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Banana Drift Diffusion in a Tokamak Magnetic Field with Ripples

K. T. Tsang

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OAK RIDGE NATIONAL LABORATORY
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BANANA DRIFT DIFFUSION IN A TOKAMAK MAGNETIC FIELD WITH RIPPLES*

K. T. Tsang
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

ABSTRACT

The drift motions of trapped particles in a tokamak field with ripples lead to a new transport process in large tokamaks in addition to the diffusion process due to particles trapped in the ripples. We study this problem by solving the bounce averaged drift kinetic equation with a model collision operator. It is shown that the "banana drift diffusion" is proportional to the collision frequency when the poloidal banana drift frequency is smaller than the effective collision frequency. This result is contrary to earlier predictions. In a reactor regime, this loss mechanism is shown to be unimportant.

1. INTRODUCTION

The discrete nature of toroidal magnetic field coils of a tokamak introduces a small, but finite, amplitude modulation on the toroidal magnetic field. These small field modulations in the toroidal direction are usually called "ripples"; their existence destroys the axisymmetry of an ideal tokamak and leads to large particle excursions and hence to particle and energy losses. There are three types of particles in such a magnetic field geometry: those trapped in the ripples which we called "ripple-trapped" particles, those trapped in the toroidal field

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modulation which are called "bananas" because of the shape of their orbits, and those untrapped in both magnetic wells. Much of the previous effort [1, 2] in understanding the effects of ripples has been devoted to the particle and energy losses by ripple-trapped particles. In a recent report [3], Davidson raised the possibility of particle and energy losses — which he claimed to be larger than the losses due to ripple-trapped particles — through the effect of ripples on the banana particles.

This new loss mechanism can be better understood by looking at the effect of ripples on the banana particles. In an axisymmetric tokamak, i.e., without ripples, the banana orbit closes by itself. After bouncing once in the toroidal field, the banana particle goes back precisely to its starting position. With the introduction of ripples, the banana orbit is no longer closed; instead, it drifts in the radial as well as in the poloidal directions. It is this kind of banana drift motion that leads to the new particle and energy transports.

By assuming that the longitudinal adiabatic invariant J is conserved while the banana is drifting, one can show that the banana drift motion has a finite amplitude proportional to the ripple size. In Section 2, we investigate the single particle drift motion by evaluation of J. Introducing collisions, we can estimate the scaling of the diffusion coefficient. We show that Davidson's calculation [3] corresponds to the limit when the effective collision frequency of the banana particle is larger than the frequency of drift motion of the banana in the ripple field. In Section 3, the drift kinetic equation is averaged over the banana bounce motion. The resulting equation is then solved by using a model collision operator. Transport coefficients valid for all collisional regimes are calculated.

2. DRIFT MOTION OF BANANA

The longitudinal adiabatic invariant J for the banana particle is defined by

$$J = \oint \upsilon_{\parallel} \, d\ell \quad ,$$

where $\upsilon_{\parallel}$ is the parallel velocity of the particle, $d\ell$ is the line element along the magnetic field, and the integration is performed between the turning points. For a particle with speed υ and magnetic moment μ, $\upsilon_{\parallel}$ can be written as $\upsilon_{\parallel} = \sqrt{\upsilon^2 - 2\mu B_o}$, where B is the magnitude of the magnetic field. For a tokamak field resulting from the use of N toroidal field coils, the B field is chosen to be

$$\vec{B} = \frac{B_0}{1 + \epsilon \cos \theta} \left[\frac{\epsilon}{q} \hat{\theta} + [1 - \delta(r,\theta) \cos N\phi] \hat{\phi} \right] + \vec{B_r} \quad , \tag{1}$$

where r, θ, and ϕ are the usual toroidal coordinates, $\delta(r,\theta)$ is the size of the ripples, $\epsilon = r/R$, R is the major radius, and q is the safety factor. The dependence of δ on r and θ varies with the shape of the toroidal field coils, the shapes of the magnetic flux surfaces we assumed, and the location of the flux surfaces with respect to the coil. For a D-shaped coil with circular flux surfaces centered at the minimum of δ, we can approximate

$$\delta(r,\theta) \simeq \delta_0 (r/a)^n \exp(-\alpha\theta^2) \quad , \tag{2}$$

where a is the minor radius of the plasma column. The radial component of B is determined by $\nabla \cdot B = 0$ which gives

$$B_r = -B_0\ \delta(r,\theta)\ K(r,\theta)\ \epsilon N \sin(N\phi) \quad ,$$

where $\delta(r,\theta)\ K(r,\theta) \equiv \int_0^1 \frac{\delta(\chi r,\theta)\chi}{1+\chi\epsilon \cos\theta}\, d\chi$.

Hence to order δ, we have

$$B \simeq B_0(1 - \delta \cos N\phi)/(1 + \epsilon \cos\theta) \ ,$$

and $d\ell \simeq Rq(1 - \delta \cos N\phi)d\theta$

$$\approx Rq(r_0)\ [1 - \delta \cos N\phi + \frac{\Delta r}{r_0} Q(r_0)] \quad ,$$

where $Q = \partial \ln q/\partial \ln r$,

$$\Delta r \simeq -r_0 q\ N \int_0^\theta \delta(r,\theta')\ K(r,\theta')\ \sin N(q\theta' + \theta_0)d\theta' \quad ,$$

and θ_0 is a constant in $\phi = q\theta + \theta_0$.

In obtaining $d\ell$, an expansion of q around the unperturbed flux surface at $r = r_0$ has been performed.

Combining all these and defining a pitch angle variable $M \equiv [\upsilon^2/2 - \mu B_0(1-\epsilon)]/2\mu B_0\epsilon$, we can evaluate J to the lowest order in $1/Nq$,

$$J = J_0 + J_1 \ , \qquad (3)$$

where

$J_0 = 8(\mu B_0\epsilon)^{1/2}\ Rq[E(M) - (1-M)\ K(M)]$,

$J_1 = 4Rq(\mu B_0\epsilon)^{1/2}\ (\cos N\theta_0)\ I$,

$I = -NqQ \int_0^\zeta d\theta \int_0^\theta \delta(r,\theta')K(r,\theta')\ \sin Nq\theta' d\theta' (M - \sin \frac{\theta}{2})^{1/2}$,

$\zeta = 2 \sin^{-1} M$, and K and E are complete elliptic integrals of first and second kind.

Note that M equals zero for deeply trapped particles, and $M = 1$ is the boundary between trapped and circulating particles.

At this point, we introduce the coordinates α and β which are determined by

$$\vec{\nabla}\alpha \times \vec{\nabla}\beta = \vec{B} \quad .$$

For the form of magnetic field given by Eq. (1), neglecting small corrections due to ϵ and δ, we have

$$\beta = q\theta - \phi = -\theta_0 \quad ,$$

and

$$d\alpha = B_0(r/q)dr \quad .$$

In the α, β coordinates, the banana drift velocity is given by [4]:

$$\begin{aligned} \bar{\dot{\alpha}} &= \frac{mc\upsilon}{e} \left(\frac{\partial J}{\partial \beta}\right)\left(\frac{\partial J}{\partial \upsilon}\right)^{-1} \\ \bar{\dot{\beta}} &= -\frac{mc\upsilon}{e} \left(\frac{\partial J}{\partial \alpha}\right)\left(\frac{\partial J}{\partial \upsilon}\right)^{-1} \end{aligned} \quad , \qquad (4)$$

where the dots over α and β represent the time derivatives, the bars the average over the banana orbit, c is the velocity of light, m and e are the mass and charge of the particle, respectively.

From Eq. (3) we have

$$\frac{1}{\upsilon}\frac{\partial J_1}{\partial \upsilon} = 2RqK(M)/(\mu B_0 \epsilon)^{1/2} \quad ,$$

$$\frac{\partial J}{\partial \beta} \approx \frac{\partial J_1}{\partial \beta} = -4RqN(\mu B_0 \epsilon)^{1/2} (\sin N\beta)\, I \quad ,$$

$$\frac{\partial J}{\partial \alpha} = \frac{q}{rB_0}\frac{\partial J_0}{\partial r} = \frac{8Rq^2}{rB_0}(\mu B_0 \epsilon)^{1/2} \left\{ \frac{\partial}{\partial r} (\ln q\epsilon^{1/2}) \, [E(M) - (1-M)\, K(M)] - \frac{K(M)}{2r}\left(M - \frac{1}{2}\right) \right\} \quad . \qquad (5)$$

Using the form of δ given by Eq. (2), I can be evaluated to the lowest order in 1/Nq.

$$I \approx \frac{Q}{n+2}\int_0^{\zeta} d\theta\left(M - \sin^2\frac{\theta}{2}\right)^{1/2}\delta_0(r/a)^n$$
$$= 2Q\delta_0(r/a)^n\,[E(M) - (1-M)\,K(M)]/(n+2) \quad . \tag{6}$$

We can now estimate the order of magnitude of the banana radial drift velocity from Eqs. (4–6):

$$\bar{\dot{r}} \sim NqQ\delta_0 v^2/R\Omega \quad ,$$

where $\Omega = eB_0/mc$.

This bounce average radial drift velocity $\bar{\dot{r}}$ is a sinusoidal function of $N\beta$. Hence the radial drift motion due to the ripples is a kind of oscillation whose amplitude is finite and proportional to δ. To estimate the frequency of such oscillation, we need the amplitude of the radial drift,

$$\tilde{r} = \int d\ell\; Br/B \sim r\delta_0 \quad .$$

Therefore, the time needed to complete one oscillation is

$$\tilde{\tau} \sim \tilde{r}/\tilde{\dot{r}} \sim rR\Omega/(NqQv^2) \quad .$$

The diffusion coefficient due to this banana drift motion can be estimated in two limits. When the effective collision time ϵ/ν for the banana is shorter than τ, i.e.

$$\nu/\epsilon > nqQv^2/rR\Omega \quad ,$$

where ν is the collision frequency, the diffusion coefficient is roughly

$$D \sim (\bar{r})^2 \epsilon^{3/2}/\nu \quad . \tag{7}$$

This is essentially Davidson's result [3]. However, when ϵ/ν is longer than τ, i.e.,

$$\nu/\epsilon < NqQ\upsilon^2/rR\Omega \quad ,$$

the step size due to banana drift motion is $\tilde{r}$ and hence the diffusion coefficient is

$$D \sim \tilde{r}^2 \nu(\epsilon)^{-1/2} \quad . \tag{8}$$

The condition under which Eq. (7) is valid can be rewritten as

$$\nu_* > Nq^2Q\upsilon/r\Omega\epsilon^{1/2} \quad , \tag{8a}$$

where $\nu_* = \nu Rqm^{1/2}/\epsilon^{3/2}T^{1/2}$ is the ratio of effective collision frequency to average bounce frequency of the banana particles. For typical reactor parameters such as $N = 20$, $q = 2.5$, $Q = 1$, $T_i = 10$ KeV, $B = 45$ kg, $r = 100$ cm, and $\epsilon = 1/3$, we need

$$\nu_{*i} > 0.68 \quad , \tag{9}$$

in order for Davidson's result to be valid. Thus, Eq. (8a) is a very restrictive condition.

3. BANANA KINETIC EQUATION

In this section, analytic calculation of the transport coefficients due to the banana drift motion is presented. Following the development in Ref. [4], we first perform gyro average and then bounce average on the Fokker-Planck equation, and obtain the following equation for the particles trapped in the toroidal field:

$$\frac{e}{mc}\frac{\partial f}{\partial t}\frac{\partial J}{\upsilon\partial\upsilon}+\frac{\partial f}{\partial\alpha}\frac{\partial J}{\partial\beta}-\frac{\partial f}{\partial\beta}\frac{\partial J}{\partial\alpha}=\frac{e}{mc}\oint\frac{d\ell}{\upsilon_{\parallel}}C(f) \quad , \tag{10}$$

where $C(f)$ is the collision operator. For equilibrium, Eq. (10) becomes

$$\frac{\partial f}{\partial\alpha}\frac{\partial J}{\partial\beta}-\frac{\partial f}{\partial\beta}\frac{\partial J}{\partial\alpha}=\frac{e}{mc}\oint\frac{d\ell}{\upsilon_{\parallel}}C(f) \quad , \tag{11}$$

which is the equation we need for the distribution function of bananas. From Eq. (5) we know $\partial J/\partial\beta$ is of the order of δ, while $\partial J/\partial\alpha$ is of the order of unity. Expanding Eq. (11) in δ, we have

$$\frac{\partial f_0}{\partial\alpha}\frac{\partial J}{\partial\beta}-\frac{\partial f_1}{\partial\beta}\frac{\partial J}{\partial\alpha}=\frac{e}{mc}\oint\frac{d\ell}{\upsilon_{\parallel}}C(f_1) \quad , \tag{12}$$

where $f_0 = N_0\left(\frac{M}{2\pi T}\right)^{3/2}\exp\left(-\frac{m\upsilon^2}{2T}\right)$ is the lowest order Maxwellian equilibrium in δ, N_0 is the density, and T is the temperature.

To solve Eq. (12), we have to assume a form of collision operator. Since it is the ions which are most susceptible to this loss, we need to solve Eq. (12) for ions only. Hence, only ion-ion collisions are included in $C(f_1)$ [5].

$$C(f_1) = \nu\frac{\upsilon_{\parallel}}{B}\frac{\partial}{\partial\mu}\mu\,\upsilon_{\parallel}\frac{\partial f_1}{\partial\mu}+\nu\,\upsilon_{\parallel}\,Pf_0 \quad ,$$

where ν is the collision frequency and P is to be determined self-consistently by conservation of momentum. The bounce average of $C(f_1)$ leads to

$$\int \frac{d\ell}{\upsilon_{\parallel}} C(f_1) = \frac{\nu}{\epsilon K(M)} \left(\frac{1}{\upsilon}\frac{\partial J}{\partial \upsilon}\right) \frac{\partial}{\partial M} \left[E(M) - (1-M)\, K(M)\right] \frac{\partial f_1}{\partial M} \quad , \tag{13}$$

in which the momentum conserving term does not contribute because its parity is odd in the parallel velocity.

Combining Eqs. (5), (12), and (13), we set

$$\frac{\partial f_0}{\partial \alpha} (\bar{\dot{\alpha}})_0 \left(\frac{E}{K} - 1 + M\right) \sin N\beta - \frac{\partial f_1}{\partial \beta} (\bar{\dot{\beta}})_0 \left[\left(\frac{q'}{q} + \frac{1}{2r}\right)\left(\frac{E}{K} - 1 + M\right) - \frac{1}{2r}\left(M - \frac{1}{2}\right)\right] = \frac{N}{\epsilon}\left[\left(\frac{\epsilon}{K} - 1 + M\right)\frac{\partial^2 f_1}{\partial M^2} + \frac{1}{2}\frac{\partial f_1}{\partial M}\right] \quad , \tag{14}$$

where $(\bar{\dot{\alpha}})_0 \simeq 2N\epsilon\upsilon^2 Q\delta_0 (r/a)^n\, mc/e\ (n+2)$,

and $(\bar{\dot{\beta}})_0 \simeq 2\epsilon\nu^2 q/(r\Omega)$.

Equation (14) can be solved by separating f_1 to

$$f_1 = f_+\, e^{iN\beta} + f_-\, e^{-iN\beta} \quad .$$

In order that f_1 be real, f_+ and f_- are complex conjugates of each other: $f_+^* = f_-$, so we need only an equation governing one of them,

$$\frac{1}{2i}\frac{\partial f_0}{\partial \alpha} (\bar{\dot{\alpha}})_0 \left(\frac{E}{K} - 1 + M\right) + \frac{iN}{r} f_+ (\bar{\dot{\beta}})_0 \left[\left(Q + \frac{1}{2}\right)\left(\frac{E}{K} - 1 + M\right) - \frac{1}{2}\left(M - \frac{1}{2}\right)\right] + \frac{\nu}{\epsilon}\left[\left(\frac{E}{K} - 1 + M\right)\frac{\partial^2 f_+}{\partial M^2} + \frac{1}{2}\frac{\partial f_+}{\partial M}\right] = 0 \quad . \tag{15}$$

Equation (15) is a second order differential equation of f_+ with complicated coefficients. We cannot proceed unless we can replace $M - 1 + E/K$ by a tractable expression. This is possible by expanding E and K in small argument M,

$$\frac{E}{K} - 1 + M \approx M/2 \quad .$$

Except near $M = 1$, which is the boundary between circulating and trapped particles, $M/2$ is a fairly good approximation of $M - 1 + E/K$. Equation (15) is then further simplified to

$$\frac{\nu}{2\epsilon} \left(M \frac{\partial^2 f_+}{\partial M^2} + \frac{\partial f_+}{\partial M} \right) + \frac{iN}{r} f_+ \, (\bar{\dot{\beta}})_0 \left[\left(Q - \frac{1}{2} \right) \frac{M}{2} + \frac{1}{4} \right] = \frac{iM}{4} \frac{\partial f_0}{\partial \alpha} (\bar{\dot{\alpha}})_0 \quad . \tag{16}$$

We have to make one more assumption before we can solve Eq. (16) analytically. In the outer half of the plasma column where the effect of ripples is most significant, Q is usually larger than 1/2. Ignoring 1/4 compared with $(Q - \frac{1}{2})\, M/2$ will not introduce an artifical sign change of the coefficient of f_+. Such an approximation would change the answer only by a factor of unity, since only when $\partial J/\partial \alpha = 0$ can a new physical entity appear, namely the superbanana [6].

Using the boundary condition that $f_+ = 0$ at the trapped circulating boundary, i.e., $M = 1$, and $\partial f_+/\partial M$ is well behaved at $M = 0$, we can solve Eq. (16) approximately by expanding f_+ into a series of Bessel functions

$$f_+ = \sum_n A_n \, j_0(\alpha_n M) \quad , \tag{17a}$$

where j_0 is the zeroth order Bessel function and α_n is the nth zero of $j_0(x)$. The coefficients A_n can be obtained by substituting Eq. (17a) in Eq. (16) and using the orthogonality condition:

$$\int_0^1 t\ j_0(\alpha_m t)\ j_0(\alpha_n t)dt = \delta_{mn}[j_1(\alpha_n)]^2/2 \quad .$$

Thus we get

$$A_n = \frac{i}{2}\frac{\partial f_0}{\partial\alpha}\frac{(\bar{\dot{\alpha}})_0}{\alpha_n j_1(\alpha_n)}\left[-\frac{\nu}{2\epsilon}\alpha n^2 + \frac{iN}{2r}(\bar{\dot{\beta}})_0\ (Q - \frac{1}{2})\right]^{-1} \tag{17b}$$

Going back to Eq. (10), it is easy to show that the flux surface averaged particle and heat fluxes in α direction are

$$\Gamma_\alpha = \int \frac{d\theta}{2\pi}\int d\vec{V}\oint\frac{d\beta}{2\pi} f\ \bar{\dot{\alpha}} \quad ,$$

and

$$\tilde{Q}_\alpha = \int \frac{d\theta}{2\pi}\int d\vec{V}\oint\frac{d\beta}{2\pi} f\ \bar{\dot{\alpha}}\frac{m}{2}\nu^2 \quad .$$

Since $\bar{\dot{\alpha}}$ is proportional to sin Nβ, we have

$$\Gamma_\alpha = \int \frac{d\theta}{2\pi}\int d\vec{V}\ (\mathrm{Im} f_+)\ (\bar{\dot{\alpha}})_0\left(\frac{E}{K} - 1 + M\right) \quad ,$$

and , (18)

$$\tilde{Q}_\alpha = \int \frac{d\theta}{2\pi}\int d\vec{V}\ (\mathrm{Im} f_+)\ (\bar{\dot{\alpha}})_0\left(\frac{E}{K} - 1 + M\right)\frac{m}{2}\nu^2$$

where $\int dV$ is the integral over the trapped particles only and after averaging over θ, it is approximately $\int \frac{d\theta}{2\pi}\int dV \simeq 2\sqrt{2\epsilon}\ \pi\int_0^\infty \upsilon^2 d\upsilon\int_0^1 dM$. Combining Eqs. (17) and (18), we have

$$\Gamma_\alpha = \sum_{m=1}^{\infty}\frac{\pi(2\epsilon)^{1/2}}{4}\int_0^\infty \upsilon^2 d\upsilon\frac{(\nu/\epsilon)[(\bar{\dot{\alpha}})_0]^2(\partial f_0/\partial\alpha)}{(\nu/2\epsilon)^2\alpha_n^4 + (N/2r)^2(\bar{\dot{\beta}})_0^2\ (Q - 1/2)^2} \quad ,$$

$$\tilde{Q}_\alpha = \sum_{m=1}^{\infty}\frac{\pi(2\epsilon)^{1/2}}{4}\int_0^\infty \upsilon^2 d\upsilon\frac{(\nu/\epsilon)(\bar{\dot{\alpha}})_0^2(\partial f_0/\partial\alpha)m\upsilon^2/2}{(\nu/2\epsilon)^2\alpha_n^4 + (N/2r)^2(\bar{\dot{\beta}})_0^2\ (Q - 1/2)^2} \quad ,$$

which are essentially Davidson's result if we neglect the $(\vec{\beta})_0$ terms in the denominators. More explicitly worked out forms for Γ and $\widetilde{Q}$ are:

$$\Gamma\alpha = -\left(\frac{2}{\pi\epsilon}\right)^{1/2} \nu_{ii} \left[\frac{Q\delta_0(r/a)^n}{(m+2)(Q-1/2)} \frac{r^2B}{q}\right]^2 \left[\frac{N_0'}{N_0} + \frac{e\phi'}{T} \lambda_1 + \left(\lambda_2 - \frac{3}{2}\lambda_1\right)\frac{T'}{T}\right]N_0 ,$$

and

$$\widetilde{Q}_\alpha = -\left(\frac{2}{\pi\epsilon}\right)^{1/2} \nu_{ii}T\left[\frac{Q\delta_0\left(\frac{r}{a}\right)^n}{(n+2)\left(Q-\frac{1}{2}\right)} \frac{r^2B}{q}\right]^2 \left[\left(\frac{N_0'}{N_0} + \frac{e\phi'}{T}\right)\lambda_2 + \left(\lambda_3 - \frac{3}{2}\lambda_2\right)\frac{T'}{T}\right]N_0$$

where

$$\lambda_n = \sum_{j=1}^{\infty} \int_0^{\infty} \frac{d\chi\ \chi^4 \xi(\chi) e^{-\chi^2} \chi^{2n}}{\chi^4 + \nu^2\alpha_j^4[\xi(\chi)]^2} ,$$

$$\xi(\chi) = \frac{1}{\chi^{3/2}}\left[\left(1 - \frac{1}{2\chi^2}\right)\mathrm{Erf}(\chi) + e^{-\chi^2}/\pi^{1/2}\ \chi\right] ,$$

$$\bar{\nu} = (\nu_{ii}/2\epsilon)/\left[\frac{N}{2r}\left(Q-\frac{1}{2}\right) \epsilon q\ T/mr\Omega\right] ,$$

$$\nu_{ii} = 2^{1/2}\pi N_0 e^4\ \ell n\Lambda/m^{1/2}T^{3/2}, \text{ and } \chi = \upsilon\,(m/2T)^{1/2} .$$

Since it is the ion species that diffuses most rapidly by this process, the neutrality condition requires

$$\left(\frac{N_0'}{N_0} + \frac{e\phi'}{T}\right)\lambda_1 + \left(\lambda_2 - \frac{3}{2}\lambda_1\right)\frac{T'}{T} = 0 .$$

Hence the resulting energy flux is

$$\widetilde{Q}_\alpha = -\left(\frac{2}{\pi}\right)^{1/2} \frac{\left[Q\ \delta_0\left(\frac{r}{a}\right)^n rB\right]^2}{\left(Q-\frac{1}{2}\right)(n+2)^2 q} \frac{N\epsilon^{3/2}T}{m\Omega}\,\bar{\nu}\left(\lambda_3 - \frac{\lambda_2^2}{\lambda_1}\right)N_0T' , \qquad (19)$$

where all the collision frequency dependences are embodied in $\chi = \bar{\nu}$ $(\lambda_3 - \lambda_2^2/\lambda_1)$. Numerical integration has been performed to evaluate χ as a function of $\bar{\nu}$. From the definition, $\bar{\nu}$ is essentially the ratio between the effective collision frequency and N times the toroidal drift velocity of the banana. The dependence of χ on $\bar{\nu}$ is plotted in Fig. 1. For $\bar{\nu}$ large, χ is proportion to $\bar{\nu}^{-1}$, which is the result obtained by Davidson [3]. However, as $\bar{\nu}$ decreases, the $1/\bar{\nu}$ dependence changes gradually to a $\bar{\nu}$ dependence. This result agrees with our phenomenological derivation in Section 2. In Fig. 1, the actual dependence of χ on $\bar{\nu}$ is represented by the solid curve. Davidson's result [3] is represented by the dash straight line. They agree with each other very well in the large $\bar{\nu}$ limit. However, at around $\bar{\nu} \approx 3$, the solid curve turns away from the dash line and approaches to a linear behavior with $\bar{\nu}$ as $\bar{\nu}$ decreases. For the same parameters just preceeding Eq. (9), this is equivalent to saying that Davidson's analysis no longer holds when $\nu_{*i} \lesssim 0.5$, which is quite close to our estimation in Eq. (9).

If we take into account the fact that trapped particles exist only when υ is larger than $\upsilon_* \equiv \nu_*^{1/4}(2T/m)^{1/2}$ [7], the lower limit of integration in the definition of λ_n should be replaced by $\nu_*^{1/4}$. For the parameters just preceeding Eq. (9), $\bar{\nu}$ is related to ν_{*i} by $\bar{\nu} = 5.88\ \nu_{*i}$. The effect of this is shown by the dash-dot curve in Fig. 1. For $\bar{\nu} < 10$, the solid curve and the dash-dot curve are practically the same. For larger $\bar{\nu}$, the dash-dot curve rapidly dips below the solid curve. This implies that by the time when Davidson's result is valid, there are not many trapped particles left.

4. CONCLUSION

The transport processes induced by the drift motion of bananas in a tokamak field with ripples have been derived by solving the banana drift kinetic equation with a bounce averaged pitch angle collision operator. The result agrees with what one would obtain by a simple physical argument. Previous calculation [3] of this effect is shown to be valid only in high collision frequency limits and suggests that it cannot be extrapolated to low collision frequency. For typical reactor parameters, this loss mechanism is probably not very significant compared to the ripple trapping loss mechanism.

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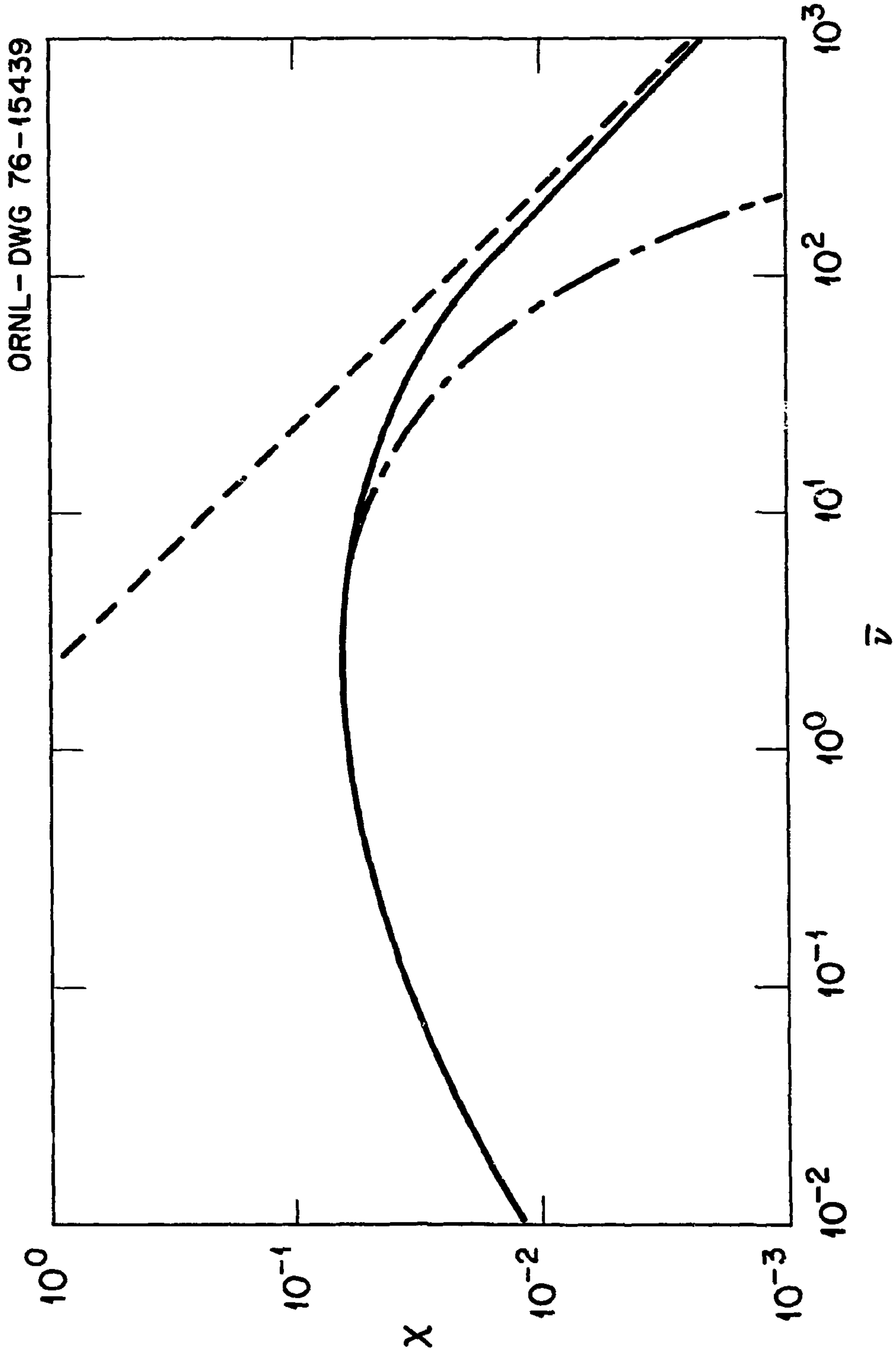
FIGURE CAPTION

Fig. 1. Comparison of present (solid line) and Davidson's (dashed line) results for the normalized ion heat conductivity due to banana drift motion as function of collisionality. The dash-dot line is the ion heat conductivity after adjustment for the fraction of trapped particles that can exist for the parameters: $N = 20$, $q = 2.5$, $Q = 1$, $Ti = 10$ KeV, $B = 45$ kg, $r = 100$ cm, and $\epsilon = 1/3$.


ORNL-DWG 76-15439
X
10^0
10^{-1}
10^{-2}
10^{-3}
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10^3
$\bar{\nu}$

Figure 1