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FINITE HOMOGENEOUS RELATIVISTIC ELASTIC SPHERE IN ITS OWN GRAVITATIONAL FIELD

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FINITE HOMOGENEOUS RELATIVISTIC ELASTIC SPHERE IN ITS OWN GRAVITATIONAL FIELD

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ABSTRACT

This paper deals with static spherically symmetric elastic matter in its own gravitational field. It is shown that in the case of a certain type of homogeneity the field equations can be reduced to an ordinary non-linear differential equation. This equation has solutions corresponding to finite bodies.

АННОТАЦИЯ

В рамках общей теории относительности обсуждается статическая упругая материя сферической симметрии, находящяя в своем гравитационном поле. Предпологая гомогенность некоторого сорта уравнения поля сводятся к несвязанному нелинейному обыкновенному дифференциальному уравнению, которое имеет решения оп[illegible]вающие конечное тело.

KIVONAT

E cikk az általános relativitáselmélet kereteiben saját gravitációs terében lévõ sztatikus, gömbszimmetrikus rugalmas anyaggal foglalkozik. Megmutatjuk, hogy bizonyos homogenitást feltéve a téregyenletek egyetlen nemlineáris közönséges differenciálegyenletre vezethetõk vissza, és ennek vannak véges testet leiró megoldásai.

1. INTRODUCTION

The problem of elasticity in general relativity has not been studied extensively. For example, the general elastic Schwarzschild interior solution is unknown, in contrast to the analogous fluid problem, which is reduced to the Tolman-Oppenheimer-Volkov integro-differetial equation [1]. Though an elastic interior solution would have much less physical importance than the fluid solutions, which are necessary for the description of the final states of stellar evolution, however this problem is of certain theoretical interest independently of its immediate utility.

In 1973, S. R. Roy and P. N. Singh found a series of solutions describing spherical symmetric elastic matter of constant density [2]. However their solutions are very special and can fulfil the boundary condition $T^1_1/r/=0$ only with dust on the surface, and have no classical limit. /In Sect. 2 we shall elaborate these statements./ For this reason a more rigorous treatment is necessary. In this paper we will deal with the simplest case: elastic Schwarzschild interior solution with a certain type of homogeneity will be treated. We shall show that this problem can be reduced to an ordinary non-linear differential equation, whose certain solutions describe finite spheres.

In the following Rayner's formalism will be used [3] with Carter and Quintana's modification [4]. In this formalism the energy-momentum tensor for the Hookean limit has the form:

$$T_{ik} = \rho u_i u_k - \tfrac{1}{2}C_{ik}{}^{rs}(h_{rs}-h^o{}_{rs}), \qquad /1.1/$$

$$h_{ik} = g_{ik} + u_i u_k, \quad u^r u_r = -1,$$

where $h^o{}_{ik}$ is a symmetric tensor of rank 3 orthogonal to the velocity with vanishing Lie derivative along u_i. It describes a fictitious undeformed /strainless/ state. The quantity C^{iklm} is a matrix of rank 6 in the pairs /ik/ and /ℓm/. It stands for the elastic coefficients of the matter. Both C^{iklm} and ρ can be expressed by material constants [5] as

$$\rho = nr_o + \frac{1}{8} C^{iklm}(h_{ik} - h^o_{ik})(h_{lm} - h^o_{lm}),$$

$$C^{iklm} = nK^{iklm}, = C^{iklm} = C^{kilm} = C^{lmik}, \qquad /1.2/$$

and the Lie derivatives of m_o and K^{iklm} vanish along u^i. The particle number density is denoted by n, which can be expressed by h_{ik}, h^o_{ik} and material constant n_o, however the relation has a complicated form, unless we introduce comoving coordinates, when

$$n = \frac{\sqrt{{}^3h^o}}{\sqrt{{}^3h}} n_o \qquad /1.3/$$

where ${}^3h$ denotes the determinant of the 3-tensor h_{IK}, I=1,2,3. The material constants may depend on certain coordinates, but must have vanishing Lie derivatives along u^i.

It can be seen that C^{iklm} has 21 independent components, as in the classical mechanics for crystals of minimal symmetry. For isotropic bodies /e.g. for macroscopic bodies without macroscopic crystallic structure/ two characteristic terms remain:

$$K^{iklm} = n^{-1}[\nu h^{o*ik}h^{o*lm} + \mu(h^{o*il}h^{o*km} + h^{o*im}h^{o*kl})] \qquad /1.4/$$

where h^{o*ik} is a matrix of vanishing Lie derivative along u^i, for which

$$h^{o*ir}h^o_{rk} = \delta^i_k - \delta^i_o\delta^o_k. \qquad /1.5/$$

These conditions do not determine it unambiguously, but the quantity $K^{iklm}(h_{lm} - h^o_{lm})$ will be unique. The quantities μ and ν are the usual elastic coefficients /the Lamé factors/ and $\underset{u}{L}\frac{\mu}{n} = \underset{u}{L}\frac{\nu}{n} = 0$ /where $\underset{u}{L}$ denotes the Lie derivative along u^i/.

2. REMARKS ON HOMOGENEITY

In general case the spherically symmetric static solutions for elastic bodies contain seven functions of the radial coordinate r, which in our case are μ, ν, g_{11}, g_{oo}, ρ, h^o_{11} and h^o_{22}. There are five equations for them, namely three of the Einstein equations and two material equations for the elastic coefficients. Thus the general solution will contain two arbitrary functions of r. The procedure of obtaining the general solution is not known.

Thus, if one looks for special solutions, two of these functions may be specially chosen.

Roy and Singh take ρ to be a constant, there is no material equation assumed, and the metric tensor of the undeformed state is given by

$$h^{o}_{ik} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\sin^2\vartheta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad /2.1/$$

because it is connected to the undeformed state. However, there is no fundamental reason for such a choice, and if h^{o}_{ik} has this form, the boundary condition $T^1_1 = 0$ is fulfilled only where $2\mu+\nu=0$. It is clear that the condition that $2\mu+\nu$ vanish somewhere is very artificial even though μ and ν are arbitrary functions. The quantities $2\mu/3+\nu$ /the compressibility/ and μ /the shear modulus/ must be non-negative because of fundamental thermodynamical principles [6], and $2\mu+\nu$ is a combination of them with positive coefficients. Thus it can vanish only where both μ both ν vanish, i.e. where there is dust. Consequently their solutions describe very special bodies, whose matter becomes continuously dust going outward to the surface. This behaviour is the consequence of the unnecessarily special form of h^{o}_{ik}.

Another difficulty about these solutions is that they have no classical limit. The deformation tensor [5]

$$\varepsilon_{ik} = \frac{1}{2}\left(h_{ik} - h^{o}_{ik}\right) \qquad /2.2/$$

has only one nonvanishing component, ε_{11}, in contrast to the classical case

$$\varepsilon_{ik} = \frac{1}{2}\left(s_{i;k} + s_{k;i}\right) \qquad /2.3/$$

where s^i is the deformation vector; ε_{ik} has three diagonal component for radial deformation:

$$\varepsilon_{11} = s_{,r};\ \varepsilon_{22} = \varepsilon_{33}\sin^{-2}\vartheta = sr; \quad s = s/r/ \qquad /2.4/$$

Thus also the energy-momentum tensor cannot have classical limit.

An alternative way is to require a certain type of homogeneity. Of course, we must not require homogeneity for the metric tensor, because in this case the solution could not describe a finite body. In this paper we will use a weaker type of the homogeneity /"material homogeneity"/, for which the definition will be the following:

a./ There be N space-like vectors K^i_A, A=1,...N, with rank 3 for the matrix K^i_A in the indices /iA/, and they be independent in the following sense: if λ^A are constants, $\lambda^A K^i_A = 0$ if and only if $\lambda^A = 0$ for every A. /If the vectors K^i_A are regarded as Killing vectors of a three-dimensional space, this condition means that the space is homogeneous/.

b./ These vectors be orthogonal to the velocity.

c./ The Lie derivatives of the "material" quantities μ, ν, ρ and $\overset{o}{h}_{ik}$ vanish along every vector K^i_A.

3. THE SYMMETRIES OF THE SOLUTIONS

We impose the conditions that the interior solutions and the material quantities be static and spherically symmetric, and that the timelike Killing vector be orthogonal to every K^i_A. Thus

$$ds^2 = e^{\lambda(r)}dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2) - e^{\psi(r)}dt^2$$

$$u^i = e^{-\frac{1}{2}\psi}\delta^i_o$$

$$h_{ik} = \begin{bmatrix} e^\lambda & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\sin^2\vartheta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad /3.1/$$

The number N of the vectors K_A must be 3, 4 or 6 because they are Killing vectors in a three-dimensional positive-definite Riemann space whose metric tensor is the space-like part of $\overset{o}{h}_{ik}$. On the other hand, the spherical symmetry is required for $\overset{o}{h}_{ik}$ i.e. the symmetry group must contain the SO/3/ group with two-dimensional transitivity as a subgroup. Clearly there are four possibilities: SO/3/ ⊗E/1/ /N=4/ and SO/4/, E/3/, SO/3,1/ /N=6/, and the form of $\overset{o}{h}_{ik}$ is:

$$\overset{o}{h}_{ik} = \begin{bmatrix} \Phi^2(r) & 0 & 0 & 0 \\ 0 & R^2 & 0 & 0 \\ 0 & 0 & R^2\sin^2\vartheta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{for } N=4, \text{ and}$$

$$h^{0}_{ik} = \begin{vmatrix} \frac{\Phi'(r)^2}{1-s\Phi^2(r)} & 0 & 0 & 0 \\ 0 & \Phi^2(r) & 0 & 0 \\ 0 & 0 & \Phi^2(r)\sin^2\vartheta & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \quad \text{for } N=6, \qquad /3.2/$$

where $\Phi/r/$ is an arbitrary function, R and s are constant, and

if the group is	SO/4/	E/3/	SO/3,1/
then s	[illegible] 0	= 0	[illegible] 0

The transformation $r'=r'/r/$, which could make Φ equal to 1 for N=4 and to r for N=6 is not compatible with the chosen form of g_{ik}, thus $\Phi/r/$ remains arbitrary.

The scalars μ, ν and ρ are constant because they are time-independent and have vanishing derivatives along the K^{i}_{Λ}-s, i.e. we need not deal with the material equations for μ and ν. The parameters ρ, μ and ν are constants with values depending on the type of matter considered.

4. THE FIELD EQUATIONS

The Einstein equation has three nontrivial components. Their left sides can be obtained from /1./, /1.2/, /3.1/ and /3.2/:

$$e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} = -\kappa c^2 \rho,$$

$$e^{-\lambda}\left(\frac{\psi'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} = -\kappa T^{1}_{1}, \qquad /4.1/$$

$$e^{-\lambda}\left(\frac{\psi''}{2} - \frac{\psi'\lambda'}{4} + \frac{\psi'^2}{4} + \frac{\psi'-\lambda'}{2r}\right) = -\kappa T^{2}_{2},$$

$$\kappa = \frac{8\pi\gamma}{c^4}\ ; \quad \gamma = 6{,}67\cdot10^{-8}\ \frac{cm^3}{gs^2},$$

where

$$T^{1}_{1} = \begin{cases} \nu-\varphi + \psi^2 e^{-\lambda}\left(\varphi - \frac{\nu r^2}{R^2}\right) & \text{for } N=4, \\ \nu-\varphi + \frac{\Phi'^2 e^{-\lambda}}{1-s\Phi^2}\left(\varphi - \frac{\nu r^2}{\Phi^2}\right) & \text{for } N=6, \end{cases} \qquad /4.2/$$

$$T_2^2 = \begin{cases} \frac{1}{2}\nu - \varphi + \frac{R^2}{r^2}\left(\varphi - \frac{\nu}{2}\frac{e^\lambda}{\Phi^2}\right) & \text{for } N=4, \\ \frac{1}{2}\nu - \varphi + \frac{\Phi^2}{r^2}\left(\varphi - \frac{\nu}{2}(1-s\Phi^2)\frac{e^\lambda}{\Phi'^2}\right) & \text{for } N=6, \end{cases} \qquad /4.2/$$

$$\varphi \quad \mu + \frac{3}{2}\nu,$$

and

φ, ν, ρ, s, and R are constant.

There are three equations for the three unknown functions $\lambda/r/$, $\psi/r/$ and $\Phi/r/$. In addition to this they have to fulfil certain initial and boundary conditions.

Since ρ=const., the matter does not vanish continuously. Thus the interior solution has to be matched to the exterior Schwarzschild solution at a certain boundary surface $r=r_o$. This is equivalent to two conditions [7]. First, there g_{ik} must be continuous. This condition is trivially fulfilled for g_{44} because ψ does not occur in the equations.

The only remaining condition for the metric tensor is:

$$e^{-\lambda(r_o)} = 1 - \frac{2m}{r_o} \qquad /4.3/$$

We shall see that this is simply the definition of m.

The second condition is that T_1^1 vanish at r_o. There are two possibilities: either T_1^1 vanishes nowhere and the solution does not describe a finite body or T_1^1 vanishes at certain values of r and one of them will be r_o /i.e. in the second case the condition does not restrict the solution/.

The initial conditions have to guarantee that there is no singularity at the center. The /11/ component of the Einstein equation can be immediately integrated:

$$e^{-\lambda} = \frac{C}{r} + 1 - \frac{8\pi\gamma}{3c^2}\rho r^2. \qquad /4.4/$$

It can be seen that for C≠0 there is singularity at the center, thus we must deal with the case C=0 only. /4.4/ shows that condition /4.3/ is simply a definition for m:

$$m = \frac{4\pi\gamma}{3c^2}\rho r_o^3 = \frac{\gamma}{c^2} M. \qquad /4.5/$$

The second equation of /4.1/ gives ψ' in terms of T_1^1 and known functions of r, and substituting it into the third equation of /4.1/, we get a differential equation for $\Phi/r/$. It is convenient to express this equation in terms of T_1^1 and T_2^2 defined by /4.2/:

$$C = 2\left(1-\frac{8\pi\gamma}{3c^2}\rho r^2\right)\left(2T^2-rT^1_{1,1}-\frac{3}{2}T_1^1+\frac{1}{2}\rho c^2\right)-\left(\rho c^2+T_1^1\right)\left(1+\frac{8\pi\gamma}{c^4}r^2T_1^1\right) \qquad /4.6/$$

Eq. /4.6/ is the fundamental equation. After integration T_1^1 and T_2^2 can be determined, and

$$\psi = \psi_o + \int\left[\frac{1}{r}\left(e^{\lambda}-1\right)-\kappa r e^{\lambda}T_1^1\right]dr. \qquad /4.7/$$

As it was mentioned, we can only accept those solutions which give vanishing T_1^1 at a finite positive value of r. On the other hand, we also require that T_1^1 and T_2^2 be finite at the center. This condition restricts the behaviour of Φ for small r, so that there are the following three possibilities:

if N=4: $\Phi/r/ = \pm\sqrt{\frac{\nu}{2\varphi}} + r^2y/r/$; y/O/ is finite,

if N=6: $\Phi/r/ = \Phi_o \pm \left[\frac{\nu}{2\varphi}(1-s\Phi_o^2)\right]^{1/2}r - \frac{s\Phi_o\nu}{4\varphi}r^2 + r^3y/r/$;

Φ_o, y/O/ and

y'/O/ are finite **Table 1.**

or $\Phi/r/ = ry/r/$; y/O/ is finite, y'/O/ = O.

The values of y/O/ and y'/O/ are the initial values for the differential equation /4.6/. There may be singularity at certain points, i.e. at the zeroes of Φ, Φ' and $1-s\Phi^2$, but these are physically extreme values of r. Every regular solution of /4.6/, which fulfils the initial conditions listed in Table 1 and has $T_1^1 = 0$ at a finite positive value of r, generates the field quantities of a regular finite elastic sphere.

Eq. /4.6/ is an ordinary but inhomogeneous non-linear differential equation for y/r/, for N=4 of first order, for N=6 of second order, and it seems to be hopeless to obtain the solution in analytic form. Of course, the numerical solution is always possible, but it is not the subject of the present paper. Here we want only to demonstrate that certain solutions of /4.6/ describe finite interpretable objects.

5. A HOMOGENEOUS MODEL OF THE EARTH

Now we are going to show that /4.6/ has a solution describing a sphere with the average parameters of the Earth. In order to see this, it is necessary to list the following parameters:

a./ Radius: $r_o = 6.4.10^8$ cm;

b./ Density: $\rho = 5.5$ g/cm^3

c./ Elastic coefficients: $\mu = 3.0.10^{12}$ g/cms^2, $\nu = 4.3.10^{12}$ g/cms^2

These are characteristic values calculated from the velocities of the longitudinal and transversal waves about the point, where ρ has its average values [8]; the fluid behaviour of the core is ignored.

These are the parameters which have immediate physical meaning. We choose the third possibility from Table 1, thus we have to take values for s and y/O/ too. The quantity s has the dimension $\gamma\rho c^{-2}$, and, if its order of magnitude is also equal to that of $\gamma\rho c^{-2}$, it does not have remarkable influence on the final result, since sr^2 would be about 10^{-9} at the boundary surface. Thus s=0 may be chosen. The free parameter which guarantees that $T^1_1/r_o/=0$ is $y/0/$.

Since eq. /4.6/ gives little hope for analytical solutions, let us introduce the power series expansion:

$$y = y(0)[1 + a_2r^2 + a_3r^3 + a_4r^4 + \ldots] \qquad /5.1/$$

The equation is too complicated to find a handy formula for a_n, thus we cannot prove the convergence, but we shall see that the last calculated term /the fourth-order one/ gives sufficiently small contributions to y/r/ and $T^1_1/r/$ /whose vanishing is necessary/.

The values of a_2, a_3 and a_4 are:

$$a_2 = -\frac{\pi\gamma\rho^2}{15(\varphi y^2_{(o)}-\nu)}[1 + \vartheta(10^{-8})]$$

$$a_3 = 0,$$

$$a_4 = -(a_2)^2 . \frac{13+17W}{14} \; ; \; W = \frac{\nu}{\varphi y(0)^2-\nu}$$

where the ignored corrections are less than 10^{-8} part of the given value if the parameters have the mentioned order of magnitude.

Having calculated T^1_1 and T^2_2 we get:

$$T^1_1 = (y(o)^2-1)\varphi - \frac{2\pi\gamma}{15}\rho^2(W+3)r^2 - \frac{\pi^2\gamma^2\rho^4(2+35W+17W^2)}{1575(\varphi y(o)^2-\nu)} r^4$$

/5.3/

$$T^2_2 = (y^2(o)-1)\varphi - \frac{2\pi\gamma}{15}\rho^2(1+2W)r^2 - \frac{\pi^2\gamma^2\rho^4(6+105W+51W^2)}{1575(\varphi y(o)^2-\nu)} r^4$$

Using the above-mentioned values for r_o, μ, ν and ρ we get:

$$y/0/^2 = 1.12$$

and the ratio of the r^4 and r^2 term of T^1_1 at the boundary surface /where it is maximal/ is 0.017, which shows that we may rely on the convergence. The function ψ can be calculated by means of this form of T^1_1, but the second term of the integrand in /4.7/ /containing T^1_1/ can be ignored because the ratio of the second and first terms is about 10^{-9}, thus

$$\psi = \left(1 - \frac{8\pi\gamma}{3c^2}\rho r_o^2\right)^{3/2}\left(1 - \frac{8\pi\gamma}{3c^2}\rho r^2\right)^{-1/2}. \qquad /5.4/$$

The quantity λ is given by /4.4/.

The maximal degree of compression at the center can be obtained as

$\frac{1}{2}(h^{or}{}_r - h^r{}_r) = 0.17$, which means 5 % linear deformation. Of course, for

such a great deformation the Hookean behaviour is generally not valid. /E.g., for steel, the critical dilatation, at which the matter breaks, is 0.2-1 %/.

Our result agrees with the classical solution up to the quadratic terms /except the factor $y_o^2 \simeq 1$ in $\varphi y_o^2 - \nu$; the classical formalism is well-defined only for the case $\varepsilon_{ik} << 1$/. The r^4 terms are not relativistic corrections; they are caused by the change of $y/r/$ because of which the constants μ and ν correspond to classical coefficients slightly depending on r. Thus we have shown that our procedure can give solutions with classical limit.

We note that the series of T^1_1, T^2_2 and Φ seem to converge faster for smaller objects, e.g. for moons and asteroids because their densities are approximately equal to that of the Earth, and their radii are less, thus the r^4 terms cannot grow up to such an extent as for the Earth.

6. REMARKS ABOUT THE ELASTIC COEFFICIENTS

We have seen that the field equations have solutions describing finite homogeneous elastic spheres. In this paper we did not want to investigate, which sets of the parameters admit finite solutions, because that would require a large amount of numerical calculations. However the questions of the possible value of the parameters and the equation of state have to be investigated.

If the elastic coefficients were too great, the velocity of sound would be greater than velocity of light. For small deformations $2\mu+\nu\leq c^2\rho$ must be required [3], for great deformations the formula of the sound velocity can be found in Ref. 3, but it should be completed by a term appearing because of Carter and Quintana's modification of C^{iklm}. On the other hand, μ and $\nu+2\mu/3$ must be non-negative, and, for the known types of matter, ν is non-negative too [6].

A further question is whether the adopted assumptions for the material homogeneity are compatible with a reasonable equation of state or not. Namely, it is known, that if the matter were fluid of constant density, either the pressure would be constant and the body would be infinitely large or the pressure would depend on r, but this is impossible for cold, one-component fluid of constant density [1]. We show that the present case differs from the mentioned one.

Consider a cold elastic matter with the equation of state given in eq. /1.2/, regarding the values of h_{ik} and h^o_{ik} as known from the solution. Since ρ, μ and ν are constants, n/r/ can be obtained from the equation of state. Now we write:

$$\nu = \frac{n/r/}{n_o/r/}\,\nu_o(n_o/r/) \qquad /6.1/$$

where the "unstrained density" $n_o/r/$ can be calculated by means of n/r/ and the determinants of the space-like parts of h_{ik} and h^o_{ik}. Thus the condition that ν do not depend on r is an equation for $\nu_o/n_o/$, which is a material equation. /Since n_o depends only on a single variable, generally such an equation is solvable./ The procedure is similar for μ. Consequently the homogeneity conditions can be fulfilled for special type of the dependence of the "unstrained" elastic coefficients on the "unstrained" density, but the denstiy must not be homogeneous. Such a case seems to be slightly unnatural, but the homogeneity of a finite body is alvays a slightly unnatural approximate assumption.

7. CONCLUSION

We have shown that there are materially homogeneous solutions of the Einstein equations describing finite elastic spheres. These solutions are generated by a non-linear ordinary differential equation, whose solutions have to fulfil certain initial and boundary conditions. These solutions are not generalizations of Roy and Singh's ones [2] because of the constancy of the elastic coefficients.

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