is a clear asymmetry of the Breit-Wigner shape as known in resonant elastic scattering[1,2]. Experimentally, the resonance must be isolated and have a smooth background ; the later is assumed to be due to direct processes. The interference is thus interpreted as a coupling of the direct inelastic scattering to a state of the compound system[3-6].

2. INTERFERENCE IN RESONANT INELASTIC SCATTERING

2.1 MOTIVATION OF THE EXPERIMENT

Let us consider a strongly deformed light even-even nucleus of which the first $J^{\pi} = 2^{+}$ state is excited through the scattering of a proton. Two kinds of excitations can be expected : the direct inelastic transition, due to nuclear plus Coulomb fields, and the compound contributions. If the compound inelastic width is not negligible ,the latter contribution appears in the well known resonant form. Since we deal with a light nucleus and low incident energy ($E_p < 4$ MeV) the states of the compound system are well spaced. Thus, in optimal phase-shift conditions, the occurrence of these two processes will give an interference between the two amplitudes. We briefly explain below how the measurement of this effect in the differential inelastic cross-section can be related to the intrinsic shape of the excited nucleus.

2.2 METHOD OF CALCULATION

The direct matrix element may be estimated by DWBA or by coupled-channel calculations.

The first $J^{\pi} = 2^{+}$ state of the light target (^{28}Si) was simply considered to be of rotational nature. Expansions of Coulomb and nuclear potentials in terms of quadrupole deformation parameters give rise, in DWBA, to the well known form factor:

$$F(r) = \left\{ \frac{3}{5} (R_0^C)^2 \frac{Z_1 Z_2 e^2}{r^3} \beta_2^C - \frac{R_0^N}{a} |V_0| \beta_2^N \left[\frac{\mathcal{E}}{(1+\mathcal{E})^2} \right]_{R = R_0^N} \right\} Y_2^0(\underset{\sim}{r}, \underset{\sim}{\Omega})$$

$$\mathcal{E} = \exp\left(\frac{r - R - R_f}{a}\right) \quad \text{and} \quad R = R_0^{C,N} \left(1 + \beta_2^{C,N} Y_2^0(\underset{\sim}{r}, \underset{\sim}{\Omega}) + \cdots \right)$$

where R_p is the projectile radius , a the diffuseness, V_0 the real value of the Woods-Saxon potential, Ω the intrinsic axis and β_2^C , β_2^N the Coulomb and nuclear quadrupole deformation parameters respectively. The direct scattering amplitude, when calculated by DWBA[7,8]), is proportionnal to β_2, the assumed common value of β_2^C and β_2^N .

More precise calculations are performed by estimating this direct inelastic scattering contribution when coupling the elastic and inelastic channels. Our analysis will be chiefly based on these calculations, performed in a macroscopic description with the program ECIS[9].) In this case the matrix element must be obtained from computer calculations since there is no longer a simple expression for it.

On the other hand, the collision matrix element relative to the compound contribution, expressed in the S matrix[11]) or R matrix[2]) formalisms for a single isolated resonance of spin J_o is :

$$\mathcal{U}^{J_o}_{L_i s_i, L_f s_f} = \exp(i\varphi_{L_i}) \left\{ \delta_{if} + \frac{i\, \Gamma^{1/2}_{L_i s_i} \Gamma^{1/2}_{L_f s_f}}{\varepsilon_R - \varepsilon - i\frac{\Gamma}{2}} \right\} \exp(i\varphi_{L_f})$$

The total scattering amplitude will be obtained by adding this expression to the direct part under the following form :

$$A(M; 0^+ \to 2^+) = -i \frac{\pi^{1/2}}{k_i} \sum_{J, l_i, l_f} (2l_i+1)^{1/2} \langle s_i\, l_i\, \nu_i\, 0 | J M \rangle \langle s_f\, l_f\, \nu_f\, m_f | J M \rangle \, \mathcal{U}^{J}_{l_i s_i l_f s_f} \, Y^{m_f}_{l_f}(\theta_f, \varphi_f)$$

with

$$\mathcal{U}^{J}_{l_i s_i l_f s_f} = \mathcal{U}^{J_o}_{L_i s_i L_f s_f} \, \delta_{J J_o} + \mathcal{U}_{\text{direct}}^{J}$$

Since U^J_{direct} is, in first order, proportional to β_2 we can see that, in favorable phase-shift conditions, interference effects may be expected in the differential inelastic cross-section. Depending on the sign of β_2, a steep descent, or a dip, will occur on one side or the other of the Breit-Wigner shape since U^J_{direct} does not change rapidly with the incident energy.

A few comments are to be made relative to the calculation of the above expression since the value of the different phases is a crucial point in such a coherent summation. Usually a certain freedom in phase definitions does not affect the calculation of cross-sections when one process is involved only. As we deal with two different routes in the inelastic transition we have to adopt the same phase conventions for both the coupled channel calculations (performed with ECIS) and the resonant term. An illustration of this problem is given by the expression $\varphi_L = \sigma_L - \phi_L$, that we had to use instead of the usual expression $\varphi_L = \sigma_L - \sigma_0 - \phi_L$ to be consistent with the calculation of the direct part[10]) (σ and ϕ being the Coulomb and hard-sphere shifts respectively). This convention was precisely checked in fitting our resonant elastic scattering data for which the Rutherford amplitude is used as an off resonant term with well known phases.

Furthermore, attention must be paid to the $\Gamma^{1/2}_{L_iS_i}\ \Gamma^{1/2}_{L_fS_f}$ product since its algebric value is a priori not known in inelastic transitions. A discussion on the estimate of the involved reduced widths will be developped following Satchler's study[13].)

The strong coupling Nilsson model was used as well as an analogy with the weak-coupling model (one-phonon state) to take into account the implied $J^\pi = 2^+$ state of the core.

Finally, the direct matrix element, as well as the reduced widths, computed in j-j coupling, are expressed in the channel spin formalism[12,13].)

2.3 EXPERIMENTAL RESULTS : $^{28}Si(p,p_0)$ and $^{28}Si(p,p_1)$ between E_p = 3.1 and 3.8 MeV

In this range of incident energy many resonances occur in both elastic and inelastic channels[14-16].) We are mainly interested in the inelastic channel and the occurrence of a clearly deformed resonance, isolated on a smooth background. Such characteristics and their experimental evidence are not usually reported because several conditions are required for an observation : low incident energy (in order to reach a single level of the compound system), non-negligible direct background, suitable phase-shifts to allow the observation.

Excitation functions in elastic and inelastic channels were measured between E_p = 3.1 and 3.8 MeV at four angles θ = 163°,150°,135°,70°. A natural silicon target ,30 $\mu g/cm^2$ thick, was used. The beam resolution was 1.5 keV. As shown in fig. 1 ,a good separation of the p_0,p_1 peaks was allowed by kinematics. A PDP 15 computer was used for peak integrations. In fig. 2 we show only the most interesting results for our purpose. A clear interference pattern occurs in the inelastic excitation function (I) in the vicinity of the E_p = 3.337 MeV resonance ($J_o^\pi = 3/2^+$ state of ^{29}P at E_x = 5.966 MeV). In part (II) is also plotted the elastic excitation function. The characteristic feature of the searched for interference is a dip on the high-energy side of the resonance, and this effect is best detected at backward angles. A search for similar examples has been performed in the same energy range with several s-d shell nuclei, but was unfruitful, except with ^{24}Mg (the weak 2.75 MeV resonance in ^{24}Mg(p,p_1) seems to exhibit such effects). A weak interference effect was also noticed a few years ago by Temmer et al[5]) concerning the E_p = 0.870 MeV resonance in ^{23}Na(p,p') ; the results have been analysed on a general point of view by Micu[4]) and by Griffy et al[6]).

In our analysis, the different characteristics of the resonance[14-16]) have been used to estimate the resonant term. Hard-sphere and Coulomb phase-shifts, as well as penetrabilities, were computed for each set of L_i, L_f angular momenta.

The square-roots of the elastic and inelastic partial widths have been computed from the expression :

$$\Gamma_{LS}^{1/2} = \left(\frac{2kR}{F_L^2 + G_L^2} \right)^{1/2} \theta_{SL}$$

in which we are chiefly concerned with the reduced widths, expressed either in the channel spin formalism (θ_{SL}) or under the form θ_{jL}[13].) In view of estimating the product of partial widths, calculations of θ_{jL} in the frame of collective models have been performed, following different studies about the first excited states of ^{29}Si or ^{29}P[17,18].) The $J^\pi = 3/2^+$ state we are concerned with, which is located at a somewhat high excitation energy, might be a member of the rotational states built either on the single particle intrinsic Nilsson orbits $|K = \frac{3}{2}^+, d\,\frac{3}{2}, \text{No.}8 >$, $|K = \frac{1}{2}^+, d\,\frac{3}{2}, \text{No.}11 >$ or $|K = \frac{1}{2}^+, s\,\frac{1}{2}, \text{No.}9 >$. A coupling with the $J^\pi = 2^+$ state of the rotating core may be involved too, in analogy with the weak coupling model. Due to the location in energy of this state we adopt the following assumption : the elastic reduced width is estimated in the frame of the $\left[\ {}^{28}Si(I_o = 0^+ ; K_o = 0) \otimes p_{|11>}\ \right]^{3/2^+}$ coupling and the inelastic reduced width by the $\left[\ {}^{28}Si(I_o = 2^+ ; K_o = 0) \otimes p_{|9>}\ \right]^{3/2^+}$ coupling. In the latter case, due to the effect of penetrabilities the $L_f = 0$ contribution is only taken into account in the discussion. Taking into account orbit No.8 does not change the final qualitative result on the sign of the reduced width product. We then use the expression[13] :

$$\theta_{jL}(K_i=0, \Omega_i=0 ; K_f \Omega_f) = \left(\frac{2(2I_i+1)}{2I_f+1}\right)^{1/2} \langle I_f K_f | I_i K_i j \Omega_f \rangle \; C_{jL}(\Omega_f)\, \theta_o$$

in which the C_{jL}'s are drawn from ref. 19. We write :

$$\theta_i = \theta_{3/2\,2}(00, 1/2\, 1/2) = \frac{1}{\sqrt{2}} \langle 3/2\, 1/2 | 0\,0\, 3/2\, 1/2 \rangle\, C_{3/2\,2}(1/2)\,\theta_o = \frac{1}{\sqrt{2}} C_{3/2\,2}(1/2)\,\theta_o$$

$$\theta_f = \theta_{1/2\,0}(00, 1/2\, 1/2) = \sqrt{\frac{5}{2}} \langle 3/2\, 1/2 | 2\,0\, 1/2\, 1/2 \rangle\, C_{1/2\,0}(1/2)\,\theta_o = - C_{1/2\,0}(1/2)\,\theta_o$$

The C_{jL}'s are found to be positive for any deformation (using the probable value $\beta = -0.3$ we find $\frac{\theta_i}{\theta_f} = -1.4$.) Transformation in the channel spin formalism through the expression :

$$\theta_{SL} = \theta_{jL} \sqrt{(2S+1)(2j+1)}\; W(I_i\, 1/2\, I_f\, L ; S\, j)^{\,j-L-1/2}$$

leads to :

$$\theta_{i\,(SL)} = -\theta_{i\,(jL)} \quad \text{and} \quad \theta_{f\,(SL)} = +\theta_{f\,(jL)}$$

Due only to vector coupling the net result is thus a positive value for the $\Gamma_i^{1/2}\ \Gamma_f^{1/2}$ product. Furthermore, with the above assumption on β, the ratio :

$$\frac{\Gamma_i}{\Gamma_f} = \left[\frac{P_{(L_i=2)}}{P_{(L_f=0)}}\right]_{R=6\,fm} \cdot \left[\frac{\theta_i}{\theta_f}\right]^2 \simeq 2.2 \times 1.9 \simeq 4.2$$

when the experimental one is $\frac{\Gamma_i}{\Gamma_f} \approx \frac{6\text{ keV}}{2\text{ keV}} \approx 3$.

Coupled-channel calculations have been performed to estimate the direct matrix element. This direct contribution was computed by fitting the angular distributions measured out of resonance, as shown in fig. 3. The background seems to be of nuclear origin chiefly, since an estimate of the Coulomb excitation gives a factor one tenth of the observed cross-section. The elastic angular distributions were fitted either by the computer programs MAGALI or ECIS.[9]) More specifically, ECIS was used for a simultaneous calculation of the elastic and inelastic direct contributions. In this case we employed a different set of optical parameters from the one used in DWBA as commented on in ref. 20. Although the angular distributions are well reproduced in shape for both channels, the calculated inelastic absolute values are somewhat small $d\sigma_{exp.}/d\sigma_{theor.} = 2.5$. Nevertheless, we assume that the corresponding complex matrix element, related to the coefficients C_f^i as defined in ref. 10 and "S matrix elements" in ECIS, needs only to be normalised for a coherent summation with the resonant term. This means that we consider that our phase conventions, as well as vector coupling coefficients, are correct and that the real and imaginary parts of the direct matrix element (C_f^i) coming from experimental comparison need only, for an unknown reason, to be multiplied by $(2.5)^{1/2}$. The typical shape, obtained at 163°, is shown in fig. 4. The results, which are to be compared with those of fig. 2, are in favour of $\beta_2 < 0$. This choice leads to the lowering of the differential cross-section experimentally present on the right side of the resonance shape, and is in agreement with the oblate intrinsic shape of ^{28}Si already known[21-24]) from reorientation experiments.

3. CONCLUSION

Among the different features of inelastic scattering displayed either with light or heavy projectiles the exhibition of interference effects in excitation functions may be quite sensitive to nuclear shapes and potential parameters. This work was motivated in bringing a new kind of experimental probe to the study of the intrinsic shapes of nuclei. Contrary to the already known interference effects occurring near the Coulomb barrier in the inelastic scattering of heavy ions the kind of interference reported here seems more directly related to the sign of the deformation parameters, even if a complete theoretical study becomes rapidly complex. Unfortunately, because of the reasons given earlier, the experimental evidence for such effects is rather scarce. We get the most favorable conditions for them when using light projectiles in the Coulomb barrier region, in order to have a simultaneous presence of direct transitions and single-resonance contributions.

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FIGURE CAPTIONS

Fig. 1 Particle spectrum obtained with protons of 3.337 MeV at a detection angle of 163 degrees on a ^{28}Si target.

Fig. 2 Excitation functions in p + ^{28}Si around 3.337 MeV at a detection angle of 163° : I) Inelastic scattering ; II) Elastic scattering.

Fig. 3 Angular distribution for protons of 3.233 MeV (I) and 3.406 MeV (II) elastically (a) and inelastically (b) scattered from ^{28}Si. Above, the solid line is a DWBA calculation with the following parameters :
r_c = 1.45 fm, V_R = -60 MeV, r_{oR} = 1.925 fm, a_R = 0.36 fm, W_I = -3.50 MeV, r_{oI} = 1.925 fm, a_I = 0.46 fm.
Below, the solid line is a coupled channel calculation for the inelastic scattering (ECIS Code) with the following parameters which also fit the elastic scattering at 3.233 MeV
r_c = 1.45 fm, V_R = -77.22 MeV, r_{oR} = 1.925 fm a_R = 0.45 fm
W_I = -3.79 MeV, r_{oR} = 1.925 fm a_I = 0.64 fm
β_2 = -0.35
At 3.406 MeV, the imaginary part of the potential is modified i.e. : -7.45 MeV (DWBA) ; -6.30 MeV (coupled channel).

Fig. 4 Shape of the resonance of 3.337 MeV in the inelastic scattering channel calculated with $\beta_2 < 0$ (solid line) and $\beta_2 > 0$ (dashed line) at 163°.

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Fig. 1

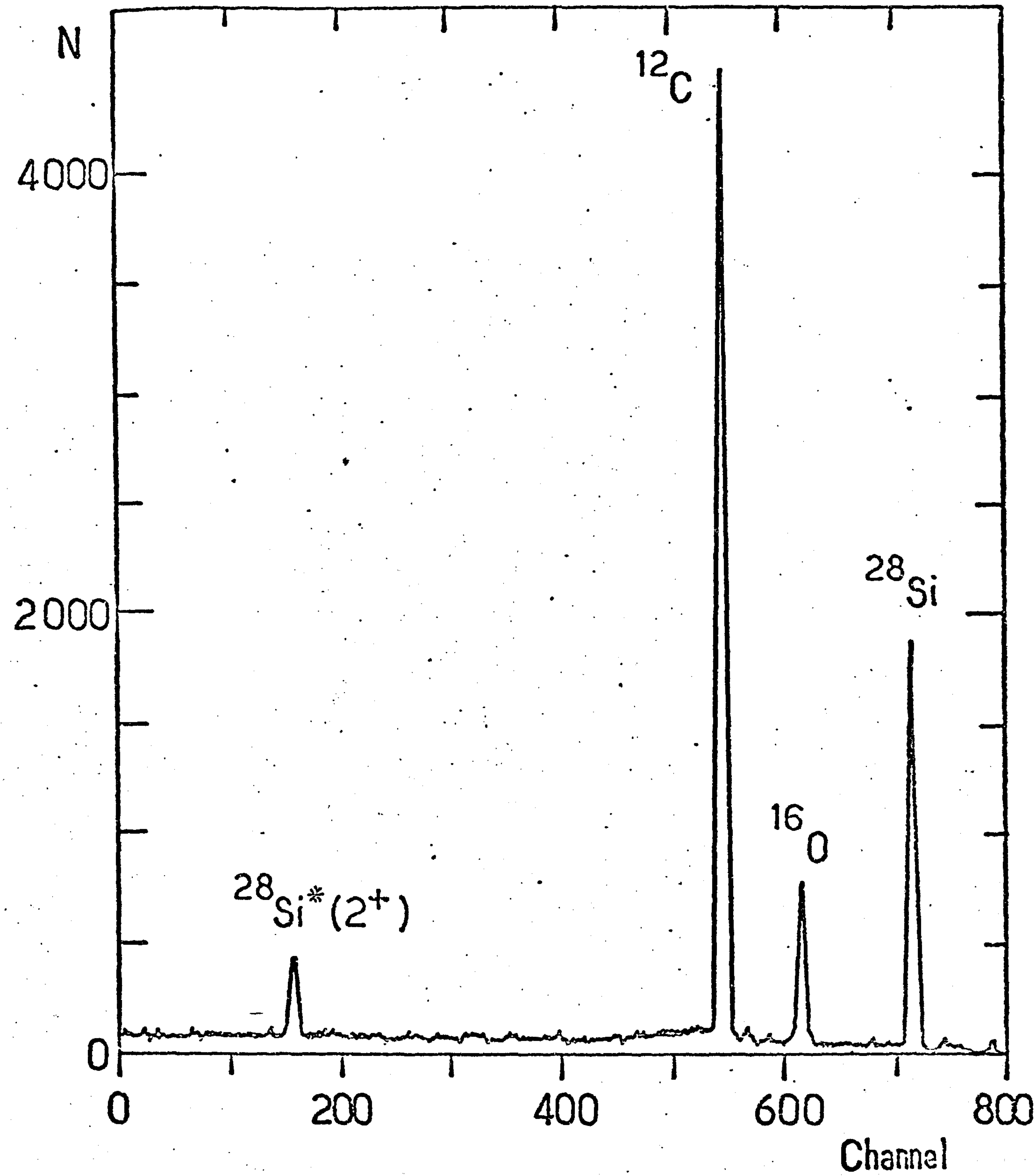

Fig. 2

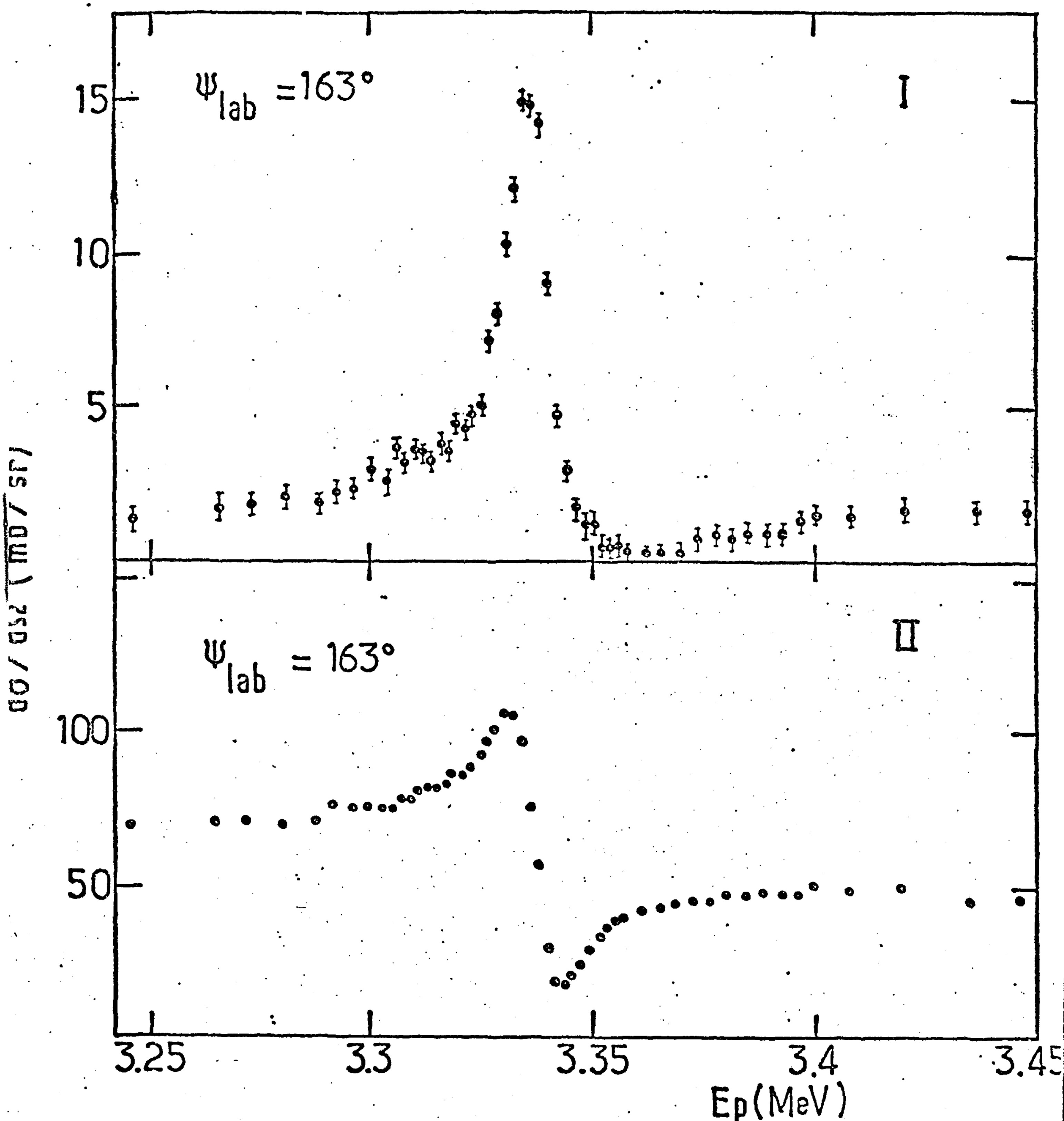

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Fig. 3

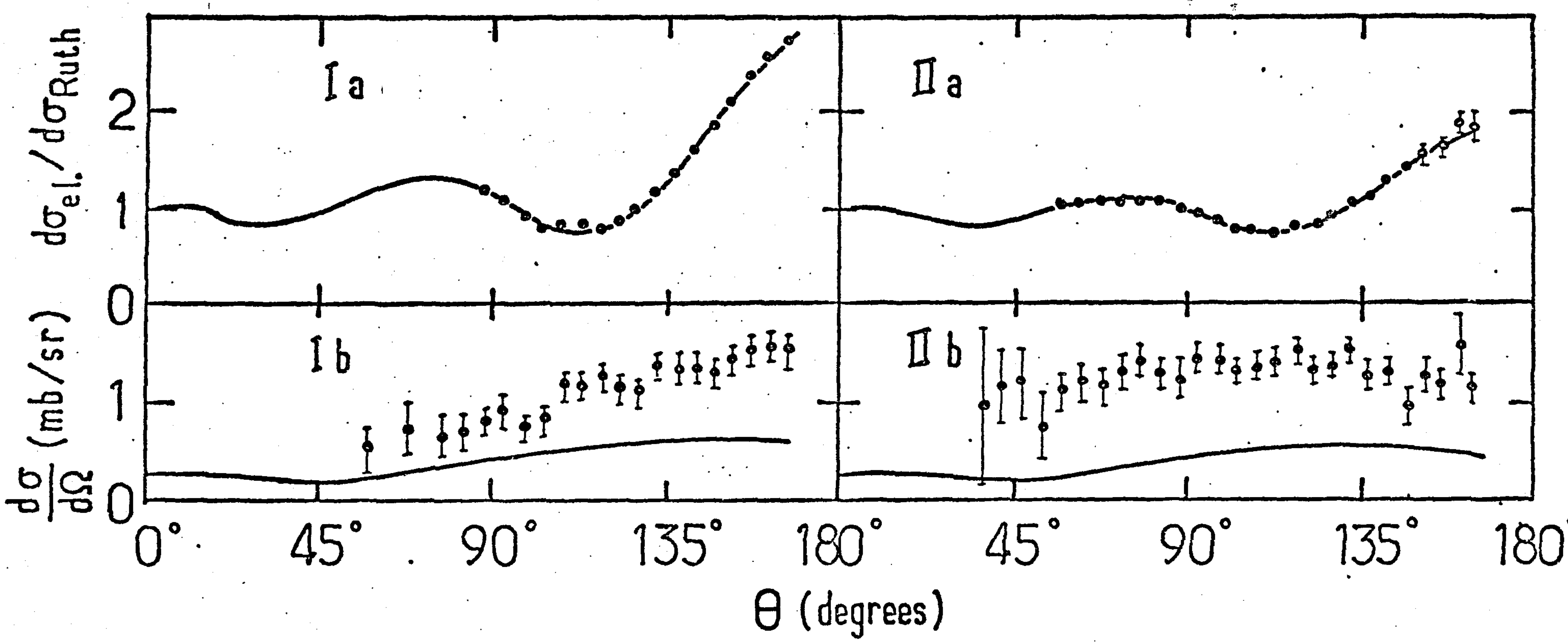

Fig. 4